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"Development of a Winter Wheat Adjustable Crop Calendar Model"

(E80-10094) DEVELOPMENT OF A WINTER WHEAT ADJUSTABLE CROP CALENDAR MODEL Final Report (Fort Lewis A&M Coll.) 83 p HC A05/MF A01 CSCL 02C

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ABSTRACT

Phenological data as reported at the crop reporting district (CRD) level, and environmental data from the National Climatological Center appear to be viable sources of information for use in development and testing of an adjustable crop calendar model for winter wheat. These data were utilized for this study, in which generalized least squares techniques were applied for parameter estimation of functions to predict winter wheat phenological stage, with environmental values as independent variables. The independent variables investigated included daily maximum temperature (T_x), daily minimum temperature (T_m), daily daylength (D_L), and daily precipitation (P_r).

After parameter estimation, tests were conducted using independent data. From these tests, it may generally be concluded that exponential functions have little advantage over polynomials. Precipitation was not found to significantly affect the fits. The Robertson "triquadratic" form, in general use for spring wheat, was found to yield good results, particularly for the emerge-joint interval, but special techniques and care are required for its use. In most instances, equations with nonlinear effects were found to yield erratic results when utilized with averaged daily environmental values as independent variables.

The particular combination of growth rate functions recommended, within the various phenological intervals is:

Plant-Emerge: $R = 0.087870 - 0.00045898$ [illegible]$_M$

Emerge-Joint: $R = 0.071784\ (D_L - 0.2796)\ \cdot$

$$\{(4.6869 \times 10^{-4})(T_X + 2.3876) -$$
$$(4.6618 \times 10^{-6})(T_X + 2.3876)^2 +$$
$$(2.3943 \times 10^{-4})(T_x + 2.3876) +$$
$$(1.7055 \times 10^{-5})(T_M + 2.3876)^2\}$$

Joint-Head: $R = \exp\ \{-6.22319 + 0.12008\ T_X$

$$+ 1.0337\ (D_L - 12) - 0.038591\ T_X\ (D_L - 12)\}$$

Head-Soft Dough: $R = 0.037980 + 0.0061145\ T_M$

Soft Dough-Ripe: $R = \exp\ \{-17.6837 + 0.53654\ T_X$

$$+ 0.33366\ T_M + 2.6105\ (D_L - 12)$$
$$-0.0080340\ T_X^2 - 0.011446\ T_M^2$$
$$-0.45441\ (D_L - 12)^2\}.$$

Variance propagation studies indicate that the results obtained utilizing these functions are consistent with the statistical uncertainties associated with the data used for parameter estimation and testing.

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INTRODUCTION

The Large Area Crop Inventory Experiment (LACIE) utilizes winter wheat crop calendar, or biometeorological time scale, information in several ways. Among these, analyst interpreters use the information in training field selection and other researchers use the information in yield modeling efforts.

To date the needed winter wheat crop calendar information as been provided in one of three ways. In the first, historical records are used to compute an average date at which the crop will reach a specific stage. A "spread", computed statistically from the data, is included to account for yearly variations. In the second, functions found by Robertson (1)* are modified for use in winter wheat. These modifications are necessary because the Robertson's model was generated for spring wheat in the Canadian prairie provinces. The most commonly used procedure used to modify the Robertson's model has been to calculate "multipliers" for winter wheat (4). A third technique (5) utilizes the "normal" winter wheat crop calendar, modified for "location specific" effects, as indicated by long term temperature averages.

These techniques suffer from serious limitations. In the first, variations in planting dates, and the large variations inherent when averaging over many years, means that considerable uncertainties result in the estimated dates. For the second, it must be emphasized that Robertson's model was generated for spring wheat, whose growth environment differs

*Numbers in parentheses throughout this report refer to the reference list in the back.

considerable from that of winter wheat. Spring wheat temperatures in general are on the rise throughout the growth and ripening of the crop. Daylengths in general increase during emergence and subsequently decrease. Winter wheat. on the other hand, is planted and emerges during a period of declining temperatures and daylengths, undergoes a long winter dormancy period, and then completes its growth under conditions of increasing temperature and daylengths after spring greenup. The third technique yields good results, but does not yield a universal model applicable at locations worldwide, being dependent upon long term temperature records.

For these reasons it was felt desirable to seek an acceptable crop calendar for winter wheat specifically, based upon phenological reports, in which weather and environmental variables could be used to predict winter wheat growth stages. This was the overall goal of the research conducted. With this goal in mind the following research objectives were formulated:

a. to define data sources which may be utilized in the formulation of such a model;

b. to generate computer programs which may be used to investigate various mathematical model;

c. to identify functional forms which are appropriate for describing winter wheat growth stages;

d. to obtain reliable estimates of parameters for these functions, and variance estimates of these parameters;

e. to identify and investigate auxilliary problem areas, such as winter dormancy criteria;

f. to test the parameter estimates obtained using independent data sets; and

g. to perform variance propagation studies to assess expected variations in applying the models, and compare these with variations realized during testing.

In the first phase of the investigation (6), activity was centered upon gathering phenological and meteorological data at the Crop Reporting District (CRD) level, reduction and editing of this data, and application of generalized least squares techniques for parameter estimation. Evaluation of the resulting functions was limited to statistical testing of residuals and variances of residuals resulting from the least squares estimation procedure.

During the second stage of the investigation, increased data were gathered and utilized in the parameter estimation programs, test programs were generated and data produced for use in these, and variance propagation studies were conducted.

FUNCTION ESTIMATION

The objective of the modeling process was to obtain a relationship predicting phenological stage numbers as a function of environmental variables. The phenological stages and the associated stage numbers used in this study are as follows:

Stage	Number
Planting	0
Emergence	1
Jointing	2
Heading	3
Soft-Dough	4
Ripe	5

The environmental variables selected were the same as those used by Robertson (1). These variables are daily maximum temperature, daily minimum temperature, and daylength, used to represent photoperiod. In addition, precipitation was investigated as a possible independent variable, in an attempt to assess the possible importance of moisture in such a model.

Data Sources

Phenological data at the Crop Reporting District (CRD) level were used in this study. Depicted in Figure 1 are the crop reporting districts used in the United States. Historical data from some Great Plains, Midwest, and Rocky Mountain states were obtained from NASA's Johnson Space Center, Earth Observations Division. Data were gathered and included in the data sets for least squares parameter estimation. A summary of the number of location-years included for each stage is shown:

Stage	Location-Years
Plant-Emerge (0-1)	37
Emerge-Joint (1-2)	53
Joint-Head (2-3)	79
Head-Soft Dough (3-4)	65
Soft Dough-Ripe (4-5)	62

For each location-year of data, meteorological data were averaged from several stations within the crop reporting district. Specific location-years used in the least squares program are summarized in Table 1. The dates used for modeling were the 50%, or median dates, converted to Julian Day Numbers, at which the crop reached each stage. Table 2 shows a typical report of the dates at which the crop reached each stage for a single year in one state. It should be noted that, throughout the United States, there exists a great variability as to winter wheat phenology reporting. As may be seen from

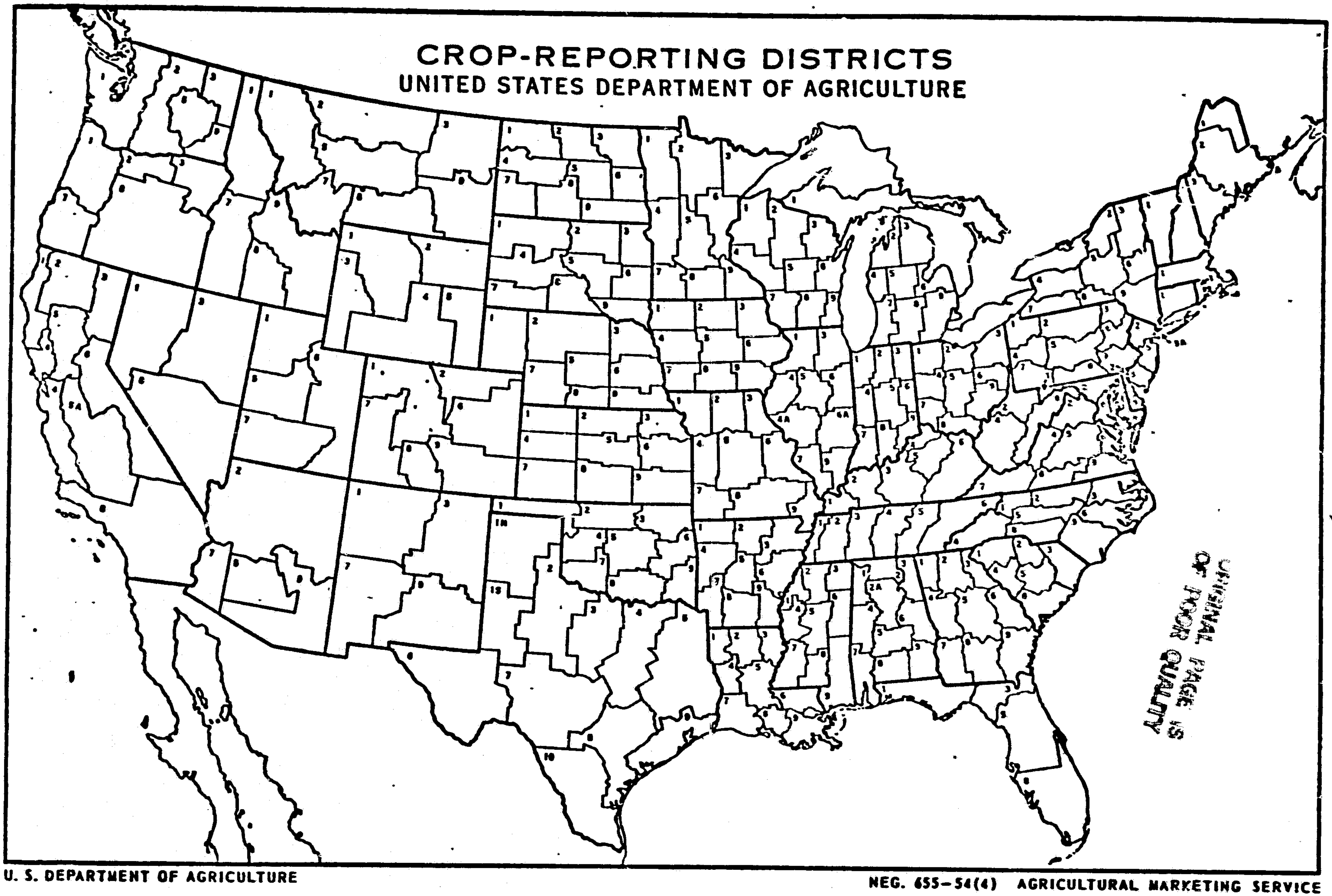

Figure I: Crop Reporting Districts in the United States

Table 1
Summary of Location Years Used
For Least Squares Parameter Estimation

Stage	State	Crop Year	C.R.D.
0-1	Colorado	1972	1,2,7,8,9
		1975	1,2,6,8,9
	Idaho	1975	1,9
	Oklahoma[1]	1965	2,3,4,5,6,7,8,9
		1966	1,2,3,4,5,6,7,8,9
		1971	1,2,3,4,5,6,7,8
1-2	Colorado	1972	1,2,6,7,9
		1974	2,6,8,9
	Idaho[3]	1975	1,9
	Montana*,[2,3]	1971	1,2,3
		1972	1,2
	North Dakota*,[2]	1970	1,4,7,8,9
		1971	1,2,4,7,8,9
		1974	1
	Oklahoma	1965	2,3,4,5,6,7,8,9
		1966	1,2,3,4,5,6,7,8,9
		1970	1,2,3,4,5,6,7,8
2-3	Colorado	1972	2,6,7,9
		1974	2,6,9
	Idaho[3]	1973	1,9
		1975	1,9
	Kansas	1964	1,2,3,4,6,7,8,9
		1965	1,2.3,5,6,7,8,9
		1968	1,2,3,4,5.6,7,8,9
	Montana[3]	1971	1,2
		1972	1,2,3
	North Dakota	1970	1,4,7,8,9
		1971	1,2,4,7,8,9
		1974	1
	Oklahoma	1965	1,2,3,4,5,6,7,8,9
		1966	1,2,3,4,5,6,7,8,9
		1969	1,2,3,4,5,6,7,8

Table 1
(Continued)

Stage	State	Crop Year	C.R.D.
3-4	Colorado	1974	2,6,9
	Idaho	1973	1,9
		1975	1,9
	Kansas	1964	1,2,3,4,6,7,8,9
		1965	1,2,3,4,5,6,7,8,9
		1966	1,2,3,4,5,6,7,8,9
		1968	1,2,3,4,5,6,7,8,9
	Missouri	1970	9
		1972	9
	Montana	1971	1,2,3
		1972	1,2
	Oklahoma	1965	1,2,3,4,5,6,7,8
		1969	1,2,3,4,5,6,7,8
4-5	Colorado	1972	6
	Idaho	1973	1,9
		1975	1,9
	Kansas	1964	1,2,3,4,6,7,8,9
		1965	1,2,3,4,5,6,7,8,9
		1966	1,2,3,4,5,6,7,8,9
		1968	1,2,3,4,5,6,7,8,9
		1969	2,3,4,5,6,7,8,9
		1974	1,2,3,4,5,6,7,8
	Missouri	1970	9
		1972	9
	Montana	1971	2,3
		1972	1,2

*Data included only for winter dormancy through spring greenup.

1.) "Acceptable Stand" reported. Emergence estimated by interpolation.

2.) Emergence not reported.

3.) "Boot" reported. Jointing estimated by interpolation.

Table 2: Typical Report of Winter Wheat Phenology for CRD's

State: KANSAS

Check One: Spring Wheat / Winter Wheat

Crop Year: 72

GROWTH STAGE DATES FOR 50% DEVELOPMENT

Month-Day-Year

CRD No.	Planting Date 1/	Emergence Date 1/	Jointing Date 2/	Heading Date 2/	Soft Dough Date 3/	Ripe Date 4/	Harvest Date [illegible]
10 N.W.	Sept. 20	Not Available	May 6	May 24	June 18	June 30	July 1[illegible]
20 W.C.	Sept. 20		May 2	May 19	June 12	June 30	July [illegible]
30 S.W.	Sept. 21		April 20	May 9	June 5	June 21	June 2[illegible]
40 N.C.	Sept. 29		May 1	May 20	June 11	June 27	July [illegible]
50 C.	Oct. 2		April 26	May 14	June 6	June 21	June 2[illegible]
60 S.C.	Sept. 28		April 15	May 6	June 1	June 14	June 2[illegible]
70 N.E.	Sept. 29		April 12	May 22	June 11	June 29	June 30
80 E.C.	Oct. 1		April 10	May 16	June [illegible]	June 24	June 29
90 S.E.	Oct. 8		April 15	April 20	May 24	June 13	June 2[illegible]

1/ Date at which 50% of crop in the CRD was planted or emerged, respectively.

2/ Date at which 50% of crop in the CRD had begun to joint or head, respectively.

3/ Date at which 50% of crop in the CRD had begun to enter soft dough stage (turning color [illegible] to yellow).

4/ Date at which 50% of crop in the CRD is in ripe (hard dough) stage [illegible]

5/ Date at which 50% of crop in the CRD has been harvested either as standing grain or cut of swath.

Table 1, few states report all the phenological stages, and several report stages which differ from the standard stages. For example, Oklahoma reports "acceptable stand" rather than emergence, and both Idaho and Montana report "boot" rather than jointing.

As independent variables, values of daily maximum and minimum temperature, daily precipitation, and daylength may be used. Values for temperatures and precipitation are available on a daily basis from the National Climatological Center, NOAA, Asheville, North Carolina. Records were obtained for the states and years for which phenological data were available. Tables 3 and 4 show typical listings of temperatures and precipitation, respectively, as reported by the National Climatological Center. The value of daylength may be calculated as a function of latitude and Julian Day Number. The empirical equation obtained by Stuff (2) was used for this purpose in this investigation in which a tangent function is utilized for latitudes less than 40°, and a tangent-squared function for latitudes above 40°.

Functional Forms

These data were used in general least squares procedures to estimate parameters for functions of the form

$$R = f(\underset{p,1}{H}, \underset{q,1}{X}) \qquad \text{Eq. 1}$$

in which R represents daily growth rate, H represents a vector of parameters, variable with the model chosen, estimated through the generalized least squares

