THEORETICAL MEAN COLORS FOR RR LYRAE VARIABLES

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ABSTRACT

A hydrodynamically pulsating 0.6 $M_\odot$ model of a typical RR Lyrae variable has been studied with a radiation transport-hydrodynamic computer program to predict theoretical T_e and colors at many phases and to find the proper methods for getting mean colors and the consequent mean effective temperatures. The variable Eddington radiation approximation method was used with gray and with multifrequency absorption coefficients to represent the radiation flow in the outer optically thin layers. Comparison between observed and computed B-V colors indicate that these low Z Population II models are reasonably accurate using King 1A composition opacities. The well known Oke, Giver, and Searle relation between B-V and T_e is reproduced. Mean colors are found by four different averaging methods. The method that gives a mean color and the mean T_e closest to the nonpulsating model was the separate intensity means of B and V, just as the case for previous studies of classical Cepheids. The best mean for T_e, which is known for all pulsation phases from four color observations of real RR Lyrae variables or from the calculated model, is a time average of T_e without any weighting function.

I. INTRODUCTION

The problem of obtaining the mean color of RR Lyrae stars, in order to get the correct non-pulsating temperature, is similar to that for Cepheids (Cox and Davis 1975). In our nonlinear dynamic models we know the original nonpulsating temperature, and by using a multifrequency snapshot approach with the appropriate filter responses, in this case for the UBV filters, we can study various ways to obtain mean colors. In this paper we study a model using nonlinear gray and multifrequency hydrodynamic transport calculations and Population II ($Y = .299$, $Z = .001$) King 1A mixture opacities. The discussion concerns the taking of color averages as well as the question, if we know the effective temperature at many phases, how best to obtain an average for the non-pulsating temperature of the model.

II. METHOD

The radiation flow for our RR Lyrae model is treated by a non-equilibrium diffusion approximation where the radiation field is not directly coupled to the material energy field as in the equilibrium diffusion approximation. The method limits to equilibrium gray diffusion in optically thick zones and to streaming in optically thin zones. The forward peaking of the radiation field is correctly described using variable Eddington factors. In our model we use a plane geometry characteristic ray calculation for the Eddington factors at each time step and for each frequency group. The multifrequency calculation is carried out for 13 frequency groups selected so that ionization edges and the Planck function emission are well resolved (Davis 1971). Some attempt has been made to include effects due to line radiation as perceived to be important by Mihalas (1969). This effect is included using a formulation proposed by Cassinelli and described in Davis (1978).

Effects on colors due to shock waves transiting the atmosphere are approximated using a number of optically thin zones outside the photosphere and the Richtmeyer-Von Neuman method of pseudoviscosity. The phase of shock transiting occurs between 0.4 and 0.5, where phase 0.5 is approximately the phase of peak luminosity. There is some evidence that a UV excess occurs during this phase to affect the continuum colors (Davis 1975). Conditions for hydrogen line emission and Ca line doubling do exist during this phase (Hill 1972).

III. MODEL

The selected mass is 0.6 $M_{\odot}$, and luminosity ($\log L/L_{\odot} = 1.6$) is equivalent to an $M_{bol} = 0.72$ where $M_{bol\odot} = 4.72$. This is in reasonable agreement with Oke, Giver, and Searle (1962) estimate for SU Dra. A $T_{eff} = 6840$ K ($\log T_e = 3.835$) then gives a fundamental period of 0.44 days. The inner radius of the model is less than 10% of the photospheric radius. No convection is allowed as appropriate for this T_e value (Deupree 1977). A zero pressure hydrodynamic boundary condition is applied.

In our structure calculations we have found that 72 zones with 5-10 zones in the optically thin atmosphere are sufficient to resolve the luminosity curve. Some noise still remains near light minimum when the ionization front has approached to 1 or 2 zones from the star's surface. The shock escaping during the phase near light minimum through light maximum results in the observation of H_{γ} line emission. In our models we have not resolved the detailed atmospheric structure during this phase (Hill 1972). The effects therefore of line emissions during the phase of light minimum to rising light are not treated exactly but the effects on the colors are expected to be small.

Spectra using the calculated structures at many phases and 30 different frequency groups are convolved with the B and V filters. Raw colors b, v are corrected for their relative transparency by the formula

$$B - V = b - v + 0.65$$

as for the classical Cepheids considered previously by Cox and Davis (1975).

Figure 1 shows the calculated M_{bol} for the model where the variation is > 1.8^m. This is larger than the usual observed range of V for RR Lyrae stars (~ 1^m). Figure 2 shows the variations of T_{eff} versus phase which go from approximately 5600 to 8700 K. In Figs. 3 and 4 we show the radius and velocity variations as calculated. The variation in radius is like 10-15% around a value of 4.5 $R_\odot$. The observed range in velocity is 62 km/s. The lower line is the velocity calculated at $\lambda = 4404$ Å at an optical depth of 0.10 (the location of the metal lines).

