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Stability Analysis for Laminar Flow Control - Part II

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Introduction

In this Report, we summarize research work done at Cambridge Hydrodynamics, Inc. under Contract No. NAS1-14907 with NASA Langley Research Center. More detailed expositions of the work described here is given in the publications cited in the References.

In Sec. 2, we summarize our work done to optimize the numerics of the SALLY computer code written by us and used at NASA LaRC and elsewhere to analyze the stability of laminar flow control wings. We have succeeded in speeding up earlier versions of SALLY by about a factor 3 while reducing the memory requirements by about a factor 2.

In Sec. 3, we summarize work done by Dr. David J. Benney of CHI on the relation between temporal and spatial stability theory in laminar flow control problems.

In Sec. 4, we describe new methods that we have developed for the solution of three-dimensional compressible flow stability problems by spectral methods.

In Sec. 5, we describe new spectral methods for the solution of boundary layer equations. These new methods are both highly accurate and efficient.

Finally, in Sec. 6, we describe our work extending the SALLY code to study the nonlinear, non-parallel stability theory of boundary layer flows.

2. Optimal Numerics for the SALLY Stability Analysis Code

The SALLY Computer code was developed for NASA Langley Research Center by CHI under contracts L-47262-A and NAS1-14427. It is essentially a 'black-box' stability analyzer for three-dimensional boundary layer flows over LFC wings. In the present work, we have speeded up the code by about a factor 3 while reducing core requirements by about a factor two. In addition, several new features and options have been added for the convenience of users. A detailed exposition of the numerical methods and results obtained by SALLY will be given in a forthcoming paper. Here we summarize the new developments.

(i) Fast Eigenvalue Solvers -- A new fast eigenvalue solver has been implemented successfully in SALLY. We use a cubically convergent inverse Rayleigh iteration[1] to improve an approximate eigenvalue to within roundoff accuracy in about 3 iterations. Practice has shown that even a relatively crude guess for the eigenvalue gives good results with this local iteration scheme.

The heart of this new eigenvalue solver is new assembly language matrix manipulation routines. Assembly language fully-pivoted LU decomposition and matrix multiply programs have been written in COMPASS for the CDC 174-176 series machines. These codes are now operational at NASA LaRC. Because of the difference between CPU speed and memory access time balances for these machines, it was found to be essential to write separate routines for each machine. For example, the Cyber 176 has a fast CPU and a fast memory cycle allowing code optimization using stackloop programming techniques. On the other hand, the Cyber 175 has a relatively slow memory cycle so the codes for this machine were specially designed to minimize the memory-access penalty. Thus, the Cyber 175 code runs about twice as fast on the 175 as the 176 code runs on the 175.

(ii) New Global Eigenvalue Routine -- The global eigenvalue routine of the original SALLY code[2] has been replaced by an improved code that both runs faster and requires less memory. This global eigenvalue routine is essential in flow regions where a good guess is not available, like near the leading edge of the airfoil or in the stable region of the LFC wing where practice has shown that the character of the least stable mode changes greatly.

(iii) New Group Velocity Code -- A new, extremely efficient code for the calculation of the group velocity has been implemented in SALLY. In the original SALLY code, the group velocity was computed in two ways, at the option of the user. A first-order finite difference approximation to the group velocity was obtainable by computing the eigenvalue at two nearby wave-vectors; alternatively, an analytic computation of the eigenvalue using the adjoint eigenfunction (determined numerically) was available.[2] The new code is based in the interesting observation that the adjoint eigenfunction used in the analytic formula for the group velocity need not be the adjoint eigenfunction of the Orr-Sommerfeld stability equation, but rather can be the adjoint eigenvector of any matrix that is similar to the matrix representation of the Orr-Sommerfeld equation.

We have implemented the latter idea by use of the adjoint vectors employed in the local Rayleigh iteration scheme. In this case, the group velocity is immediately available upon simple algebraic manipulation of the eigenvector and the matrix adjoint. The new procedure is at least a factor 10 faster than the direct use of the true adjoint eigenfunction and is as accurate; it is a factor 2 more efficient than the difference computation of the group velocity and is much more accurate.

(iv) Fast Matrix Setup -- A significant amount of computer time in SALLY is spent setting up matrices to represent the Orr-Sommerfeld equation in terms of Chebyshev polynomial (about 40% of the computer time of the original SALLY code).

