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Some Evolutionary Considerations of  
 $\beta$  Cephei Stars

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ABSTRACT

It is pointed out that the evolutionary characteristics of massive population I stellar models undergoing mass-loss and angular momentum loss do not favour an interpretation of the  $\beta$  Cephei phenomenon related to semi-convection or to the non-radial oscillations connected with semi-convection. They are not contradictory, however, to the interpretation that the phenomenon can be understood as a manifestation of Kelvin-Helmholtz instability arising from the differential rotation due to a faster rotating interior and an external layer or surface that has lost most of its angular momentum.

## I. INTRODUCTION

In a clear and comprehensive review article Lesh and Aizenman (1978) have pointed out that "an apparently ordinary class of early type stars like the  $\beta$  Cephei objects defies physical explanation despite three quarters of a century after their discovery." It appears that some of the observational features such as rotation, the mode of pulsation etc. are still subject to either uncertainties or controversy. Although a number of such objects have been discovered and studied, clear cut statements about the nature of their pulsation also appear to be difficult from a theoretical point of view. The theoretical aspects of these objects have been also comprehensively reviewed recently by Cox (1976) and Kato (1976). Thus Lesh and Aizenman (1978) conclude that "if a reasonable instability mechanism could be found that applied only to a certain evolutionary state and to one or two pulsation modes, most of the controversy concerning the interpretation of the observations would be quickly removed." We refer the reader to the excellent review articles by Lesh and Aizenman, Cox and Kato for an observational and theoretical picture of these objects. However, we would like to stress that the theoretical interpretation of these objects relied in the past on evolutionary models which did not take into account either semi-convection or mass-loss satisfactorily. Although it cannot be said at the present time that this shortcoming has been rectified fully to everyone's satisfaction, new models of evolutionary sequences of massive population I stars have been published recently by several groups which take into account semi-convection and mass and angular momentum losses as best as one can at the present time. (Chiosi, Nasi & Sreenivasan, 1978; de Loore et al., 1977; Dearborn et al., 1978; Sreenivasan & Wilson 1978a; and

Sreenivasan & Wilson, 1978b, 1978c and 1978d). Further observational studies are also available that have not been included in the review by Lesh and Aizenman, e.g. Smith and McCall (1978) and a theoretical study of the non-radial oscillations of rotating stars that has a bearing on the problem of  $\beta$  Cephei stars has also been reported recently by Papaloizou and Pringle (1978). In the light of these developments it seems pertinent to ask what evolutionary constraints are placed on the interpretation of the  $\beta$  Cephei objects.

## II. EVOLUTIONARY ASPECTS

We have recently completed a series of studies on the evolutionary aspects of massive population I stars and compared our results with those published by the Belgian group of de Loore and others. We have treated semi-convection in two different ways, and included mass-loss both in the early as well as later spectral stages. We have also taken the effect of rotation into account recently and studied the time evolution of angular momentum of these models, and further discussed the effect of differential rotation due to the faster rotating interiors and slowed-down surface layers. For details we again refer the reader to the papers mentioned in section 1. The results are summarised in the accompanying diagrams.

Figure 1 shows the evolutionary tracks of models of 15, 20 and 30 solar masses. The 15 and 30  $M_{\odot}$  tracks employed Paczynski's code as modified for our purposes. The 20  $M_{\odot}$  track used the code developed by Hofmeister et al and modified to take account of semi-convection and mass-loss by Chiosi. Chiosi treats semi-convection as described by Chiosi and Summa (1970). The mixing is treated mechanically until a stability criterion is satisfied. We

have treated semi-convective mixing as a diffusive process and described our procedure in Sreenivasan and Ziebarth (1974) and Sreenivasan and Wilson (1978a). Thus semi-convection has been treated in three different ways and except for minor quantitative differences the principal conclusion is that semi-convection almost disappears in models that are losing mass in the early spectral stages. This result has been clarified by different procedures for calculating the amount of mass lost by the models (Chiosi & Nasi, 1974, Chiosi et al, 1978 and Sreenivasan & Wilson 1978 a-d). Chiosi, Sreenivasan and Wilson therefore came to the conclusion that the  $\beta$  Cephei phenomenon cannot be attributed to the presence of semi-convection or to the non-radial oscillations that have been suggested to cause semi-convection (Gabriel et al 1976). Any possible uncertainties about either the nature of the phenomenon of semi-convection (see Spiegel for a review of this aspect: 1971, 1972) or reservations about dealing with it (Lamb et al 1976) therefore need not concern us on this account.

If these  $\beta$  Cephei objects are found to be slow rotators, and if the evidence for separate rising and falling shells and evidence for mass-loss found by Smith and McCall (1978) is a general feature of these objects, we might then ask whether this is in agreement with the evolutionary picture of stars in the mass range  $15-30 M_{\odot}$  undergoing mass-loss and angular momentum-loss. We have shown that most of the angular momentum of stars in this mass range in the outer layers is lost well before core hydrogen has been exhausted. This is more so if you take an averaged  $k^2$  for the calculation of the moment of inertia  $I = M k^2 R^2$ , where  $M$  is the mass and  $R$  the radius of the star. It is also true if you include macroturbulent pressure due to differential rotation. We would like to emphasize that the actual mass-loss

rates used may be subject to uncertainties by an order of magnitude but that does not change the qualitative result that the outer layers spin down before hydrogen has been fully converted into helium in the core.

