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Investigation of Tidal Displacements of the Earth's Surface by Laser Ranging to GEOS-3

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FIGURE CAPTIONS

Figure 1. Theoretical tidal displacements at Goddard, MD. Crosses indicate times of passes observed by Goddard laser during a three day period.

Figure 2. Theoretical differential tidal displacements between Goddard and Grand Turk Island. Crosses indicate the times of passes observed simultaneously by the Goddard and Grand Turk lasers.

Figure 3. Goddard apparent station movements in height (h), latitude (ϕ) and longitude (λ) for four 24-hour arcs and one 36-hour arc (identified by their 1975 day numbers). The reference arcs were generated using the GEM 8 geopotential model and corresponding force model parameters of Table 1. The standard error bars denote the uncertainty corresponding to the random components of the laser range residuals.

Figure 4. Goddard apparent station movements in height only for four 24-hour arcs and one 36-hour arc. The reference arcs are the same as those used for the results of Figure 3. The standard error bars denote the uncertainty corresponding to the laser range residuals after adjustment of station position. Errors of a fraction of a meter are denoted as one meter on the diagram. The character of the connecting lines identifies the 1975 day number by comparison with Figure 3.

Figure 5. Goddard apparent station movements in height only for five 24-hour arcs. The reference arcs were generated using the PGS 558 geopotential model and corresponding force model parameters of Table 1. All standard errors were less than the 15 cm denoted on the diagram. The character of the connecting lines identifies the 1975 day numbers.

Figure 6. Comparison of tenfold theoretical vertical movements (upper panel) with vertical movements recovered by the dynamic method (lower panel). All standard errors less than 0.10 m. The character of the connecting lines identifies the 1975 day number.

Figure 7. Coordinate system for proof of extremum method of station-plane identification. Ranging is done from stations A_1 and A_2 in the station-plane to the satellite at position σ. The origin is taken at the earth's center of mass.

INTRODUCTION

The ultimate objective of this investigation is the measurement of the tidal displacements of the solid earth by laser ranging to the GEOS-3 satellite. Confirmation of earth tide theory through surface measurements of gravity, tilt and strain has been difficult because of the perturbing influences of surface discontinuities, the poor distribution of stations and the lack of ocean tide information. A measurement of surface displacement by laser ranging, although not entirely immune from such effects, constitutes a more direct measurement of the tidal deformations and of the related Love numbers h_n and l_n. The accuracy of laser ranging to satellites has now reached a level of between 5 and 10 cm (Vonbun, 1977) and continuing improvements in the dynamic models for satellites and/or the distribution of laser stations should ultimately lead to the detection and measurement of the 30 to 40 cm geometric earth tide.

The present investigation is restricted to the analysis of NASA laser ranging data from three stations at Goddard Space Flight Center, Greenbelt, Maryland, Grand Turk Island, and Bermuda in the GEOS-3 "calibration area." Therefore, the necessary conditions for a purely geometric solution for relative station positions are not fulfilled (Escobal *et al.*, 1973) and the determinations of station positions will depend to some extent on the accuracy of the model for the path of the satellite. Results by Smith *et al.* (1973) for the Beacon-C satellite and a single laser station have shown that the fit of an orbit to a series of satellite passes rarely equals the quality of the laser data. Trends could be seen in the residuals showing departures of 2 or 3 meters from the predicted orbit. Errors in the gravity field, station position or other aspects of the dynamic model were suspected. Our approach has been to determine the effects of errors in the predicted orbit on the measurement of station movements and to investigate a method designed to minimize the effect of those errors.

EXPECTED TIDAL DISPLACEMENTS

In general, the measured laser station-to-station distances are affected both by the tidal displacements of the earth's surface and by the direct effect of the tidal potential on the motion of the satellite. The influence on the orbit of the Beacon-C Satellite, for instance, by the solid-earth and ocean tides (characterized by the Love number k_2 and a phase lag ϕ) were found by Smith, et al (1973) from an analysis of the perturbations in the inclination of the orbit. A subsequent fit to the laser range data, with this tidal effect included, (using k 0.245 and $\phi = 3\overset{\circ}{.}2$ derived in the previous study) showed that the inferred mean heights of the laser station from 12-hour arcs were not significantly affected. We are assuming in the present study, therefore, that any small errors in k_2 and ϕ in the implementation of GEODYNE will be a second-order effect on the relative laser station-to-satellite distances for 24-hour arcs.

The theoretical vertical and horizontal displacements of the laser stations in the calibration area due to the solid-earth tide were computed using subroutines NOMAN 1 and, with a small modification, NOMAN 2 (Harrison, 1971). The geometric earth tide in the vicinity of Goddard has a theoretical peak-to-peak amplitude of about 40 cm in the radial direction and less than 5 cm in the tangential direction (Figure 1), whereas the theoretical peak-to-peak amplitude of differential displacements between Goddard and Grand Turk, for example, is 10 to 15 cm in the radial direction and less than 4 cm in the tangential direction (Figure 2). To a first approximation the earth tide at a laser tracking station can be considered constant over the few minutes of a satellite pass. Ideally, the time sequence of satellite passes depends only on the orbital period of the satellite, the rate of rotation of the earth and the latitude of the tracking station. In practice, problems with laser ranging equipment and the weather reduce the number of usable passes.

EXPERIMENTAL RESULTS

Two different approaches to the problem of measuring tidal movements of laser tracking stations were investigated. One approach, termed "the

Figure 1. Theoretical tidal displacements at Goddard, MD. Crosses indicate times of passes observed by Goddard Laser during a three day period.

