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ALGORITHM FOR THE STABILIZATION OF MOTION A BOUNDING VEHICLE IN THE FLIGHT PHASE

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ANNOTATION

The unsupported phase of motion of a multileg bounding vehicle is examined in the work. An algorithm for stabilization of the angular motion of the vehicle housing by change of the motion of the legs during flight is constructed. The results of mathematical modeling of the stabilization process by computer are presented.

Key words: multileg vehicle, bound, motion control system, mathematical modeling.

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Introduction

The problem of stabilization of the motion of a bounding vehicle in the flight phase is investigated in the work. An algorithm for stabilization is a basic element of the motion control system of a bounding vehicle. /5*

A vehicle is considered, which consists of a housing and four or six two section legs, each of which has three degrees of freedom. The total weight of the legs is a significant portion of the weight of the housing.

The motion of the vehicle consists of alternation of two phases: support, during which all the legs stand on the supporting surface and there is quasistatic stability, and the unsupported or flight phase. In [1, 2], a mathematical model of the three dimensional motion of the vehicle is constructed for the supported and unsupported phases of motion. In the flight phase, the first integrals of the equations of motion are obtained (the motion of the center of mass of the vehicle along a ballistic trajectory and the law of conservation of the angular momentum of the vehicle about the center of mass).

In the support phase of motion of a bounding vehicle, high speeds and accelerations develop, for which great forces and power are required. Therefore, it is advisable to construct the programmed motion in the supported phase of a bound, in such a way that it can be performed with the minimum force or power developed in the leg joints. The problems of optimization of the programmed motion in the supported phase and its stabilization were investigated in [1].

In this work, the problem of performance of the programmed motion of the vehicle in the unsupported phase of a bound, constructed in [2], is considered in this work, in the case when various kinds of errors and perturbations occur. /6

The center of mass of the vehicle moves along a ballistic trajectory in the flight phase, and its motion is uncontrolled. The angular motion of the vehicle housing around the center of mass can be corrected, by changing the motion of the legs during flight. For example, by bending (unbending) the legs, their moment of inertia changes and, consequently, the effect of the transfer of the legs on the angular motion of the

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* Numbers in the margin indicate pagination in the foreign text.

housing changes by the law of conservation of angular momentum. The possibility of change of the motion of animals and man around the center of mass in unsupported motion, by movements of the limbs, has quite long been known [3-7].

The purpose of the stabilization of motion of the vehicle in the flight phase is to reach the position assigned in the motion program at the time of landing, with allowance for the shift caused by deviation of the final position of the center of mass of the vehicle from the programmed position.

The operation of the navigation and information systems is not modeled. It is considered that, in the supported phase, in the motion control system of a bounding vehicle, all current phase coordinates of the vehicle are known without error and, in the flight phase, the coordinates and angular velocities of the housing, as well as the angles and velocities of the leg joints. The control system also knows a model of the terrain. At the moment of liftoff from the supporting surface, the terrain model in the landing region corresponds to the actual supporting surface with some degree of accuracy. During flight, the terrain model in the landing region is refined, and it becomes completely equal to the actual supporting surface immediately before the time of landing.

As to the supporting surfaces, it is assumed that they differ little from a section of a plane and that their inclination to the horizon is small. The terrain as a whole can have a complex form. /7

Processing of the algorithm for stabilization of the motion of a bounding vehicle in the flight phase by mathematical modeling in a computer has permitted a quite efficient stabilization algorithm to be obtained.

Formulation of the problem of stabilization of the motion of the vehicle in the unsupported phase of a bound and a qualitative analysis of various methods of construction of the stabilization algorithm are presented in Section 1. The advisability of the use of the principle of local determination of supplementary control motion of the legs in a forward step in time is shown here. The logic of operation of the stabilization algorithm is described in Section 2. The nominal position of the vehicle at the time of landing and the basic characteristics of the forthcoming flight phase are determined (its duration, the angular momentum vector of the vehicle relative to the center of mass, etc.). The problem of reorganization of the motion of the vehicle in the concluding stage of the flight phase is considered, with account taken of more accurate information on the supporting surface obtained during flight. At the beginning and end of the flight phase, the angles in the leg joints change along the nominal trajectories of transfer of the legs during flight, which ensures a shock free liftoff and soft placement of the legs on the supporting surface. The angular motion of the housing is not stabilized, in this case. During all the remaining time, the legs participate at once in two motions: the nominal leg transfer motion during flight and the supplementary control motions, which provide the change in angular coordinates of the housing along the line of transfer, smoothly connecting the corresponding end

values of the angular coordinates and housing velocity. In Section 3, the problem of determination of the supplementary control motion of the leg is reduced to a problem of a special type of quadratic programming, the algorithm for solution of which is constructed in Section 4. Mathematical modeling of the process of stabilization of the angular
motion of a bounding vehicle in the flight phase (Section 5) and the /8
calculation results (Section 6) demonstrate the efficiency of the stabilization algorithm for various conditions of motion of the vehicle with perturbations.

The author thanks D.Ye. Okhotsimskiy for formulation of the problem and attention to the work.

1. Formulation of the Problem

We will characterize the position of the vehicle housing relative /9
to the absolute coordinate system $O_1\xi\eta\zeta$ (the $O_1\xi\eta$ plane is horizontal), by the coordinates of the center of mass of the housing ξ,η,ζ and the angles (Fig. 1): ψ yaw; θ pitch; γ bank. The axes of the Oxyz coordinate system connected with the vehicle housing are directed along the main axes of inertia of the housing.

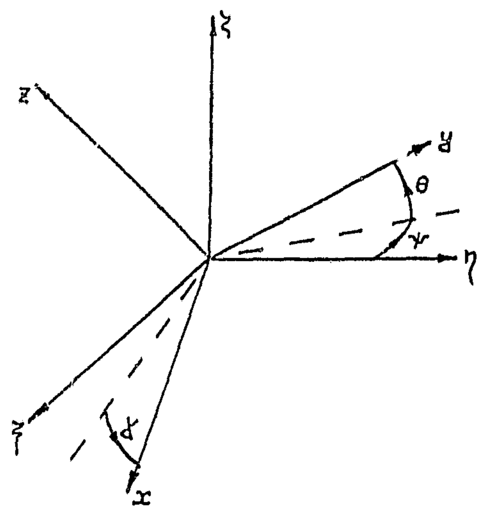


Fig. 1.

The vehicle has four or six two section weighable legs (the number of legs is designated by N). The position of the legs relative to the Oxyz axes is determined by the coordinates of the points of suspension of the thighs to the housing and by the angles (Fig. 2): α_i is the angle between the Ox axis and the plane of the legs; β_i is the angle between the negative direction of the Oz axis and the thigh; q_i is the angle between the thigh and the shank. The plane of the legs, formed by the thigh and the shank, is perpendicular to the Oxy plane.

The vehicle moves the Oy axis forward.

In [2], an algorithm for the synthesis of the programmed motion of a bounding vehicle in the flight phase is considered. From the assigned values of the housing coordinates and the absolute coordinates of the feet at the times of liftoff from the supporting surface and landing, the initial velocity of the center of mass of the housing, the initial angular velocity of the housing and the parameters of the leg transfer trajectories required to perform the forthcoming flight phase are determined. The legs are transferred in the Oxyz relative coordinate system, in such a manner that a shock free liftoff and soft placement of the legs on the supporting surface are ensured. The initial angular velocity of the housing compensates the effect of the transfer movement of the legs on the movement of the housing around the center of mass

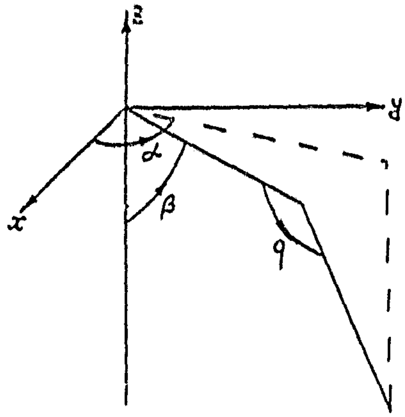


Fig. 2.

during the flight. The programmed motion in the flight phase is compiled from the first integrals of the equations of motion (motion /11 of the center of mass of the vehicle and the law of conservation of angular momentum of the vehicle relative to the center of mass).

Actually, in the unsupported phase of a bound, the vehicle deviates from the programmed motion, as a result of various kinds of perturbation. A variance of the actual dynamic and kinematic characteristics of the vehicle (dimensions, weight, moments of inertia) from their values in the motion control system is a perturbation. Such a situation arises, for example, if a load is placed on the vehicle, and this is not reported to the motion control system. Another type of perturbation involves inflight errors in processing of the leg motions. There are perturbations caused by data errors. Up to the time of liftoff from the supporting surface, the terrain model in the landing region is known with a certain degree of accuracy. During the flight, immediately before the time of landing, the model of the supporting surface in the landing region is refined. The accurate model of the terrain in the landing region is used to reorganize the motion of the vehicle in the final stage of the flight phase.

Perturbations in the supported phase by the prior flight phase data and time inaccuracy in processing the time of liftoff from the supporting surface result in errors in processing the programmed coordinates and velocities of the vehicle at the time of liftoff from the supporting surface. These errors lead to change in the distance of the bound and inflight rolling of the vehicle, and they make it necessary to recalculate the characteristics of the forthcoming phase of the flight: its duration, angular momentum vector of the vehicle relative to the center of mass and the parameters of the leg transfer trajectory, which satisfy the conditions of shock free liftoff and the required soft placement of the legs on the supporting surface. We will call the in- /12 flight leg motions, synthesized at the time of liftoff from the supporting surface, the nominal leg motion.

The center of mass of the vehicle in the flight phase moves along a ballistic trajectory [2]. Therefore, the motion of the center of mass is uncontrolled. The motion of the vehicle around the center of mass occurs according to the law of conservation of angular momentum of the vehicle relative to the center of mass. In this case, the angular motion of the housing can be controlled, by changing the transfer motion of the legs during the flight. For example, by bending (unbending) the legs, their moment of inertia changes and, consequently, the effect of the leg transfer motion on the angular motion of the housing changes. We note that, in this case, the angular momentum vector of the vehicle remains constant. Only redistribution of the angular momentum between the housing and the legs occurs.

The purpose of stabilization of the motion of the bounding vehicle in the flight phase is to reach the position assigned in the programmed motion, with allowance for a shift which causes a deviation of the final position of the center of the mass of the vehicle from that programmed. The question of determination of the nominal position of the vehicle at the time of landing will be discussed in detail in Section 2.

Angular motion of the vehicle in the flight phase which satisfies these boundary conditions is provided by inflight variation of the leg motion.

We will solve the problem of stabilization of the angular motion of a bounding vehicle in the flight phase, by considering the angular coordinates and velocities of the housing, as well as the angles and velocities in the leg joints to be known without error at the current moment of time.

The motion of the vehicle in the flight phase occurs according to the law of conservation of angular momentum of the vehicle relative to the center of mass [2] /13

$$I\bar{\omega} + I_L\dot{\bar{p}} = A\bar{G}^0_a , \tag{1.1}$$

where $\bar{\omega}$ is the angular velocity of the housing projected on the axes of the Oxyz relative coordinate system, $\bar{p}=(\alpha_1,\beta_1,q_1 . . . q_N)$ are the angles of the leg joints, A is the matrix for transfer from the $O_1\xi\eta\zeta$ absolute coordinate system to the Oxyz coordinate system connected with the housing, $\bar{G}^0_a$ is the angular momentum vector of the vehicle relative to the center of mass in the $O_1\xi\eta\zeta$ absolute coordinate system, I and I_L are matrices, the elements of which depend only on the angles of the leg joints $\bar{p}$.