Smith and McCall (1978) suggest that the  $\beta$  Cephei star  $\gamma$  Pegasi is a slow rotator and that the spectral variations in this star are produced by radial pulsation. They also cite evidence for weak mass loss.

We have plotted the observed  $\beta$  Cephei stars given in the review article of Lesh and Aizenman on the same diagram as the one depicting the evolutionary tracks. One star happens to lie on the S-bend suggested by Lesh and Aizenman (1973,4). Three stars lie below the tracks of the  $15 M_{\odot}$  model on the main sequence. All the stars in their Table 2 lie in the field occupied by the  $15$  and  $30 M_{\odot}$  tracks except for the three mentioned above. Nine of these stars are very close to tracks that are made by models with initial masses  $16$  and  $17 M_{\odot}$ . The three stars below the  $15 M_{\odot}$  track are probably in the mass range  $10-12 M_{\odot}$  and could be reached by evolutionary models showing very low mass-loss rates, lower than the threshold of observations by Copernicus. On the other hand, the minimum mass at which semi-convection appears unambiguously is in the neighbourhood of  $14 M_{\odot}$  (Sreenivasan and Ziebarth 1974) or  $13 M_{\odot}$  (Barbaro et al 1972). One can therefore argue that if the stars showing  $\beta$  Cephei phenomenon are in the mass range  $10-12 M_{\odot}$ , they would not probably possess semi-convective regions but may have very feeble mass-loss rates that do not affect drastically their evolutionary patterns. Taken together, the stars less massive than about  $15 M_{\odot}$  which do not have appreciable mass-loss and the stars more massive than  $15 M_{\odot}$  and in the mass range  $15-30 M_{\odot}$  which do have appreciable mass-loss rates, and hence no semi-convective regions, would

suggest that the  $\beta$  Cephei phenomenon may not be linked to either semi-convection or effects associated with semi-convection such as non-radial oscillations. Kato (1976) argues that a number of other mechanisms proposed do not appear to be probable either.

The only suggestion that does not conflict with evolutionary constraints is then that of Papaloizou and Pringle who suggest that the  $\beta$  Cephei phenomenon is a manifestation of Kelvin-Helmholtz instability arising from the differential rotation of the interior with respect to the surface layers. We have not examined any of our models for radial pulsation but are aware of the work of Davey (1973) which suggests that these stars are stable for radial pulsations.

It is tempting to visualize a scenario which depicts  $\beta$  Cephei stars which are in the early spectral range B0 - B2 as stars which have lost a significant amount of rotation in the surface layers, subject to Kelvin-Helmholtz instability and showing the spectral variation they do, subsequently ejecting a shell or losing mass in sufficient amount to cause shell formation. In such a picture  $\beta$  Cephei objects would be the precursors of the shell stars. But clearly more work is needed both from the evolutionary point of view as well as pulsational to clarify this interesting type of sequences of evolutionary models with initial masses 16 and 17  $M_{\odot}$  on the main sequence to cover the centre of gravity of the observed group of confirmed  $\beta$  Cephei objects in Lesh and Aizenman's Table 2. It would be equally interesting and worthwhile to investigate in greater detail the consequences of Kelvin-Helmholtz instability in these objects.

### III. CONCLUSION

Stars in the mass range  $10-12 M_{\odot}$  probably do not exhibit semi-convection. Stars more massive than  $15 M_{\odot}$  in the range  $15-30 M_{\odot}$  which undergo mass-loss do not exhibit semi-convection either. If the observation that the  $\beta$  Cephei objects are all slow rotators and subject to mass-loss is substantiated, then evolutionary constraints are consistent with the interpretation that the  $\beta$  Cephei phenomenon is a manifestation of Kelvin-Helmholtz instability. But more work is required and an examination of the evolutionary features as well as a detailed study of the instability are needed to clarify the nature of these fascinating objects.

