THE PULSATING WHITE DWARFS

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The idea that white dwarfs could be pulsationally unstable is far from new. As early as 1949 Sauvenier-Goffin (1949) calculated pulsation periods of cold, non-relativistic white dwarfs. Between 1960 and 1970 a large number of theoretical investigations of white dwarf pulsations appeared in print using increasingly realistic and increasingly complicated models for the white dwarfs (c.f. Ostriker 1971). With very few exceptions (e.g. Harper and Rose 1970) these investigations, which were made in an observational vacuum, assumed that the most likely pulsations to be excited were radial pulsations. Thus, the calculated periods were quite short, typically 2-10 sec. The first of the pulsating white dwarfs to be discovered was HL Tau-76 (Landolt 1968). A portion of the light curve is shown in Figure 1. As so often happens, the universe refused to heed our preconceived ideas. The typical interval between successive pulses in the light curve is about 750 sec, not 2-10 sec. This gross discrepancy exists for all of the variable white dwarfs discovered since HL Tau-76, and raises a number of questions. Which white dwarfs are variable? Are the variables otherwise normal white dwarfs or are they pathological in other ways as well? Are the variations actually caused by pulsations, and if so, why are the periods so long? The observational data gives unequivocal answers to these questions. The variable white dwarfs are normal, single, DA white dwarfs.

The variations are caused by pulsations, but the pulsations are non-radial rather than radial pulsations. The purpose of this paper is to summarize the data which lead to these conclusions.

A total of 12 variable white dwarfs has now been found. They are listed in Table 1 along with their spectral types, UBV colors and magnitudes, and references to the discovery paper. The colors and magnitudes have been extracted from the series of papers by Eggen and Greenstein (Eggen 1968, 1969; Eggen and Greenstein 1965; Greenstein 1969). Three white dwarfs have been reported to be variable but are not included in Table 1. According to Richer and Ulrych (1974) G169-34 varies with a period of 465 sec. We have observed this star on several occasions, but it was always constant (McGraw and Robinson 1976). Either the star has changed its properties or the variations seen by Richer and Ulrych were spurious. Hesser et al. (1976b) have given a preliminary report of variations in LFT 1679. Since the data have not yet been published and since Hesser et al. suggest that the variations were caused by eclipses, we have temporarily
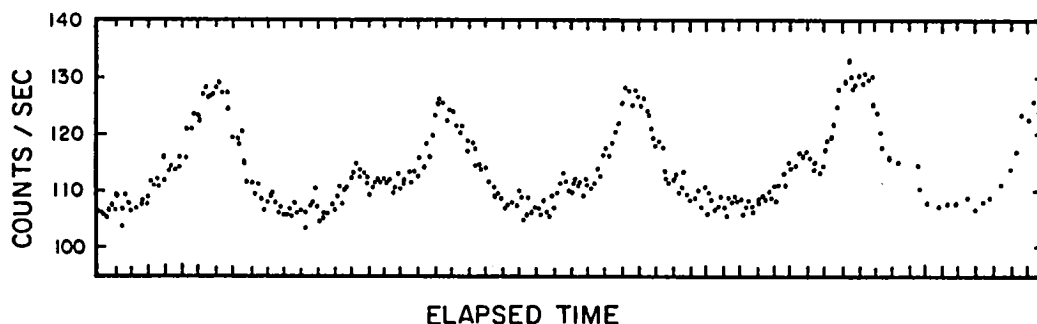

Figure 1. The light curve of HL Tau-76 on 1969 February 16 (from Warner and Nather 1970). Each point is the mean counting rate averaged over 10 seconds. The abscissa marks are minutes.

TABLE 1

THE VARIABLE WHITE DWARFS

STAR	Sp.	V	B-V	U-B	Ref.
BPM 30551	DA	15.26	+0.29	-0.58	1,2
R548	DA	14.10	+0.20	-0.54	3
BPM 31594	DA	15.03	+0.21	-0.66	4
HL Tau-76	DA	14.97	+0.20	-0.50	5
G38-29	DAs	15.63	+0.16	-0.53	6
GD 99	DA	14.55	+0.19	-0.59	7
G117 - B15A	DA	15.52	+0.20	-0.56	7,12
GD 154	DA	15.33	+0.18	-0.59	8
L19-2	DA	13.75	+0.25	-0.53	2,9
R808	DA	14.36	+0.17	-0.56	7
G207-9	DAn	14.64	+0.17	-0.60	10
G29-38	DA	13.10	+0.20	-0.65	11

REFERENCES

1) Hesser et al. (1976a)
2) McGraw (1977)
3) Lasker and Hesser (1971)
4) McGraw (1976)
5) Landolt (1968)
6) McGraw and Robinson (1975)

7) McGraw and Robinson (1976)
8) Robinson et al. (1978)
9) Hesser et al. (1977)
10) Robinson and McGraw (1976)
11) Shulov and Kopatskaya (1973)
12) Richer and Ulrych (1974)

excluded LFT 1679 from Table 1. G44-32 has been reported to be variable three times. In appendix A we give our reasons for believing that this star in fact is non-variable. Lists of non-variable white dwarfs are scattered throughout the literature. References to all of the published lists are given in Table 2.