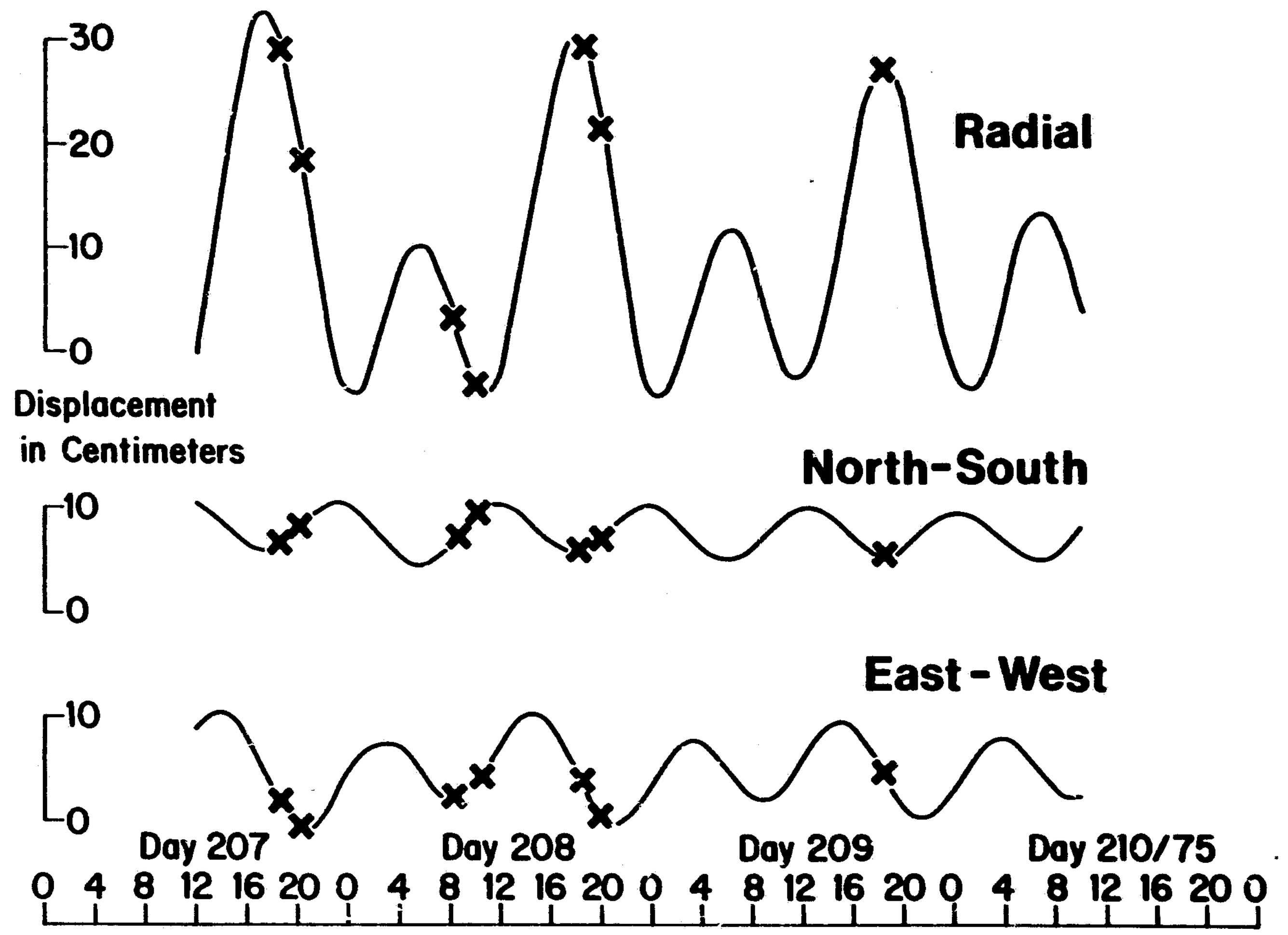
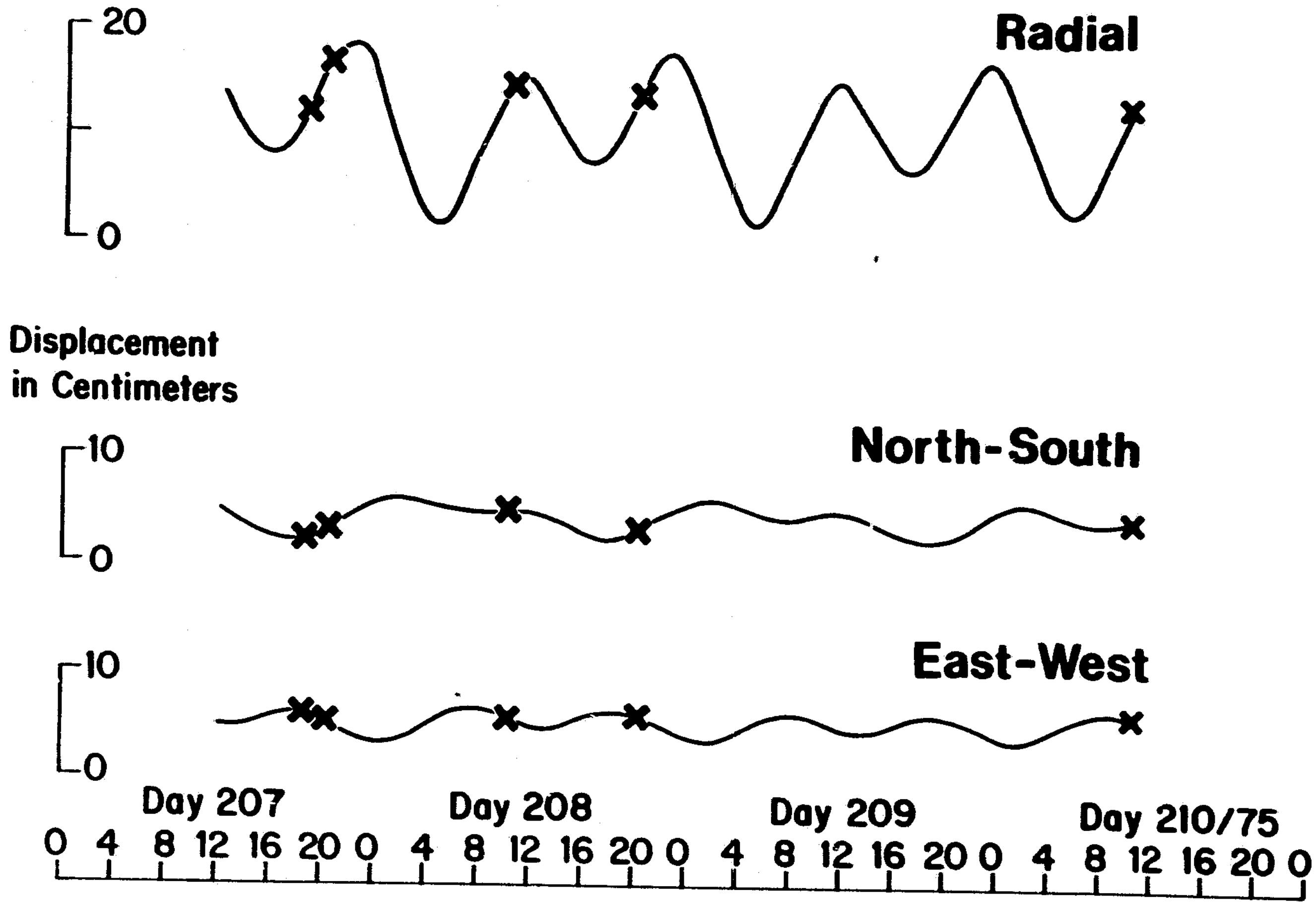


Figure 2. Theoretical differential tidal displacements between Goddard and Grand Turk Island. Crosses indicate the times of passes observed simultaneously by the Goddard and Grand Turk lasers.



dynamic method," employs 24-hour arcs as references for determining pass-to-pass changes in apparent station position. By this method the apparent station movements due to errors in the predicted satellite track as well as the tides have been investigated by an analysis of single-station ranging to GEOS-3. A second approach, termed the quasi-geometric method, attempts to minimize the effects of unmodelled satellite dynamics on the determination of tidal displacements by considering two-station simultaneous ranging to GEOS-3 at the precise time that the satellite passes through the plane defined by the two stations and the center of mass of the earth. This approach takes advantage of the geometrical constraints imposed by two-station ranging and reduces the dependence on satellite dynamics to the prediction of only the distance R_0 from the earth's center of mass to the satellite.

