THE PERIODS AND AMPLITUDES OF TU CAS

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ABSTRACT

Light curve observations of the double-mode Cepheid TU Cas obtained by 10 different sets of observers on several photometric systems over a time span of 67 years have been carefully studied to determine the fundamental and first overtone periods and their amplitudes on the V magnitude scale. The presence of a second overtone radial pulsation is discussed, and it is concluded that a previous detection of this mode was spurious due to the lack of a proper zero point correction for two groups of observations. The amplitudes of the two modes are shown to possibly vary during the entire observing period with the fundamental mode amplitude of 0.69 ± 0.03 and the overtone amplitude decreasing about 0.2 or 0.3 magnitude. If this Cepheid displays the two pulsation modes because it is mode switching, this switching time scale might be less than a hundred years.

Our motivation for this investigation is the reported existence by Faulkner (1977) of a third radial pulsation mode in the double-mode Cepheid TU Cas. At least one other Cepheid-type variable, AC And, and many of the δ Scuti variables have three radial pulsation modes, so it is perhaps not surprising to find another. There are two problems, however, TU Cas appears to be like the dozen or so double-mode Cepheids with a first overtone to fundamental mode period ratio $\cong 0.71$, unlike AC And with a ratio of almost 0.74. In addition the reported third period is abnormally long with respect to the primary and secondary periods, making it very difficult to explain in terms of reasonable stellar models. The AC And model is within the normal range of composition and structure according to Cox, King, and Hodson (1978).

Another motivation for studying TU Cas is to see if the amplitudes of the two modes are changing with respect to each other over the 67 years spanned by the available observations. Double-mode Cepheids, which might comprise one-third of all short period Cepheids (< 5 days) according to Stobie (1977), have yet to be adequately explained. Stellingwerf (1975) suggested that double-mode behavior resulted at temperatures near the red edge of the instability strip where the two modes switched toward each other. Unfortunately, Hodson and Cox (1976) found no double-mode behavior at these cooler temperatures. Currently, the only other cause of double-mode Cepheids predicted by theory is mode switching at the transition line between fundamental mode pulsation to the red and first overtone pulsation to the blue.

Observational evidence tends to support double-mode Cepheid pulsation near the transition line. Results from Rogers and Gingold (1973) on U Tr A, Schmidt (1972) on TU Cas, Pel and Lub (1978), and Cogan (1978) all arrived at temperatures placing most of these variables near the transition line. However, if double-mode behavior is the result of mode switching, we would expect to find a complete range of amplitude ratios between the two modes. As the evidence currently stands, only AX Vel has a first overtone amplitude larger than the fundamental.

Amplitude changes over a span of 67 years would support the rapid mode switching rates predicted by Stellingwerf (1975) and by unpublished results at Los Alamos, whereas no change would be more consistent with a possible slow change or just a stable permanent mixed mode.

The observations used in our TU Cas analysis are given in Table 1. The set used by Faulkner contains 290 points and consist of the four groups ranging from 1946 to 1959 (denoted by *). We attempted a period search on 302 white light observations by Osterhoff (1957), but were unsuccessful due to poor phase distribution of this data. For the amplitude analysis, we added 111 observations starting from 1962 to the present, which include 60 new visual magnitudes by Schmidt observed from 1976-78. For a more sensitive mode switching search, we converted observations by van Biesbroeck and Casteels (1914) from visual estimates to the modern V magnitude system with reasonably good results.

TABLE 1

OBSERVATIONS OF TU CASSIOPEIAE

REFERENCE	EPOCHS (JD 2400000+)	YEAR	NUMBER OF MEASURES	OBSERVING SYSTEM
Van Biesbroeck & Casteels (1914)	19198-19481	1911-1912	192	Visual
Gordon & Kron* (1947)	32039-32163	1946	28	Δm_{5000}
Oosterhoff (1957)	32424-32471	1947	265	Δm white light
	32606-32615	1948	37	
	32272-33608	1950	350	
Worley & Eggen* (1957)	33146-33206	1949	11	V_E, $(P-V)_E$
	34334-34369	1952	4	
	35063-35113	1954	102 means	
	35388-35404	1955	74 means	
Oosterhoff* (1960)	36751-36849	1959	45	UBV
Weaver, Steimetz & Mitchell (1960)*	36742-36858	1959	26	UBV
Williams (1966)	37936-37940	1962	6	UBV
Kwee & Braun (1967)	38219-38235	1963	14	UBV
Takase (1969)	38776-39079	1965	20	UBV
	39358-39433	1966	11	UBV
Schmidt (private communication)	43015-43126	1976	26	UBVRI
	43403-43500	1977-1978	34	

* Observations used by Faulkner (1977).

