# TRIPLE MODE CEPHEID MASSES

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## I. INTRODUCTION

In the preceding paper (Cox 1978) we have heard about the general problem of Cepheid masses. Large mass anomalies for beat (double-mode) Cepheids (2-6 days) have been discussed by Petersen (1973), King, et al. (1975), and Cox, King, Hodson, and Henden (1977). Ratios of these masses to evolution masses were previously found to be as little as one quarter.

The possibility that nonlinear coupling between the two principal modes in double-mode Cepheids might lead to period ratios which are sufficiently different from those predicted by linear theory as suggested by Faulkner (1977a) does not seem to solve the problem (Cox, Hodson, and King, 1977). Large amplitude mixed mode models lead to period ratios which agree with the linear values to within 0.4 percent, whereas the change required to yield masses close to the evolution value is of the order of 3.0 percent. This result is in agreement with that found by Stellingwerf (1975) in his investigation of a mixed mode model with characteristics similar to those of an RR Lyrae variable.

The suggestion that convection might change the structure in such a way as to alter the period ratios by the required amount, as suggested by Cogan (1977), has been investigated by Deupree (1977a) and by Cox, King, Hodson, and Henden (1977). It was found that for reasonable values of the ratio of mixing length to pressure scale height, convection played a very small role and was not able, in the absence of other changes, to produce the desired effect.

A possible reconciliation of low Cepheid masses obtained by using linear theory periods with the larger ones obtained by stellar evolution theory has been suggested by Cox, Deupree, King, and Hodson (1977) and Cox, Michaud, and Hodson (1978). The convection zones, presumably enriched in helium by a helium deficient Cepheid wind, change the structure of yellow giants to give the $\Pi_1/\Pi_0$ for double-mode Cepheids consistent with observations.

Until quite recently there were thought to be two cases of triple mode Cepheids. The star AC Andromedae has been discussed by Fitch and Szeidl (1976) who obtained periods of $\Pi_0$ = 0.711d, $\Pi_1$ = 0.525d and $\Pi_2$ = 0.421d. It should be noted that this star is outside of the range of periods previously indicated for the double-mode Cepheids, and there may still be some question as to whether this star is a Population II RR Lyrae star, which would be consistent with its period, or a very short period Population I Cepheid, more consistent with its spectrum.

Faulkner (1977b) has recently reported a third period for the variable TU Cassiopeiae. With periods of $\Pi_0$ = 2.14d, $\Pi_1$ = 1.52d, and $\Pi_2$ = 1.25d, this star does have fundamental and first overtone periods which place it among the other double-mode Cepheids. We have heard Hodson and Cox (1978) report in an earlier paper in this conference that the third period ($\Pi_2$) does not appear to be real. Prior to learning this models were studied in an attempt to explain the three periods. We will discuss this in Section III.

The models previously suggested had homogeneous ($Y = 0.50$ to $0.75$) convection zones which extend to 60,000 K or 70,000 K, counting the pulsation and overshooting excursions, and then a deeper 10,000 K wide buffer zone with half the surface helium enrichment. There are now at least two difficulties with those models. Possibly the large helium abundance in the atmosphere conflicts with spectral observations of the metal to hydrogen ratio for Cepheids. This difficulty must await detailed calculations of synthetic spectra to see if high helium can be tolerated. The second difficulty is that possibly the helium rich layer is very unstable due to the inverted $\mu$ gradient, and it may quickly mix by a process described by Kippenhahn (1974). Previously, the period ratios of triple-mode Cepheids AC And and TU Cas could not be correctly predicted by very thin enriched layer structures as reported by King, Cox, and Hodson (1977). Deeper enriched layers are now indicated.