1. Dynamic method

Description - This method employs 24-hour arcs fitted to laser ranging data using the dynamic model incorporated in the NASA program GEODYNE (Martin and Serelis, 1975). 24-hour arcs were chosen for the investigation because they were not inordinately expensive to compute, yet they are long enough to allow the tracking station to sample one complete tidal cycle. Each 24-hour arc comprises 14 or 15 revolutions of the satellite but only four or five passes of ranging data.

Calcuations were carried out using force-model parameters and station coordinates supplied by NASA. Table 1 lists the two sets of parameters corresponding to the two geopotential models, GEM8 (Wagner et al., 1976) and PGS558 (D. Smith, personal communication, 1977 and Lerch et al., 1977) used during this investigation. In our implementation of GEODYNE, fitting an arc to data from four or five satellite passes corresponds to solving for a set of six orbital parameters at a particular epoch and for a particular drag coefficient. Only the direct tidal perturbation at the GEOS-3 orbit is modelled in GEODYNE and not the tidal displacement of the tracking station.

A least-squares iterative procedure was employed to compute the apparent position of the station with respect to the fitted arc from the laser ranging data taken over each single satellite pass. Two different methods were adopted:

TABLE 1

SUMMARY OF FORCE MODEL PARAMETERS AND STATION COORDINATES USED IN GEODYNE

	GEM 8	PGS 558
A. Earth Gravitational Potential Coefficients (Coefficients through degree 30 and order 28)		
B. Gravitational constant, G (meter **3/ seconds **2)	3.98601300D+14	3.98600800D+14
C. Other perturbations:		
1. Lunar gravitation applied - ratio of lunar mass to earth mass	1.229997D-02	1.229997D-02
2. Solar gravitation applied - ratio of solar mass to Earth mass	3.329456D+05	3.329456D+05
3. Gravitation applied for other planets	NONE	NONE
4. Earth tides applied - lunar and solar effects included		
k2 amplitude	0.29	0.29
k2 phase angle	2.50°	2.50°
k3 amplitude	0.0	0.0
5. Drag applied (D65 JACCHIA 1965 static atmospheric density model used)		
Drag coefficient	2.3	Adjusted
6. Solar radiation pressure applied		
- 1 AU solar radiation pressure (Newtons/meter **2)	4.500D-06	4.500D-06
- Reflectivity	1.500	1.500
- Satellite Cross Sectional Area (Meters **2)	1.437	1.437
- Satellite Mass (kilograms)	3.459D+02	3.459D+02
D. Goddard station position data (station 7063)		
Coordinates - Spheroid height (meters)	9.2900	17.241
North latitude	29°1'13".8800	39°1'13".3507
East longitude	283°10'18".5000	283°10'19".7500

TABLE 1 (Cont.)

E. Bermuda station position data (station 7067)		
- Spheroid height (meters)		-24.091
North latitude		32°21'13"7636
East longitude		295°20'37"8585
F. Grand Turk station position data (station 7068)		
- Spheroid height (meters)		-19.730
North latitude		21°27'37"7762
East longitude		288°52'4"9584
Earth ellipsoid - semi major axis (meters)	6378155.00	6378145.00
Flattening	1./298.255	1./298.255

(1) The station was allowed to move in all three coordinates by solving for incremental adjustments Δx, Δy, Δz from an approximate position x, y, z by the system of linear equations:

$$2(x-x_i)\ \Delta x + 2(y-y_i)\ \Delta y + 2(z-z_i)\ \Delta z = S^2_i - P^2_i$$

where S_i is the i^{th} observed range to the satellite, P_i is the i^{th} predicted range and x_i, y_i, z_i are the predicted earth fixed satellite coordinates,

(2) The station was constrained to move in only the radial (height) direction by solving for incremental adjustments Δx, Δz by the system of linear equations:

$$2(x-x_i)\ \Delta x + 2(z-z_i)\ \Delta z = S_i^{\ 2} - P_i^{\ 2}$$

and constraint $(z/r)\Delta x - (x/r)\Delta z = 0$ where r is the radius to the station.

These methods converge to less than 1 mm in two iterations when the initial position is less than 50 m in error.

Results - During the first year of laser ranging to GEOS-3 there were a number of 24-hour periods during which several passes of the satellite were observed by the Goddard laser. Four 24-hour arcs and one 36-hour arc fitted to Goddard ranging data only were used as references for computing apparent station positions for single passes of the satellite. GEODYNE calculations using the GEM8 geopotential model, appropriate force model parameters and coordinates (Table 1) gave, during a pass, laser ranging residuals, having a small random component of amplitude about 5 cm, plus a systematic component departing 2 to 5 m from the arc. We believe that the random component indicates the precision of the laser ranging and the systematic component represents the inability of the computed satellite track to fit the laser data. The net R.M.S. residuals for the five arcs ranged from 0.45 m to 1.96 m.

Apparent station movements for Goddard were computed for all satellite passes of the five GEODYNE arcs according to the methods described in the previous section. Method 1, where three-dimensional station position adjustments are allowed for each pass, gave R.M.S. variations in Goddard station position from pass to pass of about 8 m in height and about 11 m in latitude and longitude (Figure 3). In this method the systematic components of the laser range residuals were completely absorbed into apparent station movements, leaving only a random component. The amplitude of the apparent movements depends on the amplitude of the systematic component of range residuals for the pass, on the geometry of the satellite path with respect to the laser station, and on the duration of tracking for each pass. In general, the single-pass station position is poorly determined in a direction normal to the surface containing the satellite path and the station. The resulting large apparent movements in that direction make three-dimensional station position determinations unreliable for the purposes of the present experiment. Method 2, where only changes in station height are allowed, gave significantly smaller apparent movements (Figure 4). Except for one short pass at a low elevation in arc D216/217, the four 24-hour arcs gave R.M.S. variations around 1 to 2 m in height. An offset in station height of about 15 m is seen, however, for part of the 36-hour arc. This method does not absorb the systematic components of the range residuals into station movements, yet the estimated standard errors on the apparent movements are, in general, less than 1 m.

A further decrease in apparent station height movement was achieved by Method 2 by an improvement in the GEODYNE force model. Using geopotential model PGS 558 and appropriate station coordinates (Table 1), the random component remained the same but the overall R.M.S. residuals were reduced to between 0.14 m and 0.65 m for the five arcs (the 36-hour arc D207/208 was reduced to a 24-hour arc). Figure 5 shows the resulting apparent station movements for the five arcs, plotted on an expanded vertical scale. The R.M.S. variation in station height for the results of Figure 5 is 0.80 m and the corresponding value for the theoretical tidal movements is 0.11 m. The detection of vertical tidal movements by this method clearly requires further improvements in the dynamic model for the satellite. However, the stability in station height is now good enough to allow the suitability of 24-hour length

Figure 3. Goddard apparent station movements in height (h), latitude (ϕ), and longitude (λ) for four 24-hour arcs and one 36-hour arc (identified by their 1975 day numbers). The reference arcs were generated using the GEM 8 geopotential model and corresponding force model parameters of Table 1. The standard error bars denote the uncertainty corresponding to the random component of the laser range residuals.