For our periodicity search we fitted the Faulkner data group by least squares to a sinusoidal series

$$\frac{L(t)}{\langle L\rangle} = 1 + \sum_{i=1}^{m} A_i \cos 2\pi(ift-\phi_i) \qquad (1)$$

made up of a single frequency plus m-1 harmonics taken over a specified frequency range. The quality of the fit and, consequently, the significance of any frequency is measured by σ, the standard deviation of the observations from the fit. When $m = 1$, the amplitude is related to σ by $A^2 = 2(\Delta\sigma^2)$. Equation 1 can be expanded to include two or more frequencies, not harmonically related, and their cross coupling terms. The luminosity variations for the triple mode case are fit by

$$\frac{L(t)}{\langle L\rangle} = 1 + \sum_{|i|+|j|+|k|=1}^{m} A_{ijk} \cos 2\pi[(if_0 + jf_1 + kf_2)t - \phi_{ijk}]. \quad (2)$$

The appropriate order of a fit is determined when the variance of fit, order $m + 1$, is not appreciably smaller than the variance of m^{th} order fit. In addition, the fit will yield small changes in amplitude (A_{ijk}) and phase angle (ϕ_{ijk}) with increasing order as suggested by Fitch. From Equation 2 we can directly obtain the amplitude and shape of each natural frequency.

Third order Fouriergrams on the Faulkner data group yield the same primary and secondary period values as Faulkner obtained. We used these values to construct the double-mode fit, and then searched the residual for any remaining periodicity. In Figure 1 we see, in the upper curve, the first order Fouriergram of the fourth order residual which is identical to Faulkner's results. The change in σ of the central minimum corresponds to an amplitude of ~ 0.014 for $\Pi_2 \sim 1.25247$ days. The sidelobes are due to the gap of ~ 1450 days between the 1959 and 1954-55 epochs. The lower curve is the Fouriergram of the fifth order residual, and, although the central minimum still persist, its depth gives an amplitude of ~ 0.009, down by 35% from the fourth order amplitude.

In Figure 2, we see the same Fouriergram over a larger frequency range, expanded in the direction of slightly higher, but more theoretically plausible frequencies. In the fifth order residual, we find at

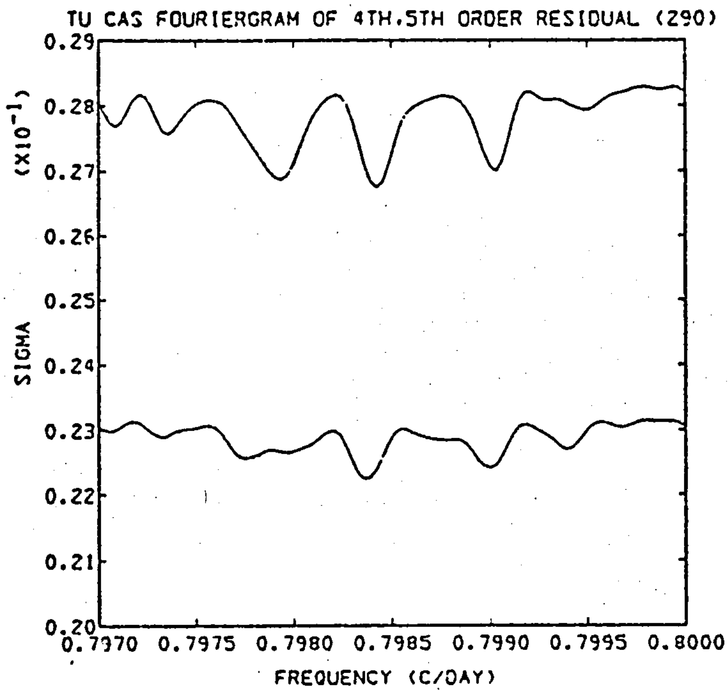

Fig. 1. First order (single frequency) standard deviation of the fourth and fifth order residuals versus frequency using the basic 290 data points.