It appears that the period ratios of AC And indicate that Cepheid wind enrichment from the surface and the instability mixing below the convection zones compete to give much deeper large Y homogeneous and transition layers than previously thought. The time spent as a yellow giant and the mass loss due to the wind, inferred from the solar wind and the relative size of the sun and the Cepheid, allow a mass fraction of perhaps $2 \times 10^{-4}$ to be enriched to maybe $Y = 0.75$. If this layer is unstable and is mixed deeper, the Y will be somewhat smaller and the temperature at the bottom will be hotter. We show that if the enrichment goes to 250,000 K ($1-q = 2 \times 10^{-4}$) with a transition zone to 300,000 K ($1-q = 5 \times 10^{-4}$) for AC And, its periods can be explained. For TU Cas a very unlikely model is required to give the three periods reported by Faulkner (1977b).

The observations of AC And are reviewed in Section II. A discussion of the periods of TU Cas has been given by Hodson and Cox (1978) and need not be repeated here. Section III gives our theoretical model data and derived masses. Conclusions are given in Section IV.

## II. AC ANDROMEDAE PERIODS

The data of Fitch and Szeidl (1976) has been analyzed by a method adapted from Lafler and Kinman (1965) and described by Cox, Hodson, and King (1977). We here, however, have adopted the Fitch and Szeidl periods with no attempt at refining them. The original data, kindly supplied by Fitch, was grouped by averaging all points in magnitude and time within 0.0142d starting at the first data point used by Fitch and Szeidl. The differing weight of these average points has been ignored in our period analysis. Component fundamental (F), first overtone (1H), and second overtone (2H) light curves are given in Figure 1. Three other periods that are found are the nonlinear coupling beats between F and 1H, F and 2H, and 1H and 2H. These final component light curves were obtained after iteratively prewhitening the mean data points with all the other 5 periods. Clearly, the second overtone is present as Fitch and Szeidl found.

The standard error of the amplitudes is 0.06 mag, much larger than obtained by Fitch and Szeidl. The reason seems to be that we have not prewhitened with so many additional periods. It may be necessary to include periods to more exactly define the distorted light curves of the three basic variations. Our curves in Figure 1 and our three nonlinear beat variations then probably do not show the exact shape of each component, but the analysis is good enough to give rather accurate amplitudes.

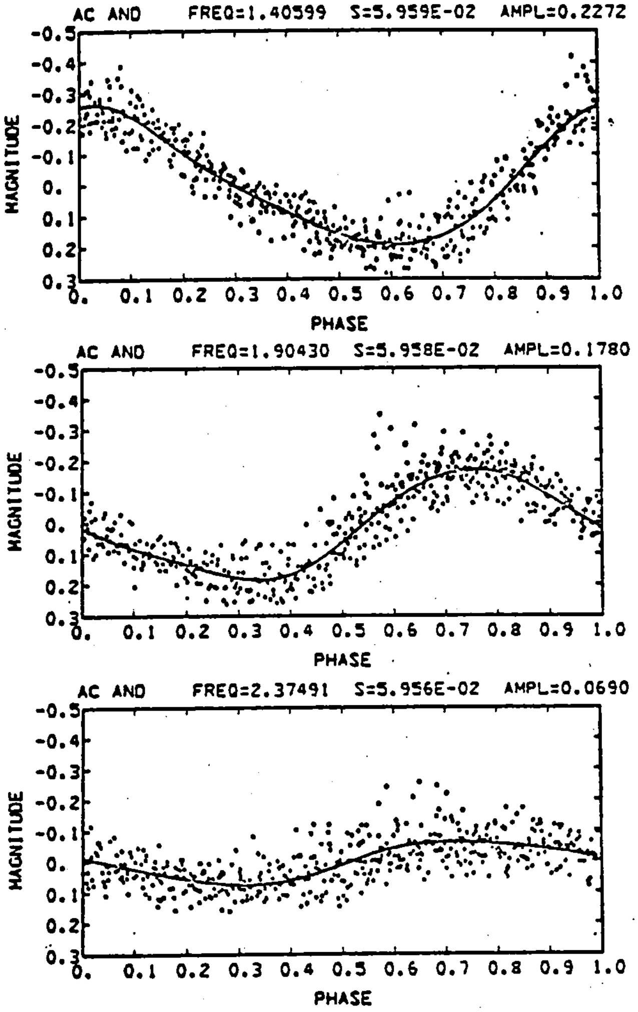

Fig. 1. Component light curves for the three periods of AC Andromedae using the data of Fitch and Szeidl. Their periods have been used. Our three amplitudes are 0.227, 0.178, and 0.069 magnitude with the standard deviation of 0.060 for all three.

