THE PERIOD STRUCTURE OF THE ZZ CETI VARIABLES

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The ZZ Ceti variables are a class of pulsating white dwarfs (cf. the previous review by Robinson) which show a large variety in the appearance of their light curves. The approximate range in amplitude, period and pulse shape is shown in Figure 1 which contains segments of the light curves of HL Tau-76 (the prototypical ZZ Ceti star), GD 154 (the variable with the longest period), and ZZ Ceti itself (R548), the most extensively observed variable. Time-
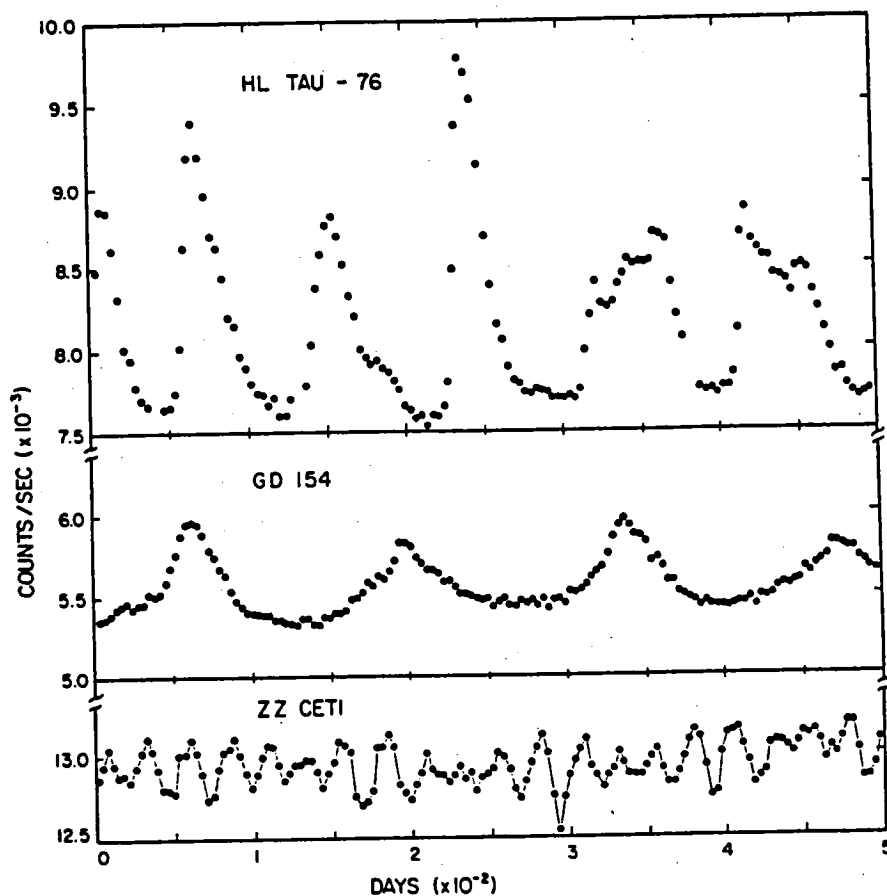

Figure 1: Segments of the light curves of HL Tau-76, GD 154 and ZZ Ceti. The ordinate is expressed in detected photons per second in "white" light, corrected for atmospheric extinction.

resolved photometry in unfiltered light has yielded a wealth
of information about the period structure of these stars.
Though we are faced by a broad range of period structure,
we believe it is possible to explain most of the features
of the light curves of ZZ Ceti variables by invoking
reasonable mechanisms; we interpret the period structure
in terms of nonradial pulsations which are modulated by
rotation of the star and which, for the large amplitude
variables, can become nonlinear. Theoretical models of
pulsating white dwarfs will ultimately confirm or reject
our suggestions. The purpose of this paper, then, is to
review the current observational status of the period
structure of the ZZ Ceti stars. We will discuss in particular
those features which appear to be the most important for
theory to explain, or which may be relevant to the directions
of theoretical development.