IV. RESULTS

The transformation formula for the conversion from B-V colors to T_{eff}, as derived by Oke, Giver and Searle (1962) for SU Dra with the assumption that the star is unreddened is $\theta = 0.62 + 0.51$ (B-V) where $T_{eff} = 5040/\theta$. We use this formula for our comparisons. In Figs. 5 and 6 we plot the OGS relationship against the calculation for the gray and multifrequency structures, respectively. There is

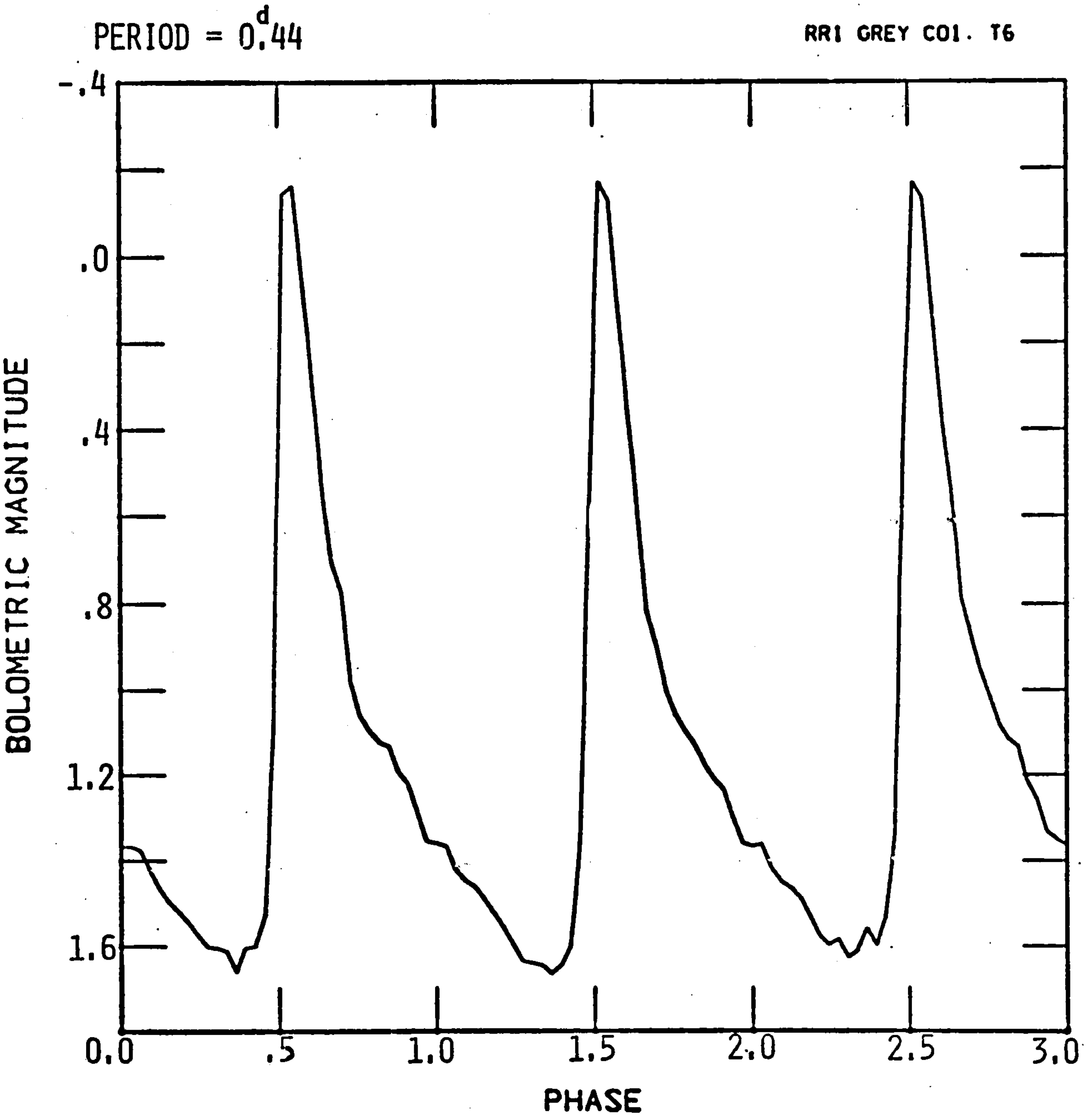

Fig. 1. Calculated bolometric magnitude for three periods ($P = 0^d.44$). Note wiggles near light minimum (see text).