TABLE 2

PUBLISHED LISTS OF NON-VARIABLE
WHITE DWARFS

Hesser and Lasker 1971
Hesser and Lasker 1972
Hesser et al. 1969
Lawrence et al. 1967
McGraw 1976
McGraw 1977
Robinson and McGraw 1976
Robinson et al. 1978
Richer and Ulrych 1974

Two characteristics of the variable white dwarfs are immediately evident from Table 1: all are DA white dwarfs, and all have colors near B-V = +0.20. Figure 2 is the color-color diagram for all of the white dwarfs which have been examined for variability and have published colors. The filled triangles represent the variable white dwarfs, and the remaining symbols, which are explained in the figure caption, represent the non-variable white dwarfs. The variable white dwarfs occupy a narrow instability strip lying between B-V colors of +0.16 and +0.29. The restricted range of colors and spectral types of the variables cannot be attributed to selection effects. A total of 136 constant white dwarfs are listed in the references given in Table 2. Of the non-variables, 60 have colors outside the limits of the instability strip; 25 have spectral types other than DA; and 34 are white dwarfs with unknown spectral types. Two of the variables, GD 154 and BPM 31594, had unknown spectral types when their variability was first detected and only afterwards were found to have DA spectra. The DA spectral type shows that a white dwarf must have hydrogen in its atmosphere in order to be variable. The exact placement of the instability strip emphasizes the importance of hydrogen. The solid line in Figure 2 is the locus of the DA white dwarfs with log g = 8 (Terashita and Matsushima 1969). The dip in the line near B-V = +0.20 is caused by a maximum in the Balmer line and continuum absorption. The variable white dwarfs cluster near this region where the Balmer absorption is strongest.

A third characteristic of the variables is that, although only 12 are known, they must be considered to be very common. Among the DA white dwarfs which have been examined for variability, there are 12 variables and 37 non-variables within the instability strip. Thus, about 25 percent of the DA white dwarfs in the instability strip are variables. This must be a lower limit for two reasons. First, variations with
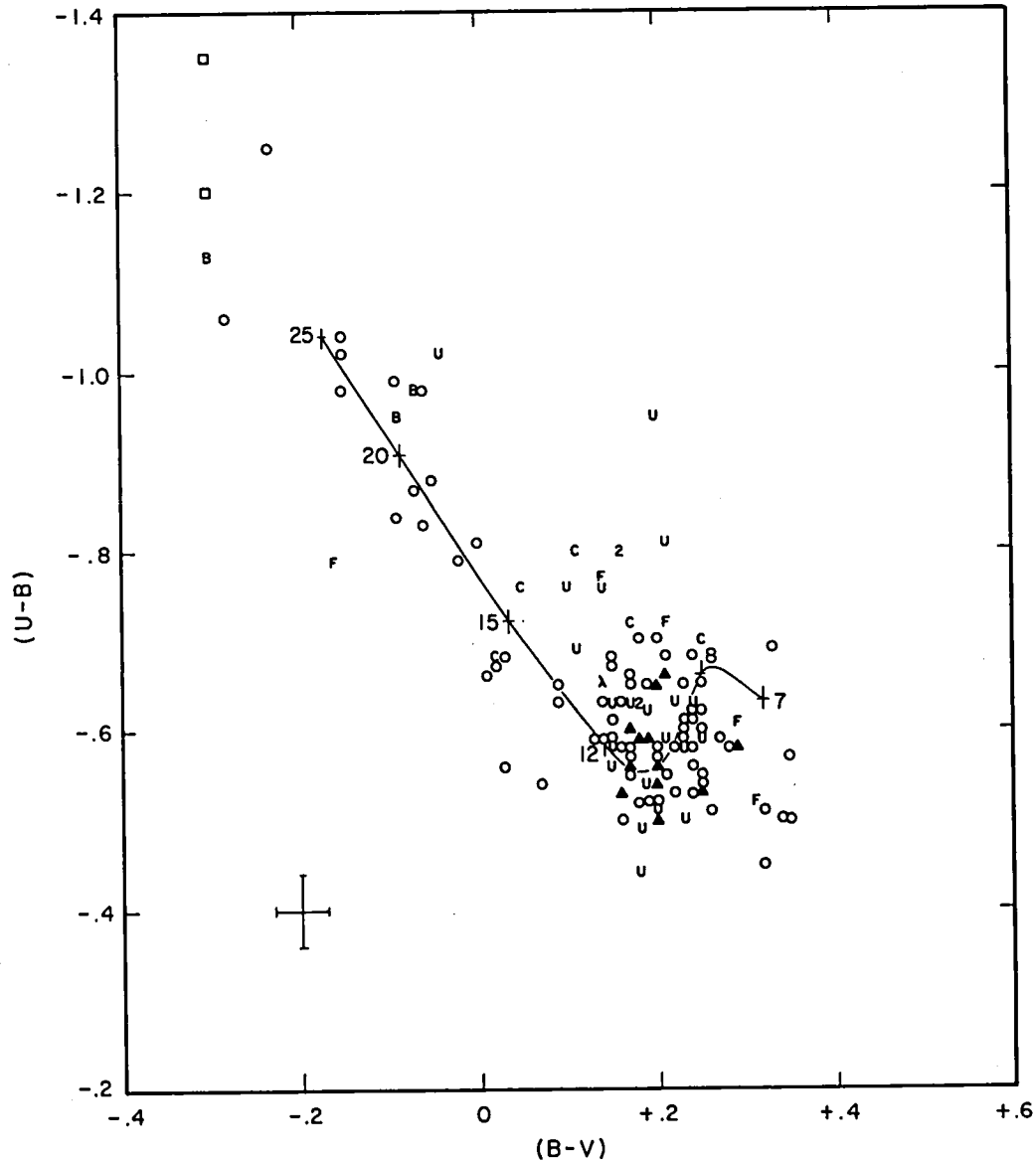
Figure 2. The Johnson two-color diagram for white dwarfs
investigated for rapid luminosity variations. The variables
are represented by filled triangles. The other symbols repre-
sent non-variable white dwarfs of different spectral types:
DA stars are open circles; B is a DB star; C a DC star; 2 a C_2
star; λ a $\lambda 4671$ star; F a DF star; DO stars are open boxes;
U is a star with unknown spectral type. Typical uncertainties
for the colors are shown by the crossed error bars. The solid
curve is the locus of the DA sequence for Log g = 8 from
Terashita and Matsushima (1969).