By careful reorganization of the code and new algorithms, we have been able to decrease this time substantially (to about 20% of the run time of the new SALLY code).

Two new programs and the fast matrix multiply program discussed above are the essential ingredients of this speedup. A new Chebyshev derivative program was developed that, for typical LFC problems, runs about 20 times faster than the original code. It is based on the algorithm of Sec. 10 of Ref. 3. Also, we have developed a new fast way to setup the matrices corresponding to non-uniform unperturbed flow. The new method is based on the convolutional structure of the equations.[3]

(v) Spatial Eigenvalue Extrapolation -- An improved technique to march from station to station across an LFC wing has been implemented. This new method is essential to get a good guess for the fast local eigenvalue solvers now used in SALLY. The idea is to use mean velocity profile information at two nearby stations to extrapolate the eigenvalue to the next station. This is done in a way that is transparent to the user of SALLY.

(vi) Optimization Options -- The original SALLY code offered the user only data on the most unstable temporal eigenvalue at a given frequency. The maximum is achieved over all wavelengths and propagation angles. Some new options have been added to the new code. They include:

a) Fixed wavelength, fixed propagation angle.

b) Fixed frequency, fixed wavelength.

c) Fixed frequency, fixed propagation angle.

In all cases, the user can determine the amplification of the given modes across the wing.

(vii) Other Modifications--In addition to the above modifications to the running SALLY code at NASA LaRC, a number of advanced developments have been made that are not yet part of the distributed version of SALLY. These include: spatial stability analysis; non-parallel flow analysis; and non-linear flow analysis. All these code modules are for three-dimensional boundary layers.

3. Relation Between Temporal and Spatial Stability Theory

We begin by reviewing the relation between temporal and spatial stability of a unidirectional, parallel, steady flow $\bar{u}(z)$. Let us study a three-dimensional mode that depends on x, y, t as $\exp(i\alpha x + i\beta y - i\omega t)$. The Orr-Sommerfeld eigenvalue problem leads to a relation between α, β, ω of the form

$$F(\alpha,\beta,\omega) = 0 \tag{3.1}$$

where α, β, and ω are, in general, complex numbers and the function F is an analytic function of its arguments. In the following discussion, we use the notation that subscripts r and i indicate real and imaginary parts, respectively; for example, $\alpha_r = \mathrm{Re}(\alpha)$.

In the temporal theory, we set $\alpha_i = \beta_i = 0$, so the mode is pure oscillatory in the streamwise and spanwise directions. Then (3.1) leads to two relations of the form

$$\omega_r = \omega_r(\alpha_r,\beta_r) \tag{3.2}$$

$$\omega_i = \omega_i(\alpha_r,\beta_r) \tag{3.3}$$

Transition prediction for LFC studies using an amplification factor criterion requires determination of the most unstable wave. To do this locally at each station on the wing, we maximize ω_i as a function of α_r and β_r. This maximum growth rate is achieved when

$$\frac{\partial\omega_i}{\partial\alpha_r} = \frac{\partial\omega_i}{\partial\beta_r} = 0 \tag{3.4}$$

The two relations (3.4) determine the mode $\bar{\alpha}_r$, $\bar{\beta}_r$ of the most unstable three-dimensional temporal instability.

The spatial stability theory for this unidirectional parallel flow is as follows. We set $\omega_i = 0$ and solve (3.1) for α_i and β_i:

$$\alpha_i = \alpha_i(\alpha_r, \beta_r, \omega_r) \tag{3.5}$$

$$\beta_i = \beta_i(\alpha_r, \beta_r, \omega_r) \tag{3.6}$$

The amplitude of this mode behaves like $\exp(-\alpha_i x - \beta_i y)$, so the maximum growth rate is achieved by maximizing $\alpha_i^2 + \beta_i^2$ with respect to α_r, β_r, and ω_r. However, the Cauchy-Riemann equations imply that either $\alpha_i = 0$ or $\beta_i = 0$. But $\alpha_i = 0$ is unrealistic. If $\alpha_i = 0$ then the wave is an 'edge' wave with its maximum growth perpendicular to the free stream.