The shortest primary period for a ZZ Ceti variable is
about 192 s, seen in L19-2, and the longest is the primary
period of GD 154, about 1186 s. In general, the light curves
of these variables are so complex that, apart from an estimate
of the primary period, little can be determined from them
directly. Power spectra of the light curves are used to
investigate the period structure in detail. Using this
technique, the light curve of every ZZ Ceti variable has
been shown to contain at least two independent periods,
that is, periods which are not simply harmonics. Figure 2
shows a power spectrum of the light curve of L19-2. The
ratio of the periods represented by the two prominent peaks
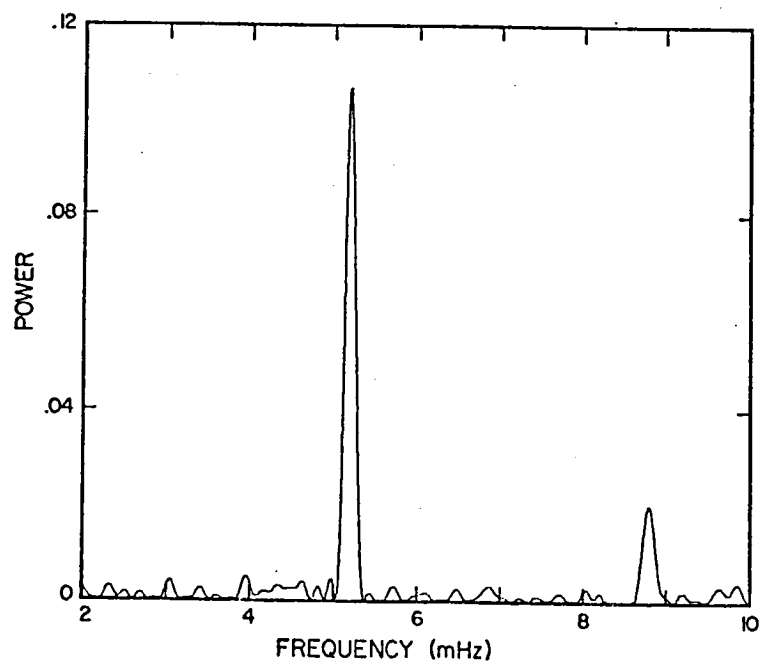in this spectrum is $P_1/P_2 \simeq 194 \text{ s}/114 \text{ s} \simeq 1.7$.

Figure 2: The power spectrum of the light curve of L19-2.
The ordinates of all power spectra in this paper are
directly comparable.

The variety of photometric complexity seen in the light curves is reflected in their power spectra. Figure 3 shows the power spectra for HL Tau-76, GD 154 and ZZ Ceti.
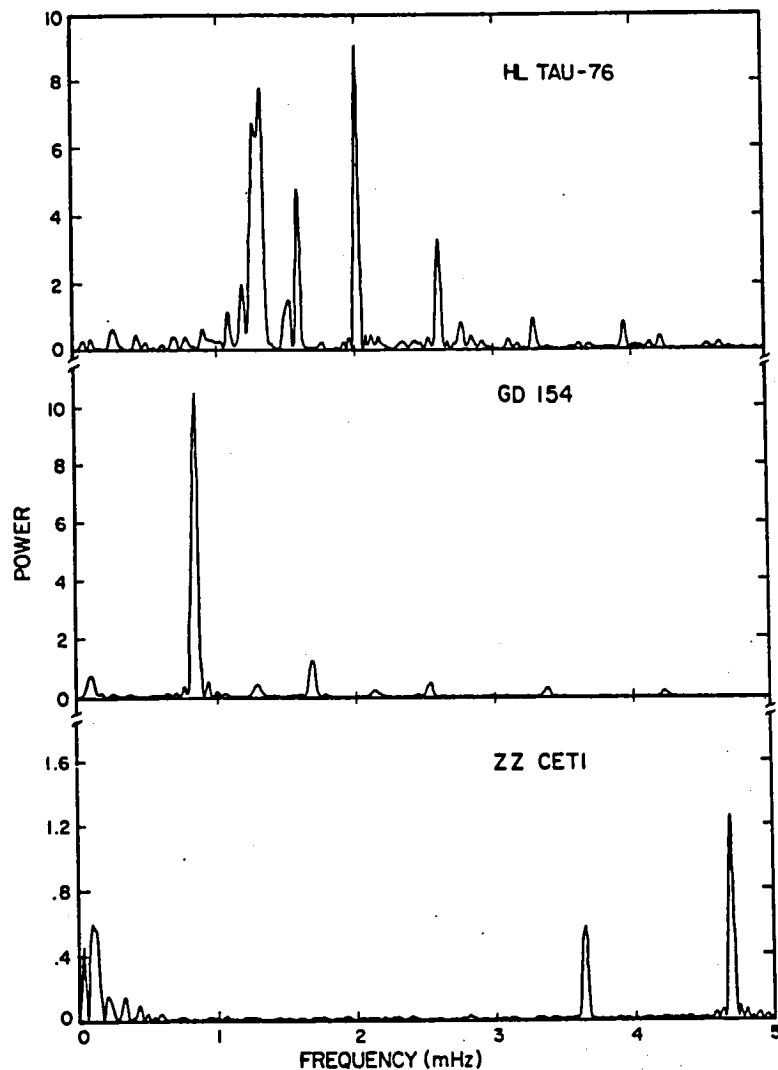