We designate the nominal transfer motion of the legs, synthesized at the time of liftoff from the supporting surface, $\bar{p}_H(t)$. We designate the angular coordinate vector of the vehicle housing $\bar{\Psi}=(\psi,\theta,\gamma)$. Let t_{c0} and t_{c1} be the moments of the start and end of the controlled stage of the flight phase, in which $t_0 \leqslant t_{c0} < t_{c1} \leqslant t_1$, where t_0 is the moment of liftoff from the supporting surface and t_1 is the time of landing.

That motion of the vehicle housing around the center of mass must be provided, which satisfies the boundary conditions

$$\bar{\Psi}(t_{c_0}) = \bar{\Psi}_{c_0} , \quad \dot{\bar{\Psi}}(t_{c_0}) = \dot{\bar{\Psi}}_{c_0} . \tag{1.2}$$

$$\bar{\Psi}(t_{c_1}) = \bar{\Psi}_{c_1} , \quad \dot{\bar{\Psi}}(t_{c_1}) = \dot{\bar{\Psi}}_{c_1} . \tag{1.3}$$

The angles of the leg joints must satisfy the boundary conditions:

$$\bar{p}(t_{c_0}) = \bar{p}_H(t_{c_0}) , \quad \dot{\bar{p}}(t_{c_0}) = \dot{\bar{p}}_H(t_{c_0}) , \tag{1.4}$$

$$\bar{p}(t_{c_1}) = \bar{p}_K(t_{c_1}) , \quad \dot{\bar{p}}(t_{c_1}) = \dot{\bar{p}}_K(t_{c_1}) , \tag{1.5}$$

and the phase limitations: /14

$$\bar{\rho}^{min} \le \bar{\rho} \le \bar{\rho}^{max}, \tag{1.6}$$

$$\bar{\rho}_v^{min} \le \dot{\bar{\rho}} \le \bar{\rho}_v^{max}, \tag{1.7}$$

$$\bar{\rho}_w^{min} \le \ddot{\bar{\rho}} \le \bar{\rho}_w^{max}. \tag{1.8}$$

Limitations (1.6)-(1.8) involve a given vehicle design. We require that condition (1.6) also ensure noninterference of the legs of the vehicle with each other.

We note that boundary condition (1.3) on the angular velocity of the housing $\dot{\bar{\psi}}(t_{c1})=\dot{\bar{\psi}}_{c1}$ is satisfied automatically, by virtue of (1.1), if boundary conditions (1.2), (1.4), and (1.5) and the first of boundary conditions (1.3) are satisfied.

We will call the difference between the actual and nominal motion of the legs the supplementary control motions of the legs

$$\bar{u}(t) = \bar{\rho}(t) - \bar{\rho}_H(t). \tag{1.9}$$

We require that the actual leg motion in stabilization of the angular motion of the housing differ to the minimum extent from the nominal motion of the legs, i.e.,

$$\Phi(\bar{u}) \to \inf. \tag{1.10}$$

where $\bar{u}$ is a certain norm of the vector. For example,

$$\Phi(\bar{u}) = \int_{t_{c_0}}^{t_{c_1}} \sum_{i=1}^{3N} (e_i u_i^2 + f_i \dot{u}_i^2)\,dt \ ,$$

or

$$\Phi(\bar{u}) = \max_{t\in[t_0,t_1]} \left\{\sum_{i=1}^{3N} (e_i u_i^2 + f_i \dot{u}_i^2)\right\},$$

where $e_i>0, f_i>0$ are weighting factors. /15

The problem of stabilization of the angular motion of a bounding vehicle in the flight phase (1.1)-(1.10) can be solved as an extreme problem with limitations of a general type. Various methods of simplification of problem (1.1)-(1.10) are possible, by means of introduction of supplementary connections. We consider qualitatively and compare some of them.

Parametric stabilization algorithm. We assume

$$\bar{u} = \bar{u}(t, \bar{a}), \tag{1.11}$$

i.e., the supplementary control motion of the legs is a fixed function of time and parameter vector $\bar{a}$ ($\bar{a}$=const). In this case, for any values of the parameter, there is

$$\bar{u}(t_{c0}, \bar{a}) = 0 \ , \ \dot{\bar{u}}(t_{c0}, \bar{a}) = 0,$$
$$\bar{u}(t_{c1}, \bar{a}) = 0 \ , \ \dot{\bar{u}}(t_{c1}, \bar{a}) = 0,$$

i.e., boundary conditions (1.4)-(1.5) are satisfied automatically. By substituting (1.11) in (1.6)-(1.8), we obtain the region of permissible values of parameters $\bar{a}$

$$\bar{a} \in V \ . \tag{1.12}$$

By integrating (1.1) with initial conditions (1.2), over segment of time $t \in [t_{c0}, t_{c1}]$, we obtain the value of the angular coordinates of the housing at time t_{c1}, with given values of parameters $\bar{a}$

$$\bar{\Psi}(t_{c1}) = \bar{\chi}(\bar{a}).$$

Then, boundary conditions (1.3) are equivalent to limitations of the type, equality to the parameter vector

$$\bar{\chi}(\bar{a}) = \bar{\Psi}_{c1} \ . \tag{1.13}$$

By substituting (1.11) in (1.10), we rewrite the optimizing functional in the form /16

$$\bar{\Phi}^*(\bar{a}) \to \inf. \tag{1.14}$$

As a result, we find that the problem of stabilization of the angular motion of a bounding vehicle in the flight phase (1.1)-(1.10) is reduced to mathematical programming problem (1.12)-(1.14).

For solution of the parametric stabilization program, a large amount of calculation work must be done. This is connected with digital integration of differential equations (1.1), with calculation of the left sides of limitations of the equality (1.13) type.

Linear transition stabilization algorithm. We require that the angular coordinates of the housing change along transition lines which smoothly connect the boundary values of the angular coordinates and velocities of the housing (1.2)-(1.3). One method of plotting the transition lines is described in Section 2. This approach permits reduction of the dimensionality of extreme problem (..1)-(1.10), which can be solved by determination of the supplementary control motion of the legs.

Local stabilization algorithm. We require that the angular coordinates of the housing also, as in the linear transition stabilization algorithm, change along transition lines. The supplementary control motion of the legs is determined at discrete moments of time τ_K(K=0,1,2...), in small forward time steps $t \in [\tau_K, \tau_{K+1}]$. We assume

$$\bar{u} = \bar{u}(t, \bar{a}_K), \text{ with } t \in [\tau_K, \tau_{K+1}], \tag{1.15}$$

where $\bar{a}_K$ is the vector of parameters, which have a constant value in a given step.

At time τ_K, such values of the parameters must be found that, with $t=\tau_{K+1}$, the angular coordinates and velocities of the housing are on the transition lines. /17

By substituting (1.15) and the values of the angular coordinates and velocities of the housing with the transition lines in angular momentum integral (1.1), we obtain a limitation of the type, equality to the values of parameters $\bar{a}_K$, which are written in the following manner

$$\bar{\chi}_K(\bar{a}_K) = 0 \tag{1.16}$$

Boundary conditions (1.4)-(1.5) and limitations (1.6)-(1.8) determine the region of permissible values of the parameters at $t \in [\tau_K, \tau_{K+1}]$

$$\bar{a}_K \subset U_K \tag{1.17}$$

We require that, at time τ_{K+1}, the actual motion of the legs differ to the minimum extent from the nominal transfer motion of the legs (i.e., instead of integral optimizing functional (1.10), we insert a local optimizing functional)

$$\sum_{i=1}^{3N} \{\alpha_i u_i^2 + \beta_i \dot{u}_i^2\}\Big|_{t=\tau_{K+1}} \to \inf. \tag{1.18}$$

By substituting (1.15) in (1.18), we obtain an optimizing functional which depends parameters $\bar{a}_K$

$$\Phi^K(\bar{a}_K) \to \inf. \tag{1.19}$$

By solving mathematical programming problem (1.16)-(1.18), we determine the values of parameters $\bar{a}_K$ and, consequently, the supplementary control motion of the legs $\bar{u}(t)$ with $t \in [\tau_K, \tau_{K+1}]$.

It can be expected that the smallness of the step will permit significant simplification of problem (1.16)-(1.18).

The basic virtue of the local stabilization algorithm is the lack of need for digital integration of the differential equations and, consequently, a very much smaller amount of calculations than in other methods of solution of the problem of stabilization of the angular motion of the vehicle in the unsupported phase of the bound. /18

The most rigid restrictions in selection of the method of construction of the algorithm for stabilization of the angular motion of a bounding vehicle in the flight phase is connected with the requirements for high speed of the vehicle motion control computer. The duration of the unsupported phase of a bound to a distance for 10 m on the surface of the earth is on the order of one and a half seconds. Therefore, the actual requirements for high speed of the computer are made only by a local stabilization type algorithm.

Qualitative analysis of various methods of construction of the algorithm for stabilization of the angular motion of a bounding vehicle in the flight phase permit it to be concluded that it is advisable to construct a local stabilization type algorithm.

2. Stabilization Logic

We designate the moment of liftoff from the supporting surface t_0 and the moment of landing t_1. The motion of the vehicle center of mass in the flight phase is uncontrolled. The vehicle center of mass moves along a parabolic trajectory [2] /19

$$\bar{R}_a = \bar{R}_a^0 + \dot{\bar{R}}_a^0 (t - t_0) - \frac{1}{2}\bar{g}(t - t_0)^2, \qquad (2.1)$$

where $\bar{R}_a=(\xi_a, \eta_a, \zeta_a)$ are the coordinates of the vehicle center of mass, $R_a^0, \dot{\bar{R}}_a^0$ are the coordinates and velocity of the vehicle center of mass at the time of liftoff from the supporting surface, $\bar{g}$ is the acceleration of gravity.

The purpose of stabilization is to reach the assigned programmed position at the time of landing, with allowance for the shift caused by deviation of the final position of the vehicle center of mass from that programmed.

We describe the procedure for determination of the nominal position of the vehicle at the time of landing. At the moment of liftoff from the supporting surface, the model of the terrain in the landing region is known with a certain degree of accuracy by the motion control system of the bounding vehicle. Let $\zeta_s(\xi, \eta)$ be the model of the supporting surface in the landing region. We require that, at the time of landing, the vertical distance from the supporting surface to the vehicle center of mass be the same as in the programmed motion, equal to $\Delta\zeta^1_a$

$$\zeta_a^1 = \zeta_s(\xi_a^1, \eta_a^1) + \Delta\zeta_a^1, \qquad (2.2)$$

where $\bar{R}^1_a=(\xi^1_a,\eta^1_a,\zeta^1_a)$ are the coordinates of the vehicle center of mass at the time of landing.