It follows that a reasonable procedure to treat spatial instability of a parallel flow is to choose $\omega_i = \beta_i = 0$ so that

$$\omega_r = \omega_r(\alpha_r, \beta_r) \tag{3.7}$$

$$\alpha_i = \alpha_i(\alpha_r, \beta_r) \tag{3.8}$$

For maximum amplification, we must then determine $\bar{\alpha}_r$ and $\bar{\beta}_r$ by

$$\frac{\partial \alpha_i}{\partial \alpha_r} = \frac{\partial \alpha_i}{\partial \beta_r} = 0 \tag{3.9}$$

In this way, all modal parameters for the most unstable eigenfunction for a unidirectional parallel flow are determined.

Now let us consider the changes in the above theory for general parallel flows in which the undisturbed flow is $(\bar{u}(z,T), \bar{v}(z,T), 0)$ in which we allow a slow time dependence on the long-time scale T. In this case, it is at first natural to use only temporal amplification. We assume a space-time dependence of the form

$$\exp(i\alpha_r x + i\beta_r y)\exp(-i\Theta_r(T) + \Theta_i(T)) \tag{3.10}$$

where

$$\omega_r = \frac{\partial}{\partial T}\Theta_r(T), \qquad \omega_i = \frac{\partial}{\partial T}\Theta_i(T) \tag{3.11}$$

The solution of the eigenvalue problem implies that

$$\Theta_{iT} = f(\alpha_r, \beta_r, T) \tag{3.12}$$

$$\Theta_{rT} = g(\alpha_r, \beta_r, T) \tag{3.13}$$

For any given wavenumbers α_r, β_r, $\Theta_{iT} = 0$ and (3.12) define a neutral surface $T = T_N(\alpha_r, \beta_r)$. The total amplification between T_N and T is

$$\sigma(\alpha_r, \beta_r, T) = \int_{T_N(\alpha_r, \beta_r)}^{T} f(\alpha_r, \beta_r, T')\,dT' \tag{3.14}$$

At any given T, the most unstable wave is determined by the equations

$$\frac{\partial \sigma}{\partial \alpha_r} = \frac{\partial \sigma}{\partial \beta_r} = 0 \tag{3.15}$$

Once $\bar{\alpha}_r$ and $\bar{\beta}_r$ are found from (3.15), (3.13) gives $\bar{\omega}_r$.

Next, let us consider a spatially varying unidirectional flow $\bar{u}(X,z)$, where X is a long-space scale in the flow direction. In this case, it is natural to study spatial amplification in which the space-time modal dependence is of the form

$$\exp(i\Theta_r(X) - \Theta_i(X))\exp(i\beta_r y - i\omega_r t) \tag{3.16}$$

The Orr-Sommerfeld eigenvalue problem implies that

$$-\Theta_{iX} = f(X, \beta_r, \omega) \tag{3.17}$$

$$\Theta_{rX} = g(X, \beta_r, \omega) \tag{3.18}$$

In this case, a neutral curve can be defined by $\Theta_{iX} = 0$ and

(3.16) so that $X = X_N(\beta_r, \omega_r)$. The total amplification from X_N to X is given by

$$\sigma(X, \beta_r, \omega_r) = -\int_{X_N}^{X} f(X', \beta_r, \omega_r)\, dX' \qquad (3.19)$$

At any given X, the most unstable wave is determined by the equations

$$\frac{\partial\sigma}{\partial\beta_r} = \frac{\partial\sigma}{\partial\omega_r} = 0 \qquad (3.20)$$

while the local wavenumber Θ_{rX} is then determined by (3.18).

Finally, let us consider a general spatially varying three-dimensional boundary layer in which the undisturbed velocity is $(\bar{u}(X,Y,z), \bar{v}(X,Y,z), 0)$. In the original SALLY code, we considered temporally amplified modes on such a flow. Here we consider a self-consistent description of spatially amplified modes. In this case, we assume the space-time dependence of the mode to be of the form

$$\exp(i\Theta_r(X,Y) - \Theta_i(X,Y) - i\omega_r t) \qquad (3.21)$$

The Orr-Sommerfeld eigenvalue problem implies that

$$\Theta_{iX} = f(X, Y, \Theta_{rX}, \Theta_{rY}, \omega_r) \qquad (3.22)$$

$$\Theta_{iY} = g(X, Y, \Theta_{rX}, \Theta_{rY}, \omega_r) \qquad (3.23)$$

The problem now is that there is no completely rational way to decide which solution of (3.22) - (3.23) to choose for maximum amplification. In particular, there is no obvious reason why the direction of growth need coincide with either the free-stream flow direction or the normal to the modal wavefronts. It seems most reasonable to study instead a wave-packet in which the evolution of a given imposed perturbation near the leading edge of the wing is followed in its motion to the leading edge. This approach will be examined in more detail in later studies. Here we concentrate on the following simplified approach.