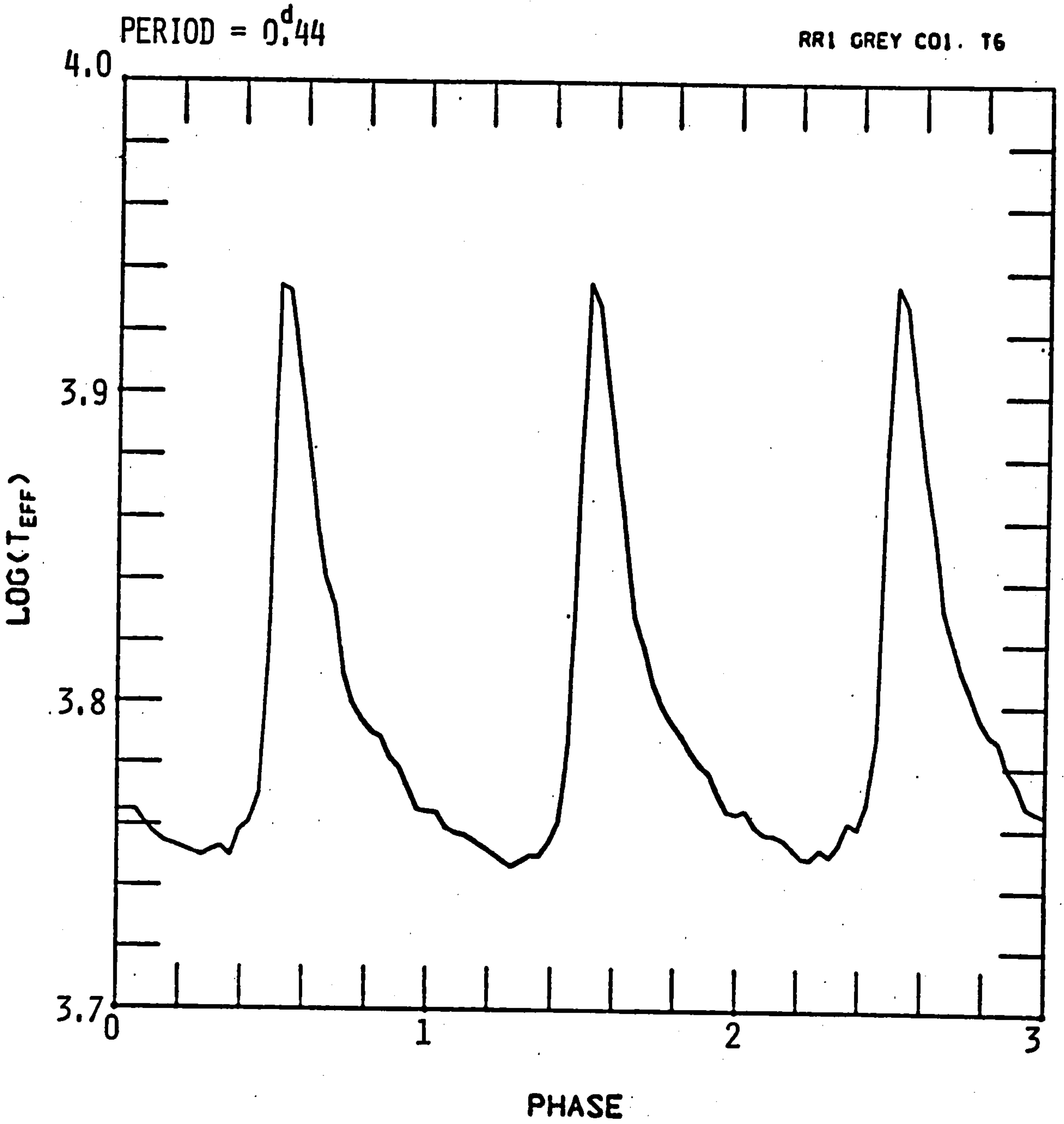

Fig. 2. Log T_{eff} versus phase with variations in T_{eff} calculated as 5600 - 8700 K.