### IV. ACKNOWLEDGEMENTS

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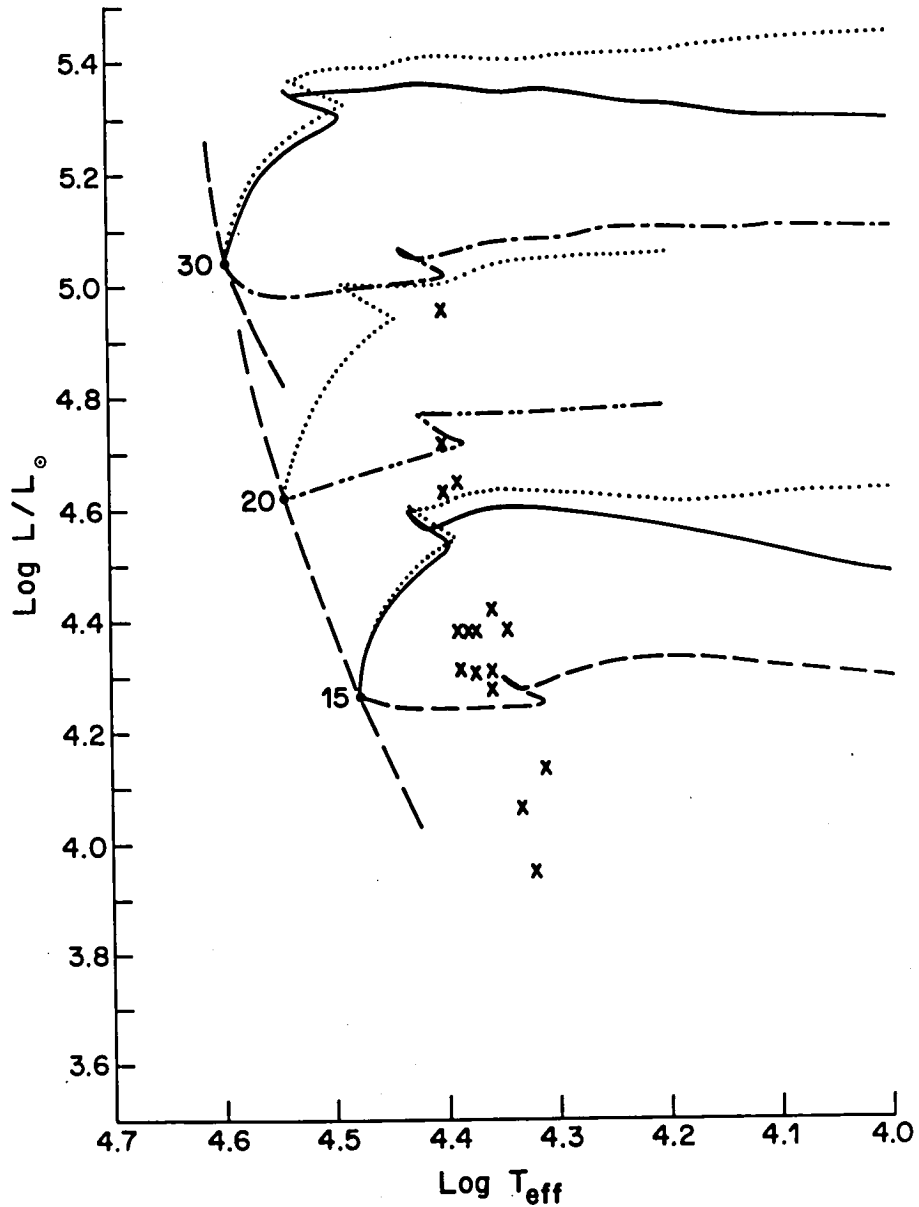


Figure 1. Evolutionary tracks for 30, 20, and 15  $M_{\odot}$  sequences near the main sequence. Solid line: evolution without mass-loss or semi-convection (sc); dotted line: without mass-loss but with sc; dashed line: with sc, and mass-loss including conservation of angular momentum; dot-dash line: with sc, and mass-loss including conservation of angular momentum; dot-to-dash line: with sc, but with mass-loss excluding rotation. The dashed line joining all sequences represents the zero age mass sequence. Crosses denote the  $\beta$  Cephei stars from Lesh and Aizenman (1978).