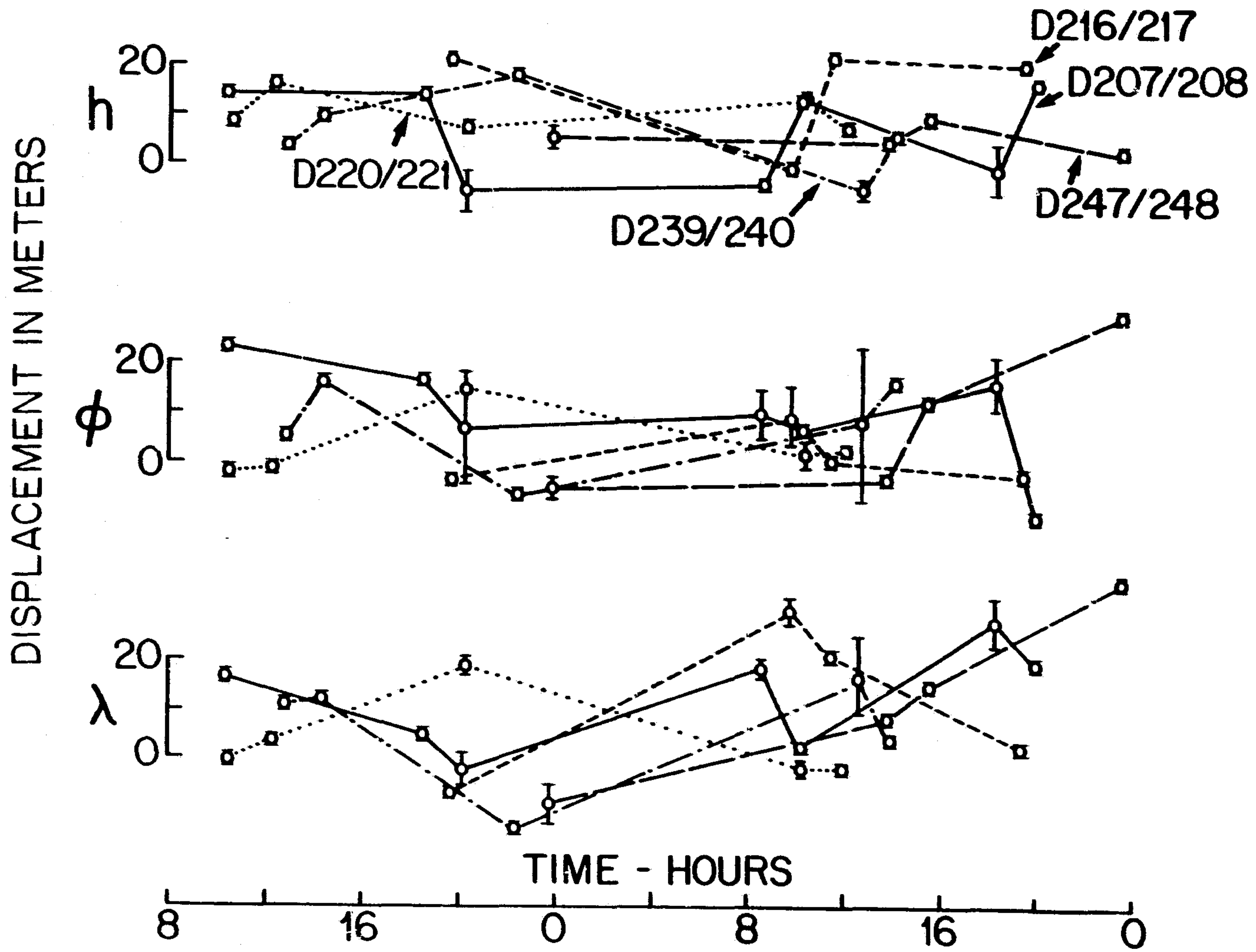


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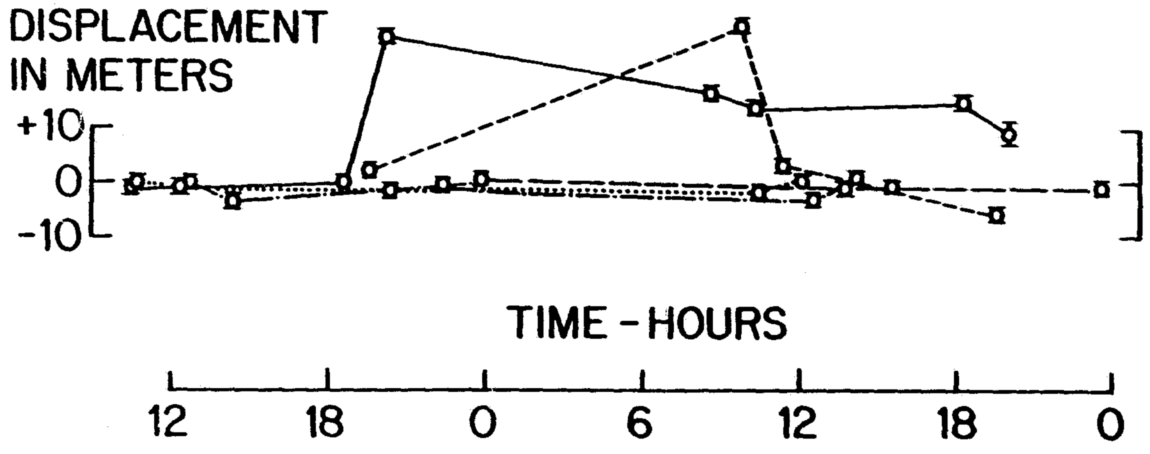
Figure 4. Goddard apparent station movements in height only for four 24-hour arcs and one 36-hour arc. The reference arcs are the same as those used for the results of Figure 3. The standard error bars denote the uncertainty corresponding to the laser range residuals after adjustment of station position. Errors of a fraction of a meter are denoted as one meter on the diagram. The character of the connecting lines identifies the 1975 day number by comparison with Figure 4.