We assume that the direction of maximum growth $-\nabla\Theta_i$ is parallel to the freestream direction $U\hat{x} + V\hat{y}$. At a neutral surface, we must require that $\Theta_{iX} = \Theta_{iY} = 0$; given the frequency ω_r and wavenumbers Θ_{rX} and Θ_{rY}, these equations determine a point X_N, Y_N. Using this fact, we can define various kinds of neutral curves. For example, those waves that initiate at angle ϕ with respect to the freesteam direction must satisfy

$$\frac{U\alpha_r + V\beta_r}{\sqrt{U^2+V^2}\;\sqrt{\alpha_r^2+\beta_r^2}} = \cos\phi \tag{3.24}$$

Eqs. (3.22) - (3.24) on the neutral surface allow elimination of α_r and β_r so we get a relation of the form

$$H_N(X,Y,\phi,\omega_r) = 0 \tag{3.25}$$

which is the equation of the neutral curve.

The total amplification between X_N, Y_N and X, Y is given by

$$\sigma = -\int_{(X_N,Y_N)}^{(X,Y)} \nabla\Theta_i \cdot d\underset{\sim}{X} = -\int_{(X_N,Y_N)}^{(X,Y)} \frac{d\Theta_i}{ds}\, ds \tag{3.26}$$

which is independent of the path connecting X_N, Y_N with X, Y. Also, since $-\nabla\Theta_i$ is assumed parallel to (U,V),

$$\frac{V}{U} = \frac{f(X,Y,\alpha_r,\beta_r,\omega_r)}{g(X,Y,\alpha_r,\beta_r,\omega_r)} \tag{3.27}$$

The maximum total amplification at a given point X, Y is achieved at a given frequency ω_r as follows. Eq. (3.27) determines β_r as a function of α_r. Then starting from X, Y we march (3.26) and determine the most dangerous mode α_r by maximizing σ. The integration in (3.26) stops as soon as the neutral curve (3.25) is reached. Once the maximum for a given frequency is found, the maximum amplification rate over all frequencies is found.

4. New Methods for Stability Analysis of Compressible Flows.

The straightforward extension of the SALLY stability analysis code to include compressible effects can lead to highly inefficient use of computer resources. While the general three-dimensional disturbance of an incompressible parallel flow satisfies the fourth-order Orr-Sommerfeld equation, no single equation describing the evolution of even two-dimensional general compressible disturbances has yet been formulated. Since SALLY applies matrix methods (in order to be a 'black box' not requiring much user interaction), the absence of a single stability equation for compressible flows implies that computer memory requirements increase by K^2 while computer time increases by K^3, where K is the number of equations entering the system that must be solved. Two-dimensional disturbances satisfy a sixth-order system that is easy to reduce to $K = 3$ second-order differential equations; three-dimensional disturbances satisfy an eighth-order system that is easy to reduce to $K = 4$ second-order equations. In both cases, the degration of computer time within the current SALLY code is unacceptable.

We have developed a new method that allows easy solution of a very general class of stability problems by the spectral methods employed in SALLY while maintaining the accuracy and efficiency that spectral methods offer relative to more conventional difference methods. The key idea is a general algorithm to reduce an arbitrary system of K first-order differential equations into a single, explicit $K^{\underline{th}}$-order differential equation. Since, in contrast to difference methods, the accuracy of spectral methods does not degrade seriously with increasing order, while the matrix methods employed by SALLY do degrade computationally with increasing number of equations, the reduction to a single high-order equation provides an attractive general way to solve the compressible flow stability problem efficiently. As a by-product of our analysis, we obtain a single equation for the evolution of general three-dimensional disturbances in a three-dimensional compressible boundary layer.