DAILY TEMPERATURES

Station		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	Average
NORTHWEST 01																																	
ATWOOD	MAX	93	86	79	72	72	87	92	89	76	89	86	91	93	87	89	86	89	82	82	77	90	87	77	77	83	87	90	85	77	88		83.8
	MIN	49	52	57	62	53	44	52	52	44	43	48	56	59	56	55	52	43	40	43	51	61	52	53	59	40	37	42	53	48	48		48.4
BREWSTER	MAX	93	92	88	69	70	84	93	79	79	82	84	90	90	88	85	84	70	81	84	79	90	90	74	75	76	83	89	86	74	86		82.4
	MIN	49	51	51	44	45	46	51	47	42	48	52	53	57	52	55	52	42	42	43	48	59	48	51	40	43	41	43	49	47	43		48.0
COLBY 1 SW	MAX	74	88	92	59	88	72	87	92	77	74	82	83	90	92	87	85	69	89	82	84	83	92	79	72	73	81	83	89	85	70		80.6
	MIN	51	51	56	44	47	47	52	48	42	45	47	56	50	50	55	52	43	43	45	47	52	51	52	40	40	41	49	49	47	45		47.9
GOODLAND WB AIRPORT	MAX	88	96	88	69	73	89	92	78	77	85	87	92	93	91	89	71	78	81	87	77	90	77	72	79	82	84	90	85	57	83		81.3
	MIN	51	50	49	43	48	51	53	46	41	51	54	56	60	57	57	46	42	46	44	45	57	48	47	38	46	43	41	51	45	48		48.0
HILL CITY FAA AIRPORT	MAX	85	93	73	72	72	87	88	76	74	80	85	89	92	86	81	69	69	79	77	88	93	87	78	76	86	84	90	86	71	89		81.6
	MIN	52	59	53	49	50	51	56	56	47	49	49	58	61	61	62	53	43	42	48	54	60	64	55	45	50	44	50	57	48	48		52.5
HOXIE	MAX	86	92	86	71	73	86	92	95	73	81	86	90	92	89	87	79	69	80	81	81	91	86	79	76	82	82	89	86	79	87		83.0
	MIN	53	55	55	48	47	48	56	50	46	43	48	55	61	58	63	55	47	43	48	55	63	62	55	46	49	44	48	58	48	47		51.7
MCDONALD	MAX	76	87	91	59	69	70	86	88	74	75	82	83	90	91	88	85	68	69	80	85	76	89	76	72	76	81	86	90	85	87		79.1
	MIN	52	53	53	43	48	46	56	49	41	45	50	56	50	57	57	50	44	43	45	58	52	48	49	40	42	41	45	54	47	46		48.4
NORTON DAM	MAX	75	82	90	63	70	69	85	87	75	72	78	81	86	89	84	78	68	68	77	78	78	91	83	79	75	82	84	88	86	70		78.9
	MIN	49	53	55	45	46	50	50	53	45	43	43	51	55	58	52	56	47	42	44	46	49	62	56	43	45	42	45	45	47	46		48.7
NORTON 8 SSE	MAX	85	72	82	73	71	80	88	78	75	81	83	80	92	87	80	71	71	79	80	80	96	87	78	79	85	86	90	89	75	89		82.9
	MIN	52	56	53	45	47	51	53	52	44	44	47	53	58	57	59	55	47	43	47	49	59	62	56	42	48	44	47	56	40	46		50.1
OBERLIN	MAX	87	92	88	71	71	87	93	85	74	82	83	90	92	90	86	79	78	81	80	78	91	89	75	77	82	85	91	87	81	87		83.5
	MIN	48	52	52	45	49	45	53	49	45	42	45	56	61	61	61	50	47	41	45	52	61	57	58	40	41	39	44	53	47	45		49.5
SAINT FRANCIS	MAX	89	95	81	72	73	86	89	84	77	86	87	92	91	92	91	75	69	79	89	83	89	76	75	75	82	84	90	86	71	86		83.1
	MIN	51	53	52	47	50	45	56	53	43	48	56	56	60	60	55	50	45	43	46	50	58	50	52	40	45	42	41	57	48	47		49.9
NORTH CENTRAL 02																																	
ALTON 6 E	MAX	86	95	93	74	72	90	91	89	75	79	81	89	95	90	87	74	73	75	77	87	93	92	85	77	85	87	88	82	76	88		84.2
	MIN	50	59	50	49	47	47	57	56	46	40	44	53	59	61	62	56	47	41	45	[illegible]	64	70	59	46	45	46	51	56	50	50		52.0
BELLEVILLE	MAX	82	91	72	70	71	85	88	79	71	75	77	82	87	86	72	70	65	75	76	81	88	87	75	75	80	79	[illegible]	46	78	86		78.7
	MIN	56	62	55	52	49	51	51	56	48	46	48	51	55	59	59	52	47	47	46	53	69	71	60	51	49	49	51	57	53	57		53.7
BELOIT	MAX	87	96	89	73	72	86	89	89	76	78	81	86	90	90	82	71	88	72	78	88	89	90	86	77	81	83	88	86	83	87		82.8
	MIN	53	53	55	52	52	52	61	60	49	44	51	51	58	60	61	57	48	47	47	56	70	71	62	50	49	48	50	55	56	57		54.4
CLAY CENTER	MAX	84	90	82	73	71	84	84	85	73	75	78	82	86	85	83	73	66	74	76	89	69	86	84	73	81	80	65	87	83	85		81.0
	MIN	54	62	45	54	46	49	62	62	53	44	45	50	57	60	59	59	47	46	48	55	70	71	64	56	45	49	49	57	52	59		54.9
CONCORDIA WB AIRPORT	MAX	83	91	74	70	70	85	86	76	70	74	77	83	88	84	74	68	65	74	76	87	88	88	75	75	82	81	86	86	81	85		79.3
	MIN	57	63	53	52	50	53	71	56	48	47	48	54	58	60	60	52	40	46	47	52	68	70	61	50	51	50	51	58	52	60		54.5
GLEN ELDER DAM	MAX	78		93	75	72	72	87	89	78	73	77	81	86	92	87	72	68	65	74	75	85	87	89	75	75	83	81	84	83	78		79.8
	MIN	53		55	49	48	50	50	56	47	42	45	49	51	58	60	61	47	46	46	47	55	65	63	48	48	47	48	52	51	56		51.3
HUNTER	MAX	92	93	87	74	73	87	90	85	75	80	82	87	92	88	76	70	67	76	81	90	90	92	84	78	84	86	87	87	83	86		83.3
	MIN	59	53	56	49	47	48	59	57	47	42	44	50	58	63	61	56	47	45	45	55	64	70	61	49	47	48	49	55	52	61		53.2
KIRWIN	MAX	77	89	93	71	72	72	88	87	77	73	80	82	89	92	85	74	67	69	76	78	81	93	90	79	77	84	84	88	89	76		80.9
	MIN	50	56	56	47	47	58	51	57	49	41	41	43	53	56	61	56	46	43	42	46	52	64	59	47	43	42	43	44	49	40		47.4
LOVEWELL DAM	MAX	79	89	91	71	70	72	66	86	76	71	75	76	82	88	86	72	66	65	75	76	79	87	87	77	75	81	81	82	86	75		78.7
	MIN	56	55	58	49	49	53	57	56	47	44	45	47	49	51	58	61	48	50	46	46	55	66	62	49	47	47	40	49	51	51		51.6
MANKATO	MAX		80	92	70	70	71	85	87	77	72	75	75	78	83	85	66	60	75	75	79	82	87	81	82	78	82	51	85	86	80		78.7
	MIN		69	63	49	49	53	55	56	46	45	47	46	46	54	59	50	46	46	48	55	52	60	52	52	48	52	52	51	52	51		52.1
MINNEAPOLIS 2	MAX	86	92	87	72	74	87	88	87	76	72	82	86	89	87	80	72	66	75	80	92	90	91	85	76	85	83	89	89	86	86		82.9
	MIN	56	63	62	52	48	51	61	61	51	46	47	51	58	59	61	59	49	44	48	56	70	72	64	53	48	51	51	56	53	58		55.5
PHILLIPSBURG 1 SSE	MAX	85	96	86	72	72	89	87	87	74	79	81	91	96	89	87	76	68	76	77	80	94	91	79	76	85	86	90	88	86	88		83.9
	MIN	51	56	62	48	50	54	56	56	48	44	47	52	61	60	62	57	49	44	46	52	65	69	59	46	50	46	47	58	48	48		53.0
PLAINVILLE	MAX	80	93	80	71	[illegible]	82	92	81	77	86	86	89	92	88	80	76	71	76	83	93	94	92	85	77	81	83	90	89	81	88		83.5
	MIN	55	58	56	48	50	48	55	56	47	42	45	46	50	57	58	50	47	48	51	55	59	60	52	51	50	46	48	60	56	54		52.2
SMITH CENTER	MAX	86	95	90	74	76	90	90	87	[illegible]	81	82	88	96	91	83	71	69	77	79	80	92	92	88	77	84	85	86	86	80	87		83.7
	MIN	50	56	56	46	46	48	58	56	45	43	46	53	76	59	62	55	48	44	46	53	64	70	58	46	45	48	51	53	50	52		52.0
WASHINGTON	MAX	83	91	88	71	71	85	86	86	74	76	80	82	87	87	82	74	63	74	75	86	88	89	85	76	82	82	83	87	85	86		81.3
	MIN	58	62	63	56	46	49	42	59	53	42	42	48	57	60	58	59	47	45	49	52	69	70	64	49	44	48	48	56	56	59		54.2
WEBSTER DAM	MAX	77	87	95	71	72	72	96	90	79	73	81	83	92	96	89	78	71	70	78	78	90	96	90	76	77	86	[illegible]	93	89	71		82.6
	MIN	56	58	57	50	49	53	52	53	50	44	44	46	55	59	62	61	50	47	45	46	52	68	55	49	49	45	[illegible]	47	48	49		51.5
NORTHEAST 03																																	
ATCHISON	MAX							86	78	72	75	77	79	81	81	80	80	65	68	71	82	81	82	76	77	79	79	81	82	81	80		78.0
	MIN							60	63	57	49	47	48	56	56	52	58	51	51	54	53	64	67	65	58	67	52	52	55	58	57		55.3
BLAINE	MAX		83	88	75	68	69	83	84	76	69	74	77	79	83	82	76	67	64	69	75	86	87	83	72	75	80	79	82	84	86		77.7
	MIN		56	60	53	45	56	50	60	52	42	48	49	52	54	53	62	46	47	49	51	54	65	61	51	46	40	58	55	55	58		52.8
CENTRALIA	MAX	88	88	80	76	73	85	85	82	71	75	77	80	83	82	80	75	65	72	73	83	86	85	82	76	80	80	80	85	82	82		79.6
	MIN	53	60	60	60	45	43	64	65	55	41	42	56	55	58	56	60	40	50	53	52	68	69	64	52	43	48	50	56	53	58		54.5
HOLTON 6 NE	MAX	78	84	83	77	68	80	82	81	71	72	76	78	80	81	79	76	65	73	72	85	87	85	78	76	77	79	81	83	82	81		78.3
	MIN	53	53	56	57	46	47	60	69	51	50	44	44	56	55	53	59	47	49	52	52	65	69	62	53	44	51	49	53	52	56		53.7
HORTON	MAX	80	87	85	79	72	83	85	81	76	75	78	80	83	82	80	82	66	71	71	86	87	85	80	77	80	79	82	84	82	82		79.9
	MIN	56	58	62	59	47	48	62	66	56	45	44	47	52	55	53	64	50	51	56	53	66	70	65	53	42	51	49	56	56	56		56.8
LEAVENWORTH 3 S	MAX																																
	MIN																																

See Reference Notes Following Station Index

Table 3: Typical Report of Daily Maximum and Minimum Temperatures

DAILY PRECIPITATION

Station	Total	Day of Month 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Table 4: Typical Report of Daily Precipitation

algorithm, and $\underline{X}$ represents the vector of independent variables chosen, in this case,

$$\underset{4,1}{X} = \left[T_X \; T_M \; D_L \; P_r\right]^t$$

where, T_X = daily maximum temperature in $^{\circ}$C.,

T_M = daily minimum temperature in $^{\circ}$C.,

D_L = daylength (representing photoperiod) in hours,

P_r = daily precipitation in millimeters.

The specific functional forms selected were of three classes. The first was a polynomial of the form

$$\begin{aligned} R = {} & H_1 + H_2T_X + H_3T_M + H_4(D_L-12) + H_5P_r \\ & + H_6T_X^{\;2} + H_7T_M^{\;2} + H_8(D_L-12)^2 + H_9P_r^{\;2} \\ & + H_{10}T_X(D_L-12) + H_{11}T_M(D_L-12) + H_{12}P_r(D_L-12) \\ & + H_{13}T_XT_M + H_{14}T_XP_r + H_{15}T_MP_r. \end{aligned} \qquad \text{Eq. 2}$$

The second functional form investigated was the exponential

$$\begin{aligned} R = \exp\,(& H_1 + H_2T_X + H_3T_M + H_4(D_L-12) + H_5P_r \\ & + H_6T_X^{\;2} + H_7T_M^{\;2} + H_8(D_L-12)^2 + H_9P_r^{\;2} \\ & + H_{10}T_X(D_L-12) + H_{11}T_M(D_L-12) + H_{12}P_r(D_L-12) \\ & + H_{13}T_XT_M + H_{14}T_XP_r + H_{15}T_MP_r). \end{aligned} \qquad \text{Eq. 3}$$

For these functions, several cases were run using computer programs generated during the first phase of the contract, in which various subsets of the equations were chosen. In all cases, terms containing daylength (D_L) were omitted for stage 0-1.

The third functional form investigated was the "Robertson model" (1), of the form

$$R = \{H_2(D_L - H_1) + H_3(D_L - H_1)^2\} \cdot \{H_5(T_X - H_4) + H_6 (T_X - H_4)^2 + H_7(T_M - H_4) + H_8(T_M - H_4)^2\} \qquad \text{Eq. 4}$$

for all stages except the plant-emerge interval. For this stage, a function of the form

$$R = \{H_2(P_r - H_1) + H_3(P_r - H_1)^2\} \cdot \{H_5(T_X - H_4) + H_6(T_X - H_4)^2 + H_7(T_M - H_4) + H_8(T_M - H_4)^2\} \qquad \text{Eq. 5}$$

was chosen.

Least Squares Techniques

For this investigation parameters for the above functions were estimated using the general constrained minimum least squares approach as published by Mikhail (3). The method, called "Simultaneous adjustment of observations and parameters," allows maximum flexibility for investigations of alternate functional forms and provides an extemely powerful tool for determining parameters of these functions and estimating variances of the parameters obtained. The following discussion provides a brief summary of the technique. For a more detailed discussion, the reader is referred to Mikhail (3).

Given a vector of n original observations

$$\underset{n,1}{X^o} = \left[x_1, x_2, \ldots x_n\right]^t \qquad \text{Eq. 6}$$

and a set of condition equations of the form

$$\underset{r,1}{F}(\underset{u,1}{H}, \underset{n,1}{X}) = \underset{r,1}{\phi} \qquad \text{Eq. 7}$$

In which there are r equations, $\underline{X}$ is the vector of n adjusted observations, $\underline{H}$ is the vector of u parameters to be estimated, and $\underline{\phi}$ is the null vector, then the following may be stated.

Since, in general, these equations will be nonlinear, a Taylor's series approximation may be written as

$$\underset{r,n}{A}\ \underset{n,1}{V} + \underset{r,u}{B}\ \underset{u,1}{\Delta} = \underset{r,1}{F^o} \qquad \text{Eq. 8}$$

in which

$$\underline{A} = \left.\frac{\partial \underline{F}}{\partial \underline{X}}\right|_{\underline{H}^o, \underline{X}^o} \qquad \underline{B} = \left.\frac{\partial \underline{F}}{\partial \underline{H}}\right|_{H^o, X^o} \qquad \text{Eq. 9}$$

are the Jacobian matrices of the functions with respect to the observations and parameters, respectively, and evaluated at observed values and parameter estimates. Further,

$$\underset{r,1}{F^o} = -\ \underline{F}(\underline{H}^o, \underline{X}^o) \qquad \text{Eq. 10}$$

is the evaluation of the functions with observed values and parameter estimates, $\underline{V}$ is the vector of residuals to be applied to the observed quantities, and $\underline{\Delta}$ is the vector of corrections to be applied to the parameters.

The solution proceeds using the least squares condition in which

$$\Phi = \underset{1,n}{V^t} \; \underset{n,n}{P} \; \underset{n,1}{V} \qquad \text{Eq. 11}$$

is to be minimized. $\underline{P}$ is the weight matrix of the original observed data. If *a priori* estimates of variance are known, then

$$\underset{n,n}{P} = \underset{n,n}{S_{L^o}^{-1}} \qquad \text{Eq. 12}$$

in which $\underline{S}_{L^o}$ is the *a priori* variance/covariance matrix of the observed quantities. If this variance/covariance matrix is unknown, then $\underline{P}$ represents some assumed weight matrix, and may be written as

$$\underline{P} = \underline{Q}_{L^o}^{-1} \qquad \text{Eq. 13}$$

in which $\underline{Q}_{L^o}$ represents some assumed cofactor matrix of relative variances and covariances. Since, in general, the $\underline{A}$ Jacobian matrix in Equation 8 will not be square, and hence, the linearized condition equations cannot be solved directly for the residuals and substituted into the least squares condition of Equation 11, the more general method of constrained minimum is used. In this method, a vector of r Lagrangian multipliers is defined,

$$\underline{K}_L = \left[k_1, k_2, \ldots k_r\right]^t. \qquad \text{Eq. 14}$$

Then, the following scalar is minimized rather than 11.

$$\Phi' = \underline{V}^t\ \underline{P}\ \underline{V} - 2\underline{K}_L^t(\underline{A}\ \underline{V} + \underline{B}\ \underline{\Delta} - \underline{F}^o) \qquad \text{Eq. 15}$$

Minimizing by taking partials of this scalar with respect ot the observations and parameters and augmenting with the original linearized condition equations resulting in the following total set of normal equations:

$$\begin{bmatrix} \underset{n,n}{P} & \underset{n,r}{A^t} & \underset{n,u}{\phi} \\ \underset{r,n}{A} & \underset{r,r}{\phi} & \underset{r,u}{B} \\ \underset{u,n}{\phi} & \underset{u,r}{B} & \underset{u,u}{\phi} \end{bmatrix} \begin{bmatrix} \underset{n,1}{V} \\ \underset{r,1}{K_L} \\ \underset{u,1}{\Delta} \end{bmatrix} = \begin{bmatrix} \underset{n,1}{\phi} \\ \underset{r,1}{F^o} \\ \underset{u,1}{\phi} \end{bmatrix}. \qquad \text{Eq. 16}$$

This set may be solved by partitioning with the following results

$$\underset{u,1}{\Delta} = \underset{u,u}{N^{-1}}\ \underset{u,1}{T} \qquad \text{Eq. 17}$$

$$\underline{K}_L = (\underline{A}\ \underline{Q}_{Lo}\ \underline{A}^t)^{-1}\ (-\underline{B}\ \underline{\Delta} + \underline{F}^o) \qquad \text{Eq. 18}$$

$$\underline{V} = \underline{Q}_{Lo}\ \underline{A}^t\ \underline{K}_L \qquad \text{Eq. 19}$$

in which,

$$\underset{u,u}{\underline{N}} = \left[\underline{B}^t(\underline{A}\ \underline{Q}_{L^o}\ \underline{A}^t)^{-1}\underline{B}\right] \qquad \text{Eq. 20}$$

$$\underset{u,1}{\underline{T}} = \underline{B}^t(\underline{A}\ \underline{Q}_{L^o}\ \underline{A}^t)^{-1}\underline{F}^o. \qquad \text{Eq. 21}$$

These equations are solved in an iterative manner until convergence. After convergence, the reference variance may be computed by

$$s_o^2 = \frac{\underline{V}^t\ \underline{P}\ \underline{V}}{r-u}\ . \qquad \text{Eq. 22}$$

Variance propagation techniques may be used to show that (3),

$$\underline{S}_x = s_o^2\ \underline{N}^{-1} \qquad \text{Eq. 23}$$

yields an estimate of the _a posteriori_ variance/covariance matrix for the parameters.

The reference variance s_o^2 represents a measure of the "goodness" of the fit, and may be used in F tests to determine the statistical significance in adding or deleting terms from the functional form under investigation, or in comparing different functional forms.

For the problem at hand the function of Equation 1, formulated in terms of Equation 7, would be for each data point

$$F = R - f(\underline{H},\underline{X}) = 0 \qquad \text{Eq. 24}$$

in which $r = 1$, $n = 5$, and u would depend upon the number of parameters required for the functional form selected. For the case using the general least squares procedure,

$$\underset{5,1}{X} = \left[R \; T_X \; T_M \; D_L \; P_r\right]^t \qquad \text{Eq. 25}$$

$$\underset{u,1}{H} = \left[h_1, h_2, \ldots h_u\right]^t. \qquad \text{Eq. 26}$$

IMPLEMENTATION

Efforts were made to formulate and execute a data reduction system subject to the constraints of the data forms available and consistent with the data requirements for application of the least squares techniques discussed in the previous section. Computer programs were written for the polynomial and exponential function forms using both the variation of parameters and the more general least squares techniques. Special techniques were required in treating the Robertson model.

Data Reduction and Preparation

Although the objective of the research was to model daily growth rates, R, as functions of environmental variables, no data could be obtained for growth rate on a daily basis. As may be seen from Table 2, phenological data reported consisted at best of dates at which the crop reached each phenological stage. With this data, only an average growth rate could be computed, of the form of

$$R_i = \frac{1}{n_i} \qquad \text{Eq. 27}$$

in which n = the number of days in the i-th phenological interval for the location/year. This average growth rate represents an average, not only over the time period for the interval, but also a spatial average for the crop over the geographic region representing the crop reporting district.

Environmental data were available on a daily basis. However, it would be inconsistent to apply daily environmental data in the least squares modeling process when only averaged growth rates were available. Therefore, in order to supply environmental data consistent with the growth rates available, the following pre-processing steps were performed.

First, within each crop reporting district several meteorological reporting stations were selected. Up to five stations were used. Temperature and precipitation data were recorded for each station on a daily basis throughout the reported phenological stage interval from the climatological records. Table 5 represents a typical data reduction form used for this purpose.

This information was keypunched onto cards and input into a data reduction program. This program computes the average position of the crop reporting district (latitude, longitude, elevation) and the associated statistics, converts the meteorological data from English to SI units, averages meteorological values (T_X,T_M,P_r) for the location/year, and computes the associated variances. A listing of this data reduction program with output from a typical run is shown in (6). For all of the phenological intervals, with the exception of the emerge-joint interval, the results for each location/year would be a single data point for input into the least squares

5	31	MTR[illegible]	1976

1-2 [illegible]	[illegible]16.323	112.217	4870.
[illegible]	47.350	114.000	5160.
[illegible]	42.400	115.533	2080.
[illegible]	47.317	119.100	2900.
[illegible]	[illegible]	114.067	2597.

1	[illegible]	5		[illegible]	15		24	.5		[illegible]	15	.02	31	[illegible]	
2	[illegible]	-1		[illegible]	2	.01	25	-2		[illegible]	6		34	6	
3	[illegible]	[illegible]		10	-1		13	-7		21	9	.05	[illegible]	[illegible]	
4	[illegible]	-2		8	-8		11	-11	.01	15	0	.02	11	-2	
5	8	-21		3	-7	.02	12	-12	.02	15	[illegible]	.01	2	-1	.04
[illegible]	14	-4		10	-1	.01	13	-5		13	[illegible]	.01	14	-1	
[illegible]	[illegible]	[illegible]	.11	[illegible]	7	.12	[illegible]	2	.01	[illegible]	[illegible]	[illegible]	[illegible]	12	.04
[illegible]	[illegible]	11	[illegible]	[illegible]	[illegible]	[illegible]	25	7		[illegible]	[illegible]	.02	[illegible]	2	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	14	[illegible]	[illegible]	19	[illegible]	[illegible]	[illegible]	.02	42	[illegible]	.10
[illegible]	[illegible]	[illegible]	[illegible]	13	[illegible]	[illegible]	[illegible]	17	.15	-11	16		-41	20	.14
[illegible]	[illegible]	[illegible]		[illegible]	-2	.11	[illegible]	[illegible]	[illegible]	12	2	[illegible]	[illegible]	[illegible]	
12	12	-12		-4	-10	.10	11	2	.18	12	-6	.13	[illegible]	11	.07
13	[illegible]	-7	[illegible]	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	11	-5	.02	24	[illegible]	
14	[illegible]	[illegible]		[illegible]	-1	.16	14	[illegible]	.15	[illegible]	[illegible]		[illegible]	[illegible]	
15	[illegible]	12		[illegible]	[illegible]	.5	24	[illegible]	[illegible]	11	[illegible]		[illegible]	[illegible]	.22
	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.02	[illegible]	[illegible]	.01
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.11	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]			[illegible]	[illegible]		[illegible]	24		[illegible]	[illegible]		[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	31	.42	[illegible]	[illegible]	.23	[illegible]	31	
[illegible]	[illegible]	[illegible]	[illegible]	-11	31	[illegible]	-11	[illegible]		[illegible]	[illegible]	.03	34	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		27	[illegible]	.34	[illegible]	14	.05	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	32	[illegible]	.01	[illegible]	11		[illegible]	[illegible]		[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]		27	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]		[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.17	[illegible]		[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.18	-2	[illegible]	.18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]		[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]		[illegible]	[illegible]		[illegible]	[illegible]		[illegible]	[illegible]		[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]		[illegible]	[illegible]		31	[illegible]	.02	[illegible]	[illegible]		[illegible]		
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	
[illegible]	[illegible]	[illegible]		[illegible]	31	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]		1	42	.21

Table 5: Data Reduction Form for a Typical Location/Year

program, containing averaged growth rate, maximum temperature, minimum temperature, daylength, precipitation, and positional information for that location/year.

Winter Dormancy and Spring Greenup

Special considerations were necessary for the emergy-joint interval. Because this interval stretches over the long winter dormancy period, no single averaged growth rate value could be considered to reasonably represent the entire interval. For the growth rates the interval was broken into three sections, as depicted in Figure 2. A fall emerge-dormant period must be defined. The crop is then assumed to have completed one half of its interval growth in this period, and a growth rate of

$$R_E = \frac{0.5}{N_E} \qquad \text{Eq. 28}$$

was assigned for this period. Over the winter dormancy period, N_D, a growth rate of zero was assumed. Lastly, a spring greenup-joint interval is defined, and a second average growth rate computed as

$$R_J = \frac{0.5}{N_J} . \qquad \text{Eq. 29}$$

These rates, and the environmental data averaged on a monthly basis were used to define several data points for each location year in this interval.