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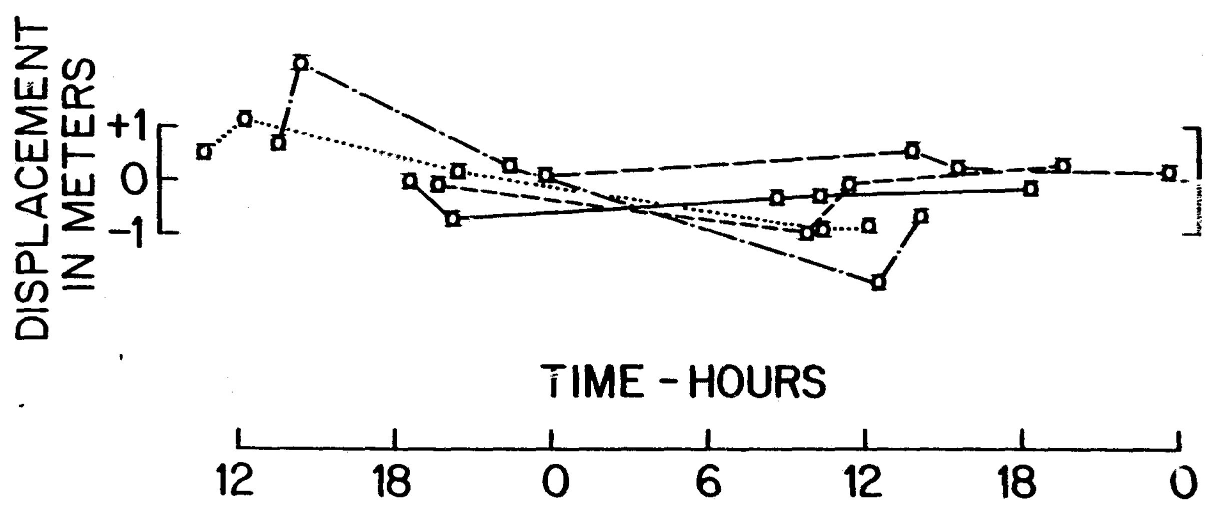
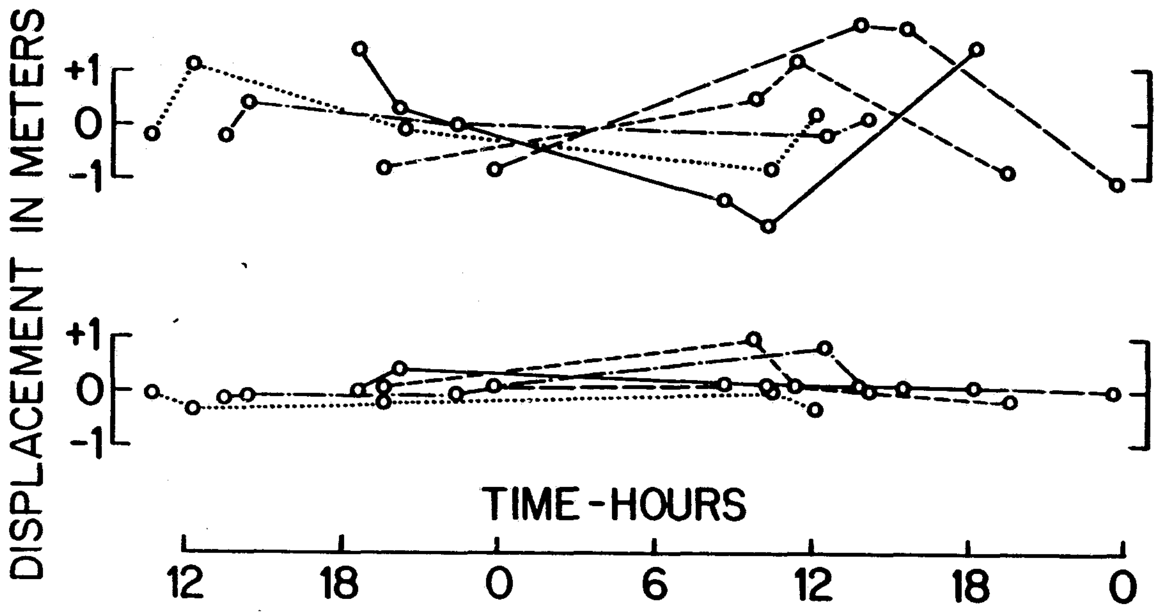


arcs to be tested. In order to investigate the tendency of 24-hour arcs to absorb the real tidal movements of the laser station, tenfold-amplified theoretical tidal movements in height were introduced into the laser ranging data before computation of the GEODYNE reference arcs. Figure 6 compares the induced height variations with those recovered by the method. Although movements up to 1 m are seen, they do not appear to be correlated with the input tides. It must be concluded that 24-hour arcs are able to absorb the geometrical tidal movements of a single tracking station and that they are therefore not suitable as reference arcs for measurement of the geometric tides by the present method. The R.M.S. amplitude of adjustments in the orbital elements and the drag coefficient necessary to absorb the theoretical tides were found to be as follows: semi-major axis, 0.09 cm; eccentricity, 0.019×10^{-6}; inclination, 4.7 milliarcseconds; right ascension of the node, 4.9 milliarcseconds; argument of perigee, 0.87 arcseconds; mean anomaly, 0.87 arcseconds; and drag coefficient, 0.28. These adjustments are equivalent to a movement of the satellite orbit in space of the order of 20 cm in both the radial and tangential directions. The changes in orbital elements are smaller by two to three orders of magnitude than the variations in orbital elements of 6-hour, four pass, arcs for the Beacon-C satellite due to direct tidal perturbations on the satellite orbit (Smith et al., 1973). Future attempts to measure the geometric tide by the method outlined will require the selection of longer reference arcs that are incapable of absorbing the tidal movements of the tracking stations.

2. Quasi-Geometric Method

Description - This is a method for determining the radial distance from the earth's center of mass to each of two laser stations which are simultaneously ranging on a satellite. The radial distance to the satellite is assumed to have been determined independently by some other means. We use the term quasi-geometric because only the satellite radial distance must be known and this only for a few seconds as it crosses the plane defined by the laser stations and the center of the earth. This is normally the best predicted satellite coordinate and has the smallest rate of change; typical rates for GEOS-3 are 5 - 10 meters per second. The method is based on a concept

Figure 6. Comparison of tenfold theoretical vertical movements (upper panel) with vertical movements recovered by the dynamic method (lower panel). All standard errors less than 0.10 m. The character of the connecting lines identifies the 1975 day number.



suggested by Vaniček (private communication, 1975) and Nesbö (unpublished manuscript, 1975) for measuring the differential earth tide (Bower, 1976) between two simultaneously-ranging stations.

With reference to Figure 7 (see Appendix) it can be shown that at the instant that the satellite crosses the station plane ($g = 0$) there is a non-linear relation involving only the parameters R_m (radius of mean station height), 2ω (angular separation of stations), $2h$ (differential radius of stations), R_0 (radial distance to satellite), and S_1, S_2 (the range distances from stations to the satellite) and not involving the parameters of satellite position x and g. By the cosine rule the laser ranges S_1 and S_2 from stations 1 and 2 to the satellite will be given by the equations:

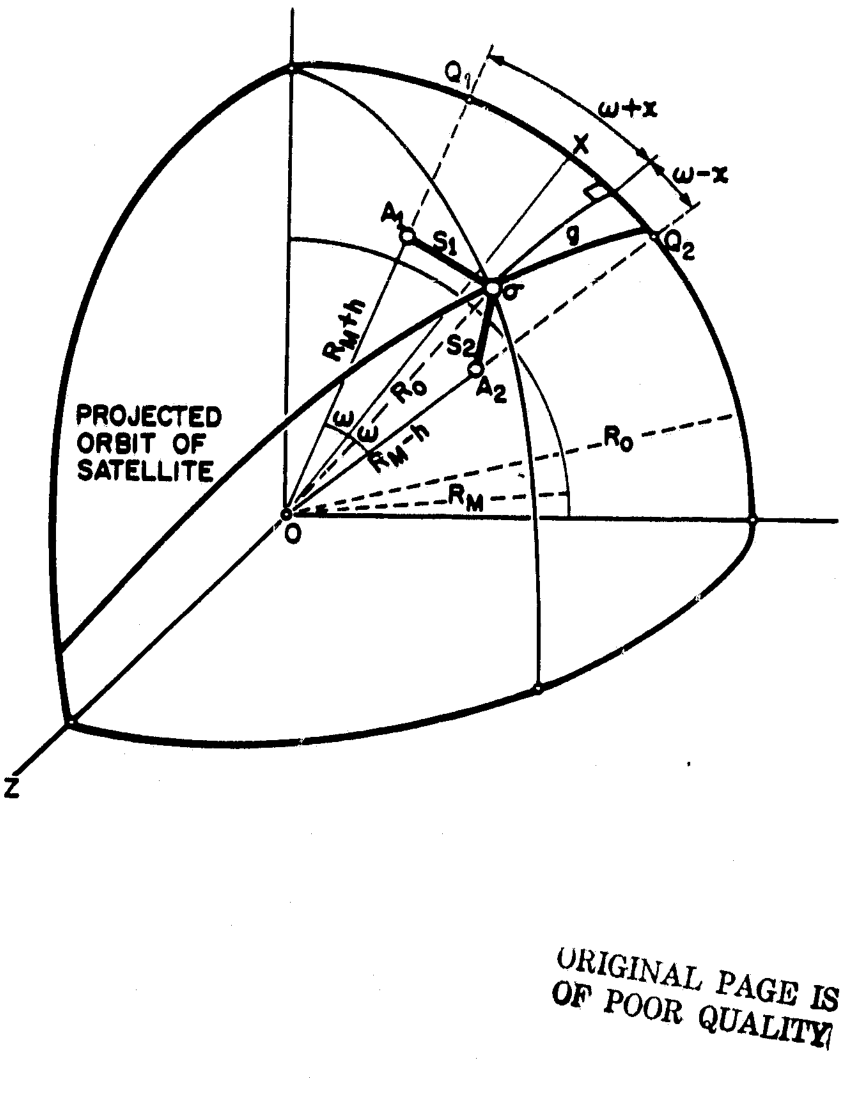
$$\begin{aligned} S_1^2 &= R_0^2 + (R_m + h)^2 - 2R_0 (R_m + h) \cos g \operatorname{Cos} (\omega + x) \\ S_2^2 &= R_0^2 + (R_m - h)^2 - 2R_0 (R_m - h) \cos g \operatorname{Cos} (\omega - x) \end{aligned} \tag{1}$$

Setting $g = 0$ and eliminating x from these equations, we obtain the following nonlinear relation in R_m, ω, h, R_0, S_1, and S_2:

$$\frac{\sin^2\omega \left[R_m\{2(R_0^2 + R_m^2 - h^2) - S_1^2 - S_2^2\} + h(S_1^2 - S_2^2)\right] + 16 R_0^2 (R_m^2 - h^2)^2}{\sin^2\omega \cos^2\omega + \cos^2\omega \left[h\{2(R_0^2 - R_m^2 + h^2) - S_1^2 - S_2^2\} + R_m(S_1^2 - S_2^2)\right]^2} = 0 \tag{2}$$

The instant that the satellite crosses the station-plane, where relation (2) holds, can be found either from a single pass fit to the range data by the program GEODYNE or by an extremum method employing the parameter h. By the extremum method, relation (2), although it only holds true at $g = 0$, can be used to generate values of the parameter h for satellite position where $g \neq 0$. It can then be shown (see Appendix) that the instant of station plane crossing ($g = 0$) is identified by the point when the computed h values reach an extremum.

Figure 7. Coordinate system for proof of extremum method of station-plane identification. Ranging is done from stations A_1 and A_2 in the station-plane to the satellite at position σ. The origin is taken at the earth's center of mass.



Before proceeding further we write the nonlinear equation (2) in terms of the non-time varying parts of the station radii, N_1 and N_2, and the tidal Love number h_2. Thus:

$$
\begin{aligned}
R_{1i} &= N_1 + h_2\tau_{1i} \\
R_{2i} &= N_2 + h_2\tau_{2i}
\end{aligned}
\tag{3}
$$

where τ_{1i} and τ_{2i} are the equilibrium tidal displacements of the stations in the radial directions, and:

$$
\begin{aligned}
R_{mi} &= (R_{1i} + R_{2i})/2 = (N_1 + N_2)/2 + h_2(\tau_{1i} + \tau_{2i})/2 \\
h_i &= (R_{1i} - R_{2i})/2 + h_2(\tau_{1i} - \tau_{2i})/2
\end{aligned}
\tag{4}
$$

Substituting expressions (4) into (2), and neglecting tidal variations in ω, we obtain for the i^{th} plane crossing a non-linear equation in R_{0i}, S_{1i}, S_{2i}, N_1, N_2, h_2 and ω which we can represent by:

$$
F_i(R_{oi}, S_{1i}, S_{2i}, N_1, N_2, \omega, h_2) = 0 \tag{5}
$$

Here R_{0i} is the radial distance to the satellite and S_{1i} and S_{2i} are the laser ranges from stations 1 and 2 to the satellite, all at the instant the satellite crosses the plane. Given four or more such plane crossings, separated sufficiently in time so that the coefficients of h_2 are not simply a linear combination of the coefficients of N_1 and N_2, we solve for N_1, N_2, ω, and h_2 by the Newton Raphson method. We make a first estimate of the parameters N_1°, N_2°, ω°, and h_2°, then improved values N_1, N_2, ω, h_2 are found by correcting the initial values by the amounts ΔN_1, ΔN_2, $\Delta\omega$, and Δh_2 obtained by solving the following system of linear equations:

$$
\begin{aligned}
&\frac{\partial F_i}{\partial N_1}(N_1^\circ)\Delta N_1 + \frac{\partial F_i}{\partial N_2}(N_2^\circ)\Delta N_2 + \frac{\partial F_i}{\partial \omega}(\omega^\circ)\Delta\omega + \frac{\partial F_i}{\partial h_2}(h_2^\circ)\Delta h_2 = \\
&\qquad F_i(R_{0i}, S_{1i}, S_{2i}, N^\circ, N_2^\circ, \omega^\circ, h_2^\circ)
\end{aligned}
\tag{6}
$$

$i = 1, 2 \ldots k$ where $k \geq 4$

Using the improved values of the unknown parameters a new set of differentials are calculated and the procedure is repeated until convergence is achieved.

Results - Laser ranging data from stations at Goddard, Grand Turk and Bermuda were examined for the presence of quasi-simultaneous measurements to GEOS-3 during plane crossings in the months of July and August in 1975 and February in 1976. Only five usable crossings were found throughout July and August and none at all in February. There were many other instances of plane crossings but, for these, laser data was not available from both stations.