Figure 3: Power spectra of the light curves of HL Tau-76, GD 154 and ZZ Ceti. Segments of the light curves from which these spectra were derived are shown in Figure 1.

This figure illustrates an approximate correlation between the amplitude of the variable and the complexity of its power spectrum: large amplitude variables tend to have complex, multi-peaked spectra while low amplitude variables have simpler spectra with fewer peaks (Robinson and McGraw 1976). Originally it was suggested that this correlation also included the period as a parameter and that large amplitude variables with complex spectra also had long

periods. GD 154, with a spectrum of intermediate complexity (cf. Figure 3), negates this earlier suggestion - amplitude and complexity appear to be the relevant parameters (Robinson et al. 1978).

Power spectra of the light curves of most of the ZZ Ceti variables change on time scales ranging from minutes to days. Two power spectra for one night's data on G29-38 are shown in Figure 4. The light curve, a run of five hours duration, was halved and each half was transformed separately. The two spectra look totally different, showing that significant changes to the period structure of this star occurred
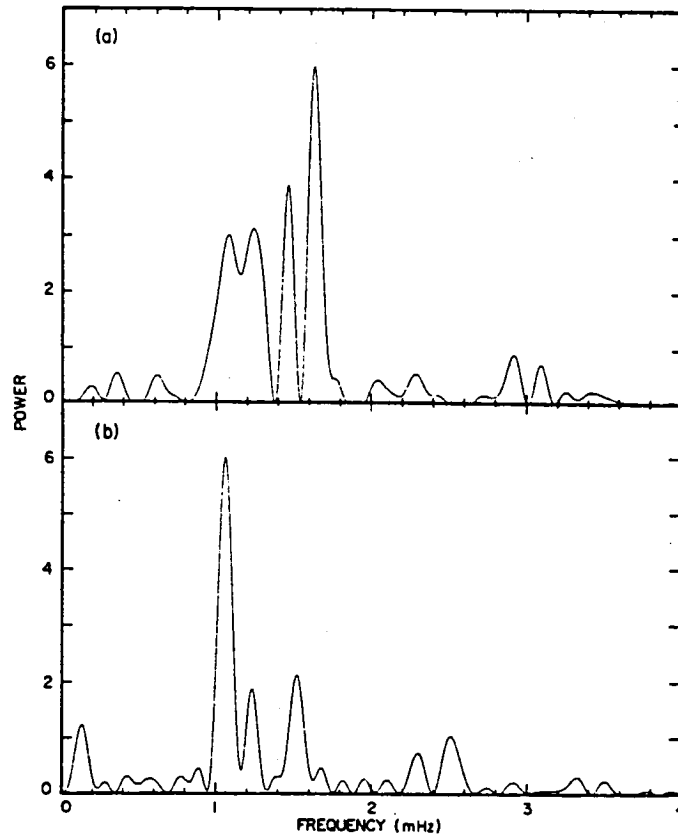

Figure 4: Power spectra of the light curve of G29-38 obtained on one night. These spectra, (a) derives from the first half of the run and (b) from the second, show typical changes in frequency and amplitude which occur during a run.