It follows from the motion of the vehicle center of mass in the flight phase that

$$
\begin{aligned}
&T=(\dot{\zeta}^0_a+\sqrt{(\dot{\zeta}^0_a)^2-2g(\zeta^1_a-\zeta^0_a)}\,)/g,\\
&\xi^1_a=\xi^0_a+\dot{\xi}^0_a T,\\
&\eta^1_a=\eta^0_a+\dot{\eta}^0_a T,
\end{aligned}
\tag{2.3}
$$

where $T=t_1-t_0$ is the duration of the flight phase. /20

We require that the angular coordinates of the housing at the time of landing be the same as in the programmed motion. Let $\bar{R}^1=(\xi^1,\eta^1,\zeta^1)$ be the position of the housing center of mass at the time of landing. We determine the leg position at the time of landing, by using a posture shaping algorithm [1]. The supporting contour projected on the horizontal plane $O_1\xi\eta$ is a rectangle of fixed dimensions, the length of which equals the distance between the suspension points of the front and rear legs. The angle between the longitudinal axis of the projection of the supporting contour on the horizontal plane and the $O_1\eta$ axis (supporting contour orientation) at the time of landing equals the angle of yaw of the vehicle housing $\psi^1_{vh}=\psi^1$. The projection of the center of the supporting contour on the horizontal plane lies on a line which connects the center of the initial supporting contour and point (ξ^1,η^1), at distance $(\zeta^1-\zeta_s(\xi^1,\eta^1)-\zeta_{min})/\max\{1,|\lambda^*|\}$ from point (ξ^1,η^1), where ζ_{min} is the minimum permissible distance from the supporting surface to the housing center of mass, λ^* is the tangent of the slope angle of the velocity of the vehicle center of mass to the horizon at the time of landing. Thus, by using the terrain model, we obtain the absolute coordinates of the feet and, consequently, the angles of the leg joints at the time of landing, as a function of the position of the housing center of mass at the time of landing $\bar{R}^1$. We designate the vector of the direction from the vehicle center of mass to the housing center of mass $\bar{\rho}$. Then, at the time of landing, $\bar{\rho}^1$ is a function of $\bar{R}^1$, and the following occurs

$$\bar{R}^1=\bar{\rho}^1+\bar{R}^1_a \tag{2.4}$$

By substituting (2.2)-(2.3) in (2.4), we obtain a system of equations relative to $\bar{R}^1$, in the computer solution of which by the modified quickest descent method, we determine the nominal position of the vehicle at the time of landing and the duration of the flight phase. /21

Because of the conditions of shock free liftoff and softness of setting the legs on the supporting surface, the angles of the leg joints change rapidly at the beginning and end of the unsupported phase of motion. Therefore, in these segments of time, we will not stabilize the angular motion of the housing. We designate the times of the start and end of operation of the stabilization algorithm t_{co} and t_{c1}, with $t_o < t_{co} < t_{c1} < t_1$.

At the time of liftoff from the supporting surface, t_o must determine the characteristics of the forthcoming flight phase, from the phase coordinates of the vehicle at time t_o, known from the navigation system readings.

The ballistic trajectory of the motion of the vehicle center of mass (2.1) is plotted. The coordinates and velocity of the vehicle center of mass at the time of liftoff from the supporting surface are calculated easily from the known phase coordinates of the vehicle, and they completely determine the motion of the vehicle center of mass in the flight phase.

The nominal position of the vehicle at the time of landing and the duration of the flight phase are determined by solution of (2.?)-(2.4).

The vehicle angular momentum vector relative to the center of mass at the time of liftoff from the supporting surface is calculated.

The leg transfer trajectory which ensures shock free liftoff and the required softness of setting the legs on the supporting surface with low absolute velocity (possibly zero), components of the fixed portion of the vehicle center of mass velocity at the time of landing, is plotted similar to the way it was done in synthesis of the programmed motion of the vehicle for the unsupported phase of the bound [2]. /22
We will call the inflight leg motion, obtained at the time of liftoff from the supporting surface, the nominal leg motion.

With $t \in [t_o, t_{co}]$ and $t \in [t_{c1}, t_1]$, the angles of the leg joints change along the nominal trajectories, and the angular motion of the housing is not stabilized.

By digitally integrating the equations of the law of conservation of angular momentum of the vehicle relative to the center of mass (1.1), from the nominal position at the time of landing, with a negative time step to $t=t_{c1}$ (two steps of integration are completed by the Runge-Kutt fourth order method), the control system determines the nominal values of the angular coordinates and velocity of the vehicle housing at time t_{c1}, which we designate $\overline{\Psi}_{c1}, \dot{\overline{\Psi}}_{c1}$ (here and subsequently, $\overline{\Psi}=(\psi,\theta,\gamma)$ are the angular coordinates of the housing).

Let, at time t_{co}, there be the following values of the angular coordinates and velocities of the housing $\overline{\Psi}_{co}, \dot{\overline{\Psi}}_{co}$. With $t \in [t_{co}, t_{c1}]$, the control system varies the nominal leg motion, in such a way that a change in angular coordinates of the housing along the transition lines which smoothly connect the boundary value of the angular coordinates and velocities of the housing $(\bar{\Psi}_{co}, \dot{\bar{\Psi}}_{co})$ and $(\bar{\Psi}_{c1}, \dot{\bar{\Psi}}_{c1})$ is ensured. The transition lines are functions of the type

$$\bar{\Psi}_{\wedge} = \bar{\Psi}_{c_0} + \dot{\bar{\Psi}}_{c_0}(t - t_{c_0}) + \frac{1}{2}\bar{a}(t - t_{c_0})^2 + \frac{1}{6}\bar{b}(t - t_{c_0})^3, \tag{2.5}$$

then,

$$\bar{a} = \frac{2}{T_c^2}\{3(\bar{\Psi}_{c_1} - \bar{\Psi}_{c_0}) - (\dot{\bar{\Psi}}_{c_0} + 2\dot{\bar{\Psi}}_{c_1})T_c\}$$

$$\bar{b} = -\frac{6}{T_c^3}\{2(\bar{\Psi}_{c_1} - \bar{\Psi}_{c_0}) - (\dot{\bar{\Psi}}_{c_0} + \dot{\bar{\Psi}}_{c_1})T_c\}$$

where $T_c = t - t_{co}$.

If, during actual motion at time $t = t^*$, the angular coordinates or velocity of the housing go outside the ε corridor of the transition lines /23

$$|\bar{\Psi} - \bar{\Psi}_{\wedge}| < \varepsilon \quad , \quad |\dot{\bar{\Psi}} - \dot{\bar{\Psi}}_{\wedge}| < \varepsilon_1, \tag{2.6}$$

a supplementary correction is made, $t_{co} = t^*$, and the transition lines are recalculated.

The ideas connected with the use of the transition lines are consistent with those reported in [8-11].

During the flight, before the time of landing, the data system refines the model of the terrain in the landing region. By using the refined terrain model, at moment t_{c1}, the control system refines the nominal position of the vehicle at the time of landing and the duration of the flight phase, by solving system (2.2)-(2.4). The nominal leg position is corrected in the corresponding manner, at $t \in [t_{c_0}, t_{c_1}]$.

In this section, the logic of operation of the algorithm for stabilization of the motion of a bounding vehicle in the unsupported phase of a bound has been constructed. The problem of determination of the supplementary control motions of the legs, which ensure a change in the angular coordinates of the housing along the transition lines at $t \in [t_{c_0}, t_{c_1}]$, is considered in the next section.

3. Problem of Determination of Supplementary Control Motion of Legs

Let $\bar{P} = (\alpha_1, \beta_1, q_1 \ldots q_N)$ be the angles of the leg joints. We designate the nominal inflight leg motion, obtained at the time of liftoff from the supporting surface, which satisfies the conditions of shock free lift off and soft setting of the legs on the supporting surface, by $\bar{P}_H(t)$, and the supplementary control motion of the legs by $\bar{u}(t)$. We then obtain /24

$$\bar{\beta}(t) = \bar{\beta}_H(t) + \bar{u}(t), \tag{3.1}$$

We will determine the supplementary control motion of the legs at discrete moments of time $\tau_K (K=0,1,2...)$ in the forward time step. We require that the accelerations of the leg joints be constant in the forthcoming step $t \in [\tau_K, \tau_{K+1}]$. In this case, we have

$$\bar{u}(\tau_{K+1}) = \bar{u}(\tau_K) + \dot{\bar{u}}(\tau_K)\Delta\tau_K + \frac{1}{2}\ddot{\bar{u}}(\tau_K)\Delta\tau_K^2, \tag{3.2}$$

$$\dot{\bar{u}}(\tau_{K+1}) = \dot{\bar{u}}(\tau_K) + \ddot{\bar{u}}(\tau_K)\Delta\tau_K, \tag{3.3}$$

where $\Delta\tau_K = \tau_{K+1} - \tau_K$ is a step in solution of the problem of determination of the supplementary control motion of the legs. The value of $\Delta\tau_K$ is selected in the following manner

$$\Delta\tau_K = \min\left\{\max\left\{\frac{\varepsilon}{\max\limits_{i=1,3N}|\ddot{u}_i(\tau_{K-1})|}, \Delta\tau_{min}\right\}, \Delta\tau_{max}\right\},$$

where $\varepsilon > 0$ is a small positive value, $\Delta\tau_{min}, \Delta\tau_{max}$ are the assigned limits of permissible values of $\Delta\tau_K$.

In the event $\ddot{\bar{u}}(\tau_K)$ differs little from $\ddot{\bar{u}}(\tau_{K-1})$, we find that $\ddot{\bar{u}}(\tau_K)\Delta\tau_K$ is a value on the order of ε^2, disregarding which in (3.2), we obtain

$$\bar{u}(\tau_{K+1}) = \bar{u}(\tau_K) + \dot{\bar{u}}(\tau_K)\Delta\tau_K, \tag{3.2*}$$

If the assumption made of the "smoothness" of $\ddot{\bar{u}}(t)$ is not justified, the error resulting from the change to approximate formula (3.2*) will lead to deviation of the angular motion of the vehicle housing from the transition lines, and it will become necessary to make a supplementary correction. By selection of parameters $\varepsilon, \Delta\tau_{min}$ and $\Delta\tau_{max}$, it can be certain that such cases will occur rarely. /25

By substituting (3.1), (3.2*), (3.3), the known nominal transfer motion of the legs and the angular coordinates and velocity of the housing, with transition lines (2.5) at time τ_{K+1}, in angular momentum integral (1.1), we obtain

$$A\,\ddot{\bar{u}}(\tau_K) = \bar{b}, \tag{3.4}$$

where

$$A = I_L(\bar{\beta}^*_{K+1}),$$

$$\bar{b} = \frac{1}{\Delta\tau_K}\left\{\bar{K}^0 - I(\bar{\beta}^*_{K+1}) - I_L(\bar{\beta}^*_{K+1})\left[\dot{\bar{\beta}}_H(\tau_{K+1}) + \dot{\bar{u}}(\tau_K)\right]\right\},$$

$$\bar{\beta}^*_{K+1} = \bar{\beta}_H(\tau_{K+1}) + \bar{u}(\tau_K) + \dot{\bar{u}}(\tau_K)\Delta\tau_K.$$

Instead of boundary conditions (1.4)-(1.5) and phase limitations (1.6), we require satisfaction of more rigid restrictions (3.5), which automatically ensures satisfaction of conditions (1.4)-(1.6).

$$\mu(t)\,\bar{U}^{min} \le \bar{u}(t) \le \mu(t)\,\bar{U}^{max},$$

where $\bar{U}^{min}$, $\bar{U}^{max}$ are fixed quantities,

$$\mu(t) = \frac{1}{2}\left(1-\cos\left(2\pi\,\frac{t-t_{c_0}}{t_{c_1}-t_{c_0}}\right)\right). \qquad (3.5)$$

In this case, the coordinate P_j (j=1,2...3N) transfer trajectory lies within the cross hatched region in Fig. 3. /26

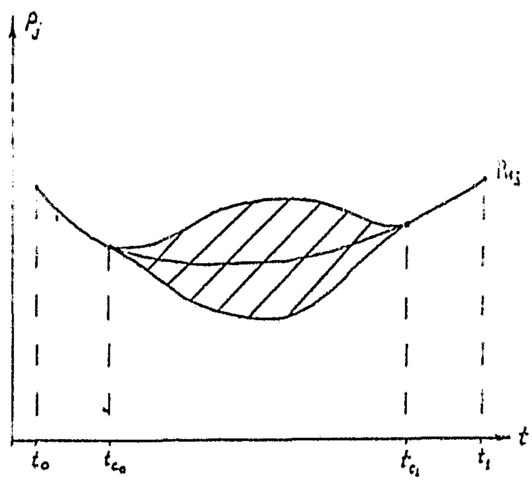


Fig. 3.