amplitudes less than 0.02 mag are difficult to detect in stars as faint as white dwarfs. Some of the non-variables could actually be low amplitude variables. Second, the measured width of the instability strip, 0.13 in B-V, is probably greater than the true width because the observational error in B-V is at least ± 0.03 mag. It is not possible to argue that all of the stars in the instability strip are variable, however. At least half, and possibly three quarters of the DA white dwarfs in the instability strip are non-variables. The solid line in Figure 2 is also the cooling sequence for DA white dwarfs. The cooling sequence passes through the instability strip. Since every DA white dwarf travels down the cooling sequence and must eventually traverse the instability strip, our statistics indicate that at least one quarter of all white dwarfs have been or will become variables. The ubiquity of the variables has an important impact on the
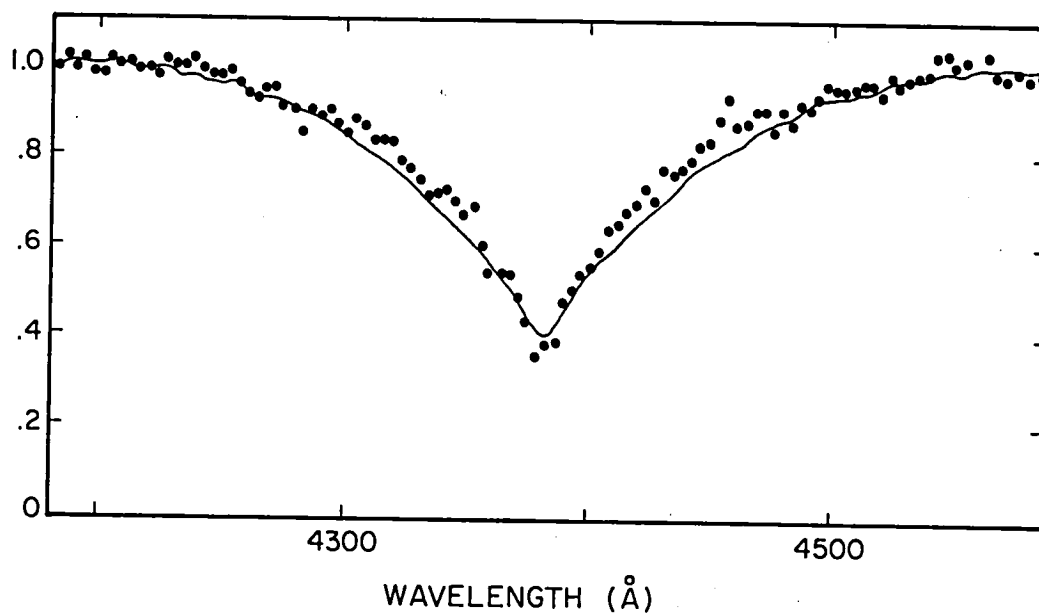

Figure 3. The Hγ line profiles of GD 154 and the DA white dwarf L47. That of GD 154 is shown by the dots, and that of L47 is shown by the solid line.

choice of physical mechanisms for producing the variations:
it is inappropriate to invoke low probability or exceptional
mechanisms of any kind.

A fourth characteristic of the variables is that, beyond
the obvious fact of their variability, they are quite normal
and completely indistinquishable from the non-variable white
dwarfs. Figure 2 demonstrates that their colors are typical
of DA white dwarfs. Figure 3 compares the $H\gamma$ line profile of
GD 154 to that of L47, a non-variable DA white dwarf with
$B-V = +0.08$. The spectra were obtained with the Tull Digicon
spectrograph under identical conditions and may be directly
compared. The line profile of GD 154 is slightly narrower and
deeper than that of L47, but the difference is consistent with
the slight difference in their colors. Thus, the spectrum of
GD 154 is normal. Trigonometric parallaxes have been mea-
sured for 5 of the variables. They are listed in Table 3
along with the derived absolute visual magnitudes. The mean
absolute magnitude of the group is 11.7 ± .2. A similar

TABLE 3

ABSOLUTE MAGNITUDES OF THE VARIABLE WHITE DWARFS

STAR	PARALLAX	ERROR	REFERENCE	M_V
R548	.014	.002	(1)	9.8 ± .3
G38-29	.013	.004	(2)	11.2 ± .8
R808	.034	.005	(2)	12.1 ± .3
G207-9	.030	.004	(3)	12.0 ± .3
G29-38	.071	.004	(4)	12.4 ± .2
		Weighted Average		11.7 ± .2

REFERENCES

1) Dahn et al. (1976)   3) Harrington et al. (1975)
2) Routly (1972)   4) Riddle (1970)

exercise for the non-variable DA white dwarfs with similar colors listed in the McCook and Sion catalogue (1977) yields M_v = 12.5 ± .2. The variables seem to be slightly brighter than the non-variables, but since their mean magnitude is heavily biased by the unusual parallax of just one variable, R548, the difference is of only marginal significance. The most accurate way to estimate the effective temperatures and gravities of DA white dwarfs is by measuring their colors in the Strömgren uvby system. The Strömgren colors of 10 of the variable white dwarfs have been measured by McGraw and Wegner (McGraw 1978) and are listed in Table 4. The colors are time averages since the colors vary as the luminosity varies. The corresponding effective temperatures and gravities are also given in Table 4 and have been derived by comparing the colors to the theoretical colors of DA white dwarfs calculated by Wickramasinghe and Strittmatter (1972). The effective temperatures range from about 10,500°K to 13,500°K and log g ranges

TABLE 4

THE EFFECTIVE GRAVITIES AND TEMPERATURES
OF THE VARIABLE WHITE DWARFS

STAR	b-y	u-b	T_e	Log g·
R548	.036	.686	12550±150	7.77±.02
HL Tau-76	.033	.634	13010±350	7.94±.02
G38-29	.063	.678	11900±1000	7.9 ±.4
GD99	.035	.587	13350±1000	8.1 ±.4
G117-B15A	.032	.556	13640±350	8.14±.05
R808	.078	.655	11730±250	7.98±.05
G29-38	.060	.614	12630±150	8.14±.02
BPM 30551	.122	.628	10315±400	7.79±.15
BPM 31594	.028	.665	12870±400	7.80±.25
L19-2	.071	.598	12520±400	8.24±.25

from about 7.8 to 8.1. None of these values is unusual for
DA white dwarfs. The finite spread in the temperatures and
gravities is real and not due to observational error. As a
by-product of this study, McGraw was able to show that changes
in the effective temperature are sufficient to account com-
pletely for the luminosity changes of the variables. This
agrees with the earlier study by Warner and Nather (1972).
Thus, the variables have normal colors, spectra, temperatures,
and gravities. In addition, Angel (1978) was unable to detect
any magnetic field in two of the variables, R548 and HL Tau-76;
and Robinson and McGraw (1977) have shown that the space motions
of the variables are undistinquished.