within this run. Figure 5 shows power spectra derived from runs on two separate nights. Again, there are significant changes in both the frequencies and amplitudes of the peaks in this spectrum.
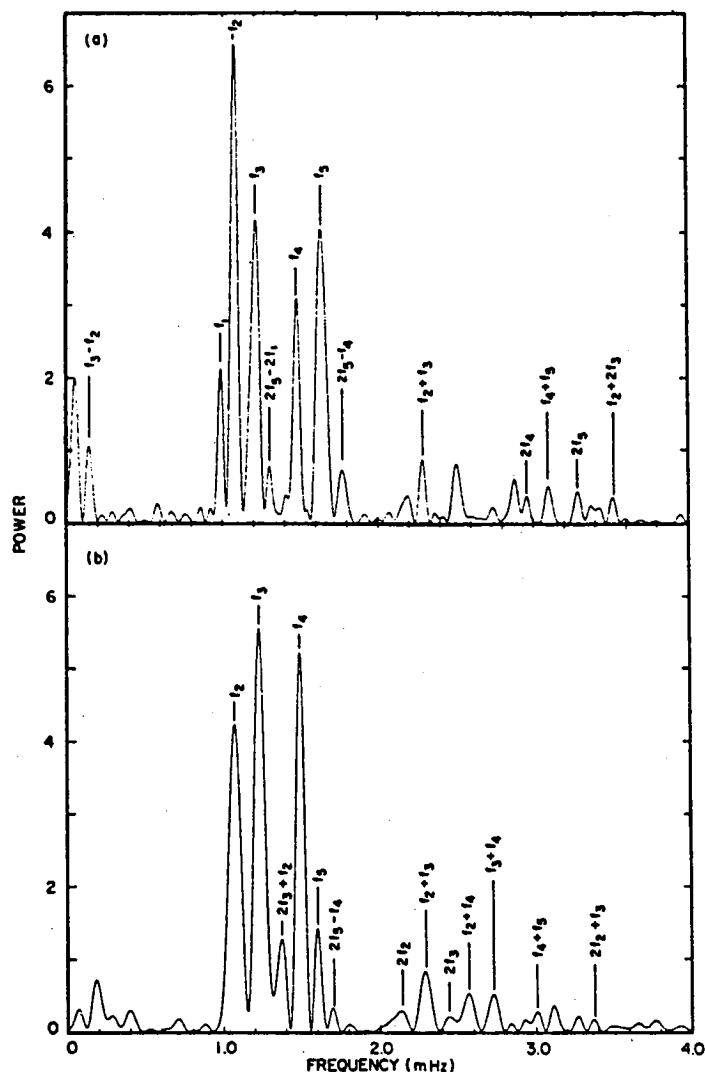
Figure 5: Power spectra of the light curve of G29-38 derived from runs obtained on consecutive nights. Five major peaks and some of the peaks identified with linear combinations of major peaks are indicated.

This figure also illustrates two numerical relationships among frequencies of peaks in the power spectra of several of the ZZ Ceti light curves. The first relationship, the occurrence of "cross-frequencies", is seen in the spectra of about half of the variables. If we pick the one to five strongest peaks in a spectrum and denote the frequencies of these peaks as primary frequencies, some, but not all, of the secondary peaks in the spectrum have frequencies, f, given by a linear combination of primary frequencies: $f = nf_i \pm mf_j$, where i and j specify primary frequencies and n and m are small integers. The second relationship, a pattern of equally spaced frequencies, occurs in the

spectra of G29-38 and two other variables. In Figure 5, the average frequency spacings $<f_3-f_2> \simeq <f_5-f_4> \sim 0.14$ mHz and $<f_4-f_3> \approx 0.26$ mHz $\approx 2 \times 0.14$ mHz can be seen.