We replace restrictions (1.7)-(1.8) by the restrictions

$$\bar{U}_V^{min} \le \dot{\bar{u}} \le \bar{U}_V^{max}, \qquad (3.6)$$

$$\bar{U}_W^{min} \le \ddot{\bar{u}} \le \bar{U}_W^{max}, \qquad (3.7)$$

where $\bar{U}_V^{min}, \bar{U}_V^{max}, \bar{U}_W^{min}, \bar{U}_W^{max}$ are fixed quantities which ensure satisfaction of (1.7)-(1.8).

In determination of the supplementary control motion of the legs at time τ_K, we will satisfy restrictions (3.5)-(3.7) at time τ_{K+1}. It may turn out that, at time τ_{K+1}, the value of u_j (j=1,2,...3N) proves to be around the boundaries of the region of permissible values of u_j (3.5), with a velocity directed towards the boundaries and, in the subsequent step, limitations (3.5)-(3.7) will prove to be incompatible. In order to exclude the possibility of the development of such a situation, instead of velocity restriction (3.6), we introduce more rigid restrictions

$$\bar{V}^{min}(\bar{u}) \le \dot{\bar{u}} \le \bar{V}^{max}(\bar{u}), \qquad (3.8)$$

where

$$\bar{V}^{max} = \begin{cases} \bar{U}_V^{max}, & \text{if } -\bar{u}+\mu\,\bar{U}^{max} > \varepsilon_V, \\ \dot{\mu}\,\bar{U}^{max} + (\bar{U}_V^{max} - \dot{\mu}\,\bar{U}^{max})(-\bar{u}+\mu\,\bar{U}^{max})/\varepsilon_V, & \text{otherwise} \end{cases}$$

$$\bar{V}^{min} = \begin{cases} \bar{U}_V^{min}, & \text{if } \bar{u}-\mu\,\bar{U}^{min} > \varepsilon_V, \\ \dot{\mu}\,\bar{U}^{min} + (\bar{U}_V^{min} - \dot{\mu}\,\bar{U}^{min})(\bar{u}-\mu\,\bar{U}^{min})/\varepsilon_V, & \text{otherwise} \end{cases}$$

$\varepsilon_V > 0$ is a fixed positive value.

Then, in approaching the boundary of the region of permissible values, at distance d_{uj} less than ε_v, the maximum permissible velocity toward the boundaries of the region of permissible values decreases in proportion to d_{uj}, to the velocity along this boundary at $d_{uj}=0$. /2

By substituting the values of $\bar{u}, \dot{\bar{u}}$ (3.2)-(3.3) in (3.5)-(3.8) and consolidating these limitations, we obtain

$$\bar{\Lambda}^{min} \leq \ddot{\bar{u}}(\tau_K) \leq \bar{\Lambda}^{max}, \tag{3.9}$$

where

$$\bar{\Lambda}^{max} = \min\{[\mu(\tau_{K+1})\bar{V}^{max} - \bar{u}^{*}_{K+1}]\frac{2}{\Delta\tau_K^2};$$
$$[\bar{V}^{max}(\bar{u}^{*}_{K+1}) - \dot{\bar{u}}(\tau_K)]\frac{1}{\Delta\tau_K};\ \bar{U}_W^{max}\},$$
$$\bar{\Lambda}^{min} = \max\{[\mu(\tau_{K+1})\bar{V}^{min} - \bar{u}^{*}_{K+1}]\frac{2}{\Delta\tau_K^2};$$
$$[\bar{V}^{min}(\bar{u}^{*}_{K+1}) - \dot{\bar{u}}(\tau_K)]\frac{1}{\Delta\tau_K};\ \bar{U}_W^{min}\},$$
$$\bar{u}^{*}_{K+1} = \bar{u}(\tau_K) + \dot{\bar{u}}(\tau_K)\Delta\tau_K.$$

We require that, at time τ_{K+1}, the actual motion of the legs differ the least from the nominal transfer motion of the legs

$$\sum_{i=1}^{3N}\{e_i[u_i(\tau_{K+1})]^2 + f_i[\dot{u}_i(\tau_K)]^2\} \to \inf, \tag{3.10}$$

where $e_i>0, f_i>0$ are weighting factors.

By substituting (3.2)-(3.3) in (3.10), we obtain

$$\sum_{i=1}^{3N}\{c_i[\ddot{u}_i(\tau_K)]^2 + d_i\ddot{u}_i(\tau_K)\} \to \inf, \tag{3.11}$$

where /2

$$c_i = \frac{1}{4}e_i\Delta\tau_K^3 + f_i\Delta\tau_K,$$
$$d_i = e_i\Delta\tau_K(u_i(\tau_K) + \dot{u}_i(\tau_K)\Delta\tau_K) + 2f_i\dot{u}_i(\tau_K).$$

As a result, the problem of determination of the supplementary control motion of the legs is reduced to quadratic programming problem (3.4), (3.9), (3.11), which, by designating $\bar{x} \underset{def}{=} \ddot{\bar{u}}(\tau_K)$, we rewrite in the form

$$\sum_{i=1}^{3N} (c_i x_i^2 + d_i x_i) \to \inf, \quad c_i > 0,$$
$$\sum_{i=1}^{3N} a_{ji} x_i = b_j, \tag{3.12}$$
$$\Lambda_i^{min} \le x_i \le \Lambda_i^{max},$$
$$(i=1,2\ldots 3N), \ (j=1,2,3).$$

In the event the restrictions of the equality and inequality types in problem (3.12) are incompatible, we will look for a solution which satisfies the inequality type restrictions and, with the maximum degree of accuracy satisfies the equality type restrictions. For this, we solve the problem

$$y_j = \sum_{i=1}^{3N} a_{ji} x_i - b_j,$$
$$\Lambda_i^{min} \le x_i \le \Lambda_i^{max}, \tag{3.13}$$
$$\sum_{i=1}^{3N} (c_i x_i^2 + d_i x_i) + \sum_{j=1}^{3} v_j y_j \to \inf,$$
$$(i=1,2\ldots 3N), \ (j=1,2,3).$$

where $v_j \gg c_i > 0$ are weighting factors, y_j are new variables.

We note that problem (3.13) is practically the same as problem (3.12). They are partial cases of the quadratic programming problem of a more general type. /30

$$\varphi = \sum_{i=1}^{n} (h_i z_i^2 + g_i z_i) \to \inf, \quad h_i > 0,$$
$$\sum_{i=1}^{n} s_{ji} z_i = z_j, \tag{3.14}$$
$$z_i^{min} \le z_i \le z_i^{max}$$
$$(i=1,2\ldots n), \ (j=1,2\ldots k), \ k<n.$$

where z_i are independent variables, $z_i^{max}=\infty$ corresponds to the case when z_i is unbounded above, $z_i^{min}=-\infty$ corresponds to the case when z_i is unbounded below.

We call the point which satisfies the equality and inequality type restrictions the initial approximation of the quadratic programming problem. In problem (3.13), the initial approximation is determined trivially

$$x_i^0 = \frac{\Lambda_i^{max} + \Lambda_i^{min}}{2}, \quad y_j^0 = \sum_{i=1}^{3N} a_{ji} x_i^0 - b_j.$$

All the independent variables in problem (3.12) are bounded above and below by finite values. Therefore, in construction of the algorithm for determination of the initial approximation of problem (3.14), we will consider only the case when all z_i^{max} and z_i^{min} have a finite value.

The surfaces of the optimizing functional level of problem (3.14), are n dimensional ellipsoids, the principal directions of which coincide with the directions of change of the independent variables. At the same time, the inequality type limitations are limitations on the limits of change of the independent variables. These conditions, which determine the special form of problem (3.14), permit construction of a simple algorithm for its solution. /31

Thus, the problem of determination of the supplementary control motions of the legs, which ensure change of the angular coordinates of the housing along the transition lines, is reduced to a special type of quadratic programming problem, the algorithm for solution of which is constructed in the next section.

4. Algorithm for Solution of Quadratic Programming Problem

An algorithm is constructed in this section, for solution of a quadratic programming problem which is a generalization of problem (3.12), (3.13), which develops in determination of the supplementary control motion of the legs in the algorithm for stabilization of the angular motion of a bounding vehicle in the flight phase /32

$$\begin{gathered}\varphi = \sum_{i=1}^{n} h_i z_i^2 + g_i z_i \rightarrow \inf \quad (h_i > 0) \\ \sum_{i=1}^{n} a_{ji} z_i = c_j \\ z_i^{min} \leq z_i \leq z_i^{max} \\ (i = 1, 2 \ldots n;\; j = 1, 2 \ldots k)\end{gathered} \tag{4.1}$$

Algorithms for solution of quadratic programming problems and the bibliographies of studies on this question can be found in [12-13]. The main bulk of calculations in algorithms for solution of quadratic programming problems of a general type involve determination of the direction of the quickest descent which does not violate the restrictions imposed. The solutions may go beyond the boundaries of the region defined by the inequality type of restrictions but, in subsequent iteration steps, it may be found that this boundary must be abandoned.

Problem (4.1) is a special type of quadratic programming problem. The surfaces of the optimizing functional level

$$\sum_{i=1}^{n} h_i z_i^2 + g_i z_i = const$$

are n dimensional ellipsoids, the principal directions of which coincide with the directions of change of the independent variables. At the same time, the inequality type restrictions are restrictions on the limits of change of the independent variables. These conditions, which define the special form of problem (4.1), permit the construction of a simple algorithm for its solution. /33

Algorithm 1, solutions of quadratic programming problem (4.1).

We designate $\overline{Z}^P$ the solution of problem (4.1), in the case when there are no inequality type restrictions. The value of $\overline{Z}^P$ is determined by the Lagrange indeterminate multiplier method

$$z_i^p = \frac{1}{2h_i}\left(\sum_{j=1}^{k} \lambda_j A_{ji} - g_i\right) \quad (i=1,2,\dots n), \tag{4.2}$$

where λ_j are Lagrange indeterminate multipliers, which are the solution of the system of linear equations

$$\sum_{l=1}^{k}\left(\sum_{i=1}^{n} \frac{A_{ji} A_{li}}{2h_i}\right)\lambda_l = z_j + \sum_{i=1}^{n} A_{ji} \frac{g_i}{2h_i} \quad (j=1,2,\dots k) \tag{4.3}$$

System (4.3) is nonsingular, if the equality type restrictions are linearly independent. If $\overline{Z}^P$ satisfies the inequality type restrictions, $\overline{Z}^P$ is the solution of problem (4.1). Otherwise, we take a provisional initial approximation $\overline{Z}^o$, which satisfies the equality and inequality type restrictions (the question of the method of determination of $\overline{Z}^o$ will be discussed below in this section). We connect $\overline{Z}^o$ and $\overline{Z}^P$ by a straight line, and we find point $\overline{Z}^1$ between them, in which the system reaches an inequality type phase restriction in motion from $\overline{Z}^o$ to $\overline{Z}^P$

$$\bar{Z}^1 = \bar{Z}^o + \mu\left(\bar{Z}^p - \bar{Z}^o\right), \tag{4.4}$$

where /34

$$\mu = \min\{\mu_1, \dots \mu_n\}$$

$$\mu_i = \begin{cases} \dfrac{Z_i^{max} - Z_i^o}{Z_i^p - Z_i^o}, & \text{with } Z_i^p > Z_i^{max} \\ \dfrac{Z_i^{min} - Z_i^o}{Z_i^p - Z_i^o}, & \text{with } Z_i^p < Z_i^{min} \\ 1, & \text{with } Z_i^{min} \le Z_i^p \le Z_i^{max} \end{cases}$$

By virtue of (4.4), point $\overline{Z}^1$ satisfies the equality type restrictions, since $\overline{Z}^o$ and $\overline{Z}^P$ satisfy them.

