Dependence of Red Edge on Eddy Viscosity Model Parameters

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ABSTRACT

The dependence of the red edge location on the two fundamental free parameters in the eddy viscosity treatment has been extensively studied. It is found that the convective flux is rather insenitive to any reasonable or allowed value of the two free parameters of the treatment. This must be due in part to the fact that the convective flux is determined more by the properties of the hydrogen ionization region than by differences in convective structure. The changes in the effective temperature of the red edge of the RR Lyrae gap resulting from these parameter variations is quite small ($\sim$150K). This is true both because the parameter variation causes only small changes and because large changes in the convective flux are required to produce any significant change in red edge location. The possible changes found are substantially less than the $\sim$600 K required to change the predicted helium abundance mass fraction from 0.3 to 0.2.

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INTRODUCTION

Recently, Deupree (1977b) calculated the location of the red edge for RR Lyrae variables and concluded that the color width of the instability strip was a sensitive function of the helium abundance. This property is emphasized by the fact that the color width appears to be virtually independent of any other property: mass, luminosity, opacity, and metal abundance (at least for normal Population I or II compositions). Comparison with the observed color width of the RR Lyrae gap in a number of globular clusters indicates that $Y \sim 0.3$ with little, if any, variation from cluster to cluster.

This result has come into conflict with the helium abundance implied ($Y \sim 0.15$) by the horizontal branch semiconvective models of Sweigart and Gross (1976) when used in comparing the number ratio of red giants to horizontal branch stars.

We have calculated models varying the parameters of the eddy viscosity approach in an effort to discern the limitations of the method and the resulting accuracy of the helium abundance determination. We shall discuss first the basis of the eddy viscosity concept and then the results of the parameter study.

THE EDDY VISCOSITY MODEL

Convection in stars is expected to be highly turbulent because of large length scales and low viscosities. A simplified picture is that "large scale" convective cells form and break up into progressively smaller cells until the size reaches a scale at which the kinetic energy of the cell can be converted into heat. Our model assumes that this conversion is the sole function of suitably small scale elements. They may thus be considered to act as an effective (or eddy) viscosity on the large scale elements. Clearly, the accuracy of this assumption should depend on the dividing line between large and small scales.

The adaption of an eddy viscosity and the restriction to two spatial dimensions effectively "define" the theory of convection. As one of our colleagues has pointed out, the theory of convection <u>is</u> the Navier-Stokes equations. Making the calculation two-dimensional understandably results in a great savings of computer time at the expense of some degree of physical reality. Since full scale three-dimensional calculations are quite rare, it is difficult to develope a feeling for how erroneous this approximation might be in any given situation. Since even people with enormous (by astronomical standards) amounts of computer time use two-dimensional codes, one can draw conclusions about 1) the great difficulty and computer requirements of the three-dimensional problem and 2) the relative success of the two-dimensional codes in obtaining meaningful results. The mood regarding two-dimensional results of most workers in this area may perhaps be defined as cautious optimism.

Our eddy viscosity model is about the simplest possible - a one parameter model. Much more sophisiticated models have been derived, but usually at the expense of physical logic and with only mixed numerical success. The one parameter is the eddy viscosity. One other parameter that is usually included in our approach is the width of the convective cell. If the mesh is fine enough to allow the convection to chose its own width (Deupree 1975), the resulting ratio of width to depth is always about the classical value of three to one (Spiegel 1960; Schwarzschild 1975). This is generally regarded as a artifact of the two dimensional restriction.

PARAMETER STUDY

Because of the uncertainty regarding the convective cell width selected by two-dimensional calculations, we regard both the cell width and the eddy viscosity as free parameters. For the eddy viscosity model to make sense, only roughly a factor of ten variation in the eddy viscosity coefficient (defined in Deupree [1977a]) is allowed. We have computed convective steady state models covering this range for a model with the properties of the Goddard model except that $M=6M_{\odot}$. The largest fraction of the energy transported by convection is given for various viscosity coefficients in Table 1. There is hardly any dependence of the convective flux on the eddy viscosity coefficient.

Table 1 - Eddy Viscosity Coefficient Effects

Eddy Viscosity Coefficient	Peak Convective Flux Fraction
0.5×10^{-5}	.21
1.5	.21
5.0	.17

The variation of convective flux as a function of horizontal cell width is examined in Table 2 for the 6500K RR Lyrae model discussed by Deupree (1977c). There is more variation in the convective flux with this variable, but the amount is not enormous. One might conclude that the red edge is $\lesssim$ 150K cooler than originally found if the cell width to depth ratio departs significantly from three to one. This would correspond to a decrease in the Y deduced of about 0.03.

Table 2 - Cell Width Effects

Cell Width to Depth Ratio	Peak Convective Flux Fraction
0.9	.08
3.0	.13
9.0	.09

DISCUSSION

The parameter study indicates that the variations of convective flux with eddy viscosity coefficient are small. This is not always true in other applications and probably results from the lack of boundary layers in the astrophysical problem. The variations with cell width are larger, but the change in location of the red edge can be expected to remain small as large changes in convective flux near the red edge produce only small changes in the effective temperature of the red edge. We would estimate that Y $\gtrsim$ .27 in the RR Lyrae envelopes and that, with the results of Deupree *et al*. (1978) probably Y $\gtrsim$ .25 for the main sequence stars in globular clusters.

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Discussion

Sweigart: The value of 0.15 that you quoted for the helium abundance when semiconvection is taken into account assumes that the ratio R of horizontal-branch to red-giant-branch stars is about 1. Renzini has recently reanalyzed the observed number counts and has found R to be typically about 1.6. This larger value for R increases the helium abundance that you obtain with semi-convection to approximately 0.22 with an uncertainty of 0.04. Such a helium abundance is in agreement with the lower limit you indicated.

Cole: Yes, when this [calculation] was done the abundance was taken from Iben's formal equations, which yield the 0.15.

Sweigart: The original Iben work gave 0.3 for R = 1. When the effects of semiconvection are included in the simplest way, i.e., by just doubling the horizontal-branch lifetime, the helium abundance is reduced to roughly 0.15.

Pel: Could you comment briefly on the observed fact that the red edge seems to move toward lower temperatures as you go to higher luminosity? In other words, the instability strip seems to get wider at the longer periods.

Cole: In our calculations, the width does not depend on the luminosity. From very well determined red and blue edges of globular clusters in the galaxy, a helium abundance of 0.3 is obtained. For other places with other helium abundances, our calculations do not have any real relevance.

J. Cox: I think it is very nice that your results for the fraction of the flux carried by convection are so insensitive to the coefficient of eddy viscosity. Are there any other parameters to which the results are insensitive?

<u>Deupree</u>: Essentially, there are five things that we input. Cole talked about two of them; the other three are essentially equivalent. One is the usual coefficient of artificial viscosity, and the other two are stability parameters which have the same effect as the artificial viscosity coefficient. So you do what you always do, turn them as low as you possibly can. You make them high enough to keep your program glued together, but otherwise you make them small. As long as you have them "small," then the results are insensitive to them. The two Cole talked about are the only ones in which you have any leeway at all. The fact that there are no boundary layers in the classical convection sense, really determines the fact that the results are insensitive to the eddy viscosity coefficient.