## III. TRIPLE MODE CEPHEID MASSES

Numerous models have been constructed and analyzed for periods and growth rates using a linear nonadiabatic program originally developed by Castor (1971). Convection with varying $\ell/H_p$ (Deupree 1977b) is allowed, but $\ell/H_p$ is limited to unity. The previous structures, which can explain the bump and double-mode Cepheids with evolutionary theory masses, were modified in ways that did not destroy their $\Pi_1/\Pi_0$ and $\Pi_2/\Pi_0$ period ratios. Thus the new revised bump and double-mode Cepheid masses remain unchanged.

One problem with the surface helium enrichment is that the structure is not stable. Helium will leak downward at a, as yet undetermined, rate. If one assumes some downward mixing to a mass level with only about $5 \times 10^{-4}$ of the stellar mass above, there is still time to enrich it by a Cepheid wind to a large Y in the few million years during a 3 $M_\odot$ first evolutionary crossing of the pulsation instability strip. Thus our AC And model is enriched to $Y = 0.48$ from the surface to 250,000 K to $Y = 0.38$ between there and 300,000 K, and finally with $Y = 0.28$, or a normal value, deeper all the way to the nuclear burning core.

Figure 2 gives a plot of $\Pi_2/\Pi_1$, $\Pi_1/\Pi_0$, and $\Pi_2/\Pi_0$ versus mass for this structure with the fundamental period for all models within a few percent of that for AC And. The observed period ratios indicate approximately 3 $M_\odot$; a more definitive value would need unknown details of the internal composition structure.

This value of 3 $M_\odot$ is the same as that suggested by Fitch and Szeidl (1976), but we do not completely agree with their techniques of getting the mass. The best data available to them was from the Cogan (1970) grid of models which was coarse and probably the models included too much convection. Fits to the pulsation constant $Q_i$ for various M and R values may not give period ratios $\Pi_{i+1}/\Pi_i$ to the accuracy of one part or less per thousand that observations merit. The error in the Fitch and Szeidl mass value was perhaps