We fix all the variables which reach the inequality type restrictions at point $\bar{Z}^1$. We transfer the terms corresponding to them to the equality type restrictions on the right side, recalculating the free terms. We exclude the fixed variables from consideration, and we will call the remaining variables free. We exclude the linearly dependent variables from the equality type restrictions, if they were formed as a result of exclusion of the fixed variables. If the number of free variables equals the number of equations in the equality type restrictions, $\bar{Z}^1$ is the solution of problem (4.1). Otherwise, we set $\bar{Z}^0=\bar{Z}^1$, and we repeat everything again for the free variables.

Theorem 1. If there exists initial approximation $\bar{Z}^0$, which satisfies the equality and inequality type restrictions of problem (4.1), algorithm 1, in a finite, smaller than n number of steps converges toward the solution of problem (4.1).

Proof. The number of variables in problem (4.1) equals n. In each step of algorithm 1, the number of free variables decreases by at least one. Consequently, algorithm 1 stops in a finite, smaller than n number of steps.

We designate L the linear multiformity determined by the equality type restrictions and Ω, the region determined by the inequality type restrictions of problem (4.1). /35

We designate the point towards which algorithm 1 converges by $\bar{Z}^*$. Without restriction of generality, it can be considered that, in this case, the first n_0 variables reached the boundary $Z_{i0}=Z_{i0}^{max}(i_0=1,2,\ldots n_0)$, and the remaining variables lie within region Ω. As was noted in the description of algorithm 1, $Z^*\in L\cap\Omega$. Consequently, in order to prove that $\bar{Z}^*$ is the solution of problem (4.1), it is sufficient to show that there do not exist directions from point $\bar{Z}^*$, which do not violate the equality and inequality type restrictions, along which the optimizing functional decreases.

We designate $P_L(\bar{a})$ as the projection of vector $\bar{a}$ in linear multiformity L.

Let variable $Z_{i_0}\ (i_0\in 1,2,\ldots n_0)$ reach the boundary of region Ω in the ℓ-th step of the algorithm. Then,

$$P_L\left(\frac{\partial\varphi}{\partial Z_{i_0}}\Big|_{\bar{Z}=\bar{Z}(\ell)}\right)<0,$$

where ϕ is the optimizing functional.

At the same time,

$$\frac{\partial\varphi}{\partial Z_{i_0}}\Big|_{\bar{Z}=\bar{Z}(\ell)}=\frac{\partial\varphi}{\partial Z_{i_0}}\Big|_{\bar{Z}=\bar{Z}^*}=2h_{i_0}Z_{i_0}^{max}+g_{i_0}; \qquad (4.5)$$

consequently,

$$P_i\left(\frac{\partial\varphi}{\partial z_{i_0}}\Big|_{\bar Z=\bar Z^A}\right)\le 0 \quad (i_0=1,2,\ldots n_0). \tag{4.6}$$

For variables which do not reach the boundary of region Ω, there is

$$P_i\left(\frac{\partial\varphi}{\partial z_{i_1}}\Big|_{\bar Z=\bar Z^A}\right)=0 \quad (i_1=n_0+1,\ldots n). \tag{4.7}$$

Let $\bar\gamma$ be an arbitrary direction along L, directed from $\bar Z^A$ toward the interior of region Ω. In this case, we have /3

$$\gamma_{i_0}\le 0 \quad (i_0=1,\ldots n_0)$$

since the first n_o variables reached the boundary $Z_{io}=Z_{io}^{max}$. The derivative of the optimizing functional along $\bar\gamma$ is

$$\frac{\partial\varphi}{\partial\gamma}\Big|_{\bar Z=\bar Z^A}=\sum_{i=1}^{n} P_i\left(\frac{\partial\varphi}{\partial z_i}\Big|_{\bar Z=\bar Z^A}\right)\gamma_i \tag{4.8}$$

It follows from (4.5)-(4.8) that

$$\frac{\partial\varphi}{\partial\gamma}\Big|_{\bar Z=\bar Z^A}=\sum_{i_0=1}^{n_0} P_i\left(\frac{\partial\varphi}{\partial z_{i_0}}\Big|_{\bar Z=\bar Z^A}\right)\gamma_{i_0}\ge 0, \tag{4.9}$$

or ϕ does not decrease along $\bar\gamma$.

Since γ is an arbitrary direction from $\bar Z^A$, which does not violate the imposed restrictions, $\bar Z^A$ is the solution of problem (4.1).

Note. Theorem 1 is not valid for a quadratic programming problem of more general form than problem (4.1). For example, we consider the quadratic programming problem for two independent variables (Fig. 4). There are no equality type restrictions, inequality type restrictions are imposed on the independent variables, and they determine the rectangular region of their change. The surfaces of the optimizing functional level are ellipses, the principal directions of which do not coincide with the directions of change of the independent variables. In Fig. 4, the initial approximation is designated $\bar Z^o$ and the point towards which algorithm 1 converges, $\bar Z^*$. The solution of this problem is point $\bar Z^\phi$, which does not coincide with $\bar Z^*$.

We proceed to the algorithm for determination of the initial approximation $\bar Z^o$, which satisfies the equality and inequality type restrictions of problem (4.1). We consider the case when the limits of change of all the independent variables in the inequality type restrictions of problem (4.1) have a finite value. In this case, /38

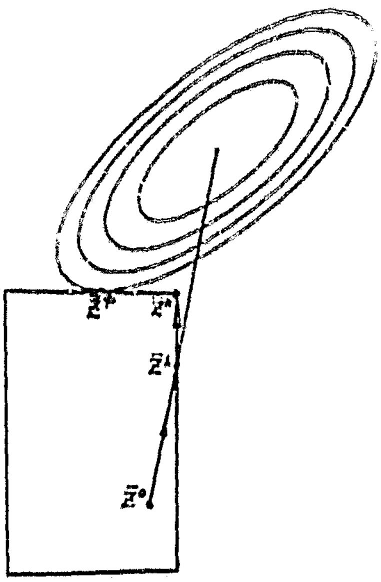


Fig. 4.

without restriction of generality, it can be considered that $Z_i^{max}=1$, $Z_i^{min}=1$ $(i=1,2,\ldots n)$. For this, it is sufficient to substitute the variables

$$\tilde{Z}_i = \left(Z_i - \frac{Z_i^{max}+Z_i^{min}}{2}\right)\frac{2}{Z_i^{max}-Z_i^{min}}.$$

Further, we will consider that $Z_i^{max}=1$, $Z_i^{min}=-1$.

The inequality type restrictions determine region Ω, which is a n dimensional cube

$$-1 \leq Z_i \leq 1 \quad (i=1,2,\ldots n), \tag{4.10}$$

and the equality type restrictions determine the linear multiformity L

$$\sum_{i=1}^{n} b_{ji} Z_i = Z_j \quad (j=1,2,\ldots k). \tag{4.11}$$

Let $\bar{Z}^*$ be the solution of the problem

$$\begin{gathered} \bar{Z} \in L, \\ \sum_{i=1}^{n} Z_i^2 \to \inf. \end{gathered} \tag{4.12}$$

The solution of problem (4.12) is determined by the Lagrange indeterminate multiplier method.

Note. If $\bar{Z}^* \in L$, $\bar{Z}^*$ is the initial approximation.

Let $\bar{Z}^* \notin L$.

We prove a series of statements, which is necessary for construction of the initial approximation search algorithm.

Lemma 1. Vector $\bar{Z}^*$ is orthogonal to linear multiformity L.

Proof. It is evident, since $\bar{Z}^*$ is a tangent of an n dimensional /39
sphere with the center at the coordinate origin $\sum_{i=1}^{n} Z_i^2 = const$ and linear multiformity L.

By H, we designate the (n-k+1) dimensional linear subspace passing through linear multiformity L and the coordinate origin, which is the center of n dimensional cube Ω. On the strength of Lemma 1, we have

$$H = \{\bar{Z} : \bar{Z} = \lambda \bar{Z}^* + \bar{Z}_L \ , \bar{Z}_L \in L, \ \lambda \in (-\infty, \infty)\}.$$

We designate Ω_H the intersection of n dimensional cube Ω and linear subspace H

$$\Omega_H = \Omega \cap H$$

Let $\bar{Z} \in H$. Then, $\bar{Z} = \lambda \bar{Z}^* + \bar{Z}_L$, where $\bar{Z}_L \in L$. We will call $|\lambda|$ the distance from point $\bar{Z}$ to linear multiformity L in subspace H. Subspace H is divided into two regions by the linear multiformity. The sign of λ determines to which of these two regions $\bar{Z}$ belongs. By the sign of λ, we will call these regions H^+ and H^-. For example, the coordinate origin is at a distance of unity from L in H, and it belongs to H^-, since $\bar{0} = -\bar{Z}^* + \bar{Z}^*$, $\bar{Z}^* \in L$.

We assign $\bar{Z}^{\Omega} = \mu \bar{Z}^* (0 < \mu < 1)$ the point of exit at the boundary of region Ω, in motion from the coordinate origin to $\bar{Z}^*$. In this case, one variable Z_{i_o} reaches the boundary of region Ω. To be definite, we assume $Z^{\Omega}_{i_o} = 1$. We designate the linear multiformity $Z_{i_o} = 1$ by G_{i_o}. Then, $G_{i_o} \cap \Omega$ is closest to linear multiformity L in H, the bound of the section of n dimensional cube Ω with linear subspace H.

Theorem 2. If $L \cap \Omega \neq \emptyset$, $L \cap G_{i_o} \cap \Omega \neq \emptyset$ (ϕ is the empty set).

Proof. We prove the contrary statement equivalent to this one: /40
If [illegible], $L \cap \Omega = \emptyset$. In this case, the bound $G_{i_o} \cap \Omega_H$ lies entirely on one side of L in subspace H, namely, $G_{i_o} \cap \Omega_H \in H^-$.
Since $G_{i_o} \cap \Omega$ is closest to L in H, the bound of the section of cube Ω with linear subspace H, Ω_H also lies on one side of L in H, $\Omega_H \subset H^-$.

Consequently, $L \cap \Omega_H = \emptyset$. By definition, $\Omega_H = \Omega \cap H$ and $H \in L$.

Then, $L \cap \Omega_H = L \cap H \cap \Omega = L \cap \Omega$. As a result, we obtain $L \cap \Omega = \emptyset$.

Algorithm 2, determination of the initial approximation of problem (4.1).

1. Assume all variables to be free.

2. Determine point $\overline{Z}^*$, which is the solution of problem (4.12) for the free variables. If $\overline{Z}^* \in \Omega$, $\overline{Z}^*$ is the solution (the stop); otherwise, execute paragraph 3.