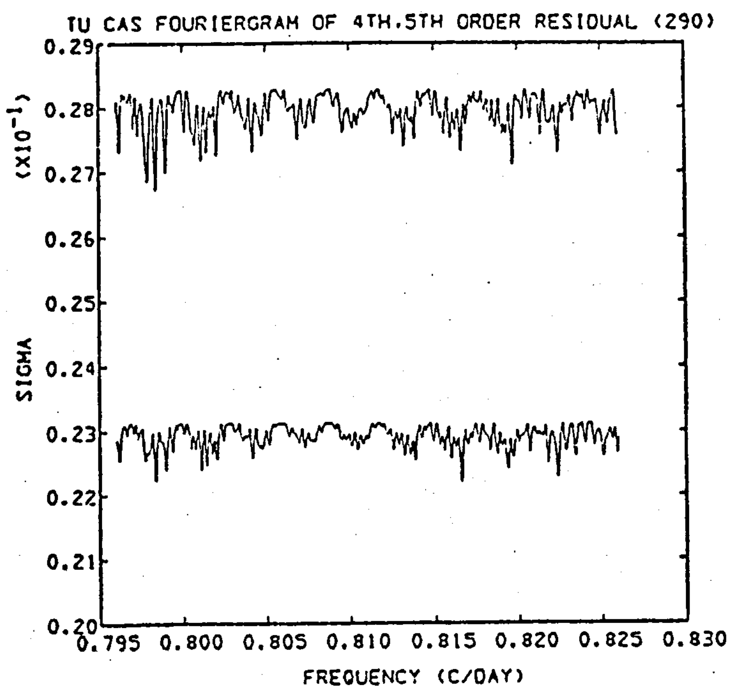

Fig. 2. First order (single frequency) standard deviation of the fourth and fifth order residuals versus frequency. This figure shows the same data as in Fig. 1 but over a larger frequency range.

least one frequency at $\sim 0.816^{-d}$ with a deeper minimum than Faulkner's in the far left portion of the curve, suggesting that perhaps his longer period may be an artifact of data noise.

To test the quality of the data, we calculated the standard deviation of each epoch, grouped by year, about a fourth-order double-mode fit through all 290 points. The results are given in Table 2. From the σ's in the first column, we see that the 1949 data have rather large scatter. The 1952 data may also be noisy, but with only four observations, its sigma value may be spurious. But the 1949 data remain suspect, especially considering the small scatter of the 1954-55 Worley and Eggen data. This epoch could be intrinsically noisy, or it may not share the same zero point with the other epochs.

We were able to confirm Faulkner's magnitude zero point adjustments between observers, and ran a subsequent check on the zero point between different observing epochs. Our check centered around Worley and Eggen

TABLE 2

EPOCH STANDARD DEVIATIONS

FOURTH ORDER DOUBLE-MODE FIT

EPOCH	No Shift (290)	1955 Shift* (290)	1949 Shift† (290)	1955 + 1949 Shift (290)	No Shift - 1949 (279)	1955 Shift - 1949 (279)
ALL (290)	0.0283	0.0271	0.0267	0.0254	0.0231	0.0214
GK 1946 (28)	.0260	.0246	.0257	.0234	.0252	.0234
WE 1949 (11)	.0835	.0841	.0713	.0724	.0000	.0000
WE 1952 (4)	.0567	.0553	.0544	.0552	.0552	.0555
WE 1954 (102)	.0205	.0180	.0212	.0187	.0208	.0180
WE 1955 (74)	.0156	.0095	.0151	.0089	.0145	.0077
O+WSM 1959 (71)	0.0323	0.0332	0.0308	0.0314	0.0299	0.0307

* 1955 Shift $\Delta V = -0.044$

† 1949 Shift $\Delta V = 0.066$

data, as it is the only group with more than one epoch. We performed various trial shifts on the three epochs in this group in an attempt to reduce the relative and total standard deviation.

From the third column, we see that by shifting the 1955 epoch by $\Delta V = -0.044$, the total σ is reduced, and σ for this epoch is down almost by a factor of two. A ΔV change of $+0.066$ in the 1949 data also gave some reduction in the total sigma, and in the fifth column, we see that the combined effects of shifting both 1949 and 1955 epochs results in an ever smaller total deviation.