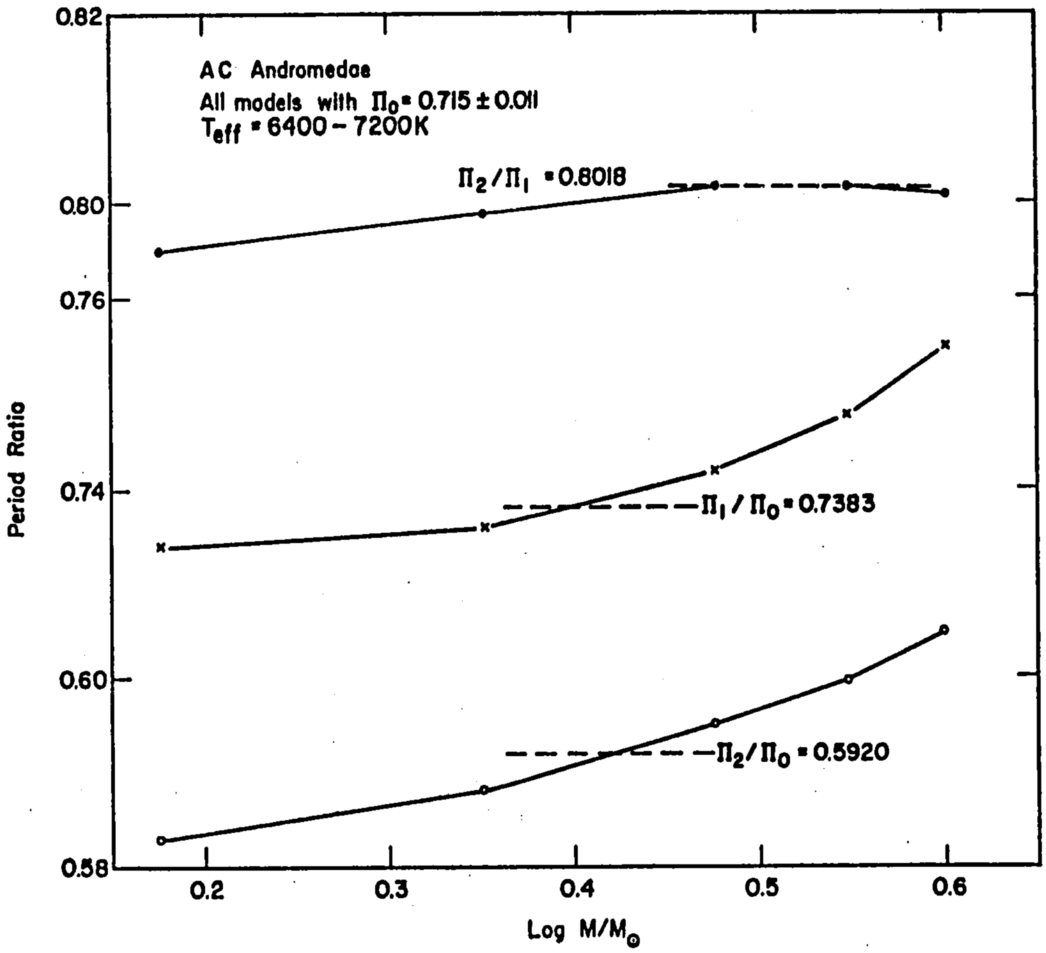

Fig. 2. Period ratios for the fundamental and first two overtones for our inhomogeneous models for AC Andromedae plotted versus log $M/M_\odot$. Luminosities range from 100-300 $L_\odot$.

30 percent or more whereas ours, using specially calculated models, is considerably smaller, assuming of course our unconventional composition structure.

As pointed out by Cox, Deupree, King, and Hodson (1977), the primary effect which leads to the correct period ratios in the inhomogeneous case is the change in the density structure of the outer envelope.