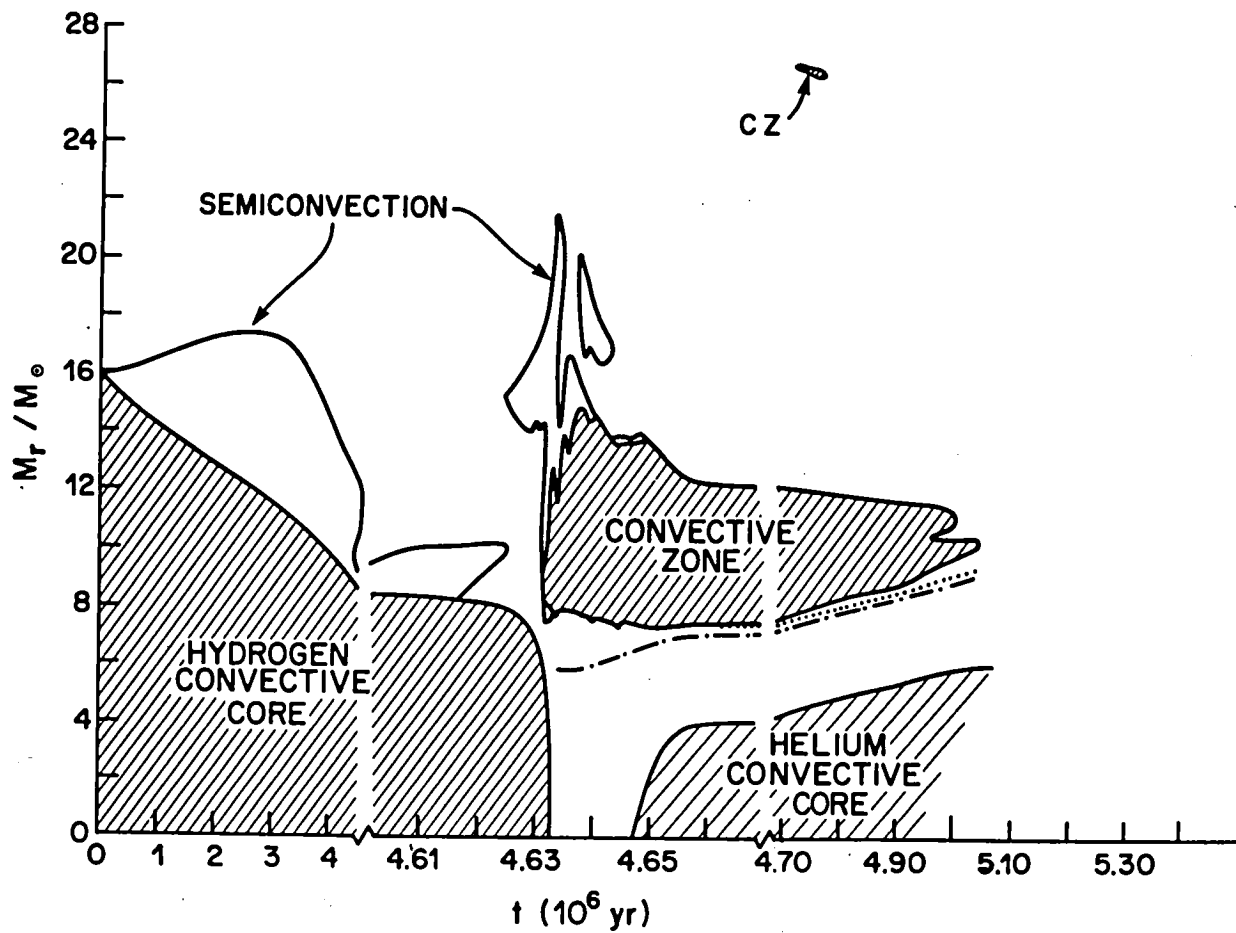


Figure 2. Location of the boundaries of fully convective zones (cz) and semi-convective zones for the  $30 M_{\odot}$  sequence without mass loss. The shell source maximum (dotted line) and the outer boundary of the helium-rich core (dash-dot line) are also shown.