Details regarding these five passes are listed in Table 2. The time shown is approximately that of the plane crossing and the columns of partial derivatives are with respect to the function F described earlier. The plane is identified by numbers referring to the stations which define the plane, where 1, 2 and 3 refer to Goddard, Grand Turk and Bermuda respectively. The columns headed d_1, d_2 and d_3 are the calculated equilibrium radial displacements in meters at the three stations due to the earth tide (i.e., $h_2 = 1.0$).

About the times of each pass, values of $R_0(t)$ were found from 24-hour arcs calculated by program GEODYNE on the basis of Goddard range measurements only (see Section 1, Dynamic Method). A linear equation of the form of equation (6) but involving the unknowns: ΔN_2, ΔN_3, $\Delta\omega_{13}$, $\Delta\omega_{23}$ and Δh_2, was obtained for each of the five plane crossings. The nominal values assumed for the station coordinates were those given in Table 1. The result of solving the five simultaneous equations is given in Table 3 for four cases, where ΔR_2 and ΔR_3 are the calculated differences between the true radial distances to the laser stations determined here and the nominal distances assumed. Similarly, $\Delta\omega_{13}$ and $\Delta\omega_{23}$ are the differences between calculated and nominal station separations expressed in terms of great circle distance. ΔR_0 represents the mean error in the predicted radial distance to the satellite for the five plane crossings. The result shown as Δh_2 is that value of the Love number h_2 which satisfies the five equations (i.e., $h_2^{\,0} = 0$).

For Solution 1 the function $R_0(t)$ was assumed correct and ΔR_0 was set equal to zero. It is known from seismological evidence however that $h_2 \approx 0.615$.

TABLE 2

DETAILS FOR THE FIVE PLANE CROSSINGS USED

(See text for explanation)

DATE	TIME (UT)	PLANE	$\frac{\partial F}{\partial R_2}$	$\frac{\partial F}{\partial R_3}$	$\frac{\partial F}{\partial R_0}$	$\frac{\partial F}{\partial \omega}$	$\frac{\partial F}{\partial S_1}$	$\frac{\partial F}{\partial S_2}$	d_1	d_2	d_3
D207/75	18:31.88	2-3	-0.094	-1.094	-1.285	-0.370	0.216	1.151	-0.167	-0.163	-0.148
D220/75	10:40.58	1-3	5.231	4.231	9.763	-8.672	5.681	6.375	-0.125	0.080	0.030
D220/75	20:21	2-3	-2.439	-3.439	-6.136	-5.532	3.492	4.178	-0.241	-0.217	-0.152
D239/75	12:42.98	1-3	-0.636	-1.636	-2.590	-1.719	1.133	1.958	0.180	0.266	0.187
D239/75	22:23.02	2-3	-0.373	-1.373	-1.928	-1.185	0.714	1.519	-0.001	0.161	0.061

TABLE 3

CALCULATED CORRECTIONS TO R_2°, R_3°, ω_{13}°, ω_{23}°, and R_0° CALCULATED h_2 AND STANDARD DEVIATIONS FOR FIVE PLANE CROSSINGS. (*denotes an enforced value).

χ	Solution 1	Solution 1A	Solution 2	Solution 2A	σ (χ)	$\frac{\partial\chi}{\partial R_o}$
ΔR_2	14.284	0.937	15.679	1.090	15.335 $\sigma(R_o)$	6.179
ΔR_3	1.148	-2.459	1.069	-2.874	1.185 $\sigma(R_o)$	1.670
$\Delta\omega_{13}$	-0.088	-0.948	0.047	-0.893	1.107 $\sigma(R_o)$	0.398
$\Delta\omega_{23}$	4.656	-0.541	5.026	-0.655	6.962 $\sigma(R_o)$	2.406
Δh_2	3.512	0.721	3.702	0.652	4.984 $\sigma(R_o)$	1.292
ΔR_o	0.0*	2.160*	0.0*	2.361*		

A second source of error in $R_0(t)$ considered was that due to the geometric effect of the earth tide on the height of the Goddard laser. Since this geometric effect was not taken into account in fitting the five orbits, at least part of the geometric tide would be reflected in $R_0(t)$. The geometric tide for Goddard is given by $h_2 d_1$ where d_1 is listed in Table 3. To determine the effect of this on our results the five simultaneous equations were adjusted on the assumption that all of the geometric tide was reflected in $R_0(t)$ and the system of equations then solved as before. The results presented in solutions 2 and 2A are the counterparts of solutions 1 and 1A after this adjustment is made. Note that the sum of the squares of the errors shown for solution 2A is slightly larger than for 1A but h_2 is closer to the theoretical value. Presumably the correct solution would fall somewhere between solutions 1A and 2A.

Estimates of the effect of random errors in $R_0(t)$ on the solutions are presented in column 6 in terms of the standard deviation of $R_0(t)$ about true values. If we suppose <u>either</u> that the nominal station coordinates adopted for this analysis are correct within 1 - 2 m <u>or</u> that the Love number is known to be 0.6 then the results suggest that $R_0(t)$ is systematically less than the true values by about 2.0 m with a much smaller random error.

CONCLUSIONS

The 5 cm precision of laser ranging measurement is certainly adequate for the observation of the 40 cm geometric earth tide, or even the 15 cm differential tide between two stations which can observe a satellite simultaneously. However, we cannot yet predict an orbit based on one tracking station and 24 hours of data which is stable enough to be used as a platform to observe the total tidal displacements.

The present dynamic model, employing gravitation field model PGS 558, fits 24-hours of laser data from a single station leaving systematic residuals during a pass of up to 1 meter, and resulting in apparent station movements in height of comparable amplitude, thus hiding the tidal variation. But even in the absence of imperfections in the dynamic model, 24-hour arcs would tend to absorb the tidal movements of a single tracking station leaving the pass-to-pass apparent station heights unchanged. Longer arcs or arcs using data from more than one laser should be less likely to absorb the geometric tides, but they would be expected to fit the laser data less well.

The quasi-geometric method is influenced significantly less than the dynamic method by errors in the predicted satellite position because this method only requires a knowledge of the radius to the satellite and it is sensitive principally to the differential tidal displacements between laser stations. Due to the stringent conditions that must be met for a usable plane-crossing to occur, however, there has been difficulty in finding a sufficient number of plane crossings for a rigorous statistical test of this method as it is presently implemented. But, for five passes over the calibration area that satisfy the criteria, a good approximation to the theoretical Love number h_2 is obtained when a systematic bias of 2.16 meters is allowed in the radial distance to the satellite. This bias is justified independently by the assumption that the nominal station coordinates are correct within 1 - 2 m. The value of h_2 appears to be reasonably insensitive to changes in the predicted radial distance to the satellite due to absorption of the tidal movements of Goddard by the GEODYNE reference arcs.