An additional feature of the period structure of the ZZ Ceti variables has been seen in the light curves of BPM 31594 (McGraw 1976) and GD 154 (Robinson et al. 1978). These stars have been observed to change their primary periods by factors of about 2 and 2/3, respectively, within 24 hours. Figure 6 shows the power spectrum derived from the light curve of BPM 31594 obtained on the discovery night (above) and that from the light curve obtained the next night (below). The primary frequency decreased by a factor of about 1.99 (significantly different from 2), but a smaller peak remained at the approximate frequency seen the first night. In addition, in the later spectra significant power appeared at frequencies near 3/2 and 5/2 the primary frequencies. GD 154 exhibited the opposite behavior. On
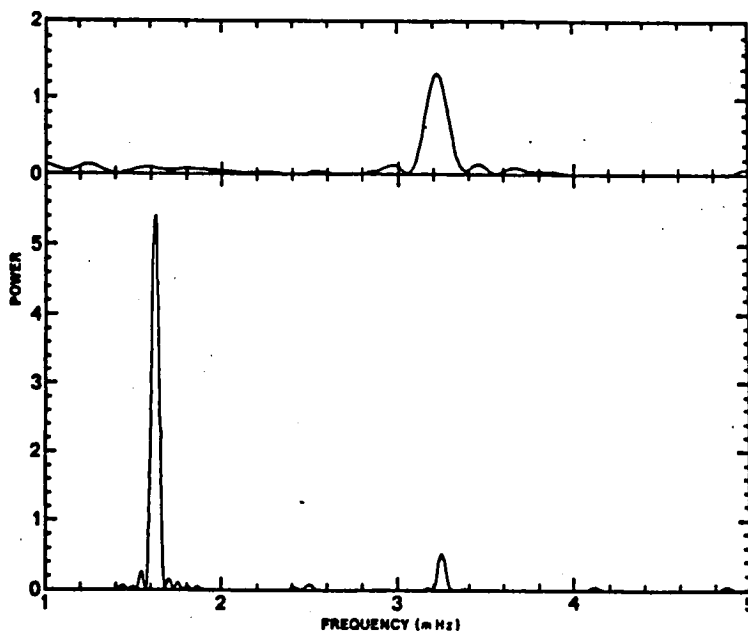

Figure 6: Power spectra of the discovery run (upper) and the run obtained the following night (lower) on BPM 31594. Note the change in frequency of the primary peak in the spectra and the appearance of peaks at frequencies near 3/2, 2, 5/2 and 3 times the frequency of the major peak in the latter spectrum.

the first nine nights it was observed it showed one principal peak and four harmonics of this frequency, plus peaks near 3/2, 5/2 and 7/2 of the principal frequency. On the tenth night it was observed, the peak near 3/2 the frequency of the original principal peak had become the dominant peak in

the spectrum. Since the original ten runs, this star has been observed in each of its "states", showing that the period change is not an isolated incident.

Amidst the photometric complexity, three low amplitude variables, ZZ Ceti, L19-2 and G117-B15A, show a refreshing simplicity and regularity. ZZ Ceti has been shown to be a variable with four very stable periods (Robinson, Nather and McGraw 1976). The periods occur in closely spaced pairs, each pair forming a single peak in a power spectrum derived from a light curve of reasonable ($\leq 6^h$) length. The close spacing causes a modulation of the amplitudes and frequencies of the two peaks in the power spectrum ("beating"). The modulation itself has a period of about 1.5 days. Stover et al. (1977) have used data from two observing seasons to show that for each of the four pulsations $Q \equiv |\dot{P}|^{-1} > 10^{11}$. Additional data from Cerro Tololo, supplied by Jim Hesser and Barry Lasker, will extend the baseline of observations to about eight years. On the assumption that $\dot{P}$ reflects the evolution of the star, these data should improve the measurement of Q by another order of magnitude.

Table 1 summarizes the observed photometric properties of the ZZ Ceti stars. GD 154 and BPM 31594 have been included as "moderately" stable pulsators because these stars have been observed to change their basic pulsational periods. When in one "state" the period stability can be very high - a $Q > 10^8$ was derived for GD 154 from an ephemeris constructed for the data gathered on the first nine nights it was observed (Robinson et al. 1978).