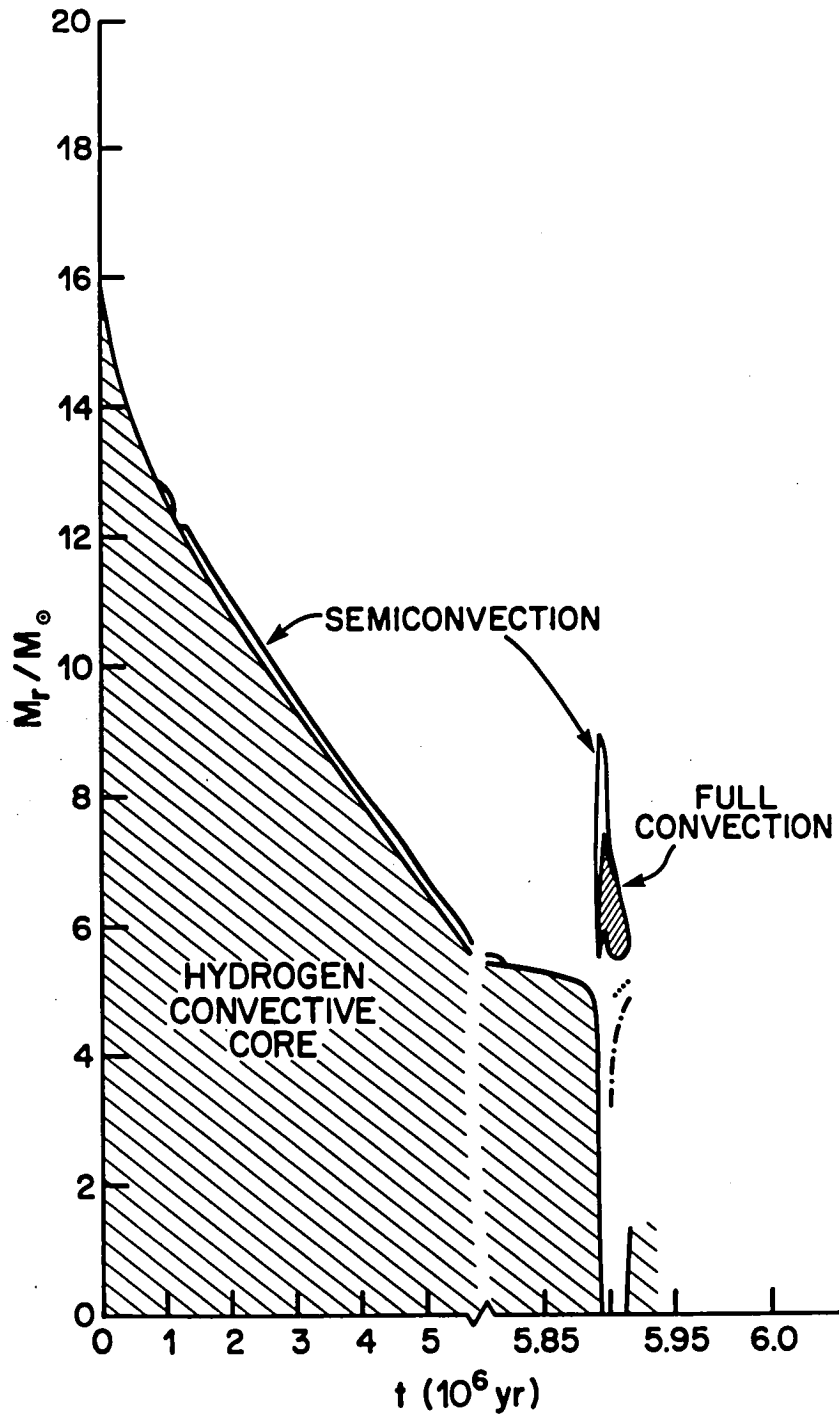


Figure 3. Boundaries of convective regions in the  $30 M_\odot$  sequence with mass-loss including turbulence and conservation of energy and angular momentum.

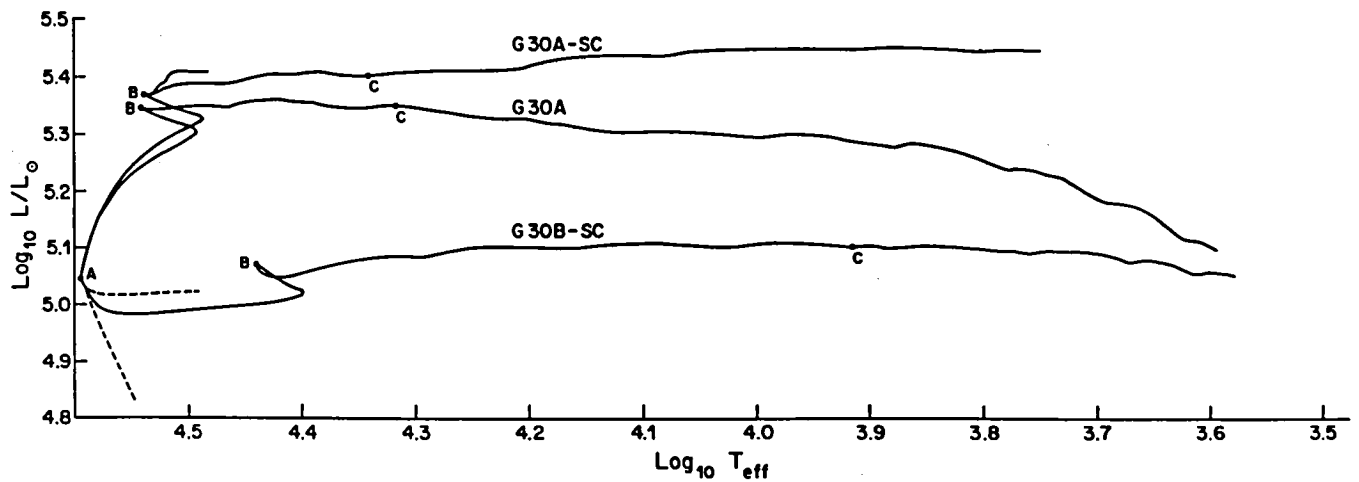


Figure 4. Evolutionary tracks for  $30 M_{\odot}$  models. 30 A-sc: evolution without mass-loss but with semi-convection. 30 A: evolution without mass-loss or semi-convection. 30 B-sc: evolution with semi-convection, and mass-loss including turbulence and conservation of energy and angular momentum.