From their normality, we conclude that most of the vari-
ables are isolated, unperturbed white dwarfs. In particular,
they are not all members of close binary systems. Any compan-
ion of the variables must be too faint to alter their absolute
magnitudes or spectra. This is not possible unless the com-
panion is either a similar DA white dwarf or is at least
2 magnitudes fainter than a white dwarf. In short, the com-
panion must be invisible when next to a DA white dwarf. It is
not impossible and perhaps even likely that a few of the
variables have close, invisible companions (e.g. Fitch 1973).
However, we have shown that the variables are very common.
The probability that more than one quarter of all white dwarfs
have invisible companions would appear vanishingly small.

We will now consider the variations themselves. McGraw
will give a detailed account of the properties of the varia-
tions in an accompanying paper, so we will only give a short
summary of the properties here. The light curve of each star
has a characteristic peak-to-peak amplitude which ranges from
about 0.02 mag to 0.34 mag among the known variables. The
light curve of R548 in unfiltered light is shown in Figure 4.
The amplitude of the variations is about 0.02 mag., the lowest

of the known variables, and the time scale of the variations is about 220 sec. Figure 5 shows power spectra of the light curve of R548 on three successive nights in 1975. At first sight two, but only two, periods are present in the light curve, one near 213 sec and one near 274 sec. Each period varies considerably in amplitude and slightly in period. A detailed analysis of the light curve demonstrates that there are four, not two, periods present (Robinson et al. 1976). Each of the two periods shown in Figure 5 is a pair of sinusoidal variations with periods so close that they are not resolved in the power spectra. Stover et al. (1978) find the periods to be

$$P_1 = 212.76840$$
$$P_2 = 213.13258$$
$$P_3 = 274.25081$$
$$P_4 = 274.77432,$$

with formal errors of about ± 2 in the least significant figure. The changes in the amplitudes and periods of the two spikes in the power spectra are due only to beating between the close pairs. These four periods reproduce the variations of R548 with an accuracy of ± 0.001 mag in amplitude and ± 10 sec in time for an interval of more than one year. The most
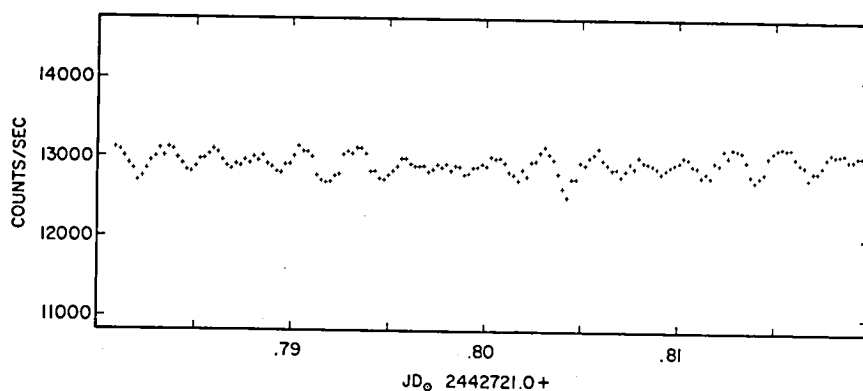

Figure 4. A portion of the light curve of R548. Each point is a 20 sec average of the photon counting rate.

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notable feature of the light curve is the remarkable stability
of the periods. Stover et al. (1978) give an upper limit of
$|\dot{P}| < 10^{-11}$ for any period changes. For comparison, the rate
of change of the period of the 71 sec variation of DQ Her is
$|\dot{P}| \sim 10^{-12}$ (Patterson et al. 1978).

The light curves of G29-38 and G38-29 in unfiltered light
are shown in Figure 6 and are typical of the light curves of
the large amplitude variables. The peak-to-peak amplitude of
the variations is about 0.21 mag in G38-29 and about 0.28 mag
in G29-38. The individual pulses are highly variable in shape,
but typically are asymmetric with a more rapid rise than decay.
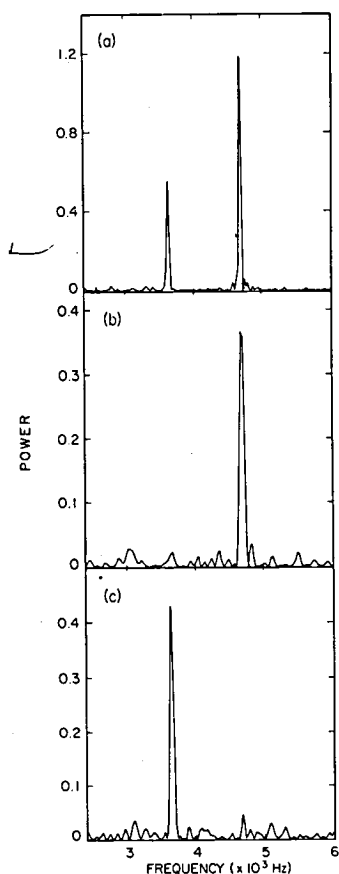The mean interval between pulses is about 850 sec, but the