Removing the 1949 epoch from the unshifted 290 points produces, by itself, a larger reduction in total sigma than does any of previous shifting. ($\sim$ 18% reduction in σ for a 4% point reduction). Removing this epoch and shifting the 1955 epoch gives the largest reduction in sigma, almost 24% lower than for the original Faulkner data.

In Figure 3 we have Fouriergrams of fourth order fit residuals in the region of Faulkner's third period for each trial listed on the table. The uppermost curve is the same Fouriergram as you saw in Figure 1, showing the central minimum surrrounded by aliases. By shifting 1955 epoch this pattern almost disappears (second curve from top). However, in the next curve, we see that 1949 epoch shift alone, produces only small changes from the original pattern of the top curve. The fourth curve shows the combined effect of the two shifts, and is similar to the second curve because of the large effect of the 1955 epoch shift. The fifth curve results from removing 1949 epoch from the data, and in spite of a substantial reduction in σ, the original pattern still persists. This pattern again disappears when we

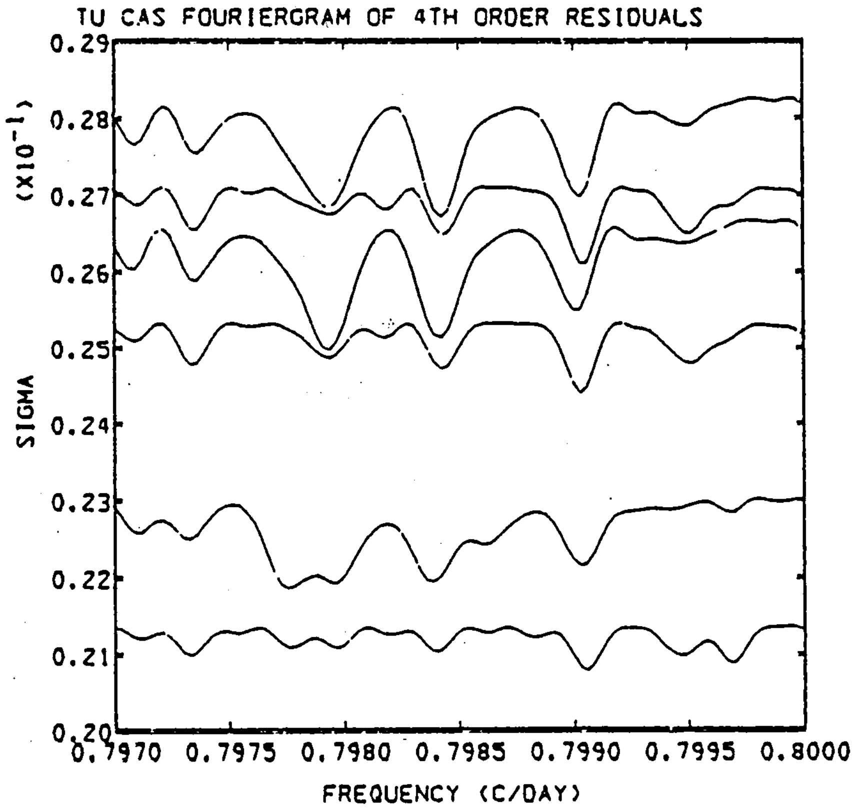

Fig. 3. First order (single frequency) standard deviation of the several fourth order fit residual cases of Table 2 versus frequency.

introduce the 1955 data shift as seen in the bottom curve.

Expanding the frequency range in Figure 4, as we did before, shows that shifting 1955 data removes the cyclic pattern of curves one, three and five, which corresponds to the 291 day interval between the 1954 and 1955 epochs. Faulkner's proposed third period, at the far left, appears to be only a deeper member of this pattern that disappears with the appropriate zero point shift.