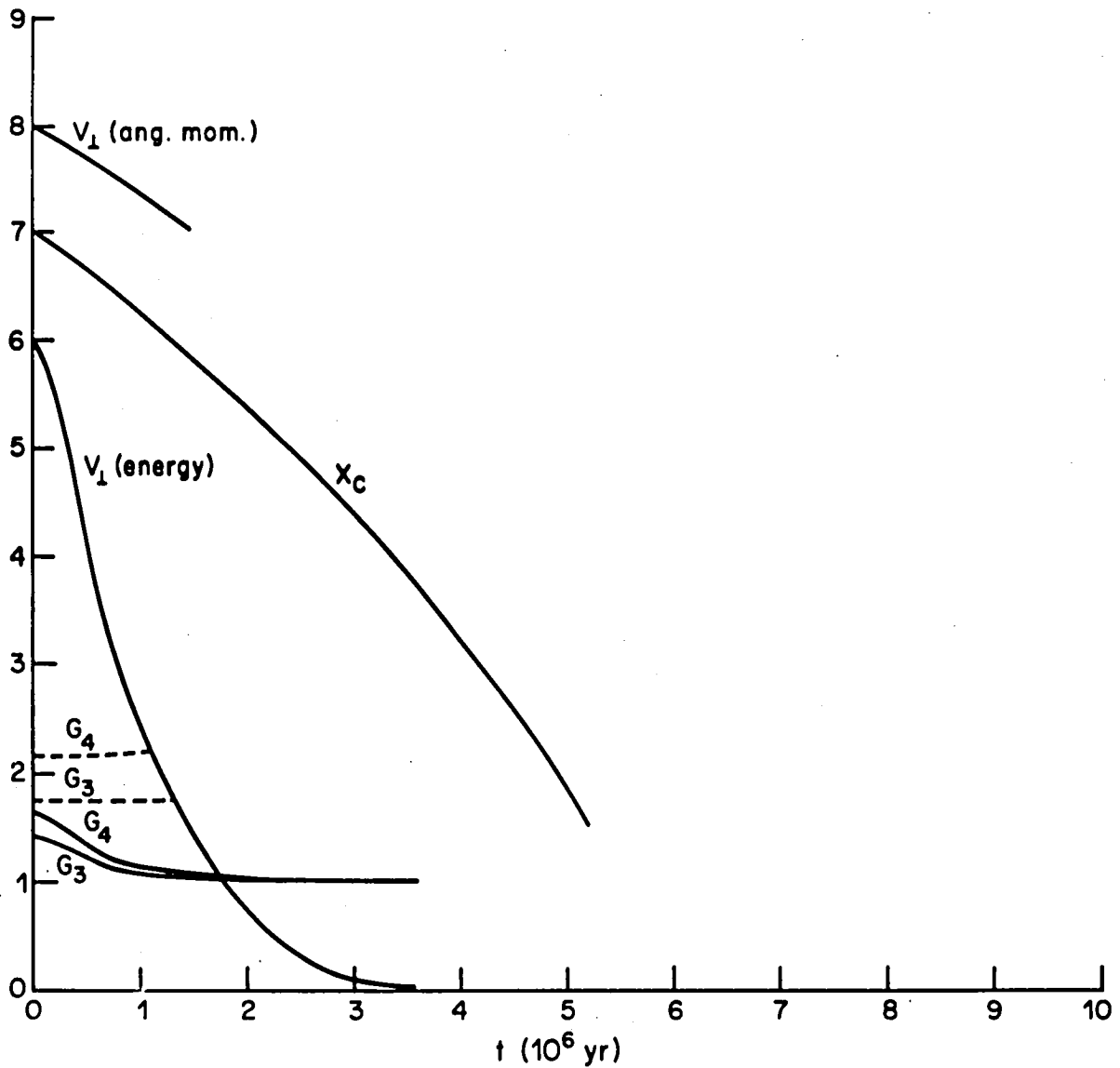


Figure 5. Plot of rotational velocity versus time for conservation of energy and angular momentum (labelled energy) for  $30 M_{\odot}$ , in units of 100 km/sec.  $G_3$  and  $G_4$  shown for the energy and angular momentum case (solid lines) and angular momentum case (dashed lines), in units of 1.0, where  $\dot{M}_4 = G_4 \cdot \dot{M}$ , and  $\dot{M}_3 = G_3 \cdot \dot{M}_2$ . The central hydrogen content,  $X_C$ , is also shown for reference, in units of 0.1.