It is easy to construct a system of K first order differential equations from the single $K^{\underline{th}}$ order linear equation

$$\frac{d^K y_0}{dx^K} + a_{K-1}(x)\,\frac{d^{K-1} y_0}{dx^{K-1}} + \ldots + a_0(x) y_0(x) = 0 \tag{4.1}$$

If we set $y_k(x) = d^k y/dx^k$ $(k = 0,1,\ldots,K-1)$ then

$$\frac{dy_k}{dx} = y_{k+1} \qquad k = 0,\ldots,K-2 \tag{4.2}$$

$$\frac{dy_{K-1}}{dx} = -a_{K-1}y_{K-1} - a_{K-2}y_{K-2} - \ldots - a_0 y_0$$

Conversely, if the K functions y_k $(k = 0,\ldots,K-1)$ satisfy the general linear system

$$\frac{d\vec{y}}{dx} = \underset{\approx}{A}_1(x)\vec{y} \tag{4.3}$$

then it is usually true (and is true for the flow stability analyses considered here) that $y_0(x)$ satisfies a $K^{\underline{th}}$ order linear equation of the form (4.1).[4] To see this, we need only introduce the differentiated systems

$$\frac{d^P}{dx^P}\vec{y} = \underset{\approx}{A}_P(x)\vec{y} \tag{4.4}$$

where

$$\underset{\approx}{A}_{P+1}(x) = \underset{\approx}{A}_P(x)\underset{\approx}{A}_1(x) + \frac{d\underset{\approx}{A}_P(x)}{dx} \qquad (P \geq 1) \tag{4.5}$$

Next, we solve (4.4) for $y_1, y_2, \ldots, y_{K-1}$ as functions of $y_0, y_0', y_0'', \ldots, d^{K-1}y_0/dx^{K-1}$ and substitute into (4.4) with $p = K$ to obtain a single $K^{\underline{th}}$ order equation for y_0.

There are some additional features that must be considered when the above algorithm is applied to compressible flow stability analysis. First, a mapping of the boundary layer direction must be made so the system of differential equations is a singular system when expressed in the form (4.3). It is better to proceed as follows. Let $\underset{\approx}{A}$ be the matrix of cœfficients of the linearized compressible Navier-Stokes equations so that the boundary layer stability problem is of the form (4.3) with $0 \leq x < \infty$ together with certain boundary conditions at $x = 0$ and $x \to \infty$. Then we make the algebraic mapping

$$z = 2x/(x+L) - 1 \tag{4.6}$$

where L is a scale parameter and $-1 \leq z < 1$. Then (4.3) becomes

$$ZD\vec{y} = \underset{\approx}{A}\vec{y} \tag{4.7}$$

where $D = d/dz$ and $Z = (1-z)^2/2L$. Then the algorithm given above constructs a single $K^{\underline{th}}$ order equation for y_0 in terms of the operator ZD. We have developed an efficient program that converts this latter equation to an equation for y_0 in terms of the differentiation operator D alone. Other new features include the automatic treatment of boundary conditions and the eigenvalue solver, to be discussed below.

We have developed a computer program that solves the general problem (4.7) with rather arbitrary boundary conditions. Here we outline the organization of this code. A user-supplied subroutine provides the coefficients of the matrix $\underset{\approx}{A}$ in (4.7). Then a subroutine calculates the coefficients f_k of the $K^{\underline{th}}$ order equation

$$(ZD)^K y_0 + f_{K-1}(ZD)^{K-1}y_0 + \ldots + f_o y_o = 0 \tag{4.8}$$

This subroutine also expresses $y_1, \ldots, y_{K-1}$ in terms of y_0 and its derivatives at the boundaries $z = \pm 1$ so that arbitrary boundary conditions in terms of $y_0, \ldots, y_{K-1}$ can be applied. This subroutine is independent of the specific equations being solved. The output of this subroutine is a single matrix representing (in Chebyshev coordinates) the effect of the differential operator (4.8) on y_0 including both boundary conditions and the reduction of the operator ZD to D.

The final step of the program is to compute the eigenvalues using the fast local eigenvalue subroutine already resident in SALLY. Since we use a local eigenvalue routine, it is necessary to provide it with an approximation to the eigenvalue. In a compressible flow problem this guess is best obtained either from a previous run or by slowly increasing the Mach number from an incompressible flow case in which a global eigenvalue routine like that employed by the incompressible SALLY code works efficiently.