3. Find $\overline{Z}^{\Omega}$ the point of exit on the boundary of n dimensional cube Ω, for motion from the coordinate origin to point $\overline{Z}^*$. Fix the variables reaching the boundary of region Ω. Transfer the terms with the fixed variables to the equality type restrictions on the right side, recalculating the free terms. If the equality type restrictions for the free variables are incompatible, the problem does not have a solution (stop). Exclude the linearly dependent equations from the equality type restrictions for the free variables. Proceed to paragraph 2.

It follows from theorem 2 that, if the intersection of linear multiformity L and n dimensional cube Ω is not empty $L \cap \Omega \neq \emptyset$, algorithm 2 converges towards point $z^0 \in L \cap \Omega$ and, if $L \cap \Omega = \emptyset$, the algorithm reports the incompatibility of the equality and inequality type restrictions of problem (4.1).

5. Stabilization Process Modeling

A block diagram for modeling the process of stabilization of motion of a bounding vehicle in the unsupported phase of the bound is presented in Fig. 5. The modeling was carried out in a BESM-6 computer in FORTRAN. /41

In block 1, from the phase coordinates of the vehicle at the time of liftoff from the supporting surface, known from the navigation system readings, and the approximate model of the terrain in the landing region, obtained from the data system, the characteristics of the forthcoming flight phase are calculated. The nominal position of the vehicle at the time of landing and the duration of the flight phase are determined. The ballistic trajectory of the motion of the vehicle center of mass is plotted. The value of the vehicle angular momentum vector and the angular velocity of the housing at the time of landing are determined. The nominal position of the legs in the flight phase, which ensure shock free liftoff and soft setting of the legs on the supporting surface, is plotted. The times of start t_{co} and end t_{c1} of stabilization of the angular motion of the vehicle housing $t_o < t_{co} < t_{c1} < t_1$ are calculated, where t_o, t_1 are the times of liftoff from the supporting surface and landing. By integrating the system of differential equations of the law of conservation of angular momentum of the vehicle relative to the center of mass, from the nominal position of the vehicle at the time of landing, with a negative time step, we find the boundary values of the angular coordinates and velocity of the housing at time t_{c1}.

The stabilization algorithm must ensure that this position is reached by the angular coordinates of the housing at time t_{e1}. Block 2, which is the algorithm monitor, coordinates the operation of its individual parts. With $t\in[t_0, t_{e0}]$ and $t\in[t_{e1}, t_1]$ in the leg joints, the nominal inflight motion of the legs is worked out. With , /43
it is required that the angular coordinates of the vehicle housing change along the transition lines which smoothly connect the boundary values of the angular coordinates and velocity of the housing at times t_{e0} and t_{e1}. The transition lines are constructed in block 5. The monitor checks the accuracy of motion along the transition lines. If the angular coordinates of the housing go outside the ε corridor of the transition lines, a supplementary correction is prescribed, and the transition lines are recalculated.

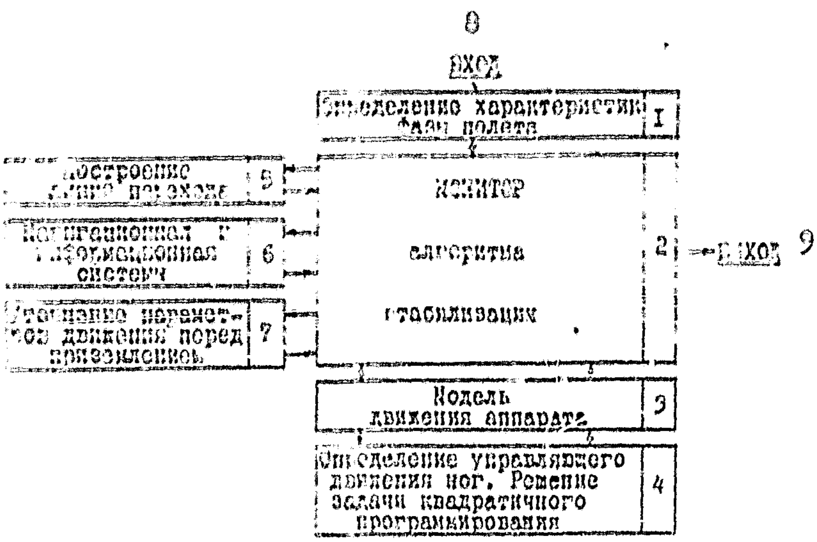


Fig. 5

Key:

1. Determination of flight phase characteristics
2. Stabilization algorithm monitor
3. Vehicle motion model
4. Determination of leg control motions. Solution of quadratic programming problem
5. Transition lines construction
6. Navigation and data systems
7. Refinement of motion parameters before landing
8. Input
9. Output

The inflight motion of the legs, which ensures change of the angular coordinates of the housing along the transition lines, is synthesized in block 4, by addition of the supplementary control motion to the nominal leg transfer motion. The control motion of the legs is determined locally in step $t \in [\tau_k, \tau_{k+1}]$, ahead in time. The problem of determination of the supplementary control motion of the legs is reduced to a special type of quadratic programming problem (see Section 3), an efficient algorithm for the solution of which is constructed in Section 4.

We note that the interval between the corrections and the times of intersection of the supplementary control motion of the legs is at least a fixed minimum time segment.

In block 7, by using more accurate information on the supporting surface obtained by the data system during the flight towards time t_{c1}, the control system refines the nominal position of the vehicle at the time of landing and the duration of the flight phase. The nominal leg motion is corrected in the corresponding manner, at $t \in [t_{c1}, t_1]$.

In block 3, the equations of the mathematical model of the three dimensional motion of a bounding vehicle in the flight phase are integrated by the Runge-Kutt method.

6. Calculation Results

Calculations were carried out for four legged and six legged vehicles with the following dynamic and kinematic characteristics. /4
Housing mass 1000 kg, housing moments of inertia (in kg·m^2) I_x=530, I_y=90, I_z=605. All the legs are the same, and they consist of two thigh and shank sections. Thigh length, 1.2 m, shank, 1.4 m. The center of mass of each section is located in the middle. Thigh weight, 30 kg, shank, 20 kg, moments of inertia of thigh (in kg·m^2): I_{x2}=3.2, I_{y2}=0.05, I_{z2}=3.2; shank: I_{x1}=3.6, I_{y1}=0.05, I_{z1}=3.6. The coordinates of the suspension points of the thigh to the housing of the four legged vehicle have the values: X_i=0.5 (i=1,2), X_i=-0.5 (i=3,4), Y_i=1.25 (i=1,3), Y_i=-1.25 (i=2,4), Z_i=0 (i=1,2,3,4); for the six legged vehicle: X_i=0.5 (i=1,2,3), X_i=-0.5 (i=4,5,6), Y_i=1.25 (i=1,4), Y_i=0 (i=2,5), Y_i=-1.25 (i=3,6), Z_i=0 (i=1,2,3,4,5,6). We recall that the vehicle moves the O_y axis forward.

The SI unit of measurement system is used. Time is measured in seconds, angles in radians. The moments developed in the leg joints are measured in thousands of Newton meters. Motion is on the surface of the earth, and the acceleration of gravity is 9.8 m/sec^2.

The integration step of the vehicle motion model is 0.005 sec.

The required accuracy of motion along the transition lines, according to the angular coordinates and velocity of the housing in the stabilization algorithm, is 0.01. The minimum permissible time interval between two successive corrections and between times of recalculation of the supplementary control motion of the legs is 0.025 sec.

The initial and final programmed positions of the vehicle are determined by the vehicle posture formation algorithm at the times of liftoff from the supporting surface and landing, the operation of which was considered in detail in [1]. The projection of the supporting /45 contour on the horizontal plane is a rectangle, the length of which equals the distance between the suspension points of the front and back legs, and the width of which equals double the lateral extension of the legs from the longitudinal axis of the supporting contour r_b. The input parameters of the posture formation algorithm, the selection of which is done, either by the driver of the bounding vehicle, or by the higher levels of the motion control system, are: D is the distance of the bound (the distance between the projections of the centers of the supporting contours on the horizontal plane $O_1\xi\eta$); ψ_f is the direction (azimuth) of motion in the flight phase; $\psi_{pl}{}^{o}$ is the orientation (azimuth) of the initial supporting contour; $\psi_{pl}{}^{1}$ is the orientation (azimuth) of the final supporting contour; λ is the tangent of the angle of inclination of the initial velocity of the center of mass, as well as of the model of the supporting surface to the horizon. At the times of liftoff from the supporting surface and landing, the vertical distance from the supporting surface to the housing center of mass is a fixed value ζ_{nom}, the pitch and roll angles equal zero ($\theta^o=\theta^1=\gamma^o=\gamma^1=0$), the yaw angle equals the azimuth of the corresponding supporting contour ($\psi^o=\psi_{pl}{}^o$, $\psi^1=\psi_{pl}{}^1$). The ξ and η coordinates at the initial and final times are determined from the condition that, in the projection on the horizontal plane $O_1\xi\eta$, these points lie on a line connecting the centers of the supporting contours, at distances $(\zeta_{nom}-\zeta_{min})/\max\{1,|\lambda^*|\}$ from the center of the corresponding supporting contour. Here, ζ_{min} is the minimum permissible vertical distance from the supporting surface to the housing center of mass, and λ^* is the tangent of the angle of inclination of the velocity of the vehicle center of mass to the horizon at the corresponding time.

For the vehicles under consideration, we assume ζ_{min}=0.5 m. It was shown in [1] that, from the point of view of minimization of the maximum power developed in the leg joints during the support phase of motion, /46 for the four legged vehicle, ζ_{nom}=1.7m, r_b=1m are best and for the six legged vehicle, ζ_{nom}=1.6 m, r_b=1.1 m. Subsequently, we will consider that ζ_{nom} and r_b have these values.

As nominal bounds on the horizontal plane we will consider distance D to be 5 and 10 m. The angle of inclination of the initial vehicle center of mass velocity to the horizon is 45°. The angle of rotation of the vehicle in the flight phase $\Delta\psi_{pl}=\psi^1{}_{pl}-\psi^o{}_{pl}$ is 0 and 0.3 radians. The direction of motion of the vehicle in the flight phase is ψ_f= $1/2(\psi_{pl}{}^o+\psi_{pl}{}^1)$. We select the absolute coordinate system in such a way that $\xi^o=0$, $\eta^o=0$, $\psi_{pl}{}^o=0$.

We investigate the efficiency of processing of various types of perturbations for these modes of motion. The required accuracy of motion along the transition lines in the algorithm for stabilization of the angular motion of the housing in the flight phase is 0.01 radian. We estimate the efficiency of the stabilization algorithm with perturbations, from the accuracy of processing of the nominal angular coordinates of the housing at the time of landing. We designate $\delta\psi^1, \delta\theta^1, \delta\gamma^1$ as the deviation of the actual angular coordinates of the housing from nominal at the time of landing. The stabilization algorithm efficiently processes inflight perturbations, if $\max\{|\delta\psi^1|, |\delta\theta^1|, |\delta\gamma^1|\} \leqslant 0.01$, i.e., the deviation of the actual angular coordinates of the housing from nominal at the time of landing does not exceed the required accuracy of motion of the angular coordinates of the housing along the transition lines. We will consider 0.03 radian (approximately 1.7°) accuracy of processing the nominal angular coordinates of the housing at the time of landing to be satisfactory.