Therefore, it was found necessary to define some criteria for winter and spring. For this purpose, a temperature criterion was selected in an attempt

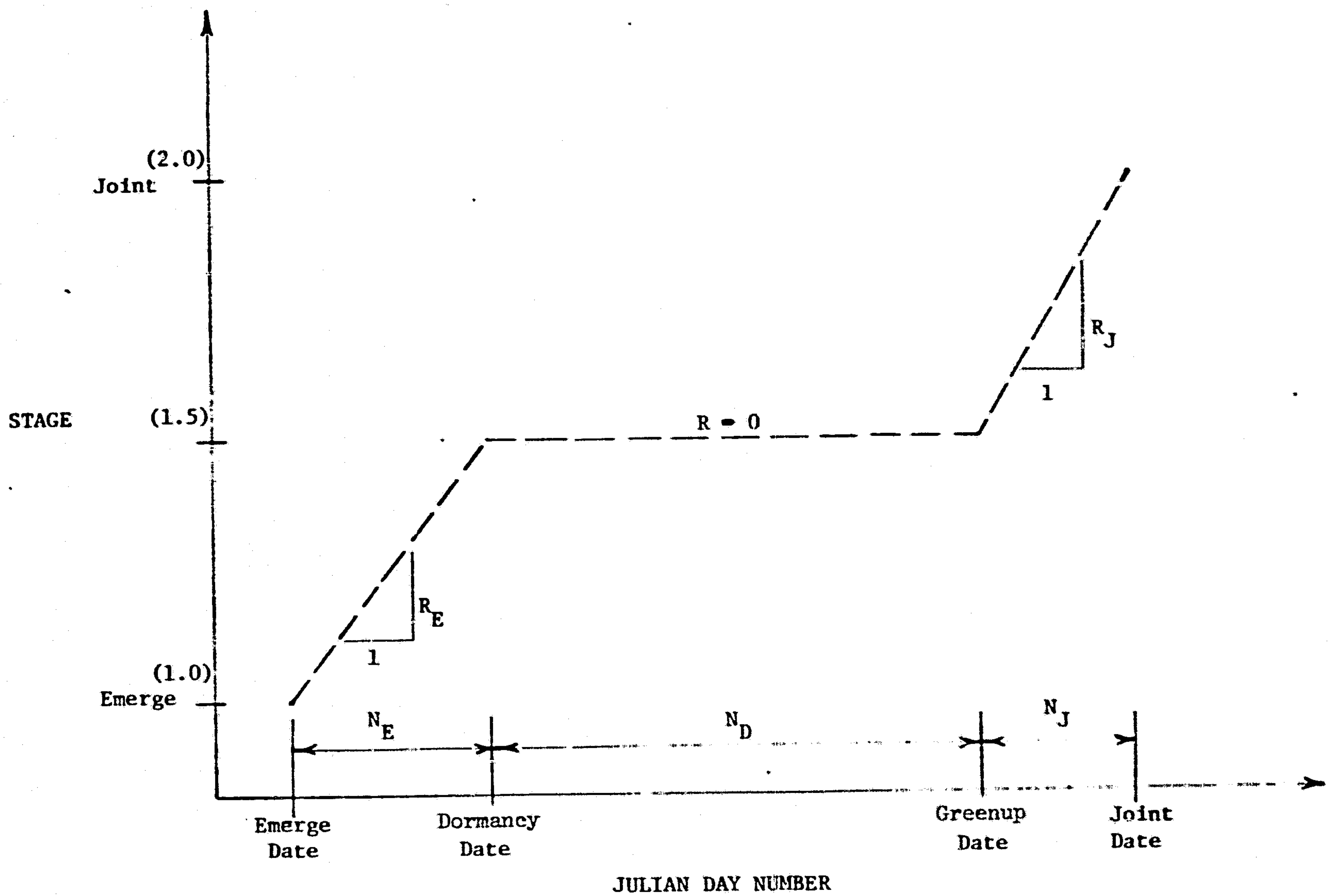

Figure 2: Emerge-Joint Growth Rate Assumptions

to predict the onset of winter and spring. Growth rates and temperatures used for stage 1-2 in the least squares programs were input to the Cate-Liebig (7) critical level analysis program. Figures 3 and 4 represent plots of growth rate versus temperature for the winter and spring runs, respectively. Critical levels computed by the algorithm were 4.69°C. (40.4°F.) for the onset of winter dormancy, and 6.14°C. (43.1°F.) for subsequent spring greenup.

Therefore, for winter, the criterion that 10 days of average temperature below 4.69°C. was used. For spring greenup, the criterion of 10 days above 6.14°C. was used.

Computer Runs

The functional forms of Equations 2 and 3 were programmed for parameter estimation using the generalized least squares algorithms. A listing of this program, with typical output is shown in (6). For the investigation, functional forms were selected and computer runs made using selected terms from Equations 2 and 3. For the polynomial, runs were made using only the constant term, using single variable linear equations in D_L, T_M, and T_X, using linear combinations of these variables with and without precipitation, and various selected combinations of linear with interaction terms and linear with squared terms. The same set of combinations was run for the exponential function.

Specific features of the programs which may be of interest include the fact that any functional form may be accomodated by changing a single statement in the EF function subprogram. Linearization is accomplished by numerical evaluation of partial derivatives using forward differences. Even greater

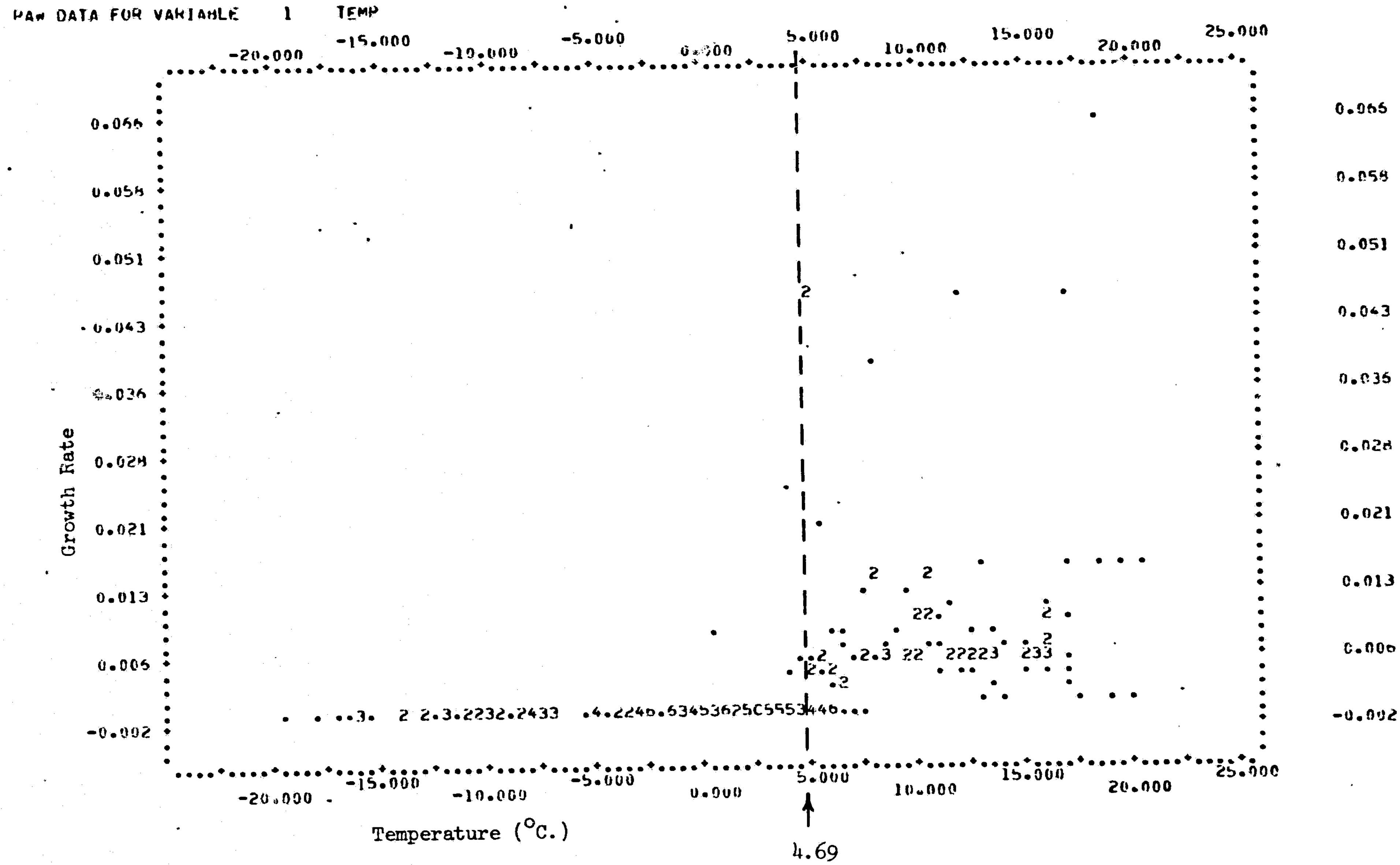

Figure 3
Plot of Data Input to Cate-Liebig
for Winter Dormancy Criteria

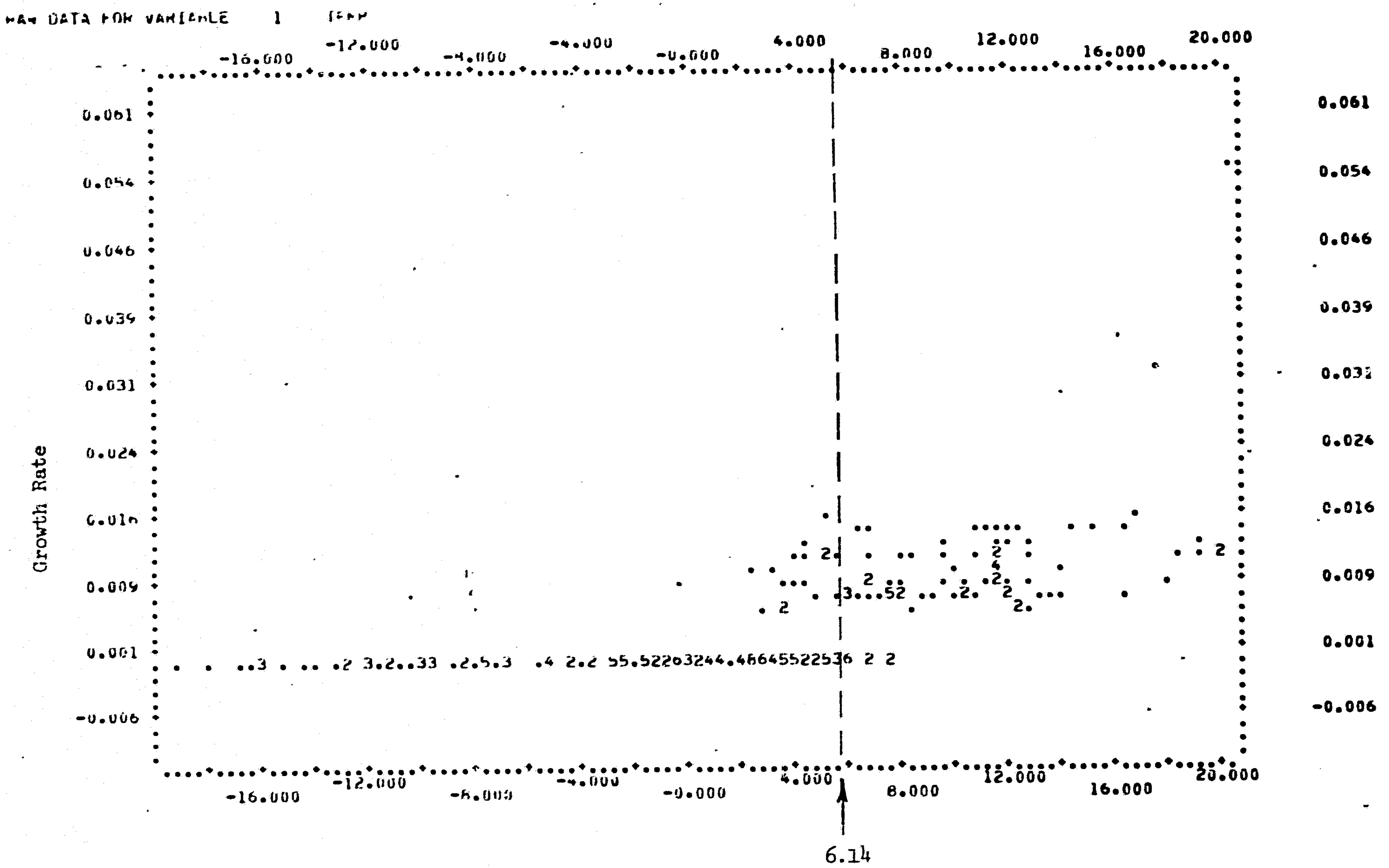

Figure 4

Plot of Data Input to Cate-Liebig
for Spring Greenup Criteria

flexibility is produced by the introduction of a parameter selection index, IPAR, which allows the inclusion or deletion of parameters within the functional form being investigated.

The use of such numerical techniques, which allows great flexibility, is not without danger. It was found in some instances that convergence of the solution did not occur when it was to be expected due to the approximations inherent in these numerical techniques. These problems occurred only for the highly nonlinear functions and is not felt to be a serious problem for reasons to be discussed subsequently.

The Robertson Model

As discussed previously, the "triquadratic" model advocated by Robertson has much to recommend it. The model, Equation 4, can accomodate both nonlinear effects and interactions with a minimum number of parameters. However, the model requires special considerations in estimating parameters. As pointed out by Robertson in his paper (1), the parameters for the model are not independent. An attempt to simultaneously estimate all of these paremeters using the generalized least squares technique verified this, for any attempt at such modeling results in a near-singular normal equation matrix.

Robertson utilized a parameter transformation technique. By expansion of each multiplicative factor separately, polynomials result which have the same number of coefficients as original parameters within each factor. Estimation may then be carried out by holding the coefficients of the transformed daylength parameters, for example, and using regression to estimate transformed temperature parameters. These temperature parameters may then be held, and transformed daylength parameters found by regression. This successive substitution technique is cycled until no change in the parameters result, and the parameters are transformed back for the original equation form. The important point here is that the temperature parameters are estimated independently from those for daylength.

The same technique was implemented more simply using the generalized least squares techniques discussed previously. The procedure is identical, except no transformation is required, since the method is not limited to regression. The method proceeds as follows. First, parameter approximations are assumed for the daylength factor. These are held as constants, and the general least squares algorithms are applied in an iterative manner until convergence for the temperature parameters. These are then held, and another computer run made to estimate new values for the daylength parameters, using these generalized least squares algorithms. The process is repeated in successive computer runs until no change is noted in the parameters.

Parameter Estimates

Parameters generated from the least squares programs are summarized in Tables 6 through 16. These parameters were then used as input for the subsequent testing and variance propagation programs. An important feature of the generalized least squares modeling technique is that a-posteriori estimates of the variance-covariance matrix for the parameters result from its application. These may then be used in subsequent variance propagation studies.

Table 6
Polynomial Coefficients Resulting from Least Squares Fit, Stage 0-1, Based Upon 37 Location Years

	Const. Only	Lin T_X Only	Lin T_M Only	Lin D_L Only	Lin P_r Only	Lin T_X, T_M, D_L	Lin T_X, T_M, D_L, P_r
$H_1(X10^{-2})$	8.431733	-0.09278631	8.786957	8.431733	9.068332	-6.452041	-8.753071
$H_2(X10^{-3})$	0.	3.6087	0.	0.	0.	7.6346	8.7159
$H_3(X10^{-3})$	0.	0.	-0.45898	0.	0.	-4.0709	-4.7988
$H_4(X10^{-3})$	0.	0.	0.	0.	0.	0.	0.
$H_5(X10^{-3})$	0.	0.	0.	0.	-3.8817	0.	1.8914

	T_X, D_L & T_XD_L	T_M, D_L & T_MD_L	Lin & Sq w/o P_r	Lin & Sq w/ P	Lin & Int w/o P_r	Lin & Int w/ P_r	All w/o P_r	All w/ P_r
$H_1(X10^{-2})$	-0.09278631	8.786957	15.28596	13.36882	-7.223039	-15.31245	44.00158	85.28296
$H_2(X10^{-3})$	3.6087	-0.45898	-11.883	-11.561	7.9903	11.773	-42.417	-77.856
$H_3(X10^{-3})$	0.	0.	-1.2308	-2.2996	-2.9555	-6.8650	15.021	20.436
$H_4(X10^{-3})$	0.	0.	0.	0.	0.	0.	0.	0.
$H_5(X10^{-3})$	0.	0.	0.	3.3961	0.	35.489	0.	-28.764
$H_6(X10^{-4})$	0.	0.	4.1978	4.4616	0.	0.	12.156	19.667
$H_7(X10^{-4})$	0.	0.	-2.0732	-2.0558	0.	0.	-0.11060	1.4981
$H_8(X10^{-4})$	0.	0.	0.	0.	0.	0.	0.	0.
$H_9(X10^{-4})$	0.	0.	0.	-1.1654	0.	0.	0.	2.1156
$H_{10}(X10^{-4})$	0.	0.	0.	0.	0.	0.	0.	0.
$H_{11}(X10^{-4})$	0.	0.	0.	0.	0.	0.	0.	0.
$H_{12}(X10^{-4})$	0.	0.	0.	0.	0.	0.	0.	0.
$H_{13}(X10^{-4})$	0.	0.	0.	0.	-0.48554	0.38845	-8.2659	-11.152
$H_{14}(X10^{-4})$	0.	0.	0.	0.	0.	-14.216	0.	13.588
$H_{15}(X10^{-4})$	0.	0.	0.	0.	0.	0.18785	0.	-3.7983

Table 7
Polynomial Coefficients Resulting from Least
Squares Fit, Stage 1-2, Based on 53 Location Years

	Const. Only	Lin T_X Only	Lin T_M Only	Lin D_L Only	Lin P_r Only	Lin T_X, T_M, D_L	Lin T_X, T_M, D_L, P_r
$H_1 (X10^{-3})$	4.859397	0.8615316	5.798746	5.634711	3.958185	2.755673	2.222651
$H_2 (X10^{-4})$	0.	3.4423	0.	0.	0.	2.1425	2.3935
$H_3 (X10^{-4})$	0.	0.	3.9145	0.	0.	1.5561	1.2110
$H_4 (X10^{-4})$	0.	0.	0.	9.9291	0.	0.14262	0.082501
$H_5 (X10^{-4})$	0.	0.	0.	0.	7.3297	0.	1.2524

	T_X, D_L & $T_X D_L$	T_M, D_L & $T_M D_L$	Lin & Sq w/o P_r	Lin & Sq w/ P_r	Lin & Int w/o P_r	Lin & Int w/ P_r	All w/o P_r	All w/ P_r
$H_1 (X10^{-3})$	2.413333	6.192233	4.749293	3.635508	1.906250	-3.485117	14.44650	-3.501598
$H_2 (X10^{-4})$	2.6036	0.	1.3716	1.9310	2.6840	6.1158	-13.507	6.3881
$H_3 (X10^{-4})$	0.	2.8102	2.1683	1.6575	-2.0061	-5.5208	18.229	-4.0224
$H_4 (X10^{-4})$	7.6379	0.78146	1.3601	1.1765	6.1203	-9.6170	-3.5812	-25.772
$H_5 (X10^{-4})$	0.	0.	0.	5.7270	0.	28.800	0.	47.081
$H_6 (X10^{-5})$	0.	0.	0.10771	4.7487	0.	0.	5.4161	0.11238
$H_7 (X10^{-5})$	0.	0.	0.83089	0.93968	0.	0.	7.5573	0.58963
$H_8 (X10^{-5})$	0.	0.	-45.599	-47.176	0.	0.	-35.466	-44.423
$H_9 (X10^{-5})$	0.	0.	0.	-7.7746	0.	0.	0.	-13.640
$H_{10} (X10^{-5})$	-5.1193	0.	0.	0.	-3.6078	5.9229	3.2799	16.239
$H_{11} (X10^{-5})$	0.	-6.3540	0.	0.	-9.5090	-17.6415	-7.2848	-19.051
$H_{12} (X10^{-5})$	0.	0.	0.	0.	0.	21.548	0.	26.190
$H_{13} (X10^{-5})$	0.	0.	0.	0.	1.6780	1.4762	-11.831	0.36190
$H_{14} (X10^{-5})$	0.	0.	0.	0.	0.	-20.016	0.	-28.796
$H_{15} (X10^{-5})$	0.	0.	0.	0.	0.	23.010	0.	31.537

Table 8
Polynomial Coefficients Resulting from Least
Squares Fit, Stage 2-3, Based Upon 79 Location Years

	Const. Only	Lin T_X Only	Lin T_M Only	Lin D_L Only	Lin P_r Only	Lin T_X, T_M, D_L	Lin T_X, T_M, D_L, P_r
$H_1(X10^{-2})$	4.589648	2.782055	2.966387	3.519618	3.906240	4.519374	1.723743
$H_2(X10^{-3})$	0.	0.74951	0.	0.	0.	-1.4123	-0.061902
$H_3(X10^{-3}$	0.	0.	1.8460	0.	0.	2.6964	1.4967
$H_4(X10^{-3})$	0.	0.	0.	4.7037	0.	4.8582	5.0642
$H_5(X10^{-3})$	0.	0.	0.	0.	2.7747	0.	2.2210

	T_X, D_L & T_XD_L	T_M, D_L & T_MD_L	Lin & Sq w/o P_r	Lin & Sq w/ P_r	Lin & Int w/o P_r	Lin & Int w/ P_r	All w/o P_r	All w/ P_r
$H_1(X10^{-2})$	-21.79267	-2.129727	53.03508	42.06301	3.805418	-21.47510	-308.7502	66.21290
$H_2(X10^{-3})$	10.621	0.	-43.989	-37.715	0.80023	-0.13505	246.80	-57.263
$H_3(X10^{-3})$	0.	5.6399	4.8274	-0.77791	-15.437	127.00	-0.62885	43.384
$H_4(X10^{-3})$	159.42	18.087	26.863	40.810	62.333	-493.18	191.53	-67.352
$H_5(X10^{-3})$	0.	0.	0.	3.6933	0.	-8.782	0.	-66.358
$H_6(X10^{-4})$	0.	0.	8.4855	7.9387	0.	0.	-31.732	11.624
$H_7(X10^{-4})$	0.	0.	0.30465	1.4016	0.	0.	50.461	11.535
$H_8(X10^{-4})$	0.	0.	-45.496	-69.655	0.	0.	196.53	-125.48
$H_9(X10^{-4})$	0.	0.	0.	4.3304	0.	0.	0.	17.122
$H_{10}(X10^{-4})$	-65.800	0.	0.	0.	-36.190	226.71	-209.48	63.258
$H_{11}(X10^{-4})$	0.	-12.542	0.	0.	29.910	-85.739.	231.95	-67.264
$H_{12}(X10^{-4})$	0.	0.	0.	0.	0.	82.147	0.	191.59
$H_{13}(X10^{-4})$	0.	0.	0.	0.	5.4502	-39.521	-57.798	-18.579
$H_{14}(X10^{-4})$	0.	0.	0.	0.	0.	31.192	0.	17.402
$H_{15}(X10^{-4})$	0.	0.	0.	0.	0.	-77.920	0.	-17.030