Fitch noted the amplitude given by Faulkner's limited triple mode fit was a factor of two larger than the amplitude predicted by the depth of the minimum in the Fouriergram (about 0.025 to 0.014). We can explain this discrepancy in Figure 5 with two plots of fundamental and first overtone prewhitened data and mean curves as a

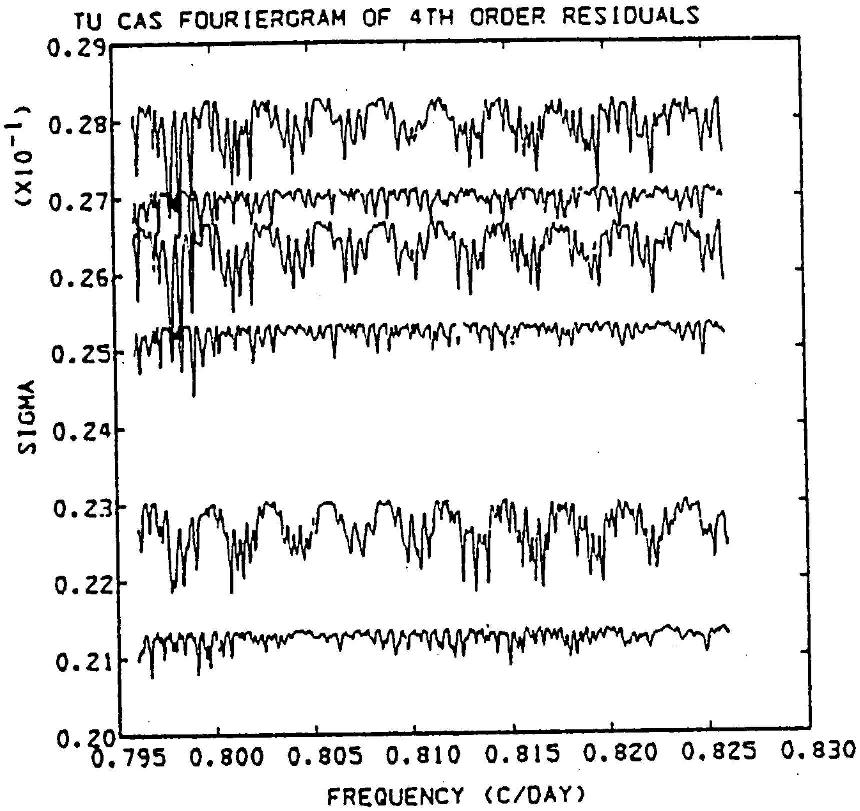

Fig. 4. First order (single frequency) standard deviation of the several fourth order fit residual cases of Table 2 versus frequency. This figure shows the same data as in Fig. 3 but over a larger frequency range.

function of phase. In both cases we have used the original, uncorrected Faulkner data set. We produced the upper plot by removing all the triple-mode fit terms except the Π_2 sine term, and phasing the residual to Π_2. This plot is identical to Faulkner's (Figure 4, 1977) and confirms his amplitude for Π_2 given by the triple-mode fit. The lower plot is the phased residual of the fourth order-double mode fit, with the amplitude estimated by a spline fit. It gives an amplitude value of ~ 0.017 which is much closer to the predicted value of 0.014, with the difference of 20% due to uncertianties in the spline fit such as the number of knots and order. In general, we have noticed when any new natural mode (real or spurious) is introduced into the multimode