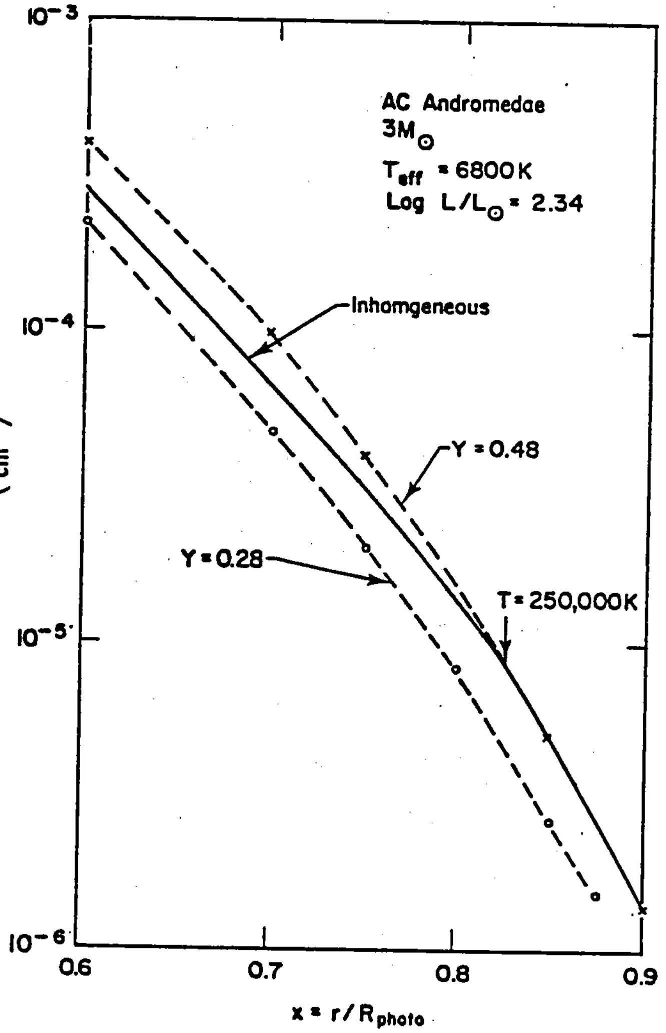

Fig. 3. Density versus fractional radius in three models for AC Andromedae. The two homogeneous models have parallel density structures and similar periods for the first three radial modes. In the inhomogeneous case, the fundamental mode feels the shallower density gradient more than the higher modes and has a larger period than for the two homogeneous cases.

Figure 3 shows the density as a function of fractional radius for a three solar mass model of AC And. This is near the derived mass as can be seen in Figure 2. Three cases are shown; two homogeneous models and the adopted inhomogeneous case. In the outer region of the star where $T \lesssim 250{,}000$ K, the density in the case for $Y = 0.28$ is about a factor of two less than in the helium enriched inhomogeneous model. As one approaches the base of the envelope the densities become more nearly equal, i.e. the magnitude of the density gradient is less steep in the inhomogeneous case. Since the total mass and radius of the models is the same, this leads to a less centrally condensed model and hence to a longer fundamental period. By allowing the envelope to be helium enriched into a depth of 250,000 K the periods of the first and second overtones are little affected since their eigenfunctions are large only in regions exterior to this and see roughly a homogeneous model. Table 1 confirms this for these cases. The lengthening of the fundamental period is about four percent whereas the first overtone and second overtone periods are increased by slightly more than one tenth of one percent.

TABLE 1

Ac And Models at 3 $M_\odot$, 217 $L_\odot$, $T_e = 6800$ K

| Model Y | $\Pi_0$ | $\Pi_1$ | $\Pi_2$ | $\Pi_1/\Pi_0$ | $\Pi_2/\Pi_0$ | $\Pi_2/\Pi_1$ |
|---|---|---|---|---|---|---|
| .48 | 0.6834 | 0.5336 | 0.4287 | 0.781 | 0.627 | 0.804 |
| Inhomogeneous | 0.7106 | 0.5277 | 0.4232 | 0.743 | 0.596 | 0.802 |
| .28 | 0.6841 | 0.5270 | 0.4226 | 0.770 | 0.618 | 0.802 |

It is of some interest to briefly discuss the models that were necessary when we were attempting to explain the reported third period of TU Cas. The ratio $\Pi_2/\Pi_1$ would be 0.8249 whereas for most models this ratio is about 0.80, as for AC And. The only model we were able to find which would give the Faulkner(1977b) $\Pi_2/\Pi_1$ had normal helium (in our case actually Y = 0.35) from the surface to 80,000K, Y = 0.70 between 80,000K and 150,000K, and then Y = 0.28 to the nuclear burning region. Using this model a mass of about $4M_\odot$ was derived. This type of structure is not needed to explain any of the other Cepheids and led us to doubt the reality of the second overtone. In the absence of the third period there is no difficulty in explaining TU Cas as just another double-mode Cepheid with the high gravity of Schmidt (1974), explained by the large surface helium abundance.

## IV. CONCLUSIONS

Unconventional composition structures are proposed to explain the periods of the triple mode Cepheid AC And. A strong Cepheid wind appears to enrich helium in the convection zones down to about 60,000 K or 70,000K. Then some downward partial mixing occurs to the bottom of a layer with about $1-q = 5 \times 10^{-4}$ of the stellar mass.