Table 9
Polynomial Coefficients Resulting from Least
Squares Fit, Stage 3-4, Based Upon 65 Location Years

	Const. Only	Lin T_X Only	Lin T_M Only	Lin D_L Only	Lin P_r Only	Lin T_X, T_M, D_L	Lin T_X, T_M, D_L, P_r
$H_1(X10^{-2})$	4.553196	4.259662	3.798018	6.377203	4.181928	6.508311	5.278253
$H_2(X10^{-3})$	0.	1.0945	0.	0.	0.	0.0054597	0.34962
$H_3(X10^{-3})$	0.	0.	0.61145	0.	0.	-0.072310	-0.30235
$H_4(X10^{-3})$	0.	0.	0.	-7.0614	0.	-7.2799	-6.0371
$H_5(X10^{-3})$	0.	0.	0.	0.	1.0661	0.	0.77576

	T_X, D_L & T_XD_L	T_M, D_L & T_MD_L	Lin & Sq w/o P_r	Lin & Sq w/ P_r	Lin & Int w/o P_r	Lin & Int w/ P_r	All w/o P_r	All w/ P_r
$H_1(X10^{-2})$	-7.890370	-1.240937	10.15888	10.43590	-30.00160	25.84509	-8.343650	-26.11011
$H_2(X10^{-3})$	5.4959	0.	6.7434	5.8296	10.388	9.2824	14.557	20.432
$H_3(X10^{-3})$	0.	6.9156	-0.67838	-1.2164	23.351	15.191	12.271	13.253
$H_4(X10^{-3})$	43.655	21.265	110.78	-10.586	60.871	68.840	-106.31	-43.059
$H_5(X10^{-3})$	0.	0.	0.	0.41741	0.	-4.1958	0.	3.1980
$H_6(X10^{-4})$	0.	0.	-1.0120	-0.79778	0.	0.	-1.1747	-0.88633
$H_7(X10^{-4})$	0.	0.	0.14672	0.33175	0.	0.	1.7558	2.1305
$H_8(X10^{-4})$	0.	0.	186.18	179.47	0.	0.	180.15	142.28
$H_9(X10^{-4})$	0.	0.	0.	0.21120	0.	0.	0.	-2.6497
$H_{10}(X10^{-4})$	-19.733	0.	0.	0.	-12.119	-17.243	1.0105	-17.363
$H_{11}(X10^{-4})$	0.	-26.766	0.	0.	-35.160	-21.568	-3.0211	12.405
$H_{12}(X10^{-4})$	0.	0.	0.	0.	0.	-30.425	0.	-40.064
$H_{13}(X10^{-4})$	0.	0.	0.	0.	-5.6042	-3.439	-6.1099	-8.5883
$H_{14}(X10^{-4})$	0.	0.	0.	0.	0.	5.169	0.	2.5037
$H_{15}(X10^{-4})$	0.	0.	0.	0.	0.	-0.86249	0.	2.2688

Table 10
Polynomial Coefficients Resulting from Least
Squares Fit, Stage 4-5, Based Upon 62 Location Years

	Const. Only	Lin T_X Only	Lin T_M Only	Lin D_L Only	Lin P_r Only	Lin T_X, T_M, D_L	Lin T_X, T_M, D_L, P_r
$H_1(X10^{-2})$	6.419849	-5.919236	3.136752	5.069346	6.555960	-6.173042	-4.706206
$H_2(X10^{-3})$	0.	4.1680	0.	0.	0.	3.5516	2.9131
$H_3(X10^{-3})$	0.	0.	2.2391	0.	0.	1.6012	2.1647
$H_4(X10^{-3})$	0.	0.	0.	4.7969	0.	-0.95581	-1.3086
$H_5(X10^{-3})$	0.	0.	0.	0.	-0.38702	0.	-0.86316

	T_X, D_L & T_XD_L	T_M, D_L & T_MD_L	Lin & Sq w/o P_r	Lin & Sq w/ P_r	Lin & Int w/o P_r	Lin & Int w/ P_r	All w/o P_r	All w/ P_r
$H_1(X10^{-2})$	-35.57323	0.7347181	-40.51947	-32.90534	0.01676347	3.788751	119.9753	-8.336963
$H_2(X10^{-3})$	15.625	0.	11.662	3.7134	0.25116	-187.21	-25.166	25.240
$H_3(X10^{-3})$	0.	3.5504	17.294	17.593	5.9644	259.48	2.7488	-28.017
$H_4(X10^{-3})$	103.02	8.6625	93.048	134.70	-30.623	-11.720	612.86	-123.31
$H_5(X10^{-3})$	0.	0.	0.	-1.6587	0.	-683.04	0.	65.054
$H_6(X10^{-4})$	0.	0.	-1.3775	-0.17400	0.	0.	-11.399	-0.65185
$H_7(X10^{-4})$	0.	0.	-5.8767	-5.8036	0.	0.	-9.8292	3.3862
$H_8(X10^{-4})$	0.	0.	-175.42	-257.73	0.	0.	-444.75	-108.80
$H_9(X10^{-4})$	0.	0.	0.	0.70022	0.	0.	0.	5.1409
$H_{10}(X10^{-4})$	-39.876	0.	0.	0.	14.491	577.64	297.80	45.052
$H_{11}(X10^{-4})$	0.	-4.7450	0.	0.	-9.7886	-71.721	-3.0017	53.668
$H_{12}(X10^{-4})$	0.	0.	0.	0.	0.	1418.5	0.	-89.873
$H_{13}(X10^{-4})$	0.	0.	0.	0.	-0.57533	-6.8993	9.1570	3.6379
$H_{14}(X10^{-4})$	0.	0.	0.	0.	0.	102.71	0.	-5.9563
$H_{15}(x10^{-4})$	0.	0.	0.	0.	0.	-106.05	0.	-18.4000

Table 11
Exponential Coefficients Resulting from Least
Squares Fit, Stage 0-1, Based Upon 37 Location Years

	Const. Only	Lin T_X Only	Lin T_M Only	Lin D_L Only	Lin P_r Only	Lin T_X, T_M, D_L	Lin T_X, T_M, D_L, P_r
H_1	-2.473168	-3.536614	-2.430835	-2.473168	-2.391737	-4.311131	-4.536354
$H_2(X10^{-2})$	0.	4.4615	0.	0.	0.	9.2731	10.315
$H_3(X10^{-2})$	0.	0.	-0.55089	0.	0.	-4.8492	-5.5545
H_4	0.	0.	0.	0.	0.	0.	0.
$H_5(X10^{-2})$	0.	0.	0.	0.	-5.3781	0.	2.0610

	T_X, D_L & T_XD_L	T_M, D_L & T_MD_L	Lin & Sq w/o P_r	Lin & Sq w/ P_r	Lin & Int w/o P_r	Lin & Int w/ P_r	All w/o P_r	All w/ P_r
H_1	-3.536614	-2.430835	-2.185849	-2.048876	-4.318064	-4.853732	1.198594	5.135680
$H_2(X10^{-2})$	4.4615	0.	-9.6644	-12.374	9.3044	11.694	-44.125	-76.589
$H_3(X10^{-2})$	0.	-0.55089	-2.4199	-2.9238	-4.7443	-10.534	13.153	14.582
H_4	0	0.	0.	0.	0.	0.	0.	0.
$H_5(X10^{-2})$	0.	0.	0.	3.3268	0.	32.338	0.	-19.285
$H_6(X10^{-3})$	0.	0.	4.0728	5.0017	0.	0.	12.659	19.093
$H_7(X10^{-3})$	0.	0.	-1.8834	-2.3159	0.	0.	-0.78802	0.19905
$H_8(X10^{-3})$	0.	0.	0.	0.	0.	0.	0.	0.
$H_9(X10^{-3})$	0.	0.	0.	-0.66833	0.	0.	0.	5.4270
$H_{10}(X10^{-2})$	0.	0.	0.	0.	0.	0.	0.	0.
$H_{11}(X10^{-2})$	0.	0.	0.	0.	0.	0.	0.	0.
$H_{12}(X10^{-2})$	0.	0.	0.	0.	0.	0.	0.	0.
$H_{13}(X10^{-2})$	0.	0.	0.	0.	-0.0044585	0.17681	-0.73298	-0.78505
$H_{14}(X10^{-2})$	0.	0.	0.	0.	0.	-1.1148	0.	1.1572
$H_{15}(X10^{-2})$	0.	0.	0.	0.	0.	-0.24084	0.	-1.0840

Table 12
Exponential Coefficients Resulting from Least
Squares Fit, Stage 1-2, Based Upon 53 Location Years

	Const. Only	Lin T_X Only	Lin T_M Only	Lin D_L Only	Lin P_r Only	Lin T_X, T_M, D_L	Lin T_X, T_M, D_L, P_r
H_1	-5.326841	-6.169140	-5.258371	-5.241116	-5.483952	-5.852482	-5.963827
$H_2(X10^{-2})$	0.	6.3917	0.	0.	0.	3.9636	4.5805
$H_3(X10^{-2})$	0.	0.	7.1782	0.	0.	3.4320	2.6651
$H_4(X10^{-2})$	0.	0.	0.	12.361	0.	-4.4626	-4.3100
$H_5(X10^{-2})$	0.	0.	0.	0.	11.799	0.	1.7655

	T_X, D_L & T_XD_L	T_M, D_L & T_MD_L	Lin & Sq w/o P_r	Lin & Sq w/ P_r	Lin & Int w/o P_r	Lin & Int w/ P_r	All w/o P_r	All w/ P_r
H_1	-6.352690	-5.095987	-9.589213	-9.757012	-5.994067	-5.948517	-6.151259	-7.765557
$H_2(X10^{-2})$	8.7016	0.	55.447	57.556	7.2653	8.7771	10.552	30.792
$H_3(X10^{-2})$	0.	7.0304	2.3567	2.7682	29.750	26.597	40.575	14.578
$H_4(X10^{-2})$	68.805	19.353	10.293	9.4373	77.991	73.821	69.655	33.945
$H_5(X10^{-2})$	0.	0.	0.	2.2833	0.	11.701	0.	49.615
$H_6(X10^{-3})$	0.	0.	-15.036	-15.699	0.	0.	-0.69791	-6.0984
$H_7(X10^{-3})$	0.	0.	0.096239	0.12464	0.	0.	7.0172	0.98346
$H_8(X10^{-3})$	0.	0.	-74.122	-71.564	0.	0.	-79.068	-71.229
$H_9(X10^{-3})$	0.	0.	0.	-7.9763	0.	0.	0.	-22.077
$H_{10}(X10^{-2})$	-3.9052	0.	0.	0.	-4.7623	-4.1519	-3.9830	-1.4824
$H_{11}(X10^{-2})$	0.	-5.6637	0.	0.	3.6255	2.1082	3.6022	-0.57119
$H_{12}(X10^{-2})$	0.	0.	0.	0.	0.	-0.18591	0.	3.3078
$H_{13}(X10^{-2})$	0.	0.	0.	0.	-1.4949	-1.6837	-2.3046	-1.1756
$H_{14}(X10^{-2})$	0.	0.	0.	0.	0.	-2.4685	0.	-4.1073
$H_{15}(X10^{-2})$	0.	0.	0.	0.	0.	6.1144	0.	7.7660

Table 13
Exponential Coefficients Resulting from Least
Squares Fit, Stage 2-3, Based Upon 79 Location Years

	Const. Only	Lin T_X Only	Lin T_M Only	Lin D_L Only	Lin P_r Only	Lin T_X, T_M, D_L	Lin T_X, T_M, D_L, P_r
H_1	-3.081367	-3.506199	-3.453945	-3.317490	-3.223841	-3.270234	-3.780026
$H_2 (X10^{-2})$	0.	1.7580	0.	0.	0.	-2.0413	0.48814
$H_3 (X10^{-2})$	0.	0.	4.1593	0.	0.	5.0967	2.8656
$H_4 (X10^{-2})$	0.	0.	0.	10.178	0.	9.7346	9.5420
$H_5 (X10^{-2})$	0.	0.	0.	0.	5.5437	0.	4.0603

	T_X, D_L & $T_X D_L$	T_M, D_L & $T_M D_L$	Lin & Sq w/o P_r	Lin & Sq w/ P_r	Lin & Int w/o P_r	Lin & Int w/ P_r	All w/o P_r	All w/ P_r
H_1	-6.223192	-3.415205	6.816540	3.000309	-5.803983	-2.465135	8.961171	7.208774
$H_2 (X10^{-2})$	12.008	0.	-86.581	-60.703	13.995	-3.3962	-122.29	-106.56
$H_3 (X10^{-2})$	0.	1.2006	2.9033	8.0928	-15.078	-14.586	54.040	106.76
$H_4 (X10^{-2})$	103.37	-1.2919	37.594	54.704	121.97	-46.937	54.625	-111.31
$H_5 (X10^{-2})$	0.	0.	0.	3.2846	0.	15.458	0.	-179.69
$H_6 (X10^{-2})$	0.	0.	16.955	12.262	0.	0.	30.434	22.617
$H_7 (X10^{-3})$	0.	0.	3.7155	-1.9390	0.	0.	3.6948	25.134
$H_8 (X10^{-3})$	0.	0.	-66.338	-89.587	0.	0.	17.316	-245.38
$H_9 (X10^{-3})$	0.	0.	0.	1.3934	0.	0.	0.	38.549
$H_{10} (X10^{-3})$	-3.8591	0.	0.	0.	-7.0349	2.0557	-3.4562	12.560
$H_{11} (X10^{-2})$	0.	1.2432	0.	0.	6.4521	-2.0114	2.2323	-17.401
$H_{12} (X10^{-2})$	0.	0.	0.	0.	0.	10.269	0.	42.530
$H_{13} (X10^{-2})$	0.	0.	0.	0.	0.21146	0.73834	-2.4683	-4.2710
$H_{14} (X10^{-2})$	0.	0.	0.	0.	0.	-1.4227	0.	6.2072
$H_{15} (X10^{-2})$	0.	0.	0.	0.	0.	0.44573	0.	-6.2186

Table 14
Exponential Coefficients Resulting from Least
Squares Fit, Stage 3-4, Based Upon 65 Location Years

	Const. Only	Lin T_X Only	Lin T_M Only	Lin D_L Only	Lin P_r Only	Lin T_X, T_M, D_L	Lin T_X, T_M, D_L, P_r
H_1	-3.089341	-3.151508	-3.263240	-2.600212	-3.168434	-2.612179	-2.923235
$H_2(X10^{-2})$	0.	0.123177	0.	0.	0.	0.22383	0.89066
$H_3(X10^{-2})$	0.	0.	1.4020	0.	0.	-0.23914	-0.67749
$H_4(X10^{-2})$	0.	0.	0.	-19.117	0.	-19.837	-14.739
$H_5(X10^{-2})$	0.	0.	0.	0.	2.2172	0.	1.5822

	T_X, D_L & T_XD_L	T_M, D_L & T_MD_L	Lin & Sq w/o P_r	Lin & Sq w/ P_r	Lin & Int w/o P_r	Lin & Int w/ P_r	All w/o P_r	All w/ P_r
H_1	-5.589585	-4.259738	-1.994119	-2.034633	-11.20194	-10.07295	-6.841593	-3.034168
$H_2(X10^{-2})$	11.369	0.	14.311	13.554	24.875	21.360	32.728	-27.975
$H_3(X10^{-2})$	0.	14.467	-1.7818	-3.1076	53.350	34.491	31.721	33.562
$H_4(X10^{-2})$	88.113	43.573	-2.2915	-2.2365	142.98	155.65	-195.34	-29.450
$H_5(X10^{-2})$	0.	0.	0.	0.98905	0.	-6.7990	0.	13.256
$H_6(X10^{-3})$	0.	0.	-2.1291	1.8677	0.	0.	-2.0501	15.378
$H_7(X10^{-3})$	0.	0.	0.46881	0.92379	0.	0.	4.4610	3.6577
$H_8(X10^{-3})$	0.	0.	385.78	380.38	0.	0.	355.94	264.71
$H_9(X10^{-3})$	0.	0.	0.	0.27780	0.	0.	0.	-4.3680
$H_{10}(X10^{-2})$	-4.1110	0.	0.	0.	-3.0201	-3.8988	-0.19754	-4.9010
$H_{11}(X10^{-2})$	0.	-5.6754	0.	0.	-7.6940	-4.9288	-0.99436	0.89450
$H_{12}(X10^{-2})$	0.	0.	0.	0.	0.	-6.1840	0.	-5.9877
$H_{13}(X10^{-2})$	0.	0.	0.	0.	-1.3176	-0.86544	-1.54581	-1.8158
$H_{14}(X10^{-2})$	0.	0.	0.	0.	0.	1.0098	0.	-0.020521
$H_{15}(X10^{-2})$	0.	0.	0.	0.	0.	-0.23377	0.	0.53834

Table 15
Exponential Coefficients Resulting from Least
Squares Fit, Stage 4-5, Based Upon 62 Location Years

	Const. Only	Lin T_X Only	Lin T_M Only	Lin D_L Only	Lin P_r Only	Lin T_X, T_M, D_L	Lin T_X, T_M, D_L, P_r
H_1	-2.745776	-4.628175	-3.243995	-2.913867	-2.725344	-4.725186	-4.496764
$H_2(X10^{-2})$	0.	6.3364	0.	0.	0.	5.4513	4.4536
$H_3(X10^{-2})$	0.	0.	3.3732	0.	0.	2.6297	3.5240
$H_4(X10^{-2})$	0.	0.	0.	5.9690	0.	-1.0206	-1.5875
$H_5(X10^{-2})$	0.	0.	0.	0.	-0.58432	0.	-1.3846

	T_X, D_L & T_XD_L	T_M, D_L & T_MD_L	Lin & Sq w/o P_r	Lin & Sq w/ P_r	Lin & Int w/o P_r	Lin & Int w/ P_r	All w/o P_r	All w/ P_r
H_1	-9.155085	-3.615012	-17.68370	-14.08316	-1.652579	8.809258	10.10662	-7.448562
$H_2(X10^{-2})$	23.795	0.	53.654	28.004	-7.9340	-25.578	-25.278	46.310
$H_3(X10^{-2})$	0.	5.1311	33.366	30.086	18.101	-52.169	13.198	-38.699
$H_4(X10^{-2})$	156.75	13.258	261.05	323.03	-158.898	-455.48	-796.65	-119.32
$H_5(X10^{-2})$	0.	0.	0.	-2.4198	0.	96.168	0.	101.24
$H_6(X10^{-3})$	0.	0.	-8.0340	-3.9726	0.	0.	-18.315	-10.860
$H_7(X10^{-3})$	0.	0.	-11.335	-9.9229	0.	0.	-15.801	3.8181
$H_8(X10^{-3})$	0.	0.	-454.41	-591.87	0.	0.	-806.70	-273.37
$H_9(X10^{-3})$	0.	0.	0.	1.0829	0.	0.	0.	7.3280
$H_{10}(X10^{-2})$	-6.0586	0.	0.	0.	6.4918	9.2975	43.473	6.3837
$H_{11}(X10^{-2})$	0.	-0.62985	0.	0.	-2.0119	19.130	-1.3173	8.8611
$H_{12}(X10^{-2})$	0.	0.	0.	0.	0.	-23.308	0.	-15.033
$H_{13}(X10^{-2})$	0.	0.	0.	0.	-0.34727	0.20559	1.2506	0.48109
$H_{14}(X10^{-2})$	0.	0.	0.	0.	0.	-0.30876	0.	0.86107
$H_{15}(X10^{-2})$	0.	0.	0.	0.	0.	-1.4927	0.	-2.7542

Table 16
Parameters for Robertson Triquadratic Model
Resulting from Least Squares Fit

	*Stage 0-1 (37 L.Y.)	Stage 1-2 (53 L.Y.)	Stage 2-3 (79 L.Y.)	Stage 3-4 (65 L.Y.)	Stage 4-5 (62 L.Y.)
H_1	-35.8697	0.2796	5.7835	21.1495	56.3622
H_2	0.029933	0.071784	-1.4036	2.6788	-0.55599
H_3	0.	0.	0.	0.	0.
H_4	48.472	-2.3876	-47.779	-822.05	12.529
H_5	0.029025	4.6869×10^{-4}	1.3137×10^{-4}	1.4005×10^{-2}	1.5414×10^{-4}
H_6	4.2075×10^{-4}	-4.6618×10^{-6}	0.	0.	0.
H_7	-0.021725	2.3943×10^{-4}	-2.3503×10^{-4}	-2.9593×10^{-6}	6.9106×10^{-5}
H_8	2.0950×10^{-4}	1.7055×10^{-5}	0.	0.	0.

*Uses form of Equation 5. All other stages use form of Equation 4.

TEST PROCEDURES

In attempting a preliminary assessment of results in the preceding phase, a simple test program was written which applied the parameters generated to daily meteorological data values. Erratic behavior was noted in using this procedure for the more highly nonlinear functions. This was believed due to the fact that, due to the nature of the phenological reports, only meteorological values averaged over the phenological period were feasible for inclusion in the least squares parameter estimation. When parameters estimated on this basis were applied to daily meteorological values, many of these weather variables were outside the range used in parameter estimation, and great fluctuations were often the result for the nonlinear functions.

When simpler functional forms were utilized, these large fluctuations were not noted. When linear functional forms were utilized, reasonable fits were produced. Figures 5 through 11 depict the "tracking" of linear functional forms driven by daily meteorological values for typical functional forms and location-years.

In an attempt to alleviate these difficulties, test programs were generated using averaged meteorological values to drive the model on a daily basis. These were of two types.