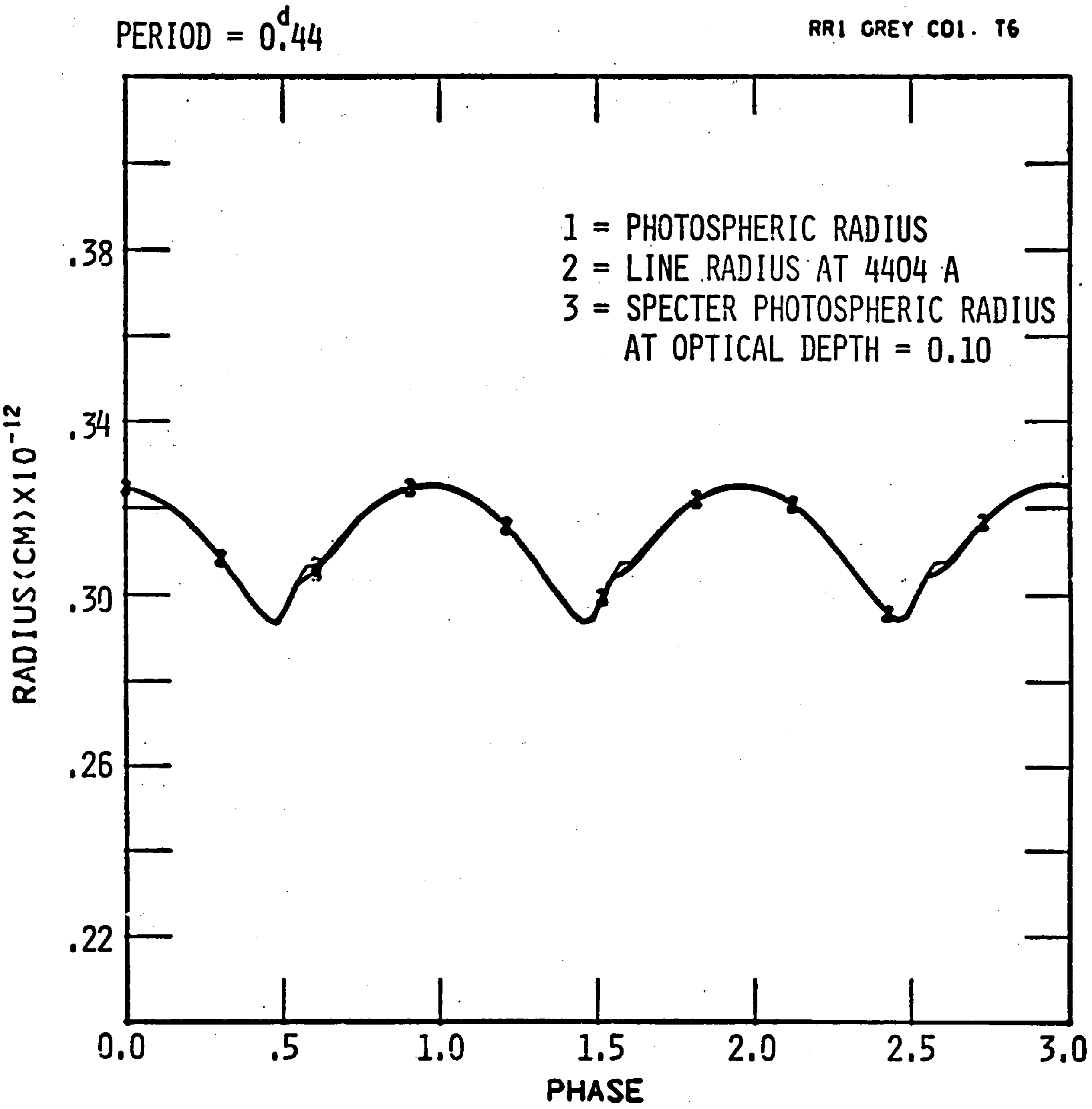

Fig. 3. Calculated photospheric radius ($R_{eq} \cong 4.5\ R_{\odot}$).