TABLE 1

PHOTOMETRIC PROPERTIES OF THE ZZ CETI STARS

Star	Basic Periods (seconds)	Mean Amplitude (magnitude)	Harmonics	Cross-Frequencies	Period Stability	Ref.
BPM 30551	823	0.18	No	No	Moderate	(12, 15)
ZZ Ceti	213, 274	0.02	No	No	$Q > 10^{11}$	(2, 14, 18)
BPM 31594	310, 617	0.21	Yes	No	Moderate	(13)
HL Tau-76	384, 494, 625, 746	0.28	Yes	Yes	Low	(1, 3, 4, 5, 6, 7, 9)
G38-29	925, 1020	0.22	Yes	Yes	Low	(8)
GD 99	350, 476, 595	0.07	Yes	Yes	Moderate	(11)
G117-B15A	216, 272	0.05	No	No	High	(11)
GD 154	780, 1186	0.10	Yes	No	Moderate	(17)
L19-2	114, 192	0.03	No	No	High	(15, 16)
R808	513, 830	0.15	Yes	Yes?	Low	(11)
G207-9	292, 318, 557, 739	0.06	No	Yes	High?	(10)
G29-38	494, 625, 746	0.28	Yes	Yes	Low	(8)

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1) Landolt 1968
2) Lasker and Hesser 1971
3) Warner and Nather 1970
4) Warner and Nather 1972
5) Warner and Robinson 1972
6) Page 1972
7) Fitch 1973
8) McGraw and Robinson 1975
9) Desikachary and Tomaszewski 1975

10) Robinson and McGraw 1976
11) McGraw and Robinson 1976
12) Hesser et al. 1976
13) McGraw 1976
14) Robinson et al. 1976
15) McGraw 1977
16) Hesser et.al. 1977
17) Robinson et al. 1978
18) Stover et al. 1978

A rather simple and self-consistent model for the general period structure of the ZZ Ceti variables, incorporating most of the data presented above, can now be proposed. The fact that multiple, highly stable periods occur in these stars definitely confirms that the luminosity variations are produced by pulsations, as has been pointed out earlier by Robinson. Even the shortest ZZ Ceti period is too long by an order of magnitude to be associated with the longest theoretically predicted radial pulsation periods for white dwarfs of reasonable mass (cf. Ostriker 1971), but periods calculated for nonradial modes match more closely. In particular, periods of nonradial g-modes calculated for white dwarfs most closely approximate the periods observed for the ZZ Ceti variables (Brickhill 1975, Osaki and Hansen 1973). The correspondence of the theoretical to the observed periods is not good enough to allow identification of individual pulsation modes, however. For linear, adiabatic pulsations, the period of a nonradial mode is specified by three integers: k, which specifies the radial overtone, ℓ, the number of surface node lines, and the degenerate parameter, m, which may assume values $|m| \leq \ell$. Periods for g-modes on white dwarfs and approximate relationships for g-mode periods as a function of k, ℓ and m are given by Brickhill (1975). For Brickhill's model which most closely resembles a ZZ Ceti ($0.6\ M_\odot$, $T_e = 13000$ K), he derives periods $P_{k\ell}$: $P_{12} = 136$ s, $P_{22} = 178$ s and $P_{32} = 218$ s. This last period approximates the shortest periods observed in ZZ Ceti variables. Note that there is no a priori or observational limit on k - with $k \lesssim 30$, theoretical periods matching the longest observed periods can be generated. There is, however, an observational constraint on ℓ. If ℓ becomes large, the surface of the star becomes dissected into many segments of varying surface brightness and the luminosity variations will be rapidly smoothed out, thus the star will not be detected as a variable.

Nonradial pulsations can also account for some of the multiple periods seen in the power spectra of ZZ Ceti variables. Multiple, independent pulsations have been suggested to explain the two principal periods (213 s and 274 s) of ZZ Ceti (Robinson, Nather and McGraw 1976) and the four strongest periods in BPM 30551 (Hesser, Lasker and Neupert 1976). For ZZ Ceti, the period ratio indicates that the 213 s period is associated with a $k = 1$, $\ell = 2$ mode and the 274 s period is associated with a $k = 2$, $\ell = 2$ mode. The period ratio of 1.7 for L19-2 might, by analogy, be associated with Brickhill's periods corresponding to modes with $k = 1$, $\ell = 1$ and $k = 1$, $\ell = 2$. Periods of other variables with these period ratios might be associated with similar modes. The point of this is that, though period ratios do recur, the period spectrum of nonradial modes is so complex that, until theory gives us reasons to choose particular modes, the identification of

the modes in which the variables are really pulsating is virtually impossible. The most likely source of multiple, independent periods in low amplitude ZZ Ceti variables is, however, multiple, independent pulsations.