Interval Tests

In these tests, various averaging techniques were applied to the independent meteorological variables used to drive the model, and statistics computed independently within each stage interval. The data used in these tests was of necessity limited to those location-years for which reports were

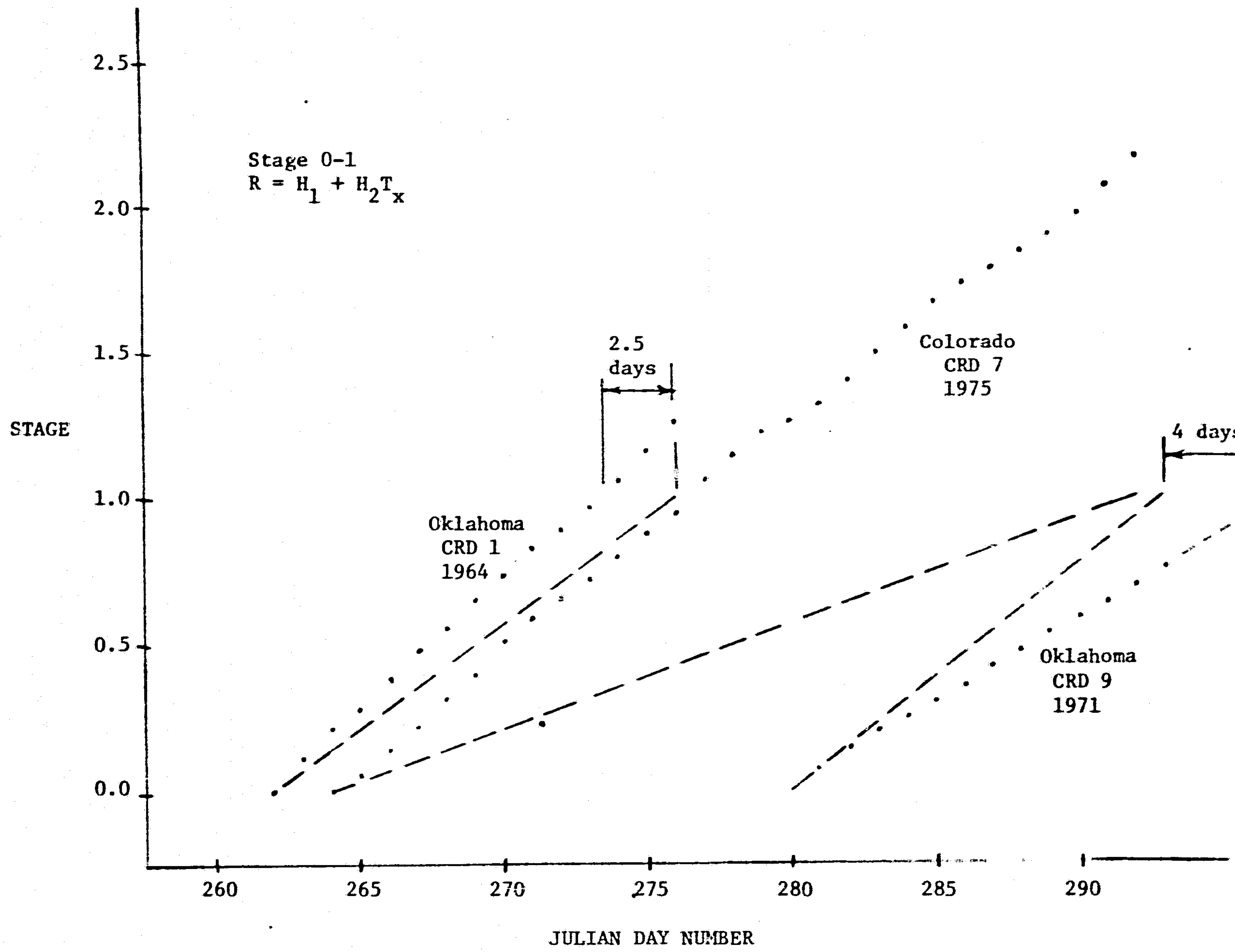

Figure 5: Tests of Linear Function, Stage 0-1

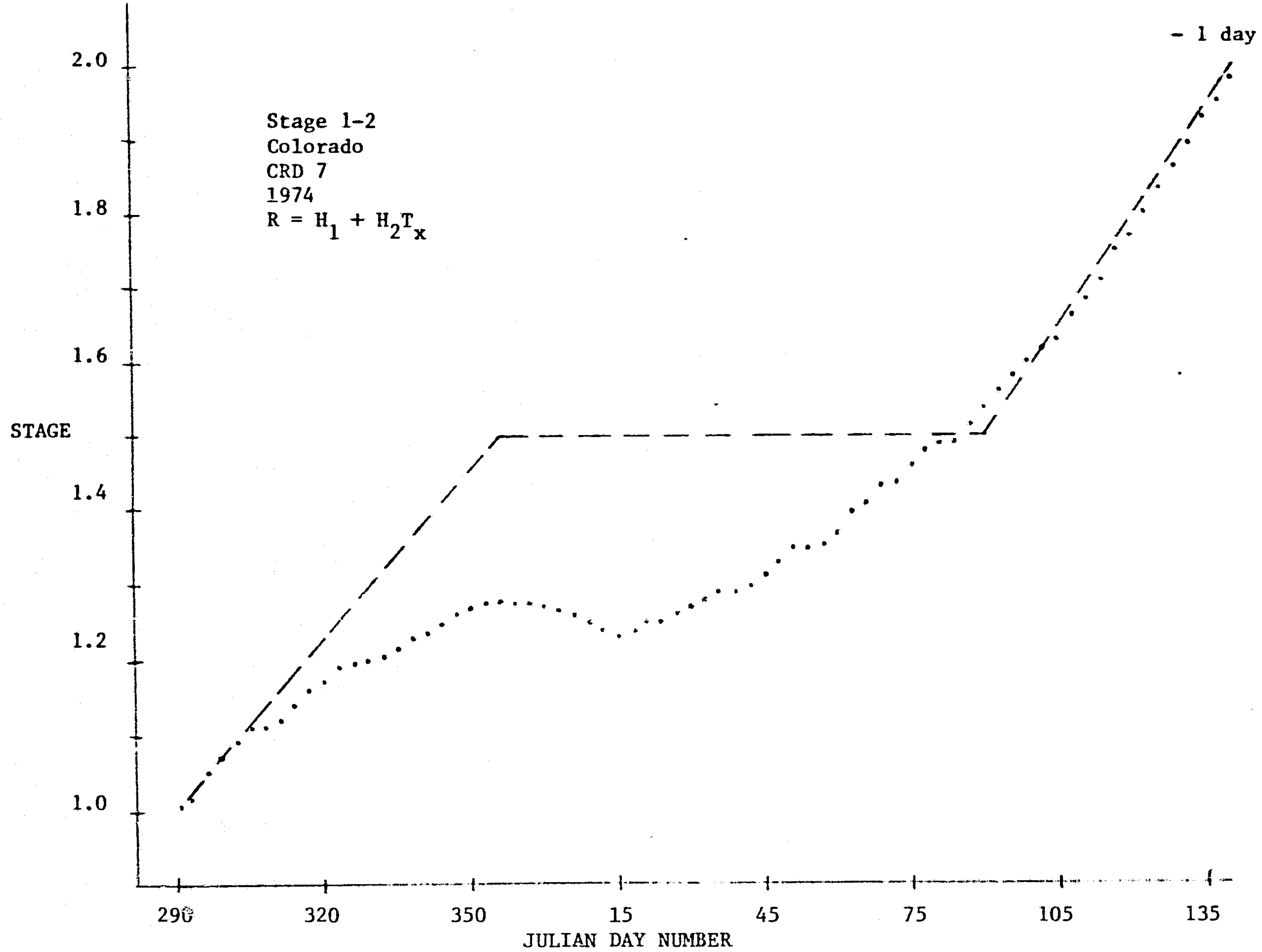

Figure 6: Test of Linear Function, Stage 1-2

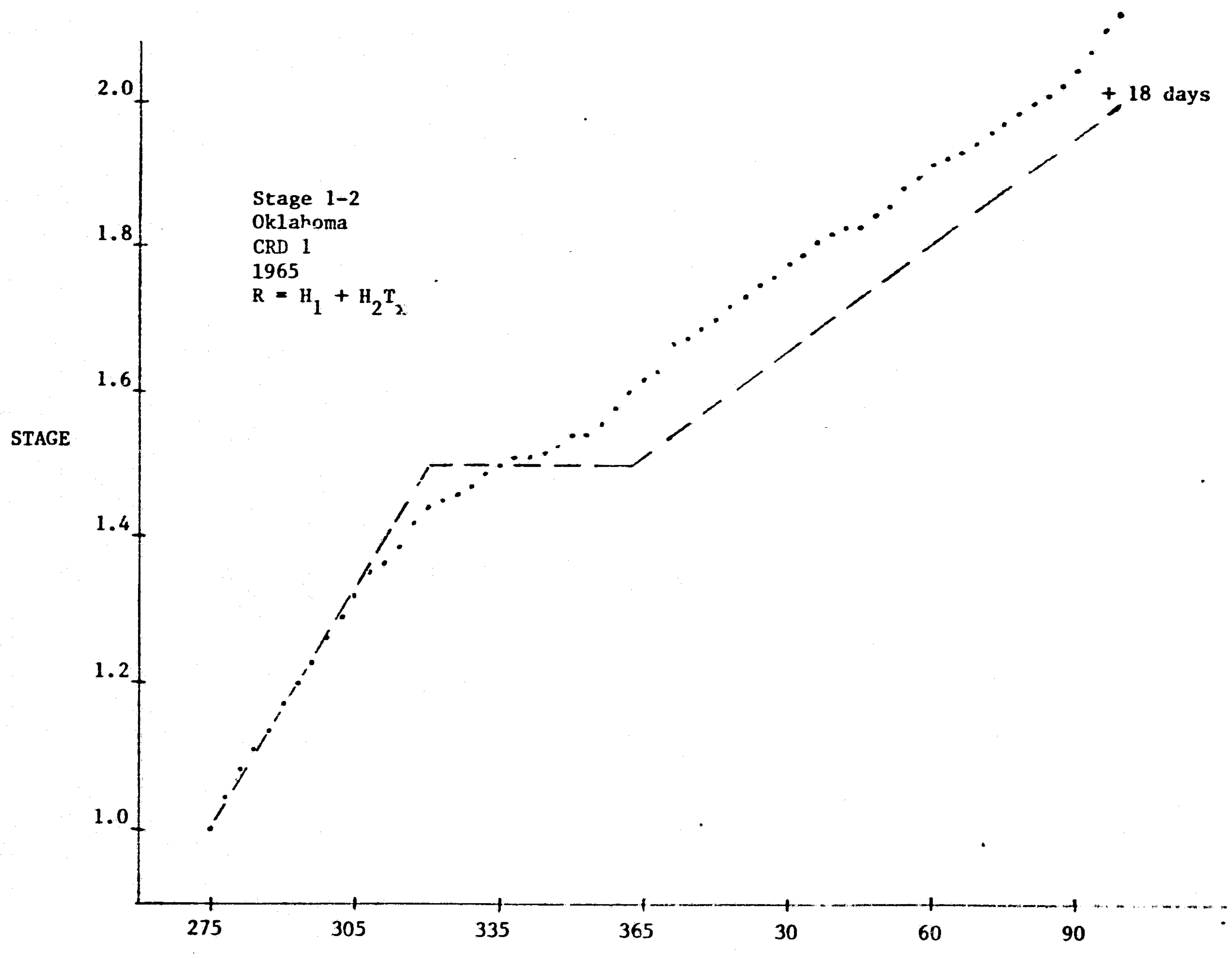

Figure 7: Test of Linear Function, Stage 1-2

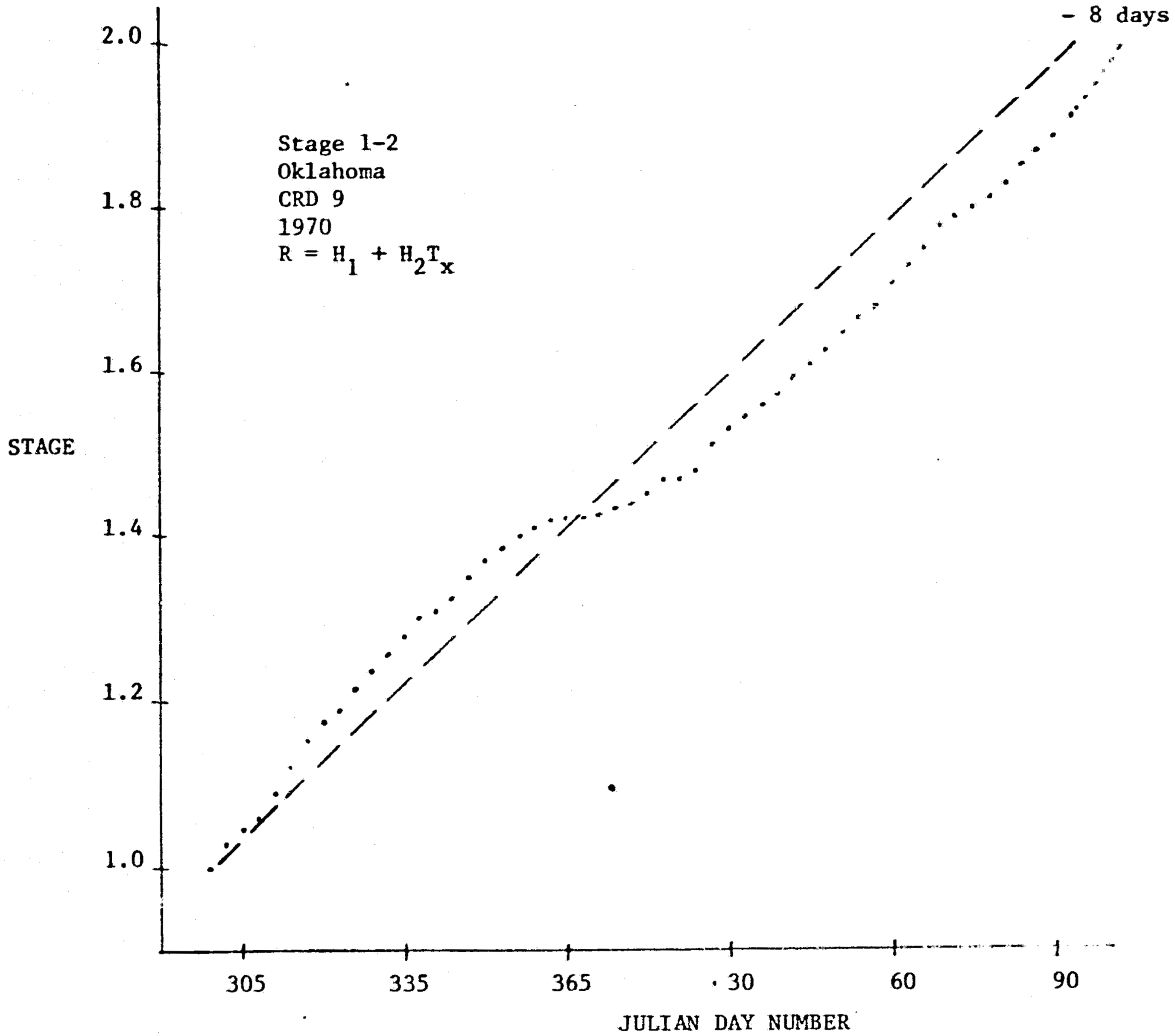

Figure 8: Test of Linear Function, Stage 1-2

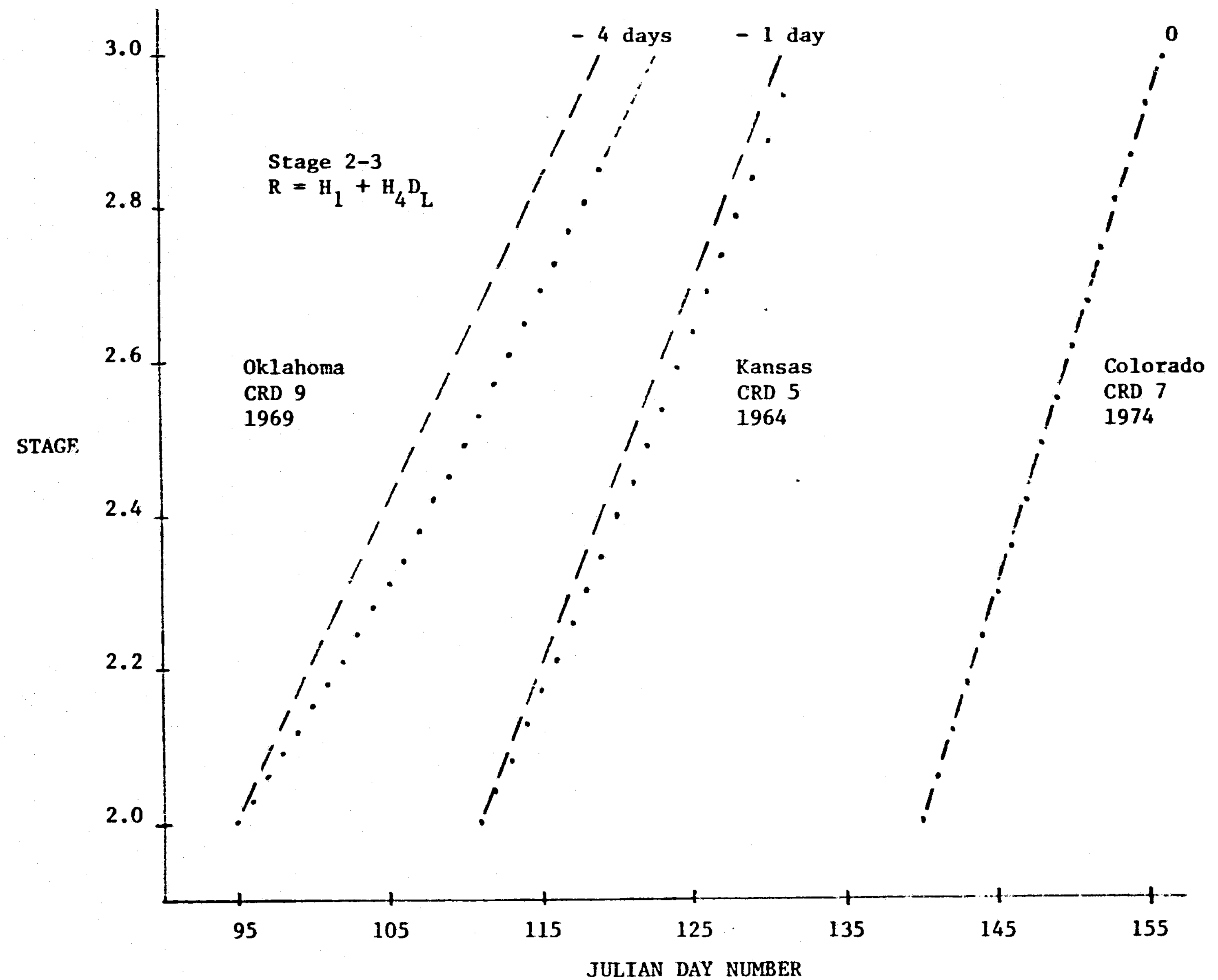

Figure 9: Test of Linear Function, Stage 2-3

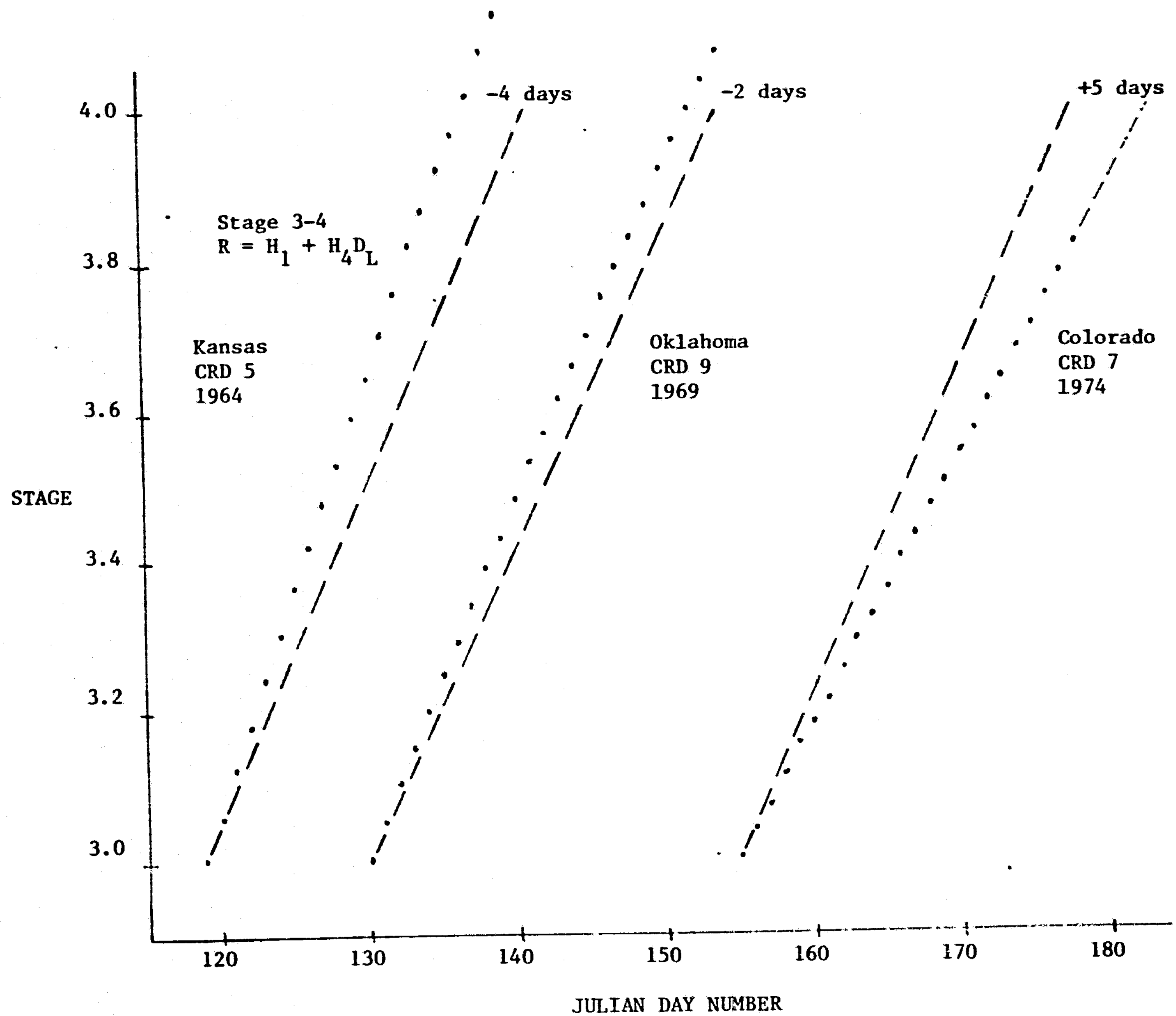

Figure 10: Test of Linear Function, Stage 3-4

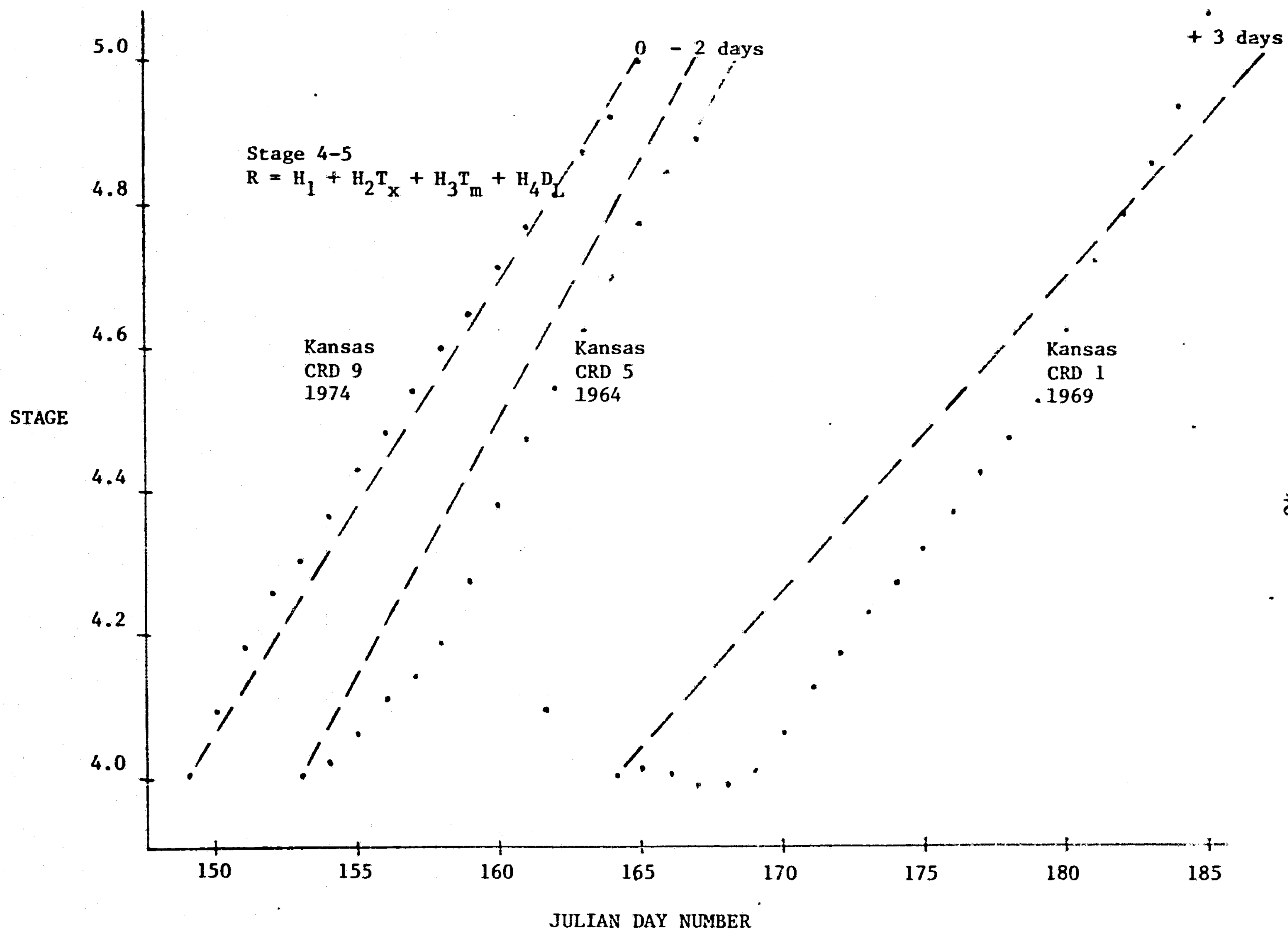

Figure 11: Test of Linear Function, Stage 4-5

available for both the beginning and ending date for the stage. This had the effect of limiting the size of the data sets, particularly for stage intervals 0-1 and 1-2, since only a few states adequately report emergence. A summary of the number of location-years included for each stage is shown:

Stage	Location-Years
Plant-Emerge (0-1)	18
Emerge-Joint (1-2)	16
Joint-Head (2-3)	33
Head-Soft Dough (3-4)	30
Soft Dough-Ripe (4-5)	25

Specific location-years included in this interval test data set are summarized in Table 17.

Running Averages. In this approach, the meteorological variables used to drive the model for any one day would be given by

$$x' = \frac{\sum_{i=j-n+1}^{j} x_i}{n} \qquad \text{Eq. 30}$$

in which x represents any of the independent variables T_X, T_M, D_L, P_r. The value n is the number of days over which running averages are to b· computed, and may be varied within the program from 1 to 9, j is the Julian day number for which the growth rate is to be computed, and x' represents the averaged variable value to be used in the equation to compute growth rate for that day.

Table 17
Summary of Location Years Used
for Interval Test Programs

Stage	State	Crop Year	C.R.D.
0-1	Colorado	1973	6
		1975	7
	Idaho	1974	9
	Oklahoma	1964	1
		1967	1,2,3,4,5,6
		1969	9
		1970	9
		1971	9
	Texas	1975	1S,1N,2N,8N,5
1-2	Colorado	1973	6
		1974	7
	Idaho	1974	9
	Oklahoma	1965	1
		1967	1,2,3,4,5,6
		1969	9
		1970	9
		1971	9
	Texas	1975	1S,2N,5
2-3	Colorado	1973	6
		1974	7
	Idaho	1974	1,9
	Kansas	1964	5
		1967	1,9
		1969	1
		1973	5
		1974	9
		1975	7
	Montana	1973	1,2,3
	North Dakota	1973	1,4,5,6,7,9

Table 17
(Continued)

Stage	State	Crop Year	C.R.D.
2-3	Oklahoma	1967	1,2,3,4,5
		1969	9
		1970	9
		1971	9
	Texas	1975	1S,2N,5
3-4	Colorado	1974	7
	Idaho	1974	1,9
	Kansas	1964	5
		1967	1,9
		1969	1
		1973	5
		1974	9
		1975	7
	Missouri	1971	9
		1973	9
	Montana	1973	1,2,3
	Oklahoma	1967	1,2,3,4,5
		1969	9
		1970	9
		1971	9
	Texas	1975	1S,2S,2N,5
4-5	Idaho	1974	1,9
	Kansas	1964	5
		1967	1,9
		1969	1
		1974	9
		1975	7
	Missouri	1973	9
	Montana	1973	1,2,3
	Oklahoma	1967	1,2,3,4,5
		1969	9
		1970	9
		1971	9
	Texas	1975	1S,2S,2N,5

Thus

$$\dot{R} = f(\underline{H}, \underline{X}') \qquad \text{Eq. 31}$$

in which the function f may represent the polynomial, exponential, or Robertson equations discussed earlier. Within any interval the stage would be computed as

$$S_K = (K - 1) + \sum_{i = j_o}^{j} \dot{R}_i \qquad \text{Eq. 32}$$

in which K represents the interval, j_o represents the reported date at which the interval began, and j represents the Julian day number at which the stage is computed.

Accumulative Averages. In this approach, meteorological values used to drive the model for any one day were computed as accumulative averages from the beginning day for the stage up to and including the day itself. Therefore, a typical averaged weather variable would be computed for a date j, as

$$x' = \frac{\sum_{i=j_o}^{j} x_i}{n} \qquad \text{Eq. 33}$$

in which x' represents the averaged weather variable used to compute growth rate for Julian date j, j_o represents the Julian date at the beginning of the phenological interval, and n represents the number of days over which averaging was done and would be computed by $n = j - j_o + 1$.

The rate for each day would then be computed by

$$R = f(\underline{H},\underline{X}') \qquad \text{Eq. 34}$$

and intermediate stage values within an interval would be computed by

$$S_K = (K - 1) + nR. \qquad \text{Eq. 35}$$

Special considerations are necessary in applying this technique over interval 1-2, which includes a winter dormancy period. If the method were applied directly, then high temperatures and daylengths carried from the fall period would have a tendency to mask the onset of winter and subsequent dormancy. A similar problem would occur in the spring greenup period, when low values included from the winter would mask the onset of spring and subsequent greenup. Thus, the winter dormancy and spring greenup criteria discussed earlier was used. The averaging algorithm was re-started when winter dormancy was indicated (10 days in which average temperature was less than 4.69°C.), and again at spring greenup (10 days greater than 6.14°C.).

Test Results. Both of the averaging techniques were applied to independent data on a daily basis. Results from the running average used with polynomial functions (TEST CC11) and used with exponential functions (TEST CC12) are summarized in Tables 18 and 19. Results from application of the running average algorithm applied to the Robertson model are shown in Table 22. Results from applying the accumulative average algorithm to polynomial (TEST CC21), exponential (TEST CC22), and Robertson functions are summarized in Tables 20, 21, and 22 respectively.

Table 18
Summary of Statistics from Interval Test Program
Using 5-Day Running Averages and Polynomial Functions (TEST CC11)

	Stg. 0-1 (18 L.Y.)			Stg. 1-2 (16 L.Y.)			Stg. 2-3 (33 L.Y.)			Stg. 3-4 (3[illegible] L.Y.)			Stg. 4-5 (25 L.Y.)		
FUNCTION	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE
Const. Only	+2.9	45.8	6.7	-27.4	1403.3	37.5	+3.5	69.8	8.4	+1.3	35.5	6.0	+9.0	138.0	11.7
Lin T_X Only	+2.8	57.3	7.6	+3.3	255.0	16.0	+1.9	55.4	7.4	+1.7	34.3	5.9	+7.7	116.8	10.8
Lin T_M Only	+2.2	41.9	6.5	-2.4	197.2	14.0	+1.6	56.1	7.5	+1.2	31.7	5.6	+8.1	123.8	11.1
Lin D_L Only	+2.9	45.8	6.8	-33.4	1556.2	39.4	+1.8	61.1	7.8	+2.3	48.8	7.0	+8.7	127.6	11.3
Lin P_r Only	+2.3	42.2	6.5	-10.1	1095.9	33.9	+2.3	63.7	8.0	+1.4	35.0	5.9	+8.9	138.2	11.8
Lin T_X, T_M, D_L	+1.9	65.1	8.1	+1.1	165.1	12.8	+1.6	61.3	7.8	+2.4	50.4	7.1	+7.5	114.4	10.7