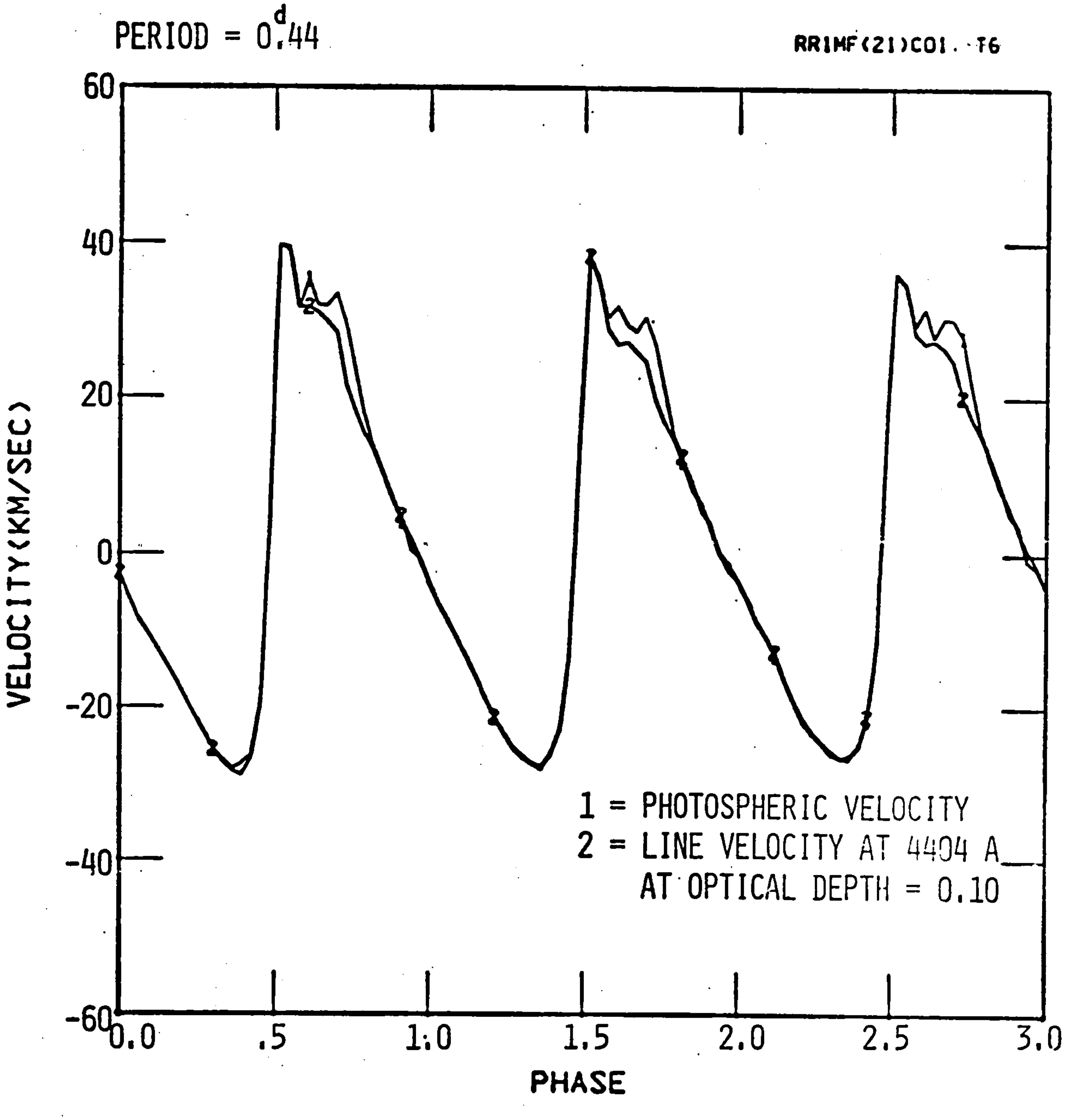

Fig. 4. Calculated velocity at $\tau = 2/3$. 2 = line velocity at 4404 Å, $\tau = 0.10$.