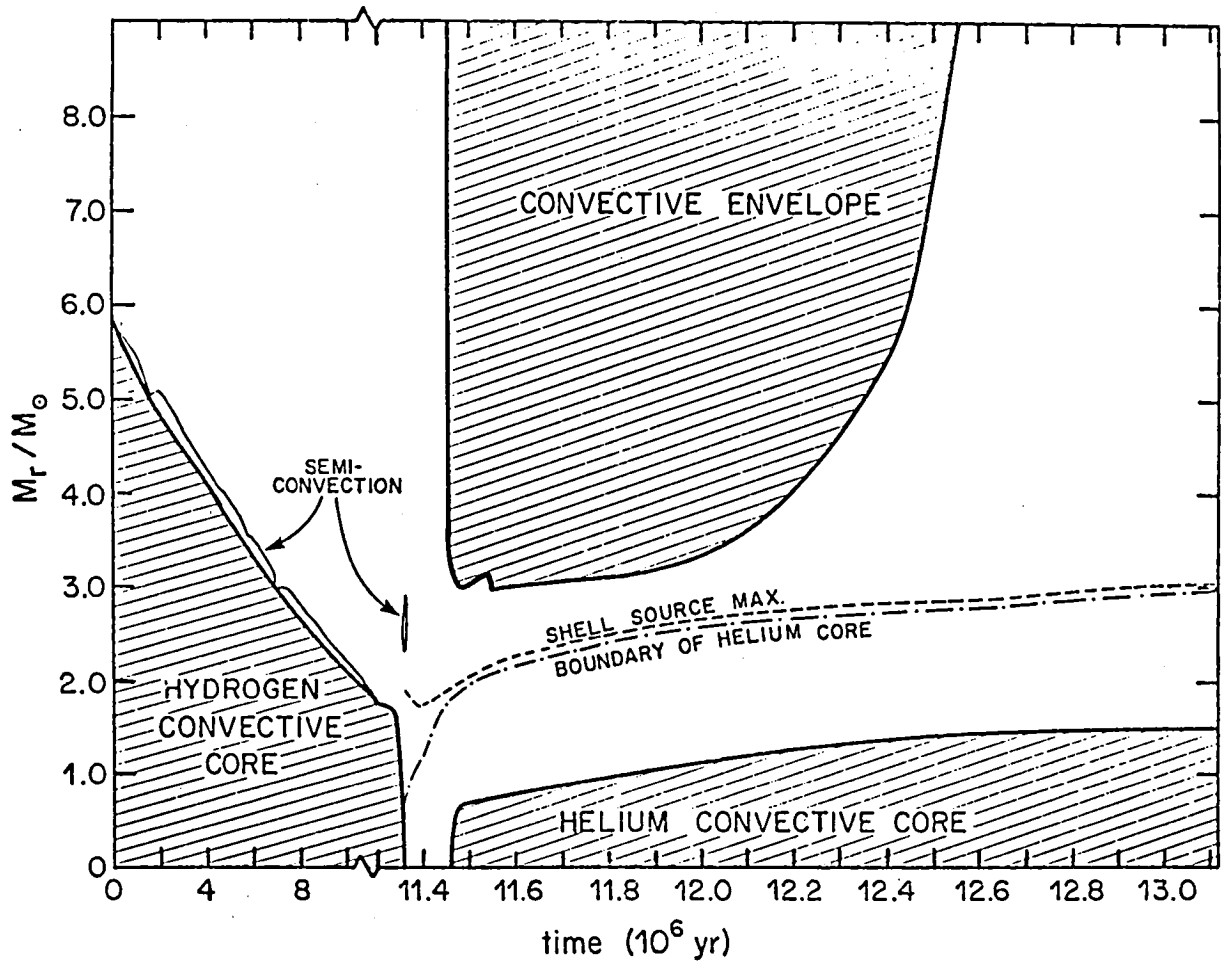


Figure 6. Locations of the boundaries of convective regions in the  $15 M_\odot$  sequence, with mass-loss including conservation of angular momentum.

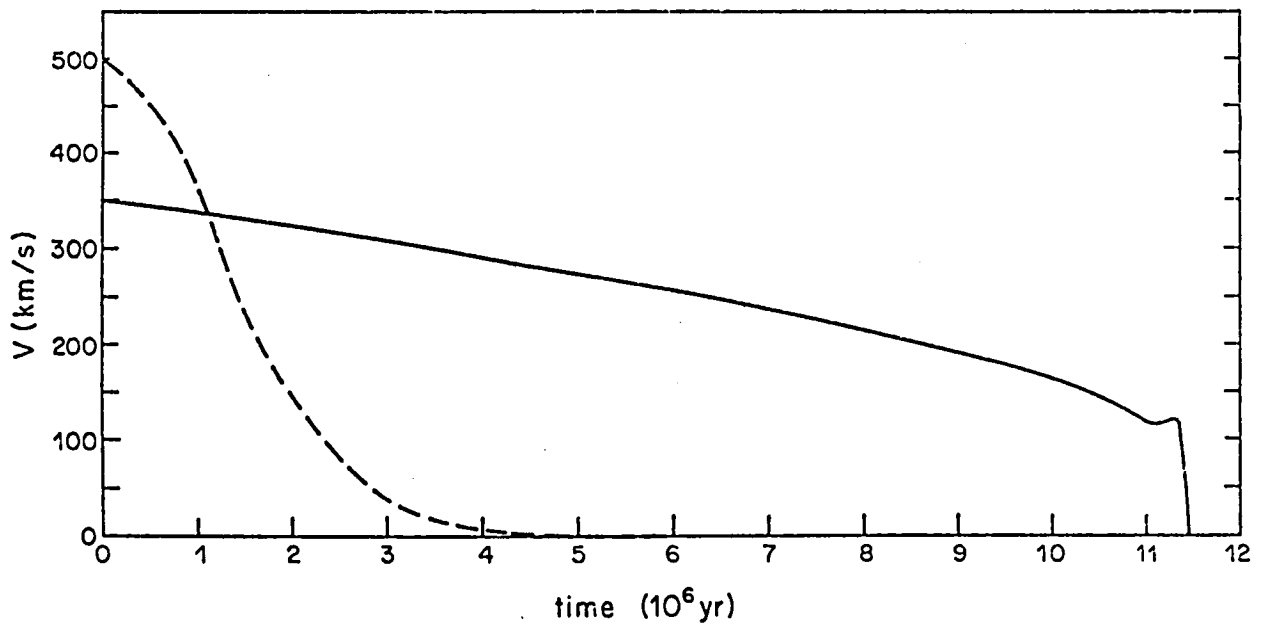


Figure 7. Plot of rotational velocity versus time for the  $15 M_{\odot}$  sequence, for mass-loss including conservation of angular momentum (solid line) and including both angular momentum and energy (dashed line). Central hydrogen exhaustion coincides with the secondary maximum in the solid line on the right hand side of the diagram.