Figure 5. Power spectra of the light curve of R548 on UT dates
(a) 1975 October 6, (b) 1975 October 7, and (c) 1975 October 8.
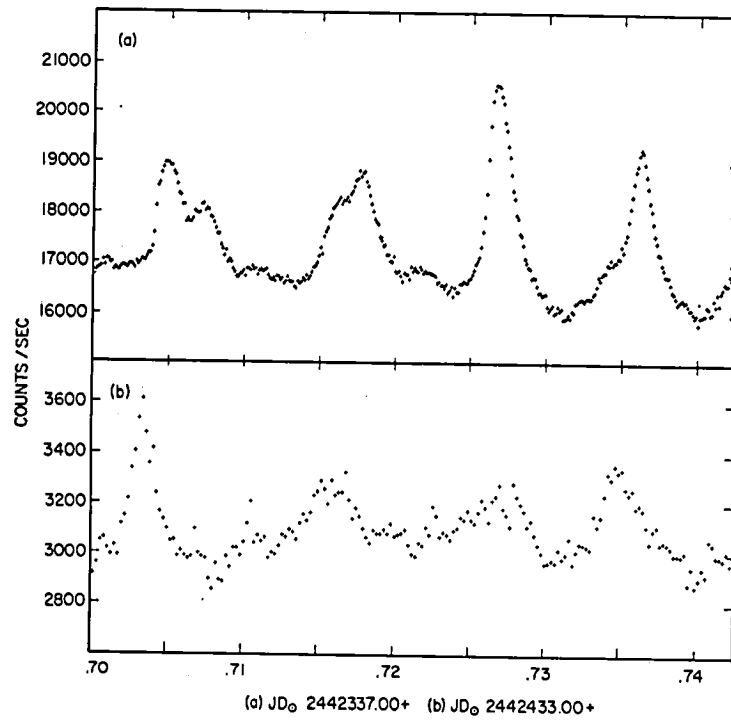
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Figure 6. (a) A portion of the light curve of G29-38. The ordinate is expressed in detected photons per second in white light reduced to outside the atmosphere. (b) The same for G38-29.

light curve is very irregular. The times of arrival of the
pulses are not predictable with any simple ephemeris. Figure 7
shows power spectra of the light curve of G29-38 on two suc-
cessive nights. The power spectra are extremely complex.
Literally scores of periods are simultaneously present, and
the power spectra change from night to night so that neither
the periods nor their amplitudes are constant. Although this
complexity has prevented any detailed understanding of the
light curve of G29-38, a few regularities can be found in the
power spectra. Periods near 694 sec, 820 sec, and 930 sec
usually are present in the spectra. Harmonics of the strongest
periods usually are present, and are reflecting the non-sinu-
soidal appearance of the pulses. Cross frequencies between
the strongest periods usually are present; if two strong
periods with frequencies f_1 and f_2 are present in the power
spectrum, $f_3 = nf_1 \pm mf_2$ is likely to be present also, where
n and m are small integers.

The properties of the pulsations of all of the variables
are summarized in Table 5. There are several important charac-
teristics of the variations. The periods are all very long.
Excluding harmonics and cross frequencies, the shortest period
is 114 sec and the longest period is 1186 sec. Without excep-
tion every variable is multi-periodic. L19-2 has at least two
periods, G117-B15A has at least three periods, and R548 has
four periods. The remaining variables all have large numbers
of periods. In the case of HL Tau-76, Desikachary and
Tomaszewski (1975) were able to identify 25 periods, but they
included only the periods with fairly large amplitudes, so
this is a lower limit to the true number. The stability of
the periods varies from extremely high in R548 to very low
in G29-38. It should be noted, however, that the apparent
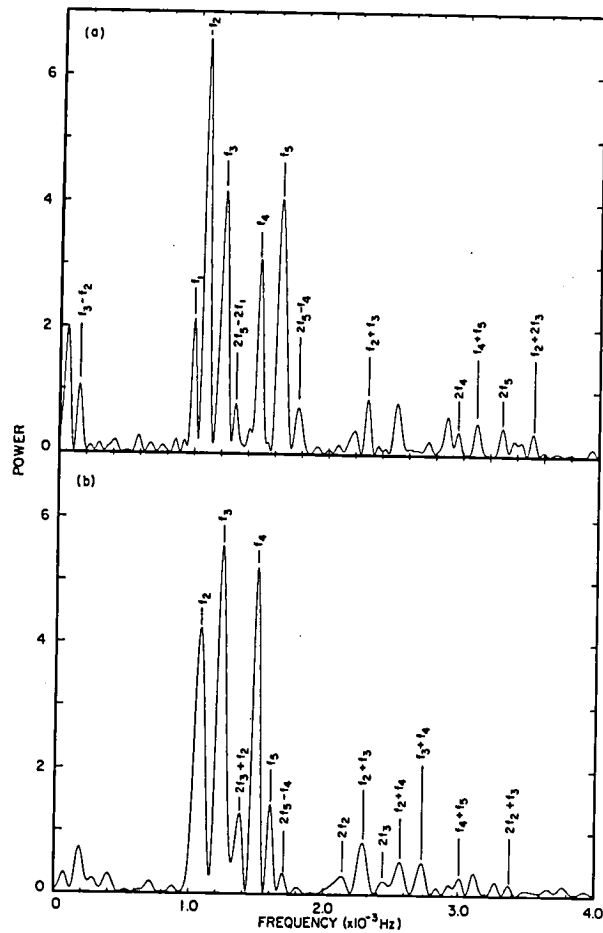
Figure 7. Two power spectra of the light curve of G29-38.
Major frequencies and a few of the frequencies identified
as harmonics and linear combinations of the major frequencies
are indicated. (a) From 1974 October 16. (b) From 1974
October 17.