In summary, our new program provides a very general solution to eigenvalue problems of almost arbitrary complexity. For the three-dimensional modes of three-dimensional boundary layers, our new general program works only about 30% slower than the current SALLY code which can handle only incompressible flows. Another nice feature of the new program is that it is written in such a way that new problems can be solved almost trivially merely by inserting new subroutines for the coefficient matrix $\underset{\approx}{A}$.

5. Spectral Methods for Boundary Layer Equations.

In this Section, we present a brief description of some work we have done applying spectral methods to nonsimilar laminar boundary layers.[5] The basic equation is[6]

$$\frac{\partial^3 f}{\partial \eta^3} + f\,\frac{\partial^2 f}{\partial \eta^2} + \beta(\xi)\left[1 - \left(\frac{\partial f}{\partial \eta}\right)^2\right] = 2\xi\left[\frac{\partial f}{\partial \eta}\,\frac{\partial^2 f}{\partial \xi \partial \eta} - \frac{\partial f}{\partial \xi}\,\frac{\partial^2 f}{\partial \eta^2}\right] \tag{5.1}$$

where f is the dimensionless streamfunction, ξ and η are the Levy-Lees transformed variables, and the boundary conditions are

$$f(\xi,0) = \frac{\partial f}{\partial \eta}(\xi,0) = 0 \qquad \frac{\partial f}{\partial \eta}(\xi,\eta) \rightarrow 1 \quad (\eta\rightarrow\infty) \tag{5.2}$$

We apply spectral-Chebyshev methods in η to the test case of Howarth's flow in which

$$\beta(\xi) = \frac{\xi}{\xi-4} \tag{5.3}$$

To apply spectral methods in ξ, we first truncate $0 \le \eta < \infty$ to $0 \le \eta \le R$ and then apply the linear mapping

$$s = 2\eta/R - 1 \tag{5.4}$$

to transform the range of the independent variable to $-1 \le s \le 1$. The boundary conditions become

$$f(\xi) = \frac{\partial f}{\partial s}(\xi,0) = 0, \qquad \frac{\partial f}{\partial s}(\xi,R) = 1 \tag{5.5}$$

We use a spectral collocation method is s based on expansion of f in a series of Chebyshev polynomials $T_p(s)$; in the ξ-direction, we use a Crank-Nicolson scheme like that employed by Keller and Cebeci.[6]

In Tables 1-2, we compare our results with those of Keller and Cebeci.[6] It is apparent that we achieve limiting accuracy in the spectral direction η with $R = 6$ and only 19 Chebyshev

TABLE 1. SPECTRAL METHODS FOR HOWARTH'S FLOW

$$A = \frac{\partial^2 f}{\partial \eta^2}(\xi = 0.7, \eta = 0) \qquad B = \frac{\partial^2 f}{\partial \eta^2}(\xi = 0.894,\ \eta = 0)$$

Separation point is $\xi = 0.901344$.

Truncation $0 \le s \le R$	6	6	8	8	8	8	8	8
# of Chebyshev polynomials in s	19	23	23	23	23	23	23	23
# of grid points in ξ	6	6	6	11	16	21	51	$\left\{\begin{matrix}11\\6\end{matrix}\right\}$ Richardson extra-polate in ξ
Total # of degrees of freedom	114	138	138	253	368	483	1173	253
A	0.2123946	0.2123947	0.2123939	0.2091297	0.2079484	0.2076701	0.2072421	0.2071836
B	0.0445471	0.0445470	0.044542	0.0347061	0.0324852	0.316549	0.0307245	0.0306378

TABLE 2. Finite Difference Results[6] for Howarth's Flow

Truncation R	6	6	6
# pts in η	19	61	121
# pts in ξ	16	16	51
Total # pts	304	976	6171
A	0.21063	0.207232	0.207227
B	0.41647	0.030932	0.030832
Best Richardson extrapolation on this grid: # of pts used in extrapolant	346	1708	9282
A	0.20735	0.207410	0.2071427
B	0.03511	0.031213	0.0305307

polynomials. It is also apparent that the spectral method achieves results of at least the accuracy achieved by the extrapolation methods with an order-of-magnitude fewer grid points. In particular, the results obtained by Richardson extrapolation in the ξ-direction of the spectral scheme uses only 253 points and is nearly as accurate as that obtained by Keller and Cebeci with 9282 points. The number of points used in the spectral calculation could be reduced by choosing $R = 6$ and using only 19 polynomials (a total of 209 degrees of freedom) without any expected loss of accuracy.