### Discussion

Baker: Didn't Smak get a rather secure mass estimate of about  $10 M_{\odot}$  for one of these stars --  $\alpha$  Vir? (Note added in proof: The actual number he got was  $9 \pm 1 M_{\odot}$ . See Acta Astro. 20, 75, 1970.)

Sreenivasan: Possibly. If it is  $10 M_{\odot}$ , then there is no semiconvection in these objects. That is all I was saying. I was not claiming that the mass should be a certain value, but if it started on the main sequence as a  $10 M_{\odot}$  star, and then if there is any mass-loss, it will be too low to affect any evolutionary considerations.

Baker: I was just wondering what the actual tracks are.

Sreenivasan: I think  $10$  to  $15 M_{\odot}$  is the mass range that is quoted. On the other hand, if the star is losing a significant amount of mass, it may have started higher on the zero-age main sequence and come down to this region. A  $30 M_{\odot}$  star can lose about 40% of its mass by the time it exhausts hydrogen, and a  $15 M_{\odot}$  star loses 2 or  $3 M_{\odot}$ .

Lesh: I think the measurement Dr. Baker is talking about was made not by Smak but by Hanbury-Brown and his coworkers in Australia, using the intensity interferometer. Their result for  $\alpha$  Vir was  $11 \pm 1 M_{\odot}$ . I think that this error estimate is a bit optimistic, and the star may actually be a little more massive than that. But in general, the range of masses attributed to  $\beta$  Cephei stars in  $10 - 15 M_{\odot}$ .



J. Cox: Can you say that it takes a significant amount of mass-loss to get rid of the semiconvection zone? Have you had a chance to investigate this relation?

Sreenivasan: Yes. We have investigated the  $15 M_{\odot}$  and  $20 M_{\odot}$  objects, as I showed you. The mass-loss on the main sequence is of the order of  $10^{-7} M_{\odot}$  per year for a  $15 M_{\odot}$  star. And if it has rotation, there is a centrifugal force with reduction of gravity, so that increases the mass-loss. It could be several times  $10^{-7} M_{\odot}$  per year, but it is certainly less than  $10^{-6} M_{\odot}$  per year. On the other hand, for a  $30 M_{\odot}$  star, it is of the order of  $10^{-6} M_{\odot}$  per year, and these figures are well within the observational limits imposed by the Copernicus satellite measurements. I think this is consistent with the observational information we now have from Copernicus. So even a few times  $10^{-7} M_{\odot}$  removes semiconvection in a  $15 M_{\odot}$  star. In other words, the type of mass-loss that is allowed by the theory of Castor et al., with proper  $\alpha$  and  $K$  values, takes away semiconvection. This is a very interesting result, because many people are not quite sure what kind of a "beast" semiconvection is, and how to treat it; so if it disappears, that solves one of the problems. And the fact that it disappears is borne out by everybody who has looked at this problem. So we don't have to worry about what it is, but all we can say is that you can't blame the  $\beta$  Cephei phenomenon on semiconvection.

Aizenman: The mention of differential rotation is also interesting because the original Chandrasekhar and Lebovitz mechanism, which Janet mentioned earlier, was examined by Maurice Clement as a possible explanation of the beat phenomenon. Clement ran into problems explaining the phenomenon by

this mechanism because of the very large rotation velocities required to match the observed beat periods. In 1967, he wrote a paper assuming a Stokely differential rotation law, and he found that in that case, he could match the observations on this type of phenomenon. While no mechanism was involved, this type of differential rotation law did allow one to obtain the observed beat periods.

Sreenivasan: In fact, Papaloizou and Pringle invoked differential rotation as well. We have looked at two models of a  $15 M_{\odot}$  star starting out at 500 km/s on the main sequence, which has a ratio of centrifugal force to gravity of about  $2/3$ , and at 350 km/s. From observations of angular momentum alone, all this surface rotation is gone by the time hydrogen is exhausted in the core; but if you take energy conservation into account as well as angular momentum conservation, the surface rotation is gone in about 5.5 million years. But of course, if it is differential rotation, the interior layers will be rotating faster, and this is consistent with what Papaloizou and Pringle said. All I'm saying is that if there is mass-loss, there is angular momentum loss, and thus differential rotation is enhanced. Semiconvection disappears, so you can't blame it on semiconvection, but you could blame it on differential rotation and a Kelvin-Helmholtz instability. We have not investigated this Kelvin-Helmholtz instability in detail, but we hope to.