TABLE 5

PROPERTIES OF THE PULSATIONS

Star	Peak to Peak Amp(Mag)	Typical Pulse Interval (Sec)	Periods (Sec)	Period Stability	Ref.
R548	0.02	220	213 + 274	$\|\dot{P}\| < 10^{-11}$	(1,2,3)
L19-2	0.04	190	114 + 192	High	(4,5)
G117-B15A	0.06	215	216 + 272 + 308	High	(6)
G207-9	0.06	300	292 + 318 + 557 + 739	High?	(7)
GD 154	0.10	1200	780 + 1186 + Others	$\|\dot{P}\| < 10^{-8}$	(8)
GD 99	0.13	350	260 + 480 + 590 + Others	Moderate	(6)
R808	0.15	850	513 + 830 + Others	Low	(6)
BPM 30551	0.18	750	607 + 745 + 823 + Others	Moderate	(9,4)
BPM 31594	0.21	600	311 + 404 + 617 + Others	Moderate	(10)
G38-29	0.21	850	925 + 1020 + Others	Low	(11)
G29-38	0.28	850	694 + 820 + 930 + Others	Low	(11)
HL Tau-76	0.34	750	494 + 626 + 661 + 746 + Others	Low	(12,13,14,15)

REFERENCES

1) Lasker and Hesser (1971)
2) Robinson et al. (1976)
3) Stover et al. (1978)
4) McGraw (1977)
5) Hesser et al. (1977)
6) McGraw and Robinson (1976)
7) Robinson and McGraw (1976)
8) Robinson et al. (1978)
9) Hesser et al. (1976a)
10) McGraw (1976)
11) McGraw and Robinson (1975)
12) Warner and Robinson (1972)
13) Page (1972)
14) Fitch (1973)
15) Desikachary and Tomaszewski (1975)

low stability of many of the variables may be due to incomplete data. Finally, there is a strong correlation between the amplitude of the variations and their remaining properties. The higher amplitude variables have more periods in their light curves and the periods are more unstable.

These last characteristics are sufficient to limit the physical mechanism for producing the variations to non-radial pulsations. Relatively few ways are known for producing periods as stable as those in R548. They are orbital motion, rotation, and pulsation. Only pulsations can produce multi-periodic variations with high stability. It is true that differential rotation can give multi-periodic variations, but the 213 sec and 274 sec sets of periods in R548 differ by about 30 percent. The shear across the surface of a star with differential rotation this large will destroy the period stability. We are left with pulsations, and thus we are left with the original discrepancy between the observed periods

and the periods expected for radially pulsating white dwarfs. The only way radial pulsations can have long periods is if the mass of the white dwarf is very low, but a low mass white dwarf will also have a low surface gravity which is inconsistent with the observed normal gravities. An easy and natural way out of this dilemma was suggested by Warner and Robinson (1972). Stars are not necessarily restricted to pulsations in purely radial modes, non-radial pulsations are also possible (c.f. the review by Cox 1976). Among the non-radial pulsations at least one group, the g^+ modes, can have periods long enough to match the observed periods. The non-radial modes also provide a natural framework in which to understand the large number of closely spaced periods in the light curves of the larger amplitude variables. Even if we consider only the low order non-radial pulsation modes, a very large number of modes, and thus periods, could conceivably be excited.

The g-mode interpretation has never been seriously challenged, although some interesting modifications of the g-mode model have appeared (e.g. Wolff 1977, Dziembowski 1977). It is also fair to say that the g-mode pulsation model has not been proven. Such a proof would require a demonstration that there is a detailed correspondance between the observed periods and the theoretical periods expected in a white dwarf. Such a demonstration has not been made for two reasons. The first reason is observational. Only one variable, R548, has had its period structure completely deciphered, and none of the variables have accurately known masses and chemical compositions. Without these data the comparison between observed and theoretical periods cannot be made. The second reason is theoretical. The theory of non-radial pulsations is still in its infancy. Only the results from the linear theory are available, and of these results, it is still true that no theoretical model of a white dwarf has been found to be pulsationally unstable. Thus, it is not possible to decide which pulsation modes are

most likely to appear. Nevertheless, there is an encouraging qualitative agreement between the observed and theoretical periods. The periods of R548 can be made to agree with Brickhill's (1975) periods of g-mode pulsations. One of the possible specific identifications of the excited modes is the 213 sec pair as the $\ell = 2$, $k = 1$ (= fundamental) mode; and the 274 sec pair as the $\ell = 2$, $k = 2$ (= first overtone) mode. The splitting of the modes into close pairs could be caused by slow rotation. A similar qualitative agreement between theoretical and observational periods has been found for HL Tau-76 (Desikachary and Tomaszewski 1975), BPM 30551 (McGraw 1977), G29-38, and G38-29 (McGraw and Robinson 1975). The agreement can only be called qualitative, however. The 1186 sec period in GD 154 requires the pulsation mode to be a very high overtone ($k \sim 10$-30) if the variations are caused by g-mode pulsations (Robinson et al. 1978). It is not clear that pulsations in such high overtone modes are acceptable. A more serious problem is that neither the mode assignments nor the basic pulsation model is uniquely specified by the data. McGraw (1977) and Hesser et al. (1976b) give completely different mode assignments to the periods of BPM 30551. Wolff (1977) suggests that the variations arise from the rotation of non-linearly coupled g-mode oscillations. In his model the period structure is determined by the rotation period of the white dwarf, not by the basic pulsation frequency. His model also agrees qualitatively with the observed periods.