Let the values of the dynamic characteristics (masses and moments of inertia) of the leg sections of the vehicle be loaded into the motion control system with errors. We consider the effect of perturbations due to these errors. Let m_{ji}, I_{ji} be the actual masses and moments of inertia of the j-th section of the i-th leg (j=2 corresponds to the thigh, j=1, to the shank), and m_{ji}^u, I_{ji}^u be the masses and moments of inertia of this section loaded into the control system. We consider the case when the magnitudes of the relative errors are the same with respect to the masses and moments of inertia of all sections of the legs and equal to /47

$$\delta m_H = \frac{m_{ji}^u - m_{ji}}{m_{ji}} = \frac{I_{ji}^u - I_{ji}}{I_{ji}} \quad (j=1,2), (i=1,2\ldots N).$$

For the modes of motion listed above, errors of ±5% ($|\delta m_H| \leqslant 0.05$) are processed efficiently. In this case, the accuracy of processing the nominal angular coordinates of the housing at the time of landing, practically, is not over 0.01 radian. With relative error δm_H of -15% to 25%, the accuracy of processing the nominal angular coordinates of the housing at the time of landing is not over 0.03 radian. The maximum error arises in pitch angle $\delta\theta^1$, and the nominal values of the bank and yaw angles are processed practically ideally. Errors $\delta m_H > 0$ are processed better than negative errors. In other words, it is better if the motion control system considers that the legs are heavier than actual, and not the contrary. These results are valid for the four legged and six legged vehicles. The accuracy of processing the nominal angular coordinates of the housing at the time of landing vs. δm_H, for a bound to a distance of 10 m and an angle of rotation in the flight phase $\Delta\psi_{p1}$=0.3 radian is shown in Table 1.

We consider another type of perturbation, connected with the inaccuracy of processing the leg motion in the flight phase. It is

TABLE 1

δm_k	a. конечная ошибка					
	b. 4-х ногий аппарат			c. 6-ти ногий аппарат		
	$\delta\psi^1$	$\delta\theta^1$	$\delta\gamma^1$	$\delta\psi^1$	$\delta\theta^1$	$\delta\gamma^1$
0.05	*	*	*	*	*	*
0.075	*	*	*	*	*	*
0.10	*	-0.012	*	*	-0.011	*
0.125	*	-0.013	*	*	-0.014	*
0.15	*	-0.015	*	*	-0.016	*
0.175	*	-0.019	*	*	-0.020	*
0.20	*	-0.019	*	*	-0.021	*
0.25	*	-0.020	*	*	-0.021	*
0.30	*	-0.032	*	*	-0.027	*
-0.05	*	*	*	*	*	*
-0.075	*	0.015	*	*	0.016	*
-0.10	*	0.023	*	*	0.023	*
-0.125	*	0.028	-0.011	*	0.027	-0.010
-0.15	*	0.025	-0.013	*	0.024	-0.010
-0.175	*	0.038	-0.013	*	0.027	-0.010
-0.20	*	0.070	-0.024	*	0.032	-0.011

* - error less than 0.01

Key: a. Final error
b. 4 legged vehicle
c. 6 legged vehicle

considered that, at $t\in[t_{k_0},t_k]$ the error in processing the required accelerations in the leg joints is an unbiased normal distribution of a random quantity. We recall that the minimum permissible step of recalculation of the leg motion is 0.025 sec. Let $\ddot{P}_j^{\,n}(t)(j=1,2,\dots 3N)$, where $P_j=\alpha_1,\beta_1,\dots,q_N$, be the required acceleration of the leg joints at time t. We set the actual accelerations of the leg joints equal to $\ddot{p}(t)=\ddot{p}^n(t)(1+\nu)$, where ν is the normal distribution of a random quantity with a mathematical expectation of zero and a root mean deviation of σ. The calculation results show that, for the modes of motion listed above, an unbiased random error of the required acceleration of the leg joints, on the order of their nominal value (with $\sigma\leq 1$), has practically no effect on the accuracy of processing the nominal angular coordinates of the housing at the time of landing $\max\{|\delta\psi^1|,|\delta\theta^1|,|\delta\gamma^1|\}\leq 0.01$. /49

At the time of liftoff from the supporting surface, information on the supporting surface in the landing region is known with a certain degree of accuracy to the motion control system of the bounding vehicle. During the flight, the model of the terrain in the landing region is refined. The precise model of the terrain is used to reorganize the vehicle motion in the concluding stage of the flight phase, at $t\in[t_{k_0},t_k]$. We investigate the effect of data errors on the accuracy of processing the nominal angular coordinates of the housing at the time of landing. We consider bounds on a horizontal plane without turning in the flight phase ($\Delta\psi_{p1}=0$), to distance D between 5 and 15 meters, to be nominal motion. The angle of inclination of the initial vehicle center of mass velocity to the horizon is 45°. During the flight, the motion control system determines that the altitude of the supporting area differs from the nominal value by the amount δh. A data error on the supporting area altitude δh for these modes of motion lead only to error in processing the nominal pitch angle at the time of landing $\delta\theta^1$, and the bank and yaw angles are processed practically ideally. The areas of values of data errors δh, in which $|\delta\theta^1|$ does not exceed 0.01, 0.02 and 0.03 radian are shown in Fig. 6, for a four legged vehicle. The permissible errors δh depend essentially on bound distance D. We designate D_f the distance between the front legs of the vehicle at the time of liftoff from the supporting surface and the back legs at the time of landing. For the vehicles considered in this section, D_f=D-2.5 m. For the four legged vehicle, we obtain a value of $|\delta\theta^1|\leqslant 0.01$, with $-0.016\leqslant \delta h/D_f \leqslant 0.028$, i.e., such data errors have practically no effect on the accuracy of processing of the nominal angular coordinates of the housing at the time of landing. At $-0.065\leqslant\delta h/D_f\leqslant 0.08$, there is satisfactory accuracy in processing the nominal angular coordinates of the housing at the time of landing, $|\delta\theta^1|\leqslant 0.03$. Similar results, obtained for a six legged vehicle, are presented in Fig. 7. Here, the permissible data errors δh are somewhat less than for the four legged vehicle. This is because the total weight of the legs is 1.5 times greater in the six legged vehicle and, consequently, the errors of processing the nominal angular coordinates of the housing at the time of landing, due to reorganization of leg motion of the vehicle in the concluding stage of the flight phase, are greater. /52

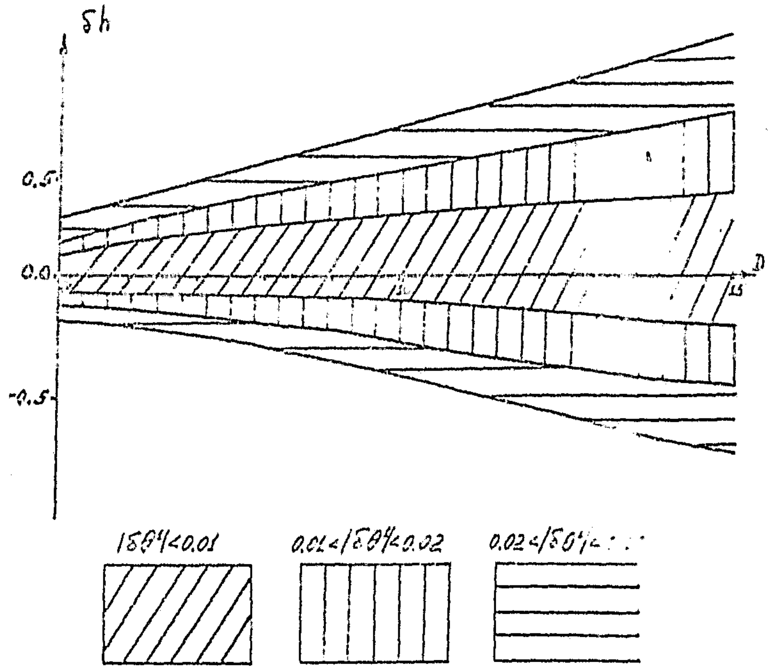


Fig. 6

To bring about the supported phase of the motion of a bounding vehicle, in this case, the active perturbations cause errors in processing the programmed coordinates and velocity of the housing at the

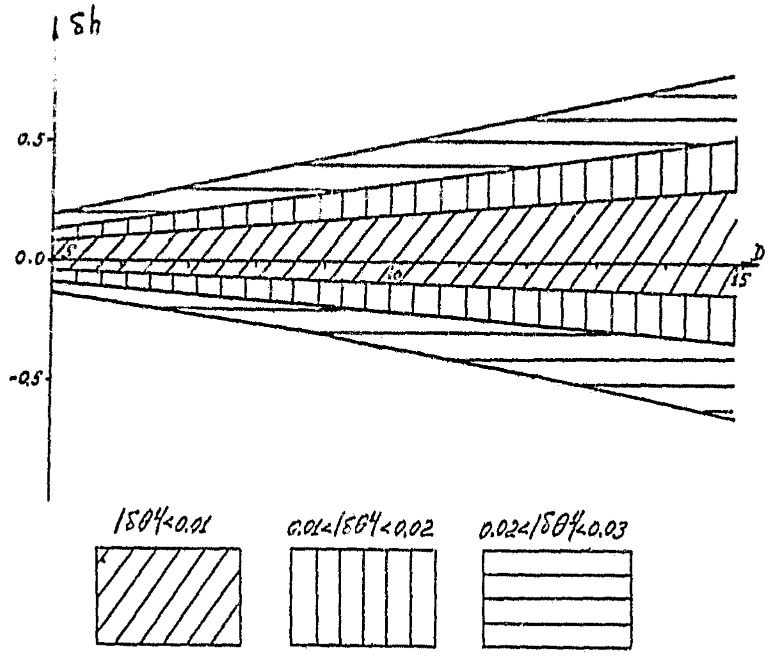


Fig. 7.

time of liftoff from the supporting surface. We investigate the question of the effect of these errors on the accuracy of processing the nominal angular coordinates of the housing at the time of landing. /54
We consider a bound on the horizontal plane to a distance of 10 m as an unperturbed motion. The angle of rotation of the vehicle in the flight phase is 0.3 radian, the angle of inclination of the initial vehicle center of mass velocity to the horizontal is 45°. We will assign an error of one angular coordinate or the velocity of the housing at the time of liftoff from the supporting surface and determine the accuracy of processing the nominal angular coordinates of the housing at the time of landing, by the stabilization algorithm. The calculation results for four legged and six legged vehicles are presented in Table 2. In the event the initial errors of the angular coordinates of the housing do not exceed modulus 0.05 radian, and the initial errors of angular velocities of the housing do not exceed modulus 0.1 rad/sec angle of pitch or 0.15 rad/sec angles of yaw and bank, the stabilization algorithm processes the nominal angular coordinates and velocity of the housing at the time of landing to within 0.01 radian.

We consider in detail the process of stabilization of the angular motion of a bounding vehicle in the unsupported phase of the bound, in the case when the unperturbed motion is a bound to a distance of 7.5 m in the horizontal plane. The final supporting area is turned by 0.2 radian relative to the initial one, and the deviation of the direction of motion of the vehicle from the azimuth of the initial supporting area is 0.1 radian. The bound is made at an angle of 45° to the horizon. A 100 kg load (10% of the weight of the housing) is placed on the vehicle. The motion control system does not know of its presence.