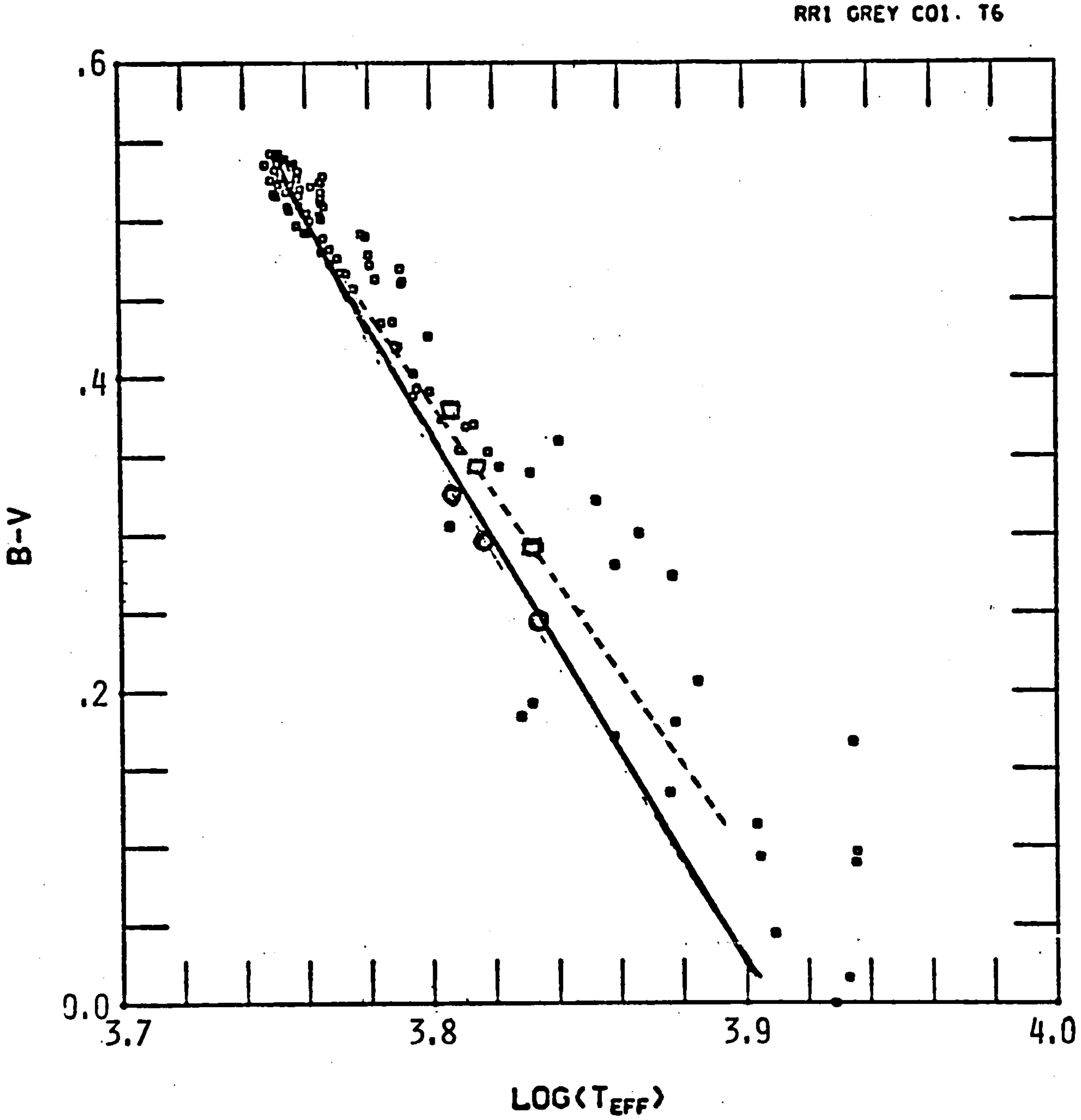

Fig. 5. B-V versus log T_{eff} calculated using gray transport structure. Solid line shows Oke, Giver, and Searle relation. Calibration values (O) for static models at $L = 38\ L_{\odot}$ and $T_{eff} = 6800$, 6500 and 6350 K are plotted. Squares (□) are calibration values with convection. Dashed line is fitted to Kurutcz's latest results (unpublished).

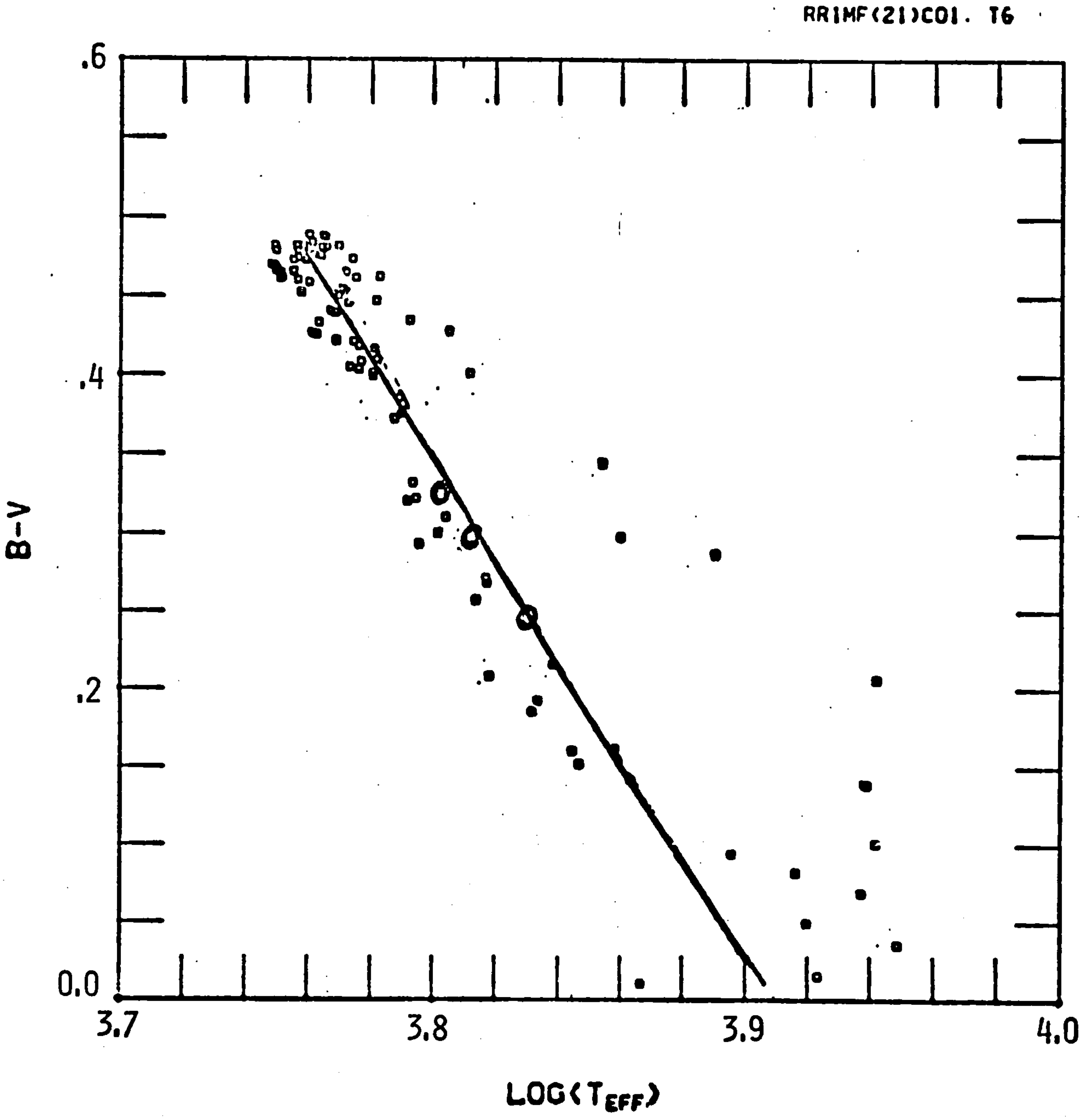

Fig. 6. B-V versus log T_{eff} calculated using MF/G transport structure. Solid line shows the Oke, Giver, and Searle relation. Calibration values (0) for static models at $L = 38\ L_\odot$ and T_{eff} = 6800, 6500, and 6350 K are plotted.

an improvement in slope for the multifrequency structure as anticipated by Cox and Davis (1975).