Petersen (1978b) has suggested that AC And may be a c-type RR Lyrae variable pulsating only in the first, second, and third overtones. Fitch and Szeidl indicate, however, a Population I composition. We have two major objections to the Petersen models. First, we find that if nonadiabatic periods are calculated instead of his adiabatic ones, the period ratios are too small by as much as 4 percent for $\Pi_2/\Pi_1$ and somewhat less for $\Pi_3/\Pi_1$ and $\Pi_3/\Pi_2$. This leads to an unacceptable solution for the masses given by Petersen.

Second, we find that the second and third overtones for either a Population I or II mixture are stable at all surface effective temperatures for these low masses. The rapid kinetic energy decay rates, like 25 percent per period, give the few percent difference between the adiabatic and nonadiabatic periods. At 0.6 and 1.0 $M_\odot$ the first overtone is sometimes pulsationally unstable, but not at the $T_e$ value of 7100 K suggested by Jakate (1978). For our inhomogeneous models with more surface helium, the fundamental and first two overtones are all naturally unstable for $T_e$ between 6400 and 7000 K.

We note that AC And is not unlike the anomalous Cepheids recently discussed by Zinn and Searle (1976) and Deupree and Hodson (1977). Masses of between one and two solar masses are suggested, however, and the population is more likely type II.

It is worthwhile noting that the double-mode Cepheids, such as U Tr A, can still be explained with our proposed enrichment below the homogeneous convection zones. A case where Y is 0.50 from the surface to 150,000 K ($1-q = 2 \times 10^{-4}$), 0.39 from there to 200,000 K, and then 0.28 to the nuclear burning core gives the proper $\Pi_1/\Pi_0 = 0.7105$ for the double-mode Cepheid U Tr A (2.57d) at about 4 $M_\odot$. Thus we do not destroy the explanation for any of the double-mode Cepheids.

The bump Cepheids, however, cannot have enrichment below the convection zones or the ratio $\Pi_2/\Pi_0$ and its variation [also the Hertzsprung (1926) light curve bump variation] with phase will be upset. Note that in this case the mass level of about $10^{-4}$ of the stellar mass is right at the bottom of the lower He II convection zone.

We finally note that some of the B stars also have helium enrichment, according to Osmer and Peterson (1973), which has been discussed by S. Vauclair (1977). The problem of stabilizing a helium enriched layer, established in B stars by downward diffusion in the presence of a stellar wind, and in Cepheids by a Cepheid wind, seems to be an important problem to be solved.

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## Discussion

J. Wood: Could the instability be caused by a magnetic field?

King: I've no idea.

J. Wood: What's the status on magnetic fields in models? Can they be neglected?

King: They have been neglected, but whether they can be is questionable. If they are really as large as indicated, perhaps they should be included.

A. Cox: Georges Michaud, who has worked with me on helium enrichment, has great hope for magnetic fields, but I don't. However, they would have to be stronger that what you [J. Wood] measured. That's why I don't believe they will be effective.

Mullan: How do you estimate the ammount of mass loss?

King: We take the solar value scaled by the ratio of the surface areas. We don't know anything more than that. It may exceed that value.

Mullan: There was some discussion here at an IAU Colloquium on stellar winds arising in chromospheres. Wouldn't those be better values? If the supersonic point moves down into the chromosphere, large mass loss rates can occur.

King: I forget the exact values, but if you have too high a rate and lose a large amount, it is unnecessary to use inhomogeneous models to explain the anomalies.

Mullan: It amounts to not more than 10%.

King: Ten percent is not enough to explain the anomalies.

A. Cox: Let me just remark, we have no idea what the Cepheid wind is like. But it has been pointed out that if it is too strong, it will carry away the helium also. We need to have helium left.