APPENDIX: PROOF OF THE EXTREMUM METHOD OF STATION-PLANE IDENTIFICATION

Let us consider a reference system of spherical coordinates with origin at the center of the earth 0 and axis OZ perpendicular to the plane $Q_1A_1OA_2Q_2$ passing through the center of the earth. The radial directions to the ground stations A_1 and A_2 are extended to points Q_1 and Q_2 such that $OQ_1 = OQ_2 = R_0$, the radial distance of the satellite at any instant from the center of the earth. The axis of meridional reference OX in this plane bisects the angle Q_1OQ_2. Then, with g as the perpendicular arc from the position σ, of the satellite to this plane and x as the arc from the foot of this perpendicular to the bisector OX, the coordinates of the satellite can be denoted as (R_0, g, x). Also, A_1 and A_2 can be represented in the same system of reference by the coordinates $(R_M + h, 0, -\omega)$ and $(R_M - h, 0, \omega)$ respectively, where R_M is the mean radius to the ground stations, 2h is their elevation difference and 2ω is the angle they subtend at the center of the earth. Then laser ranges S_1 and S_2 which are linear distances from the satellite σ to ground stations A_1 and A_2 respectively will be given by the equations

$$\left.\begin{aligned} S_1^{\,2} &= R_0^{\,2} + (R_M + h)^2 - 2R_0(R_M + h)\cos g \cos(\omega + x) \\ S_2^{\,2} &+ R_0^{\,2} + (R_M - h)^2 - 2R_0(R_M - h)\cos g \cos(\omega - x) \end{aligned}\right\} \qquad (A1)$$

These are the basic equations which are used to evaluate h from the observations S_1 and S_2. In developing our method of solution for h, we assume that ω and R_M are constants which are known before-hand. Range observations S_1 and S_2 as well as the radial distance to the satellite R_0 which vary with time are assumed to be available at discrete instants of time.

The problem is not solvable in its present form since, corresponding to n given sets of (S_1, S_2, R_0) values, we have n pairs of equations of the type (A1) involving 2N + 1 unknowns viz., n difference g's, n different x's (as g and x vary with time) and a constant h. Thus, the number of unknowns being more than the number of equations by one, no unique solution for h will be possible until and unless an additional condition for the problem is made available.

To obtain this, we consider a second pair of equations

$$\left.\begin{aligned} S_1^{\,2} &= R_0^{\,2} + (R_M + H)^2 - 2R_0(R_M + H)\ \cos(\omega + X) \\ S_2^{\,2} &= R_0^{\,2} + (R_M - H)^2 - 2R_0(R_M - H)\ \cos(\omega - X) \end{aligned}\right\} \qquad \text{(A2)}$$

where H, unlike h, and X, different from x, vary with time.

Eliminating X between the two equations in (A2), we can write

$$\begin{aligned} 16R_0^{\,2}(R_M^{\,2} - H^2)^2\sin^2\omega\cos^2\omega = {} & \sin^2\omega[R_M\{2(R_0^{\,2} + R_M^{\,2} - H^2) - S_1^{\,2} - S_2^{\,2}\} + \\ & H(S_1^{\,2} - S_2^{\,2})]^2 + \cos^2\omega[H\{2(R_0^{\,2} - R_M^{\,2} + H^2) - \\ & S_1^{\,2} - S_2^{\,2}\} + R_M(S_1^{\,2} - S_2^{\,2})]^2 \end{aligned} \qquad \text{(A3)}$$

from which H can be obtained when other quantities are known. We have developed a subroutine which computes H iteratively, starting from the initial value of H = 0.

If we assume the x-eliminate between the equations in (A1) can be formally written as

$$h = F(S_1,\ S_2,\ R_0,\ \cos g) \qquad \text{(A4)}$$

then, the similar equation for H will be

$$H = F(S_1,\ S_2,\ R_0,\ 1) \qquad \text{(A5)}$$

which is another form of (A3).

Differentiating (A4) with respect to time t and remembering that h is independent of t, we have

$$0 = \frac{\partial F}{\partial S_1}(S_1,S_2,R_0,\cos g)\,\dot{S}_1 + \frac{\partial F}{\partial S_2}(S_1,S_2,R_0,\cos g)\,\dot{S}_2 +$$

$$\frac{\partial F}{\partial R_0}(S_1,S_2,R_0,\cos g)\,\dot{R}_0 + \frac{\partial F}{\partial \cos g}(S_1,S_2,R_0,\cos g)\,(-\sin g)\,\dot{g} \qquad \text{(A6)}$$

Substituting in (A6) $t = t_0$ corresponding to $g = 0$, we have

$$0 = \frac{\partial F}{\partial S_2}(S_1,S_2,R_0,1)\,\dot{S}_1 + \frac{\partial F}{\partial S_2}(S_1,S_2,R_0,1)\,\dot{S}_2 + \frac{\partial F}{\partial R_0}(S_1,S_2,R_0,1)\,\dot{R}_0 \qquad \text{(A7)}$$

which, when compared with the equation obtainable from differentiation of (A5) with respect to t, yields:

$$\dot{H} = 0 \text{ when } g = 0. \qquad \text{(A8)}$$

Consequently, from comparison of (A1) and (A2), we find that when $\dot{H} = 0$ corresponding to $g = 0$, $H = h$.

Thus, the equation (A8) is the additional relation that is needed to obtain h uniquely.

In practical computation, values of H are computed iteratively from each set of (S_1,S_2,R_0) values, using the subroutine based on (A3). The plot of these values of H against time would show a smooth curve with an extremum (i.e., $\dot{H} = 0$) occurring at the instant when the satellite crosses the vertical plane through the ground stations (i.e. $g = 0$). For precise computation of this instant, four consecutive values of H are selected such that $H(t_{k+1})$ and $H(t_{k+2})$ are either both greater than or both less than $H(t_k)$ and $H(t_{k+3})$. A third-degree polynomial in time is then fitted to these values of H to obtain the extremum value for H and this is the value of h we require. This part of the computation is implemented by a second subroutine.

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16. Abstract

An analysis of NASA laser ranging data from three stations located at Goddard Space Flight Center, Greenbelt, MD, Grand Turk Island and Bermuda respectively was carried out in an attempt to measure the geometric earth tide. Two different approaches to the problem were investigated:

1) A "dynamic" method that computes pass-to-pass apparent movements in stations height relative to short arcs fitted to several passes of data from the same station by the program GEODYNE, and
2) A "quasi-geometric" method that reduces the dependence on unmodelled satellite dynamics to a knowledge of only the radial position of the satellite by considering two-station simultaneous ranging at the precise time that the satellite passes through the plane defined by two stations and the center of mass of the earth.

The "dynamic" method was applied to 24-hour arcs in an attempt to study the total tidal displacement at Goddard but, it was found to be unsuitable for two reasons:

a) An R.M.S. variation in apparent station-height of 0.8 m was obtained due to unmodelled perturbations of the satellite orbit when geopotential model P6S 558 was used.
b) Introducing tenfold theoretical tidal displacements into the Goddard laser data showed that, even in the absence of errors in the dynamic model, 24-hour arcs would tend to absorb the tidal movements of a single tracking station.

The "quasi-geometric" method, on the other hand, is significantly less influenced by these effects because it is sensitive principally to the differential tidal displacements between laser stations. Using this method a good approximation to the theoretical Love number h_2 was obtained from five passes over the GEOS-3 calibration area when a systematic bias of 2.16 meters was allowed in the radial distance to the satellite.

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