TABLE 2

d		a					
		b			c		
		$\delta\psi'$	$\delta\theta'$	$\delta\gamma'$	$\delta\psi'$	$\delta\theta'$	$\delta\gamma'$
$\delta\theta^\circ$	0.05	*	*	*	*	*	*
	0.075	*	*	*	*	*	*
	0.IO	0.037	*	-0.0I4	0.067	*	-0.0I4
	-0.05	*	*	*	*	*	*
	-0.075	-0.040	*	*	-0.036	*	*
	-0.IO	-0.069	*	*	-0.095	-0.0I7	-0.0I2
$\delta\psi^\circ$	0.05	*	*	*	*	*	*
	0.075	0.029	*	-0.026	0.047	*	-0.030
	0.IO	0.036	0.0I0	-0.054	0.I00	*	-0.052
	-0.05	*	*	*	*	*	*
	-0.075	-0.087	*	0.0I7	*	*	*
	-0.IO	-0.I32	*	0.033	-0.054	*	0.0I9
$\delta\gamma^\circ$	0.05	*	*	*	*	*	*
	0.075	-0.022	*	0.05I	-0.0I2	*	0.029
	0.IO	-0.02I	*	0.090	-0.029	*	0.069
	-0.05	*	*	*	*	*	*
	-0.075	0.0I3	*	-0.030	0.0I3	*	-0.047
	-0.IO	0.042	0.0I3	-0.I63	0.043	*	-0.II4
$\delta\dot{\theta}^\circ$	0.IO	*	*	*	*	*	*
	0.I5	-0.024	0.048	0.0I5	*	*	*
	0.20	-0.039	0.030	0.020	-0.0I2	0.035	*
	-0.IO	*	*	*	*	*	*
	-0.I5	-0.0I7	-0.024	*	*	-0.0I4	*
	-0.20	-0.02I	-0.043	0.0I8	0.0I8	-0.028	*
$\delta\dot{\psi}^\circ$	0.IO	*	*	*	*	*	*
	0.I5	*	*	*	*	*	*
	0.20	*	*	*	*	*	*
	-0.IO	*	*	*	*	*	*
	-0.I5	*	*	*	*	*	*
	-0.20	-0.027	*	*	*	*	*
$\delta\dot{\gamma}^\circ$	0.IO	*	*	*	*	*	*
	0.I5	*	*	*	*	*	*
	0.20	*	*	*	*	*	*
	-0.IO	*	*	*	*	*	*
	-0.I5	*	*	*	*	*	*
	-0.20	0.02I	*	-0.0I4	*	*	*

* - error less than 0.01

Key: a. Final error c. 6 legged vehicle
b. 4 legged vehicle d. Initial error

In this case, the moments of inertia of the housing change in the following manner (in $kg \cdot m^2$): $\delta I_x=25$, $\delta I_y=5$, $\delta I_z=25$. The perturbations introduced by the load cause deviations from the transition line and make it necessary to make supplementary corrections, both in the supported phase of motion, and in the flight phase. The vehicle coordinates and velocities at the time of liftoff from the supporting surface were obtained by the operation of the motion control system in the supported phase. The perturbations introduced by the load in the supported phase caused inaccuracy in processing the programmed coordinates and velocities of the housing at the time of liftoff from the supporting surface by the algorithm for stabilization of the motion of a bounding vehicle in the supported phase. The process of stabilization of the motion of a bounding vehicle in the flight phase is shown in Figs. 8-11. The time dependence of the angular coordinates of the vehicle housing are presented in Fig. 8. The stabilization algorithm efficiently processes the perturbations, and the angular coordinates of the housing at the time of landing scarcely differ from the nominal /59

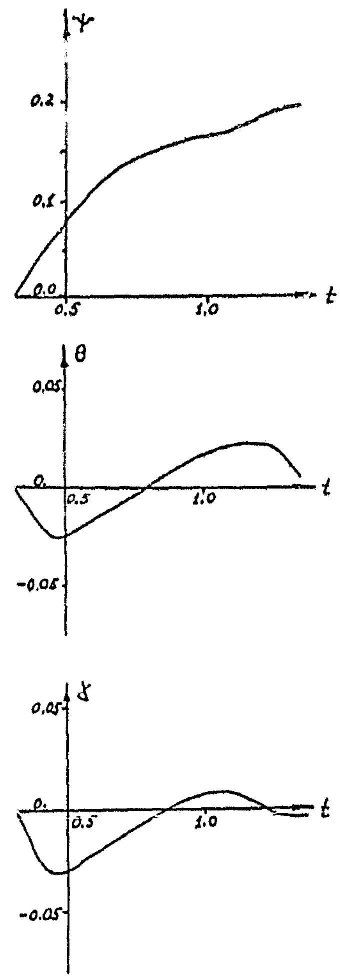


Fig. 8.

values. The angles of the leg joints and their velocities change continuously during motion of the vehicle (Fig. 9-10). The time dependence of the forces developed in the leg joints (in thousands of Newton meters) is presented in Fig. 11. At the start and end of the flight phase, the angular motion of the housing is unstabilized, and the legs participate only in the basic transfer motion of the legs (see Section 2). The maximum moments developed in the leg joints in these time segments are scarcely less than the maximum moments developed in the leg joints during the flight. Consequently, the necessity for completing the bound in the unsupported phase, with the basic transfer motion of the leg, the additional control motion of the legs does not result in an increase in the forces developed in the leg joints. The moments developed in the leg joints in the supported phase of motion were obtained in [1]. In the flight phase, the forces developed in the leg joints are approximately 5 times less than in the supported phase of motion.

By using the similarity and scale methods in the mechanics [14], the results obtained can be generalized. With a proportional ℓ times /60
change of all linear dimensions and a m times change in the weights of all the components of the bounding vehicle, the basic characteristics of motion of the vehicle change in the following manner:

flight phase duration, $\sqrt{\ell}$ times;

angular velocity of housing and velocities of leg joints, $1/\sqrt{\ell}$ times;

accelerations of leg joints, $1/\ell$ times;

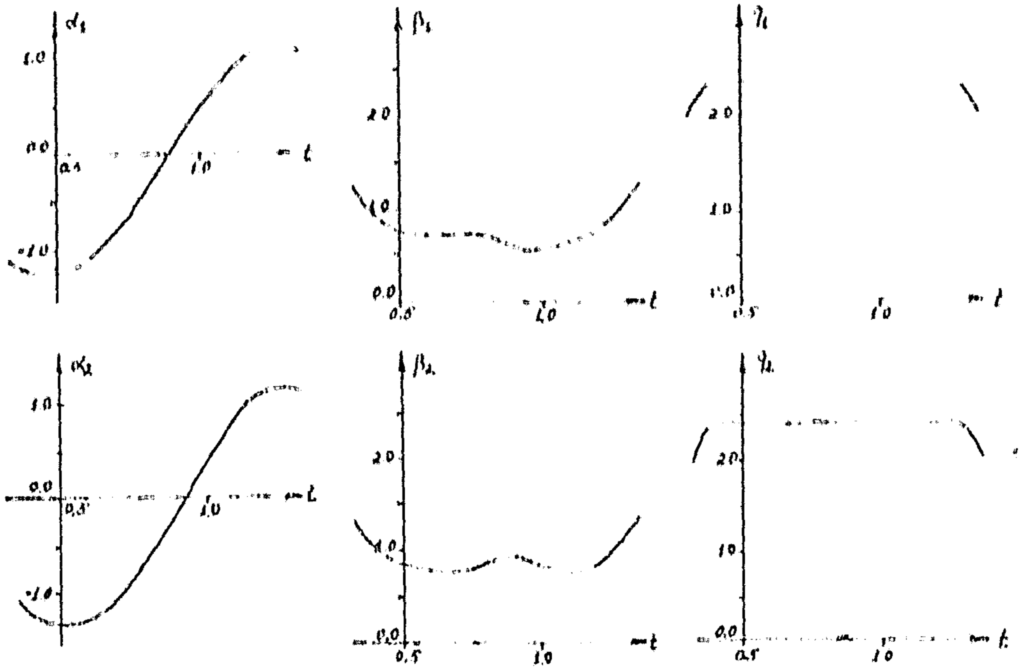
moments in leg joints, $m\ell$ times.

For example, with a twofold decrease in linear dimensions, let the mass decrease in proportion to volume (the vehicle has the same specific weight). Then, the moments in the leg joints decrease 16 times.

The calculation results show the efficiency of the algorithm for stabilization of the angular motion of a bounding vehicle in the flight phase, for various modes of motion with perturbations.

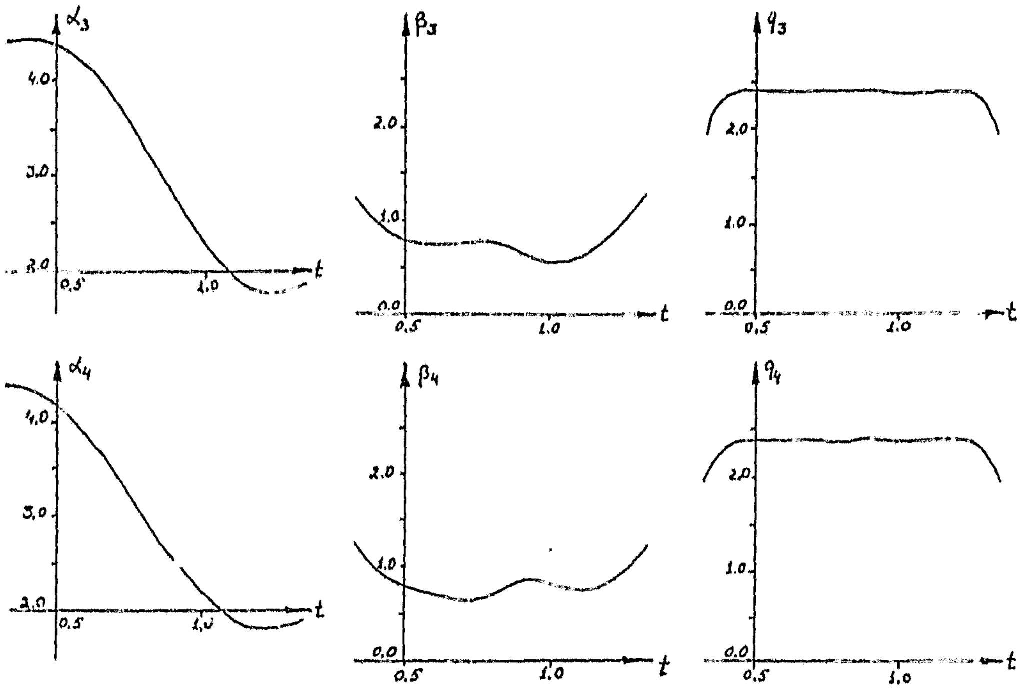
Conclusion

The problem of stabilization of the angular motion of a bounding vehicle in the flight phase is investigated in the work. Control of /61



α_1
β_1
q_1
α_2
β_2
q_2
t

Fig. 9.



α_3
β_3
q_3
α_4
β_4
q_4
4.0
3.0
2.0
1.0
0.0
0.5
t

Fig. 10

Fig. 11.

the angular motion of the housing in the unsupported phase of the bound is accomplished by a supplemental control motion of the legs. The following conclusions can be drawn from the results of the work:

1. the advisability of the use of the principle of local determination of the supplementary control motion of the legs in the stabilization algorithm was demonstrated;

2. an algorithm for stabilization of the angular motion of a bounding vehicle in the flight phase was constructed and utilized by computer;

3. the results of mathematical modeling of the process of stabilization of the motion of a bounding vehicle in the flight phase show the efficiency of the stabilization algorithm, for various modes of motion of the vehicle with perturbations;

4. the magnitudes of the moments in the leg joints, developed in the flight phase, are approximately 5 times less than in the supported phase of motion;

5. the necessity for making inflight supplementary control motions of the legs does not lead to an increase in the maximum moments in the leg joints.

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