Lin T_X, T_M, D_L, P_r	+2.1	70.7	8.4	+1.7	180.5	13.4	+1.4	58.7	7.7	+2.1	45.2	6.7	+7.5	116.5	10.8
T_X, D_L & T_XD_L	+2.6	54.2	7.4	+6.4	344.9	18.6	+0.2	125.2	11.2	+0.5	41.4	6.4	+8.4	139.6	11.8
T_M, D_L & T_MD_L	+2.2	41.9	6.5	+0.8	207.1	14.4	+0.2	59.5	7.7	+0.7	36.4	6.0	+8.0	123.0	11.1
Lin & Sq w/o P_r	+2.6	61.7	7.9	-1.4	249.4	15.8	+4.0	80.6	9.0	+4.9	72.9	8.5	+6.1	96.3	9.8
Lin & Sq w/ P_r	+2.4	70.9	8.4	-9.4	390.7	19.8	+4.0	76.4	8.7	+4.8	67.4	8.2	+6.5	105.1	10.3
Lin & Int w/o P_r	+1.9	65.1	8.1	+2.8	344.8	18.6	+1.8	52.1	7.2	-1.6	33.8	5.8	+7.6	117.6	10.8
Lin & Int w/ P_r	+1.7	62.9	7.9	+2.8	344.8	18.6	+1.7	49.2	7.0	-1.6	33.8	5.8	+7.6	117.6	10.8
All w/o P_r	+3.3	58.8	7.6	+4.6	234.1	15.3	-5.0	512.7	22.6	+2.2	145.7	12.1	-0.2	244.4	15.6
All w/ P_r	+3.0	66.9	8.2	+73.0	6166.0	78.5	-50.1	9109.4	95.4	+3.3	45.0	6.7	-16.0	646.3	25.4

Table 19
Summary of Statistics from Interval Test Program
Using 5-Day Running Averages and Exponential Functions (TEST CC12)

	Stg. 0-1 (18 L.Y.)			Stg. 1-2 (16 L.Y.)			Stg. 2-3 (33 L.Y.)			Stg. 3-4 (30 L.Y.)			Stg. 4-5 (25 L.Y.)		
FUNCTION	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE
Const. Only	+2.9	45.8	6.8	-27.4	1403.0	37.5	+2.5	63.8	8.0	+2.1	38.7	6.2	+9.0	138.0	11.7
Lin T_X Only	+2.9	56.1	7.5	+2.2	229.9	15.1	+1.9	55.4	7.4	+1.7	34.4	5.9	+8.0	119.9	11.0
Lin T_M Only	+2.2	41.9	6.5	-3.6	228.8	15.1	+1.6	55.3	7.4	+1.2	31.8	5.6	+8.2	124.1	11.1
Lin D_L Only	+2.9	45.8	6.8	-31.4	1491.9	38.6	+1.9	62.9	7.9	+2.6	54.3	7.4	+8.7	128.8	11.3
Lin P_r Only	+2.4	43.7	6.6	-9.2	970.3	31.1	+2.5	63.5	8.0	+1.4	35.0	5.9	+9.0	138.2	11.8
Lin T_X, T_M, D_L	+2.3	64.2	8.0	+3.3	218.0	14.7	+1.5	58.6	7.6	+2.7	55.3	7.4	+7.6	115.8	10.8
Lin T_X, T_M, D_L, P_r	+2.6	69.2	8.3	+4.2	249.8	15.8	+1.5	55.2	7.4	+2.3	46.5	6.8	+7.7	119.0	10.9
T_X, D_L & T_XD_L	+2.9	56.1	7.5	+30.6	2617.1	51.2	+1.4	51.2	7.2	+1.0	40.2	6.3	+8.7	144.3	12.0
T_M, D_L & T_MD_L	+2.2	41.9	6.5	+6.6	1320.6	36.3	+1.0	55.5	7.4	+1.1	37.6	6.1	+7.9	119.4	10.9
Lin & Sq w/o P_r	+2.8	63.4	8.0	-8.0	208.2	14.4	+6.1	121.4	11.0	+5.5	84.9	9.2	+4.8	75.0	8.7
Lin & Sq w/ P_r	+2.7	67.4	8.2	-10.9	281.1	16.8	+4.0	77.9	8.8	+5.4	79.2	8.9	+5.4	88.4	9.4
Lin & Int w/o P_r	+2.6	54.2	7.4	+6.4	344.9	18.6	+0.2	125.8	11.2	+0.5	41.4	6.4	+8.4	139.6	11.8
Lin & Int w/ P_r	+1.8	69.2	8.3	+116.0	17502.3	132.3	+6.4	118.3	10.9	-3.3	41.6	6.4	-3.5	117.3	10.8
All w/o P_r	+3.6	67.4	8.2	*+1.2	298.3	17.3	+8.2	170.8	13.1	+3.5	74.0	8.6	+4.4	100.2	10.0
All w/ P_r	+1.3	86.8	9.3	125.8	18193.7	134.9	-68.3	11405.7	106.8	+11.4	156.9	12.5	-35.7	18349.4	135.5

Table 20
Summary of Statistics from Interval Test Program
Using Accumulative Averages and Polynomial Functions (TEST CC21)

	Stg. 0-1 (18 L.Y.)			Stg. 1-2 (16 L.Y.)			Stg. 2-3 (33 L.Y.)			Stg. 3-4 (30 L.Y.)			Stg. 4-5 (25 L.Y.)		
FUNCTION	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE
Const. Only	+2.8	42.9	6.5	-27.4	1403.3	37.5	+3.6	69.1	8.3	+1.1	35.5	6.0	+9.0	138.9	11.8
Lin T_X Only	+2.4	54.9	7.4	+5.2	561.2	23.7	+1.8	53.3	7.3	+1.6	34.7	5.9	+7.6	126.1	11.2
Lin T_M Only	+2.1	38.7	6.2	-2.9	448.1	21.2	+1.1	55.5	7.4	+1.1	30.8	5.5	+8.0	122.0	11.0
Lin D_L Only	+2.8	42.9	6.5	-50.0	3045.5	55.2	+1.6	58.5	7.6	+2.4	50.6	7.1	+8.7	127.6	11.3
Lin P_r Only	+1.8	40.2	6.3	-18.3	1271.5	35.7	+2.5	69.4	8.3	+1.2	33.7	5.8	+9.0	138.4	11.8
Lin T_X, T_M, D_L	+1.4	74.0	8.6	+2.8	409.8	20.2	+1.3	61.6	7.8	+2.4	53.9	7.3	+7.2	120.9	11.0
Lin T_X, T_M, D_L, P_r	+1.3	82.4	9.1	+4.4	453.7	21.3	+.7	61.3	7.8	+2.2	46.7	6.8	+7.3	122.6	11.1
T_X, D_L & $T_X D_L$	+2.4	54.9	7.4	+9.8	651.9	25.5	-.1	199.4	14.1	+0.6	42.6	6.5	+8.3	149.0	12.2
T_M, D_L & $T_M D_L$	+2.1	38.7	6.2	+1.[illegible]	389.9	19.7	-1.4	97.8	10.0	+0.4	38.9	6.2	+8.0	120.6	11.0
Lin & Sq w/o P_r	+1.9	59.7	7.7	-3.9	653.0	25.6	+2.4	122.1	11.0	+5.4	80.8	9.0	+5.3	110.8	10.5
Lin & Sq w/ P_r	+1.8	67.1	8.2	-6.6	656.2	25.6	-.9	183.8	13.6	+5.0	74.1	8.6	+5.8	115.7	10.8
Lin & Int w/o P_r	+1.3	74.0	8.6	+3.3	918.6	30.3	-1.9	225.6	15.0	-2.8	83.7	9.1	+7.3	123.5	11.1
Lin & Int w/ P_r	-57.1	48175	219.5	+3.1	1030.8	32.1	-64.1	23330.	152.7	+22.9	556.6	23.6	+23.6	615.9	24.8
All w/o P_r	+2.8	56.6	7.5	-2.9	699.5	26.4	-69.5	68764.	262.2	+4.5	78.9	8.9	-5.9	650.7	25.5
All w/ P_r	+3.4	70.7	8.4	-7.2	569.2	23.9	-8.8	1582.7	39.7	+1.7	53.7	7.3	+6.3	140.4	11.8

Table 21
Summary of Statistics from Interval Test Program
Using Accumulative Averages and Exponential Functions (TEST CC22)

	Stg. 0-1 (18 L.Y.)			Stg. 1-2 (16 L.Y.)			Stg. 2-3 (33 L.Y.)			Stg. 3-4 (30 L.Y.)			Stg. 4-5 (25 L.Y.)		
FUNCTION	BIAS	VAR.	RMSE	BIAS	VAR	RMSE	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE
Const. Only	+2.8	42.9	6.5	-27.4	1403.3	37.5	+2.6	62.8	7.9	+2.1	38.4	6.2	+9.0	138.9	11.8
Lin T_X Only	+2.5	53.7	7.3	+0.9	617.4	24.8	+1.7	52.3	7.2	+1.6	34.3	5.9	+7.9	125.9	11.2
Lin T_M Only	+2.1	39.4	6.3	-9.4	683.2	26.1	+1.2	54.1	7.4	+1.1	30.8	5.5	+8.0	122.7	11.1
Lin D_L Only	+2.8	42.9	6.5	-42.6	2400.0	48.9	+1.6	58.1	7.6	+2.6	56.8	7.5	+8.7	129.8	11.4
Lin P_r Only	+2.2	42.6	6.5	-16.3	1271.4	35.7	+2.7	77.4	8.8	+1.1	33.9	5.8	+9.0	138.4	11.8
Lin T_X, T_M, D_L	+1.8	66.4	8.2	+3.8	620.4	24.9	+1.1	55.1	7.4	+2.7	58.3	7.6	+7.5	119.3	10.9
Lin T_X, T_M, D_L, P_r	+1.9	69.4	8.3	+5.8	699.2	26.4	+1.0	56.5	7.5	+2.3	47.3	6.9	+7:6	123.2	11.1
T_X, D_L & T_XD_L	+2.6	54.8	7.4	+36.9	4587.3	67.7	+0.5	47.0	6.9	+1.0	41.3	6.4	+8.7	154.8	12.4
T_M, D_L & T_MD_L	+2.1	39.4	6.3	+0.4	2762.9	52.6	+0.5	50.3	7.1	+0.9	36.6	6.1	+8.0	119.8	10.9
Lin & Sq w/o P_r	+2.2	60.8	7.8	-0.9	389.1	19.7	+5.8	175.7	13.3	+6.0	95.9	9.8	+4.2	83.8	9.2
Lin & Sq w/ P_r	+2.3	65.4	8.1	-1.6	350.2	18.7	+2.8	117.7	10.8	+5.7	87.4	9.3	+4.7	92.7	9.6
Lin & Int w/o P_r	+1.8	66.4	8.2	+13.1	1138.2	33.7	-0.1	53.9	7.3	-1.4	39.6	6.3	+7.5	118.4	10.9
Lin & Int w/ P_r	+2.1	67.3	8.2	+28.3	3319.1	57.6	+2.5	100.0	10.0	-1.6	23.6	4.9	+7.8	153.0	12.3
All w/o P_r	+2.8	60.9	7.8	+1.0	615.0	24.8	+7.8	233.4	15.3	+4.9	79.9	8.9	+2.8	112.5	10.6
All w/ P_r	+4.3	76.6	8.8	+16.3	2008.7	44.8	+4.7	191.3	13.8	+8.9	125.6	11.2	+7.0	125.9	11.2

Table 22
Summary of Statistics from Interval Test Programs for Robertson Model

Using 5-Day Running Averages

Stage	Bias	Variance	R.M.S.E.
0-1	+2.4	66.8	8.2
1-2	+1.1	141.3	11.9
2-3	+1.4	60.8	7.8
3-4	+2.3	49.0	7.0
4-5	+7.6	115.2	10.7

Using Accumulative Averages

Stage	Bias	Variance	R.M.S.E.
0-1	+2.0	64.5	8.0
1-2	-1.4	551.0	23.5
2-3	+1.2	61.8	7.9
3-4	+2.3	50.0	7.1
4-5	+7.2	120.9	11.0

Crop Year Tests

In applying the above testing procedures, data were limited within each stage to location-years for which both beginning and ending dates were reported for the stage. In order to overcome this limiting aspect as to location-years, a crop year test program was written, in which the equations generated from least squares were applied throughout the entire winter wheat crop year.