The ultimate hope in studying the pulsating white dwarfs is that they will provide a direct observational test of models for the structure and evolution of white dwarfs. This hope has not yet been fulfilled. The reasons are not hard to find. Until the theoretical models can predict which pulsation modes should be excited and which should not, there are too many degrees of freedom available in comparing observation to theory. Van Horn will discuss the theoretical problems in dealing with white dwarfs, and we will not pursue this point. There is

also clear room for improvement of the observations. The period structure is known in detail for only one variable, R548. More are needed. The light curves of the low amplitude variables L19-2 and G117-B15A should be comparatively easy to decipher, but considerable effort will be needed to decipher the complex variations of the large amplitude variables. We have shown that at least one quarter of the white dwarfs with hydrogen rich envelopes that have temperatures near 12,000°K and gravities near log g = 8.0 are variable. It is equally true that three quarters of them are not variable or vary with undetectably low amplitudes. Apparently it is necessary to specify at least one more physical property of a white dwarf to insure that it is a variable. Since we have not been able to discover this additional parameter, it probably is some property that we have not yet measured. Two likely candidates are the exact hydrogen content of the atmosphere of the white dwarfs and the rotation period of the white dwarfs. Neither of these properties is easy to measure. A differential comparison of the variables to non-variables with similar colors might find differences in, if not actual values of, the rotation periods and hydrogen contents.

APPENDIX A

G44-32 has been classified as a DC white dwarf by
Greenstein (1970), although weak hydrogen absorption lines
may be present so it could be a very weak lined DA white dwarf.
Eggen and Greenstein (1965) give V = 16.5, B-V = +0.29, and
U-B = -0.58 for its magnitude and colors. G44-32 has been
reported to be variable three times, first by Giclas,
Slaughter, and Burnham (1959), then by Lasker and Hesser
(1969), and finally by Warner et al. (1970). Nevertheless,
we believe that the evidence that G44-32 varies is not con-
vincing and have not included it in Table 1. We give our
reasons in this appendix.

G44-32 has been observed twice at McDonald Observatory,
once in 1970 and once in 1973. Warner et al. (1970) discuss
only the 1970 data. According to them, G44-32 displayed two
types of variability during their observation. It showed low
amplitude continuous variations, and it showed a flare with
an amplitude of 0.61 mag which lasted for several minutes.
We have re-examined the original data and the log book from
1970, and we have discussed the data with one of the original
observers (GWvC). We find that the photometer used to acquire
the data was definitely malfunctioning: high voltage was
audibly arcing in the base of the photomultiplier tube during
the observation. Therefore the 1970 data are unreliable and
cannot be used to determine the properties of G44-32. The
1973 data consists of high speed photometry in unfiltered
light taken with the 82-inch telescope. Conditions were good
and the length of the light curve was 12,500 sec. Visual
inspection of the light curve reveals no variability. The
power spectrum of the light curve is shown in Figure A-1 and
has several notable features. There are significant periodi-
cities at 120 sec and 60 sec due to the periodic drive error
in the 82-inch telescope. There are no other significant

periodicities between 40 sec and 30 min; the upper limit on
the amplitude of any periodicity in this range is .002 mag.
The mean power level increases at low frequencies. This
increased power is not intrinsic to G44-32. Low frequency,
or "red," noise is ubiquitous in nature and is always present
in high speed photometry. The noise is introduced into light
curves primarily by variations in sky background and atmos-
pheric transparency. For an example of red noise in high
speed photometry of a star known to be constant, see Figure 6
in Robinson (1973). In sum, G44-32 was constant during the
1973 observation.

Lasker and Hesser (1969) observed G44-32 on five nights
in 1969. According to them, G44-32 varied at periods of
27.3 min, 13.7 min, and 10.0 min with amplitudes of 1.8, 1.2,
and 1.5 percent respectively. However, only three of their
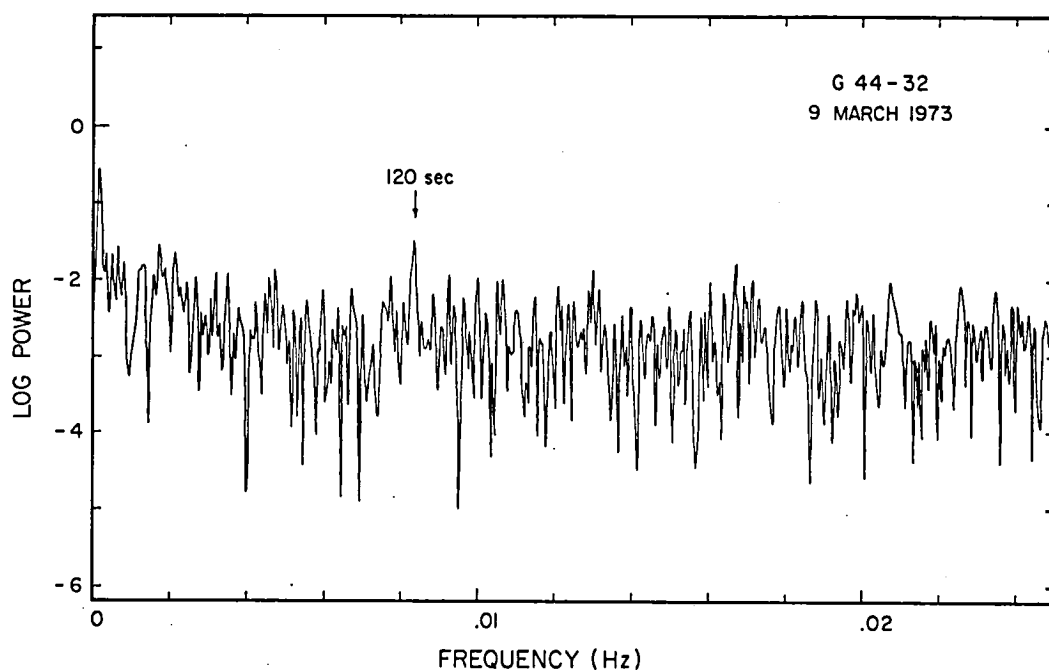light curves were long enough to be analyzed: runs 2, 4, and

Figure A-1. The power spectrum of the light curve of G44-32
on 1973 March 9. The 120 sec periodicity is spurious and is
caused by a periodic drive error in the telescope.