The various averages of B-V that we obtained are shown in Table 1. Averages over the three periods, for the intensity means $\langle B\rangle_{int} - \langle V\rangle_{int}$, for the gray structure which is essentially the same for the multifrequency structure is 0.264. The average T_{eff} calculated from the OGS relation is 6678 K within 162 K of the model T_{eff} (6840 K). The other means are at least 600 K low. The static fine zoned radiative model gives a (B-V) of .244 and an OGS T_{eff} of 6770. In Fig. 6 we also plot calibration points for static models with $L = 38\ L_{\odot}$, and 6800, 6500 and 6350 K effective temperatures.

Direct averages of T_{eff} were obtained using 100 phases over three periods for a more reasonable amplitude variation of RR Lyrae ($\sim 1.1^m$) and for our King 1A large amplitude model (1.8^m). T_{eff} is determined from the relationship:

$$L = 4\pi R_*^2\, T_{eff}{}^4$$

where L is the multifrequency transport calculated luminosity and R_* the photospheric radius determined at $\tau = 2/3$. The weightings used were none, direct weighting on L and a weighting similar to that used by Lub (1977), i.e.,

$$\theta_{eff}^{eq} = [(L/\langle L\rangle)^{1/2}\ \theta_{eff}^2]^{1/2} .$$

The results for the 1.1^m model (Fig. 7), averaged over three periods are: T_{eff} = 6800, 7115, and 6670 K, respectively. Only the direct luminosity weighting is well out of line, being high by 300 K, but