In this way, cumulative error throughout the growing season could be estimated. Further location-years for which phenological dates were not known in every interval could be included. For those intervals not having known phenological dates, the unknown error when a stage was reached was carried forward, and included within the cumulative error at the next stage at which a phenological date was reported. This program utilized the running average technique.

A data set, consisting of 28 location-years of complete crop year meteorological values was prepared for this program. The specific location-years included, and the stages reported for each, are shown in Table 23.

Results of these tests are shown in Tables 24 and 25. The Robertson and a "combination" model are included in Table 24. In general, the crop year tests resulted in errors less than would be expected from the interval tests. The Robertson model, in particular, resulted in relatively small biases and variances.

A virtually infinite number of combinations, utilizing different functional forms may be investigated using this program. The results from one such combination, selected based upon bias and variance values from interval tests, are shown at the bottom of Table 24. This combination consists of the following functions:

Table 23
Location Years Included in
Crop Year Test Data Set

State	C.R.D.	Crop Year	Stages Reported
Colorado	6	1973	1,2,3,4,5
Idaho	1	1974	2,3,4,5
	9	1974	1,2,3,4,5
Kansas	1	1967,1969	2,3,4,5
	7	1975	2,3,4,5
	9	1967,1974	2,3,4,5
Missouri	9	1971,1973	3,4,5
	9	1974	3,5
Montana	1	1973	2,3,4,5
	2	1973	2,3,4,5
	3	1973	2,3,4,5
North Dakota	1	1973	2,3,5
	4	1973	2,3,5
	6	1973	2,3,5
	7	1973	2,3,5
	9	1973	2,3,5
Oklahoma	1	1967	1,2,3,4,5
	3	1967	1,2,3,4,5
	5	1967	1,2,3,4,5
	9	1969,1970,1971	1,2,3,4,5
Texas	2N	1975	1,2,3,4,5
	2S	1975	1,2,3,4,5
	5	1975	1,2,3,4,5

Table 24
Summary of Statistics from Yearly Test Program Using
5-Day Running Averages and Polynomial Functions (YEAR TEST 1) (28 L.Y.)

	Stage 0-1			Stage 1-2			Stage 2-3			Stage 3-4			Stage 4-5		
FUNCTION	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE
Const. Only	+1.9	22.3	4.7	-2.4	1460.7	38.2	-5.4	1308.0	36.2	-9.3	897.3	30.0	+15.7	862.0	29.4
Lin T_X Only	+1.3	30.7	5.5	-8.5	651.7	25.5	-9.8	615.2	24.8	-2.5	423.9	20.6	+1.5	463.5	21.5
Lin T_M Only	+1.4	19.4	4.4	-4.3	316.3	17.8	-3.6	301.8	17.4	+0.7	216.7	14.7	+5.0	372.8	19.3
Lin D_L Only	+1.9	22.3	4.7	-14.3	959.0	31.0	-16.2	904.1	30.1	-19.3	761.6	27.6	+0.4	219.5	14.8
Lin P_r Only	+1.1	19.5	4.4	+5.3	874.0	29.6	+10.5	712.7	26.7	+6.1	578.6	24.1	+18.8	789.6	28.1
Lin T_X, T_M, D_L	+0.6	29.5	5.4	-6.5	494.0	22.2	-8.3	387.0	19.7	-1.3	280.8	16.8	0.0	424.2	20.6
Lin T_X, T_M, D_L, P_r	+0.6	32.1	5.7	-6.4	548.8	23.4	-8.7	431.9	20.8	-1.8	305.3	17.5	+0.6	443.0	21.0
T_X, D_L & T_XD_L	+1.3	30.7	5.5	-12.9	1208.9	34.8	-11.4	670.0	25.9	-2.9	148.8	12.2	+4.3	235.4	15.3
T_M, D_L & T_MD_L	+1.4	19.4	4.4	-6.3	591.4	24.3	-9.9	384.2	19.6	-3.6	232.4	15.2	+7.1	300.7	17.3
Lin & Sq w/o P_r	+1.5	23.6	4.9	-20.7	1431.1	37.8	-14.8	1007.4	31.7	+0.4	543.1	23.3	+9.5	267.1	16.3
Lin & Sq w/ P_r	+1.2	25.5	5.1	-28.1	1921.3	43.8	-17.4	871.2	29.5	-3.2	252.2	15.9	+6.8	248.9	15.8
Lin & Int w/o P_r	+0.5	29.6	5.4	-14.8	1153.7	34.0	-13.1	1109.4	33.3	-0.3	252.6	15.9	+8.5	444.1	21.1
Lin & Int w/ P_r	+0.8	31.4	5.6	-7.5	1736.3	41.7	+5.4	987.4	31.4	-2.5	452.1	21.3	+11.4	858.5	29.3
All w/c P_r	+2.1	20.6	4.5	-9.7	935.5	30.6	+9.9	1178.9	34.3	+26.8	1725.0	41.5	+44.4	3571.6	59.8
All w/ P_r	+3.2	27.9	5.3	-10.3	1069.3	32.7	-11.8	889.3	29.8	-5.2	362.3	19.0	+4.9	389.9	19.7
Robertson	+1.5	26.3	5.1	-2.0	258.5	16.0	-3.3	200.5	14.2	-0.2	205.6	14.3	+1.8	305.5	17.5
Combination	+1.4	19.4	4.4	-3.4	262.8	16.2	-5.5	186.6	13.7	-2.7	125.7	11.2	-2.9	197.8	14.1

Table 25
Summary of Statistics from Yearly Test Program Using
5-Day Running Averages and Exponential Functions (YEAR TEST 2) (28 L.Y.)

	Stage 0-1			Stage 1-2			Stage 2-3			Stage 3-4			Stage 4-5		
FUNCTION	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE	BIAS	VAR.	RMSE
Const. Only	+1.9	22.3	4.7	-1.4	1456.8	38.2	-5.4	1308.0	36.2	-9.3	897.3	30.0	+14.7	831.7	28.8
Lin T_X Only	+1.5	28.9	5.4	-1.6	348.5	18.7	-3.8	334.4	18.3	-.1	320.1	17.9	+2.7	396.9	19.9
Lin T_M Only	+1.4	19.4	4.4	-1.2	199.1	14.1	-1.0	217.9	14.8	-0.8	198.1	14.1	+5.6	294.3	17.2
Lin D_L Only	+1.9	22.3	4.7	-9.3	1066.3	32.7	-11.9	983.8	31.4	-15.9	747.4	27.3	+6.2	336.3	18.3
Lin P_r Only	+1.3	18.0	4.2	+13.8	1098.6	33.1	+0.2	4224.0	65.0	+3.4	2830.0	53.2	+36.5	3049.1	55.2
Lin T_X, T_M, D_L	+0.8	27.2	5.2	-1.8	356.7	18.9	-3.4	263.8	16.2	+0.9	321.6	17.9	+3.7	397.0	19.9
Lin T_X, T_M, D_L, P_r	+1.0	27.9	5.3	-1.2	429.0	20.7	-3.5	317.7	17.8	+0.9	372.6	19.3	+5.2	415.6	20.4
T_X, D_L & T_XD_L	+1.5	28.9	5.4	-13.4	3457.1	58.8	-10.1	877.3	29.6	-0.6	698.8	26.4	+8.0	639.5	25.3
T_M, D_L & T_MD_L	+1.4	19.4	4.4	-22.2	4410.3	66.4	-5.2	1198.6	34.6	+7.4	1024.5	32.0	+13.6	1079.7	32.9
Lin & Sq w/o P_r	+1.5	24.2	4.9	-14.7	1057.2	32.5	-3.3	452.9	21.3	+5.9	474.5	21.7	+4.7	341.2	18.5
Lin & Sq w/ P_r	+1.5	24.7	5.0	-9.8	532.7	23.1	-7.6	355.3	18.8	-0.5	332.6	18.2	+2.7	281.6	16.8
Lin & Int w/o P_r	+0.8	27.2	5.2	-10.1	1127.8	33.6	-9.9	976.0	31.2	-1.9	427.1	20.7	+4.7	656.2	25.6
Lin & Int w/ P_r	+1.1	27.8	5.3	-38.0	10075.8	100.4	-32.0	8536.4	92.4	+2.9	2133.4	46.2	+5.7	3341.7	57.8
All w/o P_r	+1.9	21.0	4.6	-16.4	1925.1	43.9	+3.8	1038.7	32.2	+14.1	1291.1	35.9	+25.5	2100.3	45.8
All w/ P_r	+3.5	30.4	5.5	-19.7	5600.0	74.8	-7.9	2083.5	45.6	+4.9	1913.9	43.7	+13.7	1872.3	43.3

Stage	Function	Terms in Equation
0-1	Polynomial	Linear T_M Only
1-2	Robertson	Triquatratic
2-3	Exponential	T_X, D_L and $T_X D_L$
3-4	Polynomial	Linear T_M Only
4-5	Exponential	Linear and Sq. w/o P_r.

This particular combination worked well, resulting in low variances, relative to the other cases tested, although biases exceeded slightly those from the Robertson model.

Figure 12 shows, in graphical form, the general behavior of the bias values resulting from these crop year tests, for typical functions, and Figure 13 shows a similar plot for variance values.

VARIANCE PROPAGATION

In order to assess the acceptability of the variances resulting from the testing procedures, the technique of variance propagation may be used as an independent check. The generalized variance propagation relation provides a powerful tool to determine the resulting variances of quantities computed from parameters and independent variables subject to statistical variation.

Mathematical Model

The basic relationship utilized in variance propagation is

$$\underset{c,c}{Q_s} = \underset{c,n}{J_{sy}} \; \underset{n,n}{Q_y} \; \underset{n,c}{J^t_{sy}} \quad \text{Eq. 36}$$

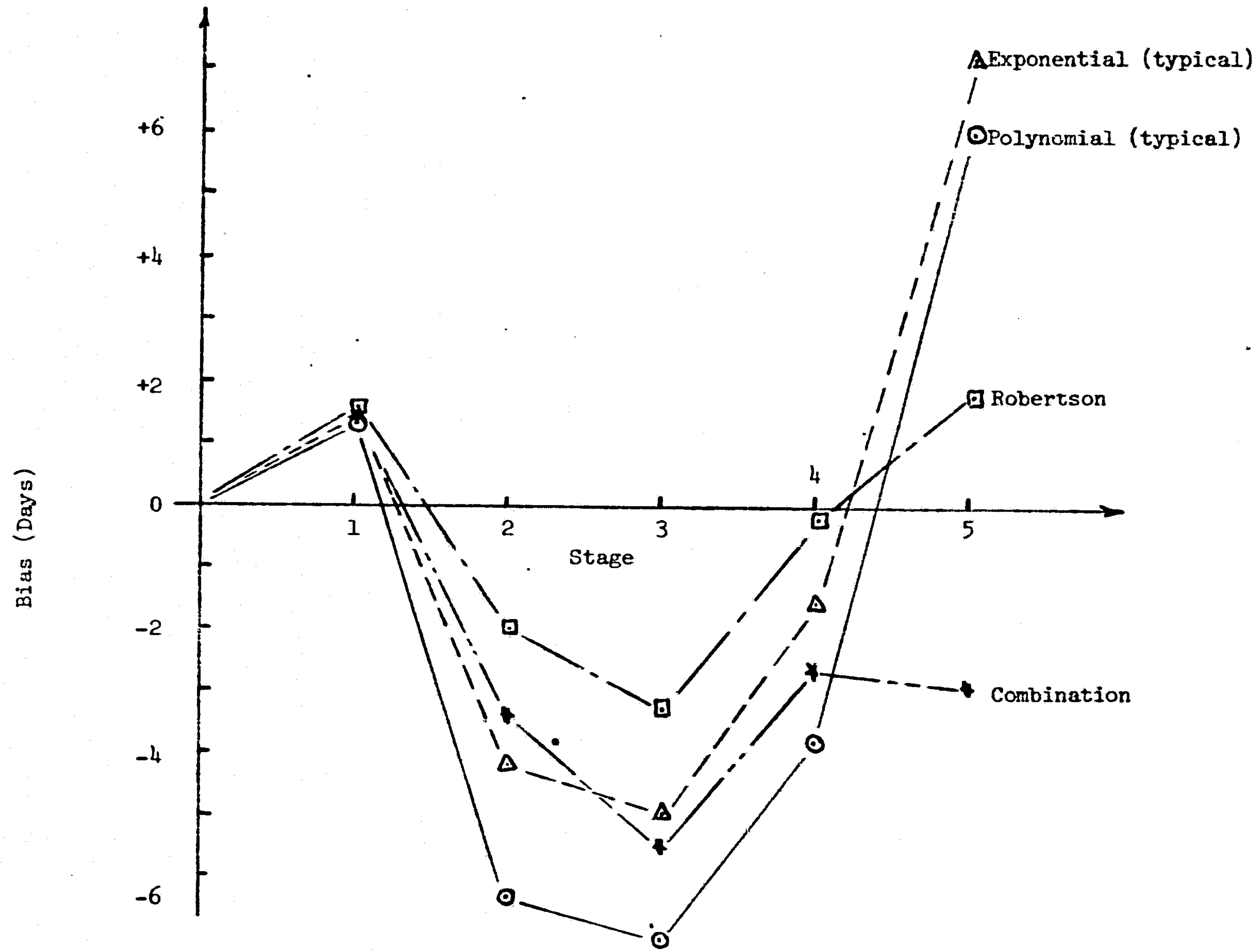

Figure 12
Plot of Bias from Crop Year
Tests for Various Cases

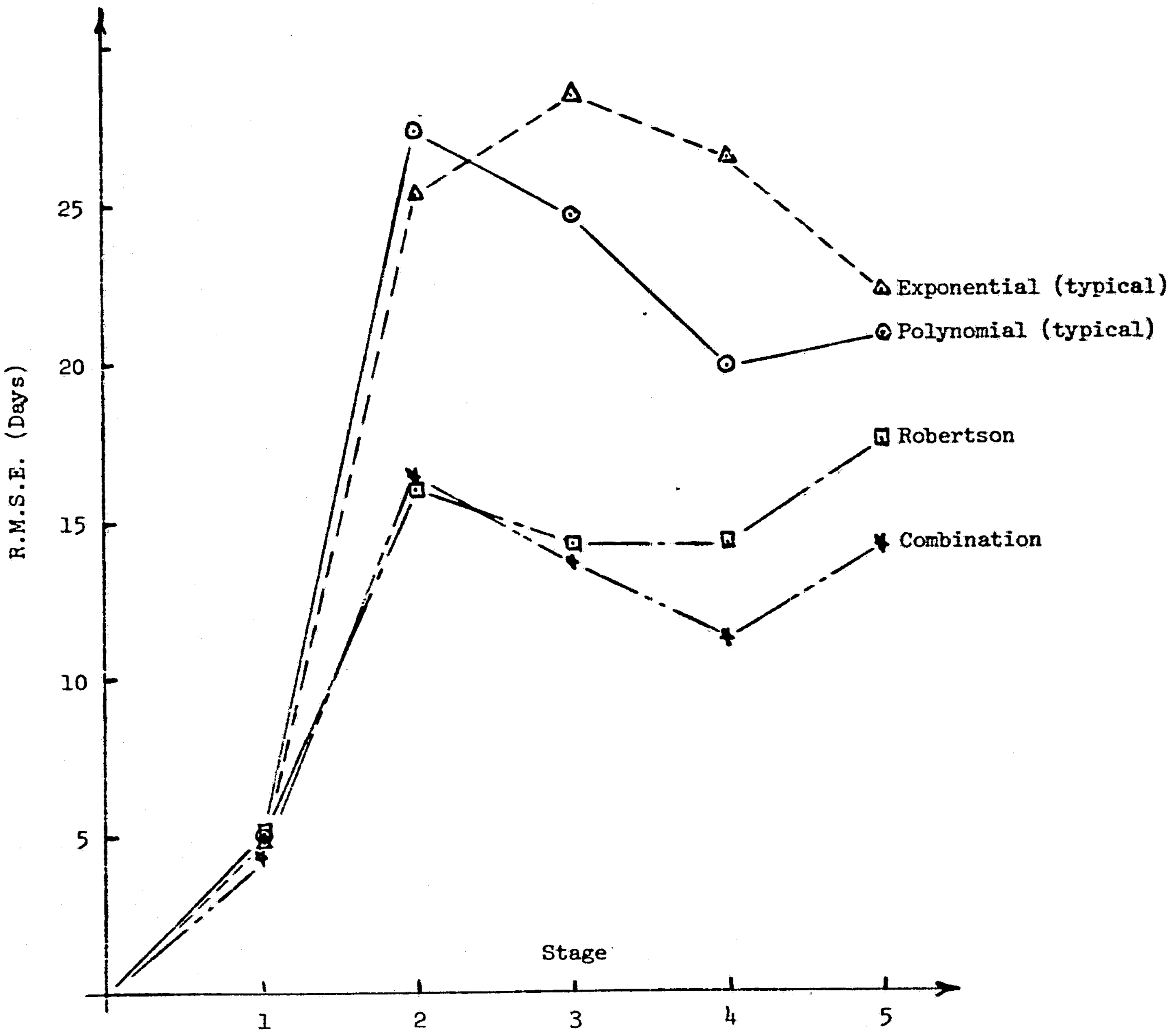

Figure 13
Plot of Variance from Crop Year
Test for Various Cases

In this equation, $\underline{Q}_s$ represents a c x c variance-covariance matrix for the c x 1 vector of functions

$$\underset{c,1}{\underline{s}} = \underset{c,1}{\underline{f}}\ \underset{n,1}{\underline{(y)}} \quad \text{Eq. 37}$$

The n x 1 vector $\underline{y}$ represents the quantities (parameters and independent variables) upon which computation of $\underline{s}$ is based, through the functional relationships $\underline{f}$, and $\underline{Q}_y$ is the n x n variance-covariance matrix of these quantities.

For the problem under consideration

$$\underset{1,1}{\underline{s}} = R = f(\underset{p,1}{\underline{H}}, \underset{q,1}{\underline{X}}) \quad \text{Eq. 38}$$

$$\underset{n,1}{\underline{y}} = \left[\underset{1,p}{\underline{H}^t}\ \underset{1,q}{\underline{X}^t}\right]^t \quad \text{Eq. 39}$$

where n = p + q, $\underline{H}$ is the vector of parameters estimated from least squares, and $\underline{X}$ is the vector of independent weather related variables defined previously.

Then Equation 36 may be applied to growth rate as

$$q_R = \underset{1,19}{\underline{J_{Ry}}}\ \underset{19,19}{\underline{Q_y}}\ \underset{19,1}{\underline{J^t_{Ry}}} \quad \text{Eq. 40}$$

where q_R is the variance of daily growth rate, and

$$\underset{19,19}{\underline{Q_y}} = \begin{bmatrix} \underset{15,15}{\underline{Q_H}} & \underset{15,4}{\underline{0}} \\ \underset{4,15}{\underline{0}} & \underset{4,4}{\underline{Q_X}} \end{bmatrix} . \qquad \text{Eq. 41}$$

The key to application of the relationship is to obtain values for the parameter variance-covariance matrix $\underline{Q}_H$. One of the attributes of the generalized least squares technique used is that propagation is carried out during parameter estimation, and reliable estimates of the parameter variance-covariance matrix result from the least squares estimation process (3,6).

After estimation of the variance of a typical growth rate within a given stage, q_R, other variances may be computed. The number of days within a stage may be approximated by

$$\hat{n} \cong 1/\hat{R} \qquad \text{Eq. 42}$$

in which $\hat{n}$ is the estimated number of days in that stage, and R is a typical growth rate within that stage given by

$$\hat{R} = f(\underline{H}, \hat{\underline{X}}) \qquad \text{Eq. 43}$$

where $\hat{\underline{X}}$ represent typical values of weather related variables within the stage, e.g., average or median values. Using Equation 42 and applying the variance propagation relationship of Equation 36 results in

$$q_{\hat{n}} = \frac{q_R}{\hat{R}^4} \qquad \text{Eq. 44}$$

and a standard deviation of

$$\sigma_{\hat{n}} = \sqrt{q_{\hat{n}}} \,. \qquad \text{Eq. 45}$$

These represent the estimates of variance and standard deviation of the number of days within each stage, which may be compared to those values computed by the interval tests. To estimate the cumulative variance, in days, at the end of any stage j, the relationship

$$\hat{N}_j = \sum_{i=1}^{j} \hat{n}_i \qquad \text{Eq. 46}$$

may be used. Applying the variance propagation relationship of Equation 36 to this function results in

$$q_{\hat{N}_j} = \sum_{i=1}^{j} q_{\hat{n}_i} \qquad \text{Eq. 47}$$

and

$$\sigma_{\hat{N}_j} = \sqrt{q_{\hat{N}_j}} \,. \qquad \text{Eq. 48}$$

Test Results

The above relationships were programmed and run on the computer. Variance-covariance matrices for parameters were taken directly from the least squares program. The interval test data set was used to compute typical weather-related variables, in this case averages, as well as variances for and correlations between these variables. The results revealed a high correlation between the T_X, T_M, and D_L variables, and these correlations should not be neglected in further work.