5. We assume here that what they call run 3 in the text must actually be run 2 in their Table 1. If not, their analysis is vitiated because the run from February 21-22 is far too short to be reliably analyzed for long periods. Lasker and Hesser themselves admit that G44-32 was constant during run 4, so the claim for periodic variability relies on runs 2 and 5 alone. The combined power spectrum for runs 2 and 5 is shown in their Figure 1. The spectrum does indeed have three high points at their three periods, but the real question is whether these points are significantly greater than the mean noise level in the power spectrum. In other words, are the three points just noise? Hesser and Lasker do not address this question and do not estimate the mean noise level. Suppose that two power spectra have been averaged together and that the mean level of noise in the averaged spectrum at frequency ν is P_0. The probability that a single measurement of the noise power at ν will give a value greater than P is $(1+2P/P_0)$ exp $(-2P/P_0)$. If N independent samples of the power are made at N independent frequencies, we expect $N(1+2P/P_0)$ exp $(-2P/P_0)$ of the samples to be greater than P. The mean noise level at low frequencies is very large in the spectrum shown by Lasker and Hesser since the spectrum clearly displays the red noise phenomenon. We estimate that in the frequency range of interest $N \sim 20$, and $P_0 \sim (0.01 \text{ mag})^2$. Then for $P = (0.018 \text{ mag})^2$ we expect 0.23 high points; for $P = (0.015 \text{ mag})^2$ we expect 1.2 high points, and for $P = (0.012 \text{ mag})^2$ we expect 4.4 high points. Thus, there is a significant probability that the spectrum shown by Lasker and Hesser is caused by random noise rather than periodicities. Furthermore, the data has been pre-selected by the exclusion of run 4, so the probability that their spectrum is due to pure noise is yet higher. We note that even Lasker and Hesser characterize their results as "marginal and speculative."

Giclas, Slaughter, and Burnham (1959) observed G44-32 as part of the Lowell Observatory proper motion survey. Although they suspected that G44-32 varied, the data are photographic and the star is near the limiting magnitude of the survey. Non-intrinsic variability is common under these conditions.

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Discussion

Belserene: Have you looked for variability among the DA's?

Robinson: Yes, the width and the exact placement of the strip are not well defined. These stars are pretty faint, and the colors are only accurate to a couple of hundredths of a magnitude. So it's hard to know whether or not any individual star is actually in the strip. But I think it's fair to say that there are definitely non-variable DA white dwarfs in the strip. It's not sufficient just to pick out a DA white dwarf with the right color, in order to know that it's variable.

Belserene: Then there are no observable characteristics to distinguish them?

Robinson: Not that we have found so far.

A. Cox: You implied very strongly that the cause of the pulsation is the appearance of hydrogen.

Robinson: I was trying to state it a little less strongly than that. I said that you must have hydrogen in the atmosphere in order to see variability.

A. Cox: I was wondering whether that has any bearing on Wolff's theory, because if he's going to have these things rotate, it doesn't matter whether they have any hydrogen or not, or does it?

Wolff: You need something to drive the pulsations; otherwise, hydrogen content is irrelevant to my theory.

A. Cox: But you find that none of them pulsate unless they have hydrogen, is that right?

Robinson: That's right.

Wesselink: What is the period of the DA star in compound binary white dwarfs?

Robinson: Five to ten seconds.

Aizenman: Your Table V definitely implied a relationship between amplitude and number of periods. Have you got anything more specific than that -- some sort of curve?

Robinson: Yes, there is a very definite relationship between the amplitude and some of the other characteristics of the variables. As the amplitude goes up, you tend to see more periods in the light curve, and the light curve seems to get less stable. The canonical explanation -- at least in Texas -- is that the relationship between amplitude and the other properties is just caused by non-linearities. The low-amplitude ones are linear variables, and there's very little coupling between the modes. The variations are sinusoidal. As you drive up the amplitude, you're more likely to be coupling between modes, and to excite other modes, and so on.

Keeley: If you could look at the amplitude of an individual mode in a star with many modes, would you say that the amplitude of that individual mode is greater than it would be if it were the only mode? Is the higher total amplitude caused by the presence of many modes, or is the intrinsic amplitude of each mode higher?

Robinson: I can't answer that question. It's an interesting one.

Shipman: In terms of composition, the composition that Rob's talking about is the exterior composition.

Robinson: Oh, yes. It's just the photosphere.

Shipman: And the only thing that we know about the interior is that they're not hydrogen. [Laughter] We don't know what they are -- probably carbon. Now a question: how many (what fraction) of the helium white dwarfs have been searched for variability?

Robinson: I don't know the fraction. We've looked at a total of 136 stars which did not vary, of which I believe 25 were non-DA, and another 34 had no known spectral types -- but their colors and their proper motions placed them as white dwarfs. And among those, there are a fair number of DB's.

Nather: I think it's fair to say that the likelihood of a DB being variable is small, because among those that have been looked at, none has been known to vary.

Sion: Even though the variability seems to be unique to single white dwarfs, can you rule out the possibility that in certain close binaries where you have minimal interaction and no appreciable accretion disk, some of the optical variability could be due to a ZZ-Ceti type instability? So maybe it could operate in certain close binaries where the accretion rate is low.

Robinson: That's probably right. Also, I didn't want to rule out inter-acting binaries altogether. My main point was that a least some of the

ZZ-Ceti stars are single stars. So that the basic mechanism for exciting

them is intrinsic to a single star. If you put in another star, you're going

to modulate that, and it's going to get more complicated.

J. Cox: First a comment, and then a question. The comment is that it

worries me a little when I see all these non-linearities. These are actually

fairly small amplitudes, I think.

Robinson: Three tenths of a magnitude.

J. Cox: Anyway, there is evidence for non-linearity. The question is, is

the $\dot{P}$ consistent with the cooling rate of these stars?

Robinson: It is consistent. There are various ways that you can change

the period of a white dwarf. You can change the temperature, or the chemical

composition. Temperature changes lead one to expect a $\dot{P}$ of the order of

10^{-15}, 10^{-16}, or 10^{-17}, depending on the chemical composition.