Spectral methods for boundary layer problems offer much promise. They seem to require an order-of-magnitude less computer memory than comparable finite-difference schemes and at least several times less computer time to achieve accurate results. Efficient methods to solve the spectral equations are discussed by Orszag and Jayne.[5]

6. Nonlinear Nonparallel Stability Analysis

In Sec. 2 of Ref. 2, the equations of two-dimensional nonlinear, nonparallel disturbances in an incompressible boundary layer are formulated. We have extended these equations to three-dimensions and have developed computer programs, consistent with SALLY to solve the resulting equations.

Several simplifications in the equations of Ref. 2 have been made. As discussed in Sec. 2 of this report, it is not necessary to use the true adjoint eigenfunction of the Orr-Sommerfeld equation to impose solvability conditions, like that used to derive the group velocity. Instead, the matrix adjoint available from the subroutine EIGREV works just as well as is available with no extra computing effort. The run time for a typical three-dimensional nonlinear, nonparallel stability calculation is about twice the computer time of the linear version of the SALLY code.

To date, we have run two sets of calculations with the full nonlinear, nonparallel version of SALLY. First, we have tested the formulation of the nonparallel flow terms by comparison with the results of Saric and Nayfeh.[7] Agreement with the published data was achieved. Second, we have compared the results of SALLY with the results obtained by Antar and Collins[8] for the nonlinear three-dimensional evolution of a Blasius boundary layer. Good numerical agreement was obtained. We have compared these results on the flat plate boundary layer with direct numerical simulations of transition in this flow.[9] The nonlinear stability theory calculations predict a somewhat slower growth of disturbances than given by the full Navier-Stokes calculations.

It is recommended that future runs with the SALLY code produce the nonlinear, nonparallel information available from the present extension of the code. In particular, it has been shown by Orszag[9] and Orszag and Kells[10] that three-dimensional nonlinear effects are crucial in the transition process.

References

1. A. Ostrowski, "On the convergence of the Rayleigh quotient iteration for the computation of characteristic roots and vectors." Arch. Rat. Mech. Anal. 4, 153 (1959).

2. S.A. Orszag and D.J. Benney,"Stability Analysis for Laminar Flow Control. Part I." NASA CR-2910 (1977).

3. D. Gottlieb and S.A. Orszag, Numerical Analysis of Spectral Methods. SIAM, Phila. (1977).

4. C.M. Bender and S.A. Orszag, Advanced Mathematical Methods for Scientists and Engineers. McGraw-Hill, New York (1978).

5. S.A. Orszag and L.W. Jayne, "Spectral Methods for Boundary Layer Equations." Cambridge Hydrodynamics Report No. 14, August 1978.

6. H.B. Keller and T. Cebeci, "Accurate Numerical Methods for Boundary Layer Flows. Two-Dimensional Laminar Flows". Proc. Second Intl. Conf. on Numerical Methods in Fluid Dynamics, Springer, Berlin (1971), p. 92.

7. W.S.Saric and A.H. Nayfeh, "Nonparallel stability of of boundary layer flows." Phys. Fluids 18, 945(1975).

8. B.N. Antar and F.G. Collins, "Numerical calculation of finite amplitude effects in unstable laminar boundary layers. Phys. Fluids 18, 289 (1975).

9. S.A. Orszag, "Turbulence and transition: A progress report." Proc. Fifth Intl. Conf. on Numerical Methods in Fluid Dynamics, Springer, Berlin (1976),p.32.

10. S.A. Orszag and L.C. Kells, "Transition to Turbulence in Plane Poiseuille and Plane Couette Flow." J. Fluid Mechs., vol. 96, Jan. 16, 1980, pp. 159-205.

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16. Abstract

This report summarizes work done by Cambridge Hydrodynamics, Inc. on the following problems: (i) optimization of the numerics of the SALLY stability analysis code; (ii) relation between temporal and spatial stability theory; (iii) compressible flow stability calculations; (iv) new numerical methods for the boundary layer equations; and (v) numerical study of nonlinear, nonparallel stability of incompressible flows.

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