The results of these computations are, for selected cases, shown in Table 26. The figures shown are the variances and standard deviations predicted by variance propagation for the cumulative mismatch in days, which may be compared directly to the results from the crop year test sequence. Figures 14, 15, 16, and 17 show these comparisons, in graphical form, for the polynomial, exponential, Robertson, and combination models.

Table 26
Cumulative Variances and Standard Deviations
Predicted by Variance Propagation for Selected Cases

FUNCTION	Stage 0-1		Stage 1-2		Stage 2-3		Stage 3-4		Stage 4-5	
	Var (Days^2)	Std. Dev. (Days)	Var (Days^2)	Std. Dev. (Days)	Var (Days^2)	Std. Dev. (Days)	Var (Days^2)	Std. Dev. (Days)	Var (Days^2)	Std. Dev. (Days)
Polynomial										
Const. Only	7.34	2.71	125.66	11.21	125.66	11.21	125.89	11.22	125.95	11.22
Lin T_X Only	4.71	2.17	660.49	25.70	662.03	25.73	662.03	25.73	666.16	25.81
Lin T_M Only	3.67	1.92	712.89	26.70	716.63	26.77	741.47	27.23	742.02	27 24
T_X, D_L & $T_X D_L$	4.71	2.17	449.86	21.21	501.31	22.39	501.31	22.39	503.55	22.44
T_M, $D_{.}$ & $T_M D_L$	3.67	1.92	454.54	21.32	468.29	21.64	468.72	21.65	470.02	21.68
Lin T_X, T_M, D_L	3.87	1.97	506.25	22.85	523.49	22.88	523.95	22.89	529.92	23.02
Lin T_X, T_M, D_L, P_r	7.89	2.81	924.77	30.41	974.06	31.21	979.69	31.30	1000.46	31.63
Lin & Sq w/o P_r	8.05	2.84	338.56	18.40	349.69	18.70	352.31	18.77	355.70	18.86
Lin & Sq w/ P_r	13.66	3.69	1730.56	41.60	2043.04	45.20	2055.72	45.34	2058.44	45.37

Table 26
(Continued)

FUNCTION	Var (Days^2)	Std. Dev. (Days)	Var (Days^2)	Std. Dev. (Days)	Var (Days^2)	Std. Dev. (Days)	Var (Days^2)	Std. Dev. (Days)	Var (Days^2)	Std. Dev. (Days)
<u>Exponential</u>										
Const. Only	7.35	2.71	126.11	11.23	126.34	11.24	126.56	11.25	126.70	11.25
Lin T_X Only	5.30	2.30	1073.87	32.77	1075.18	32.79	1076.00	32.79	1079.78	32.86
Lin T_M Only	3.69	1.92	1086.36	32.96	1090.98	33.03	1091.64	33.04	1092.39	33.05
T_X, D_L & T_XD_L	5.30	2.30	3237.61	56.90	3242.16	56.94	324.44	56.96	3249.01	57.00
T_M, D_L & T_MD_L	3.69	1.92	3099.15	55.67	3103.60	55.71	3209.22	56.65	3210.36	56.66
Lin T_X, T_M, D_L	3.68	1.92	921.12	30.35	922.94	30.38	923.02	30.38	929.64	30.49
Lin T_X, T_M, D_L, P_r	5.86	2.42	1357.19	36.84	1395.02	37.35	1400.26	37.42	1405.50	37.49
Lin & Sq w/o P_r	8.58	2.93	1042.64	32.29	1047.17	32.36	1053.65	32.46	1055.60	32.46
Lin & Sq w/ P_r	17.44	4.18	1031.05	32.11	1060.15	32.56	1064.06	32.62	1068.64	32.69
<u>Robertson</u>	35.90	5.99	1046.52	32.35	1048.46	32.38	1050.41	32.41	1069.94	32.71
<u>Combination</u>	3.67	1.92	992.25	31.50	997.30	31.58	1022.08	31.97	1025.28	32.02

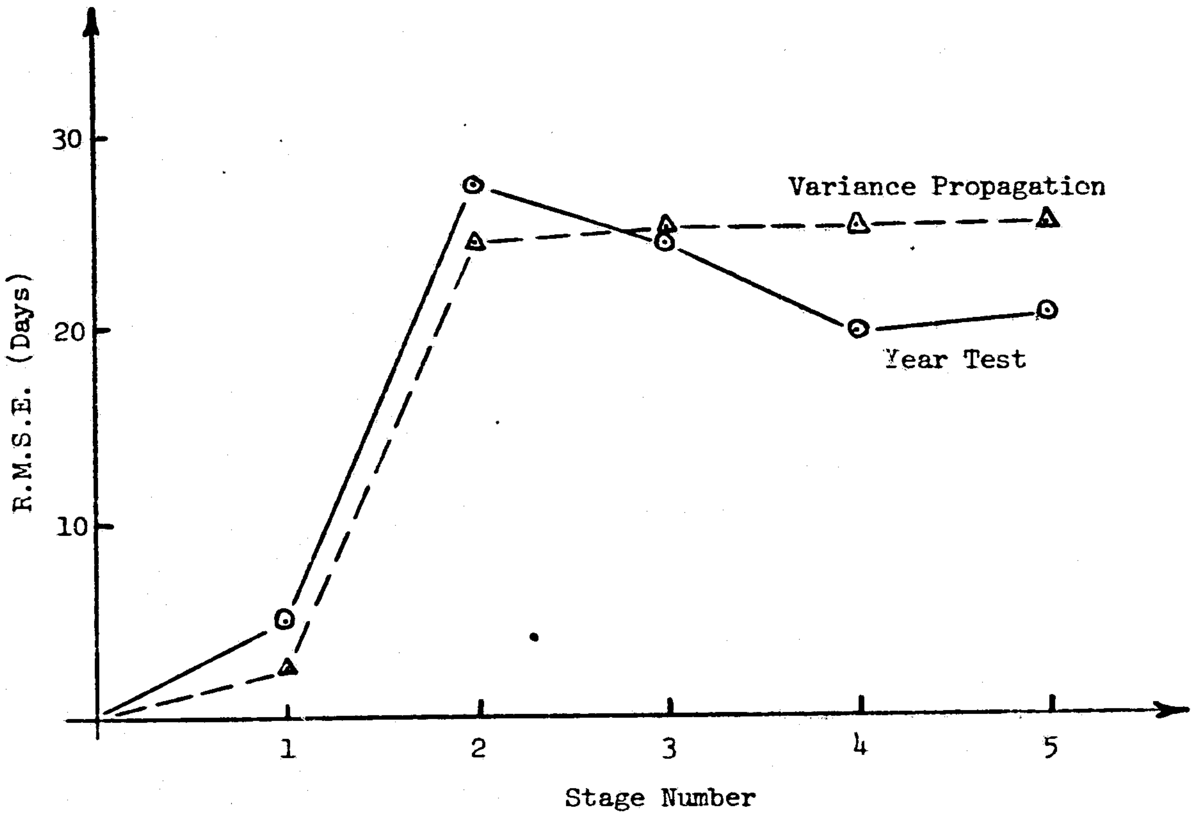

Figure 14
R.M.S.E. from Year Test Compared to those Predicted from Variance Propagation for Typical Polynomial

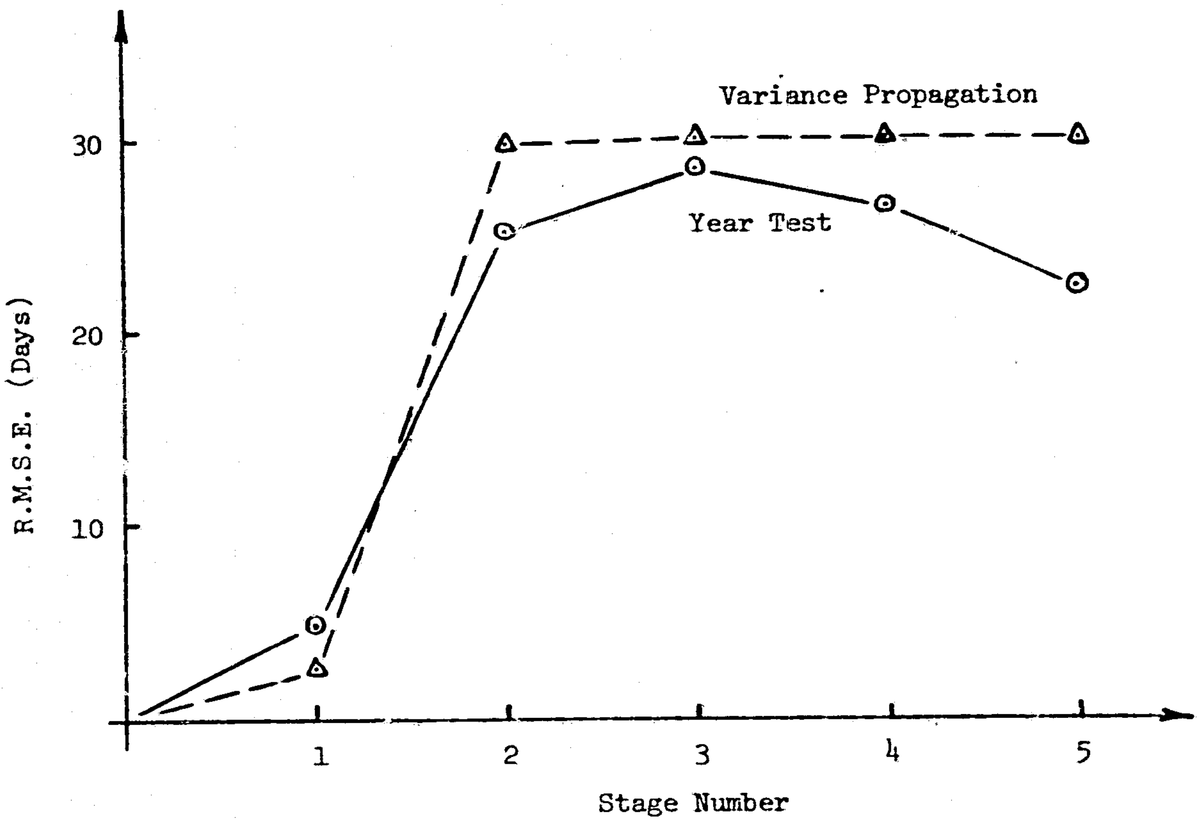

Figure 15
R.M.S.E. from Year Test Compared to those Predicted from Variance Propagation for Typical Exponential

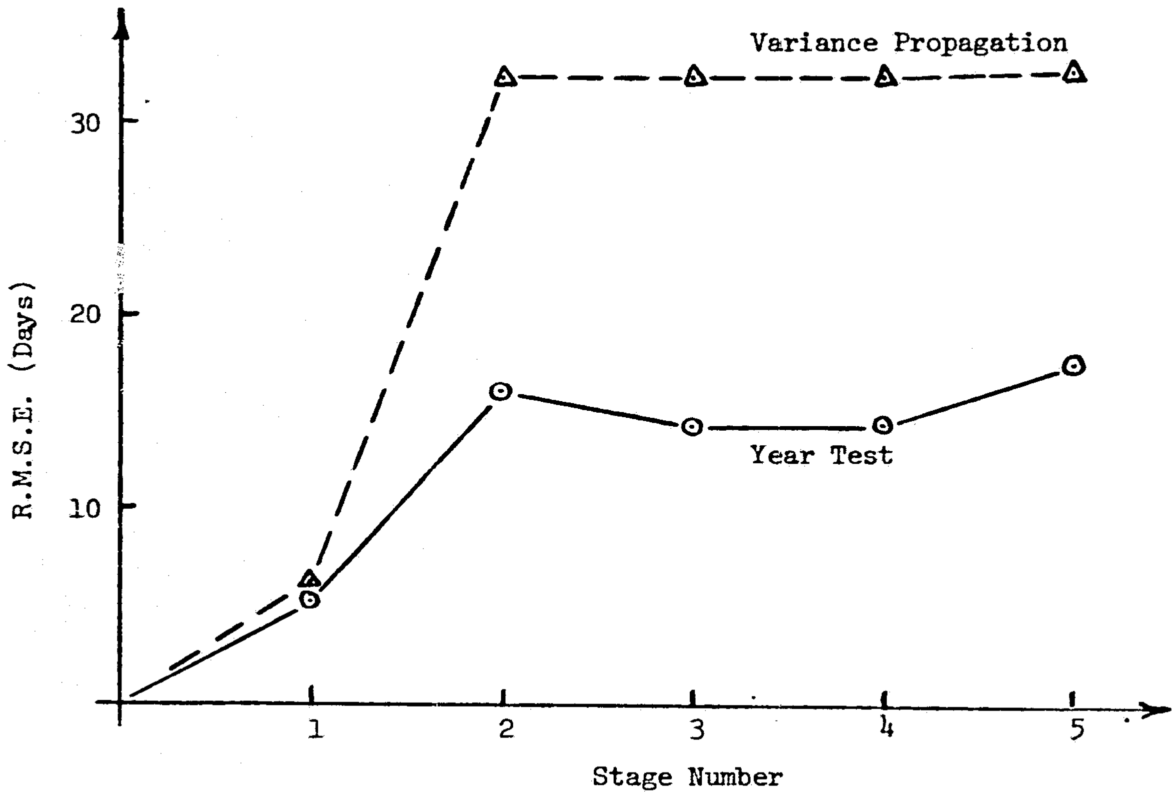

Figure 16
R.M.S.E. from Year Test Compared to those Predicted from Variance Propagation for Robertson Model

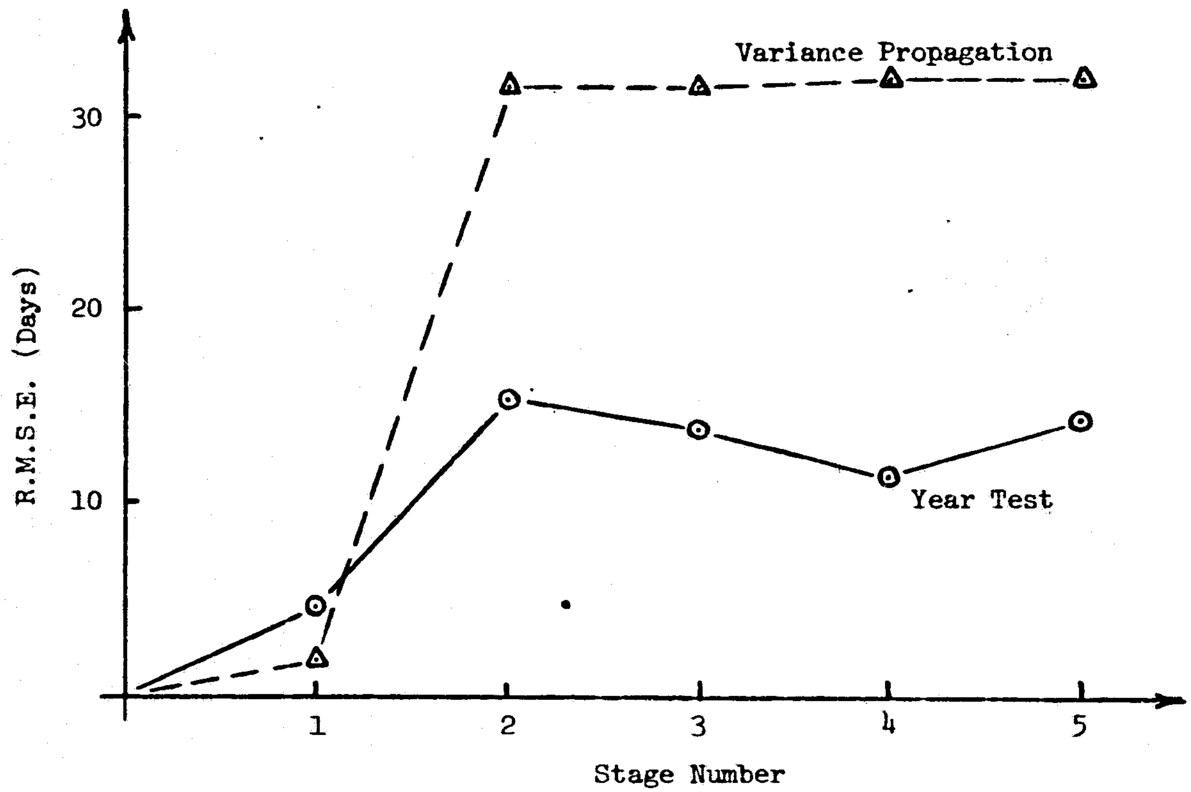

Figure 17
R.M.S.E. from Year Test Compared to Those Predicted from Variance Propagation for Combination Model

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

Winter wheat phenological information was gathered for crop reporting districts located in the Great Plains and Rocky Mountain regions of the United States. Corresponding meteorological data were obtained from the National Climatological Center for these location-years, for values of daily maximum temperature, minimum temperature, and precipitation. Daylength values were computed.

These data, after editing and reduction, were utilized in least squares fitting programs using the method of simultaneous adjustment of observations and parameters (generalized least squares). Parameters were estimated using these techniques for various forms of polynomial, exponential, and the Robertson "triquadratic" functions. The functions resulting from these fits were then tested in a predictive mode by applying them to data sets withheld from the least squares procedures. Variance propagation studies were conducted in order to predict expected variances of growth rates computed from the parameters estimated.

Conclusions

The investigations conducted support the following general conclusions:

1.) The use of phenological reports from crop reporting districts was troublesome. Although use of such information requires no separate data gathering, as it is accomplished within the U.S.D.A. reporting service, the data have several disadvantages. Few states report all phenological stages, and many report other than the "standard" stages (plant, emerge, joint, head, soft dough, ripe). No states, to this investigator's knowledge, report information concerning dormancy or spring greenup.

2.) The data from crop reporting districts are "noisy", and in some instance contained outright mistakes. This caused relatively high variances in parameter estimates, and, in turn, relatively large uncertainties in predicted phenological stages.

3.) Because only end points are available, at best, in the phenological reports, only average growth rates may be computed and utilized for parameter estimation within each stage interval for each location-year. This, in turn, necessitates that the corresponding environmental values be averaged spatially and temporally over the CRD and the stage interval.

4.) For the emerge to joint interval special considerations are necessary. This interval includes the long winter dormancy period, and some criteria must be established in data preparation which reflects this fact. Later testing procedures indicated that the behavior of the resulting models were relatively insensitive to minor variations in the dormancy assumptions made.

5.) The generalized least squares modeling techniques, combined with numerical methods for linearization and function evaluation, appear to provide a powerful and flexible tool for parameter estimation within a variety of function types. While some difficulties were encountered in obtaining convergence for models with higher order terms, due to the approximations involved, the problems were relatively minor, and are certainly reconcilable. Fairly good parameter approximations are necessary in using the model. Further, the method lacks efficiency in testing of variance for inclusion of terms when compared with stepwise regression.

However, the flexibility of the technique, including its ability to accommodate nonlinear models, and configurations within each model is a great advantage. The fact that the method recognizes and includes computationally the statistical variability within all observed quantities, and minimizes with respect to all these, is felt to result in more realistic parameter estimates than those resulting from regression analysis;

6.) In virtually no instance was the inclusion of precipitation within the model found to be significant.

7.) Tests of functions on independent data revealed that the more highly nonlinear functional forms produced very erratic results when driven with daily environmental variables. This was not unexpected, since parameter estimates were generated using averaged data, and the daily values used for testing often lay outside the range of these averaged values, resulting in greatly magnified results for those points.

8.) Attempts to alleviate the difficulties of 7.) above through the use of running and accumulative averages was only partially successful.

9.) Variance propagation techniques appear to be a useful device for predicting expected uncertainties. In this study, these techniques were used to verify that the uncertainties noted during testing were reasonable for the primary data set used to generate and test parameters.

Recommendations

The following recommendations are made based upon the investigations conducted.

1.) The best combination of functions tested was the following:

<u>Stage 0-1</u>: $R = 0.87870 - 0.00045898\ T_M$

<u>Stage 1-2</u>: $R = 0.071784\ (D_L - 0.2796) \cdot \{(4.6869 \times 10^{-4})(T_X + 2.3876) - (4.6618 \times 10^{-6})(T_X + 2.3876)^2 + (2.3943 \times 10^{-4})(T_M + 2.3876) + (1.7055 \times 10^{-5})(T_M + 2.3876)^2\}$

<u>Stage 2-3</u>: $R = \{\exp -6.22319 + 0.12008\ T_X + 1.0337(D_L - 12) - 0.038591\ T_X(D_L - 12)\}$

<u>Stage 3-4</u>: $R = 0.37980 + 0.0061145\ T_M$

Stage 4-5: $R = \exp\{-17.6837 + 0.53654\ T_X + 0.33366\ T_M + 2.6105(D_L - 12) - 0.0080340\ T_X^2 - 0.011335\ T_M^2 - 0.45441(D_L - 12)^2\}$.

2.) Future attempts at parameter estimation should be based upon better, less "noisy" data, particularly as concerns phenological reports. Current efforts at JSC to collect data bases formulated from intensive test sites and blind sites represent a desirable first step in this direction.

3.) The generalized least squares technique is recommended for parameter estimation of any sort, rather than the more standard linear regression techniques. The method can accomodate both linear and nonlinear model forms, and provides useful variance information concerning parameter estimates.

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