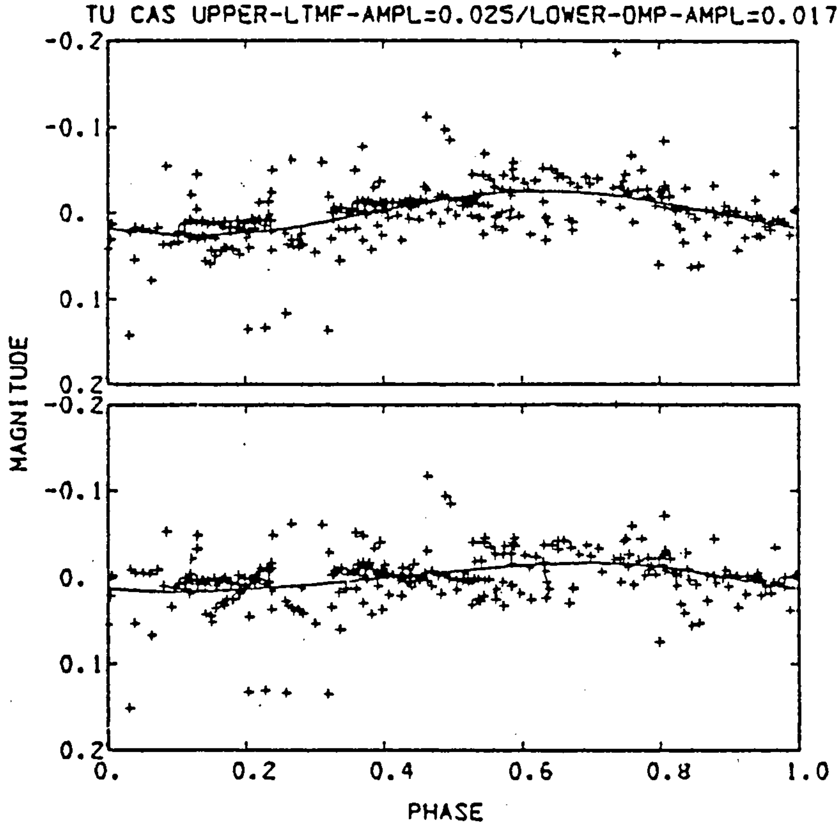

Fig. 5. The upper panel shows the Π_2 single frequency variation of the 290 point Faulkner triple-mode fit and the points obtained by prewhitening by all the Π_0 and Π_1 terms of his fit. The lower panel gives the variation of the residuals of a double-mode fit with the proposed Π_2 phase together with a spline fit through the points.

fit, it interacts in varying degrees with the other natural modes, resulting in a substantial amplitude for this mode. Hence, one cannot reliably determine the significance of a variation by its amplitude size given by multimode fitting.

In searching for amplitude changes in TU Cas, we selected the longest time base possible with the hope of revealing the more conspicuous variations in modal amplitude. A summary of our results is given in Table 3.

The 1911-12 epoch by van Biesbroeck and Casteels was converted from a visual estimates system to V magnitudes. Initially, our

TABLE 3

MODE AMPLITUDES OF TU CASSIOPIAE*

Epoch	No. of Observations	A_0	A_1	σ	A_1/A_0
1911-12†	192	.70	.45	.13	.64
1911-12††	192	.72	.54	.19	.74
1911-12†††	192	.86	.53	.14	.62
1946-1977	390	.71	.27	.04	.38
1946-1959	279	.71	.29	.03	.41
1962-1977	111	.67	.24	.03	.35

* Third order double-mode Fourier Series fit

† Adopted magnitudes by Van Biesbroeck and Casteels

†† Revised magnitudes from HD catalog

††† UBV magnitudes by Henden (private communication, 1978)

primary uncertainty in converting these observations resulted from the absence of any recorded UBV measurements of the six comparison stars used for this study. Van Biesbroeck and Casteels estimated these magnitudes from a relative scale used to determine the variable's changing magnitude. We attempted to improve on these magnitudes using the best revised estimates from the HD catalog (as suggested by Bidelman, private communication). Eventually we were able to obtain UBV magnitudes for the six comparison stars from Arne Henden at Indiana University (private communication, 1978)

The ratio of first overtone to fundamental mode amplitude (A_1/A_0) for this earliest epoch averages ~ 0.67 and is appreciably higher than for the 1946-1977 era (~ 0.38). To check this last ratio we split the modern era into two ~ 15 year intervals to determine if this trend still persisted. The decrease of the ratio from 0.41 to 0.35 is suggestive, but probably within the range of uncertianty.

Due to the relatively larger uncertainty of the 1911-12 data, we are unable to conclude that the first overtone mode really decayed substantially since 1911. However, observations after 1945 span nearly 32 years and comprise a long time base of quality UBV measures. With the inclusion of new observations the decrease of A_1/A_0 within the modern epoch may possibly be confirmed.

Our conclusions are that TU Cas is not a triple-mode pulsator, and that maybe the first overtone pulsations have decayed since 1911.

We wish to thank R. F. Stellingwerf for locating the van Biesbroeck and Casteels data and providing it to us.

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Discussion

Wesselink: How do you identify a mode? How do you know whether it is the fundamental or an overtone?

Hodson: Hopefully, the one with the longest period with largest amplitude is the fundamental. Sometimes that turns out to be the 1st or 2nd overtone. But that's the nomenclature I use.

J. Cox: In 1977, I think, Stobie pointed out that there was no observational evidence for amplitude changes over at least a 50-year time span. Do you think this is because no one has looked, or is there more substantial evidence?

Hodson: The evidence we have suggests that this might be happening. We have 32 years of observations in the "modern" era. We need quite a few more "modern" observations to resolve the problem.