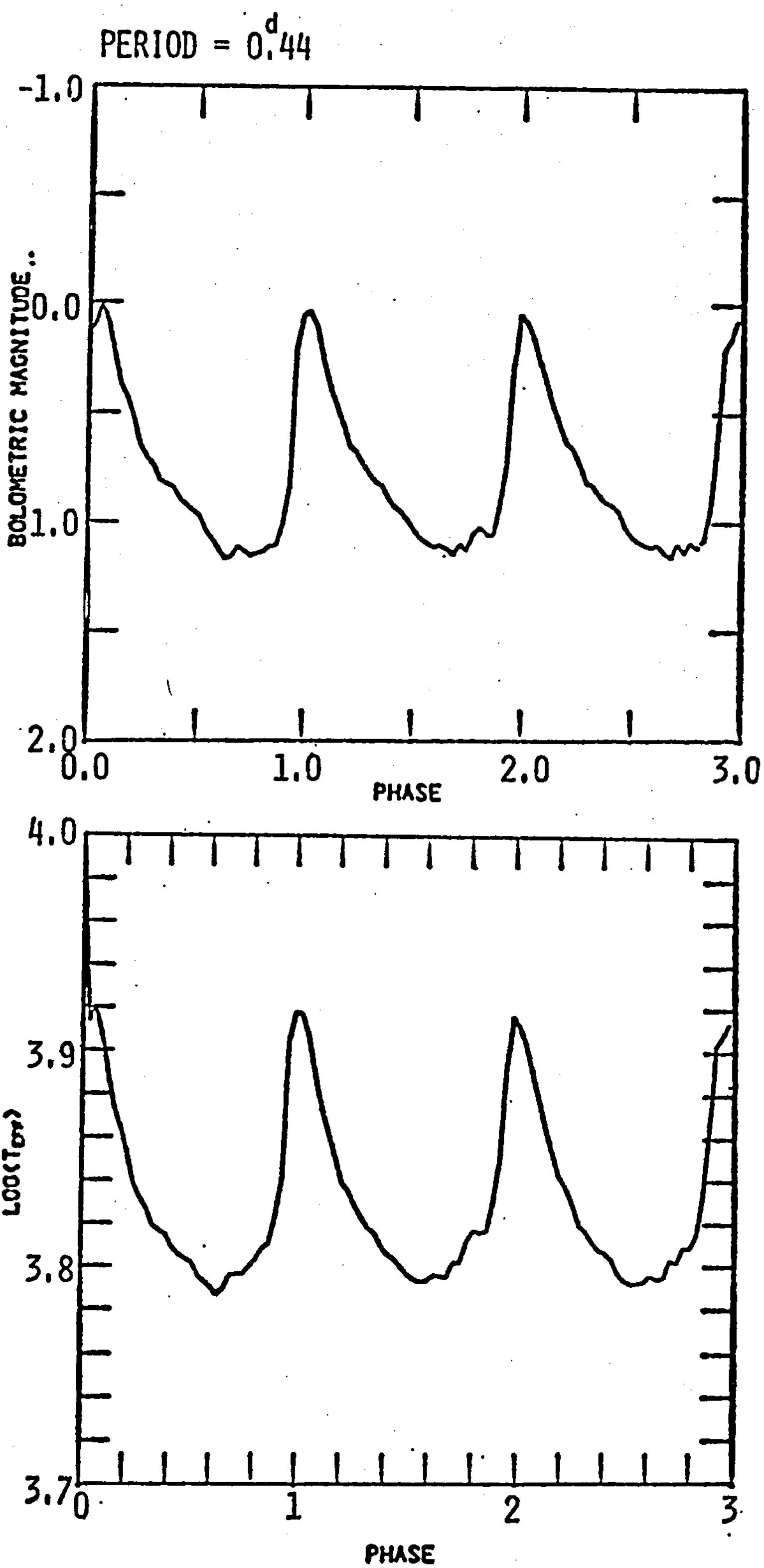

Fig. 7. Bolometric magnitude and log T_{eff} for an "observed amplitude" RR Lyrae used to obtain averages of T_{eff} over phase. (This model used opacities that we believe are too high).

TABLE 1

	B-V	T_{eff}(K)	T_{eff}(K)
$\langle B-V\rangle_{mag}$	.3907	6151	-689
$\langle B\rangle_{int} - \langle V\rangle_{int}$	.2640	6678	-162
$\langle B-V\rangle_{int}$	.4018	6108	-732
$-\langle V-B\rangle_{int}$	.4532	5925	-915

using the Oke, Giver, and Searle relation

$\theta_e = 0.62 + 0.51(B-V)$ where $T_{eff} = 5040/\theta_e$

the unweighted mean is the best. For our larger amplitude King 1A model the direct T_{eff} average is 500 K low and the Lub average was not attempted.

V. CONCLUSIONS

The slope of our (B-V), log T_e relation is close to that given by Oke, Giver, and Searle if one uses a full transport solution for the atmospheric structure. It appears that an intensity mean on B and V is the most appropriate mean to use for RR Lyrae stars as for Cepheids. From $\langle B\rangle_{int} - \langle V\rangle_{int}$ we obtain a calculated T_{eff} within about 160 K of the known nonpulsating T_{eff}. A model calculated that agrees in amplitude variations with RR Lyrae implies that a direct time average of T_e is preferable to any other weighting.

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Discussion

Baker: Can you say anything in a general way about how the Cox-Davis opacities compare to the Cox-Stewart opacities?

Davis: They seem to about double the opacities in the region below 5000°K, and we're not ready to say what the reasons are. But it appears to be a difference in the treatment of the line wings from that used in the original Cox-Stewart opacities. The Cox-Davis opacities are a great improvement because molecules are included, so that the line blanketing treatment for Cepheids is improved. We found good agreement, where we didn't without the Cox-Davis opacity, in Cepheids. Here for RR Lyrae stars we didn't expect it to make much difference, and it made a big difference, so we were surprised. We're studying the question of what really went into the Cox-Davis opacities.

Spangenberg: That opacity effect might be subject to the effect of zoning when you're doing the low photoshere. Your temperature structure could be quite a bit different if you had significantly more zones in one case than in the other. But if you kept the number of zones the same, then you would lose resolution in the one case, but the opacity would be adding a lot of temperature-dependent features which would get lost in the zones. If you changed the atmospheric zoning, you would understand these opacity effects better.

Davis: The zoning was done in the same manner as for the Goddard Cepheid model -- we used 72 zones with a 10% inner radius. Art has looked at the opacities in this region and there is a difference of a factor of two. Your point is well taken concerning the position of these effects.

Spangenberg: When you're trying to estimate the optical depth in order to get T_{eff}, it could be quite zone-dependent.

A. Cox: You're right, Bill, but unfortunately we seem to have gotten into a glitch and we hope it will be straightened out soon.

Davis: The new opacities did improve the amplitude . . . I wish we could keep those opacities.

A. Cox: Do you think your amplitudes will decrease if you use a non-zero boundary pressure?

Davis: No. It just disturbs my light curve. I did not try the Castor boundary condition.

A. Cox: I'd like to elicit something from Pel about criticisms we have on how you take your temperature means.

Pel: What we did is very simple. As soon as you have the temperature and radius variation, it is clear how you have to average. If you assume the luminosity mean over the cycle is exactly the time average over the luminosity curve, and the radius mean is approximately the time average over the radius curve, then the Stefan-Boltzmann relation tells you what the temperature mean is. Where we may come out with different results is that our definition of the mean radius is not exactly where the equilibrium radius was. I think that's all the play there is in the definitions, and I'm a bit surprised that there is a difference of about 190°K for the RR Lyrae stars. Did I hear that correctly?

Davis: 170°K cooler than T_{eff}.

A. Cox: He took three means. One was the time average, one weighted with the luminosity (which weights the higher temperatures more), and then your technique.

Pel: You would prefer the intensity time average of the individual bands and we would prefer the average, not in the colors, but in the temperature and gravity curves, working back to see what that meant in the colors. That is a little bit closer to the straight time average of the color itself.

A. Cox: I should tell the audience that he [Pel] is talking about Cepheids, whereas Davis was talking about RR Lyrae stars.

Pel: Yes, but this recipe works for both.