The fact that the power spectra of the light curves of ZZ Ceti stars change with time can be partially explained by nonradial pulsations on a slowly rotating star. Rotation destroys the spherical symmetry of a star, thus removing the degeneracy of the index m. This results in "rotational splitting" of modes with periods $P_{k\ell}$ into additional modes $P_{k\ell m}$: $P_{k\ell m} = 1/\sigma_{k\ell m} = |\sigma_{k\ell} - m (1 - C_{k\ell})\Omega|^{-1}$, where $C_{k\ell}$ is a constant which depends on the structure of the star and can assume values $0 \leq C_{k\ell} \leq [\ell(\ell + 1)]^{-1}$, and Ω is the rotational frequency (cf. Brickhill 1975). The closely spaced periods produced by this mechanism can modulate the light curve. For ZZ Ceti, the changes with time seen in the power spectra occur because the period of modulation is greater than the length of a photometric run. If closely spaced periods occur in the power spectra of other ZZ Ceti variables, they too will change with time, as is observed. If observing runs longer than the beat period could be obtained, the power spectra of these stars may appear stable. Robinson et al. (1978) suggest that rotational splitting of the $\ell = 2$ g-mode periods creates the closely spaced pairs of periods in ZZ Ceti. It is reasonable that white dwarfs rotate; therefore, this mechanism almost certainly contributes to the changes in the power spectra. This mechanism is not unique, however. Any periods sufficiently closely spaced, arising for example from two independent pulsation modes, will result in a modulation of the power spectra. Another possibility, which I would rather not admit, is that some of these stars, especially the large amplitude variables, are fundamentally unstable in their period structure.

The period changes observed in the moderate amplitude variables BPM 31594 and GD 154 have been interpreted as a transfer of pulsational energy from one mode to another by (weak) nonlinear coupling (McGraw 1976, Robinson et al. 1978). For radial pulsations, Ledoux and Walraven (1958) derive nonlinear coupling coefficients between modes with frequencies f and f_0: $k(f,f_0) \propto (f - 2f_0)^{-1}$. In the absence of similar theory for nonradial modes, we generalize this to $k(f_i,f_j) \propto (nf_i - mf_j)^{-1}$. The coupling can become very efficient near the resonances. The suggestion is, then, that the observed changes occurred between modes where n = 1 and m = 2. In terms of the indices k, ℓ and m, changes in ℓ and m do not readily account for the observed period changes, but if k is allowed to change value by at least 3, modes near this resonance may be found (McGraw 1976). For larger amplitude variables which are presumably more nonlinear, the coupling is more efficient and periods corresponding to

many resonances can appear. This can explain the appearance of the cross-frequencies observed in the power spectra of larger amplitude variables. In addition, nonlinear pulsations produce nonsinusoidal pulses in the light curve (cf. Figure 1). The pulse shape will then contribute to the frequency structure in the power spectrum. Elementary Fourier analysis tells us that the first additional frequencies to appear will be harmonics of the pulse repetition frequency. These two effects, harmonics and cross-frequencies, both of which are related to increasing nonlinearity, are responsible for the complexity/amplitude correlation. Apparently, ZZ Ceti variables of low amplitude are linear pulsators but large amplitude variables are highly nonlinear.

In summary, the light curves of ZZ Ceti variables range from simple to very complex, but even the most complex can apparently be explained by several simple effects. Multiple nonradial modes, probably corresponding to different radial overtones, may be simultaneously excited in each star. The excitation energy of individual stars is distributed among permitted modes by nonlinear resonant coupling. In addition, "rotational splitting" of the nonradial modes can produce closely spaced periods which results in modulation of the light curve. The amplitude/spectral complexity correlation results from the appearance in the power spectrum of harmonics and cross-frequencies which are the effects brought on by increasing nonlinearity of the pulsations.

When theoretical models of these stars are done, the rewards are likely to be great. Certainly we will increase our understanding of the fundamental evolution of white dwarfs. A program already underway is to directly measure the cooling times of linearly pulsating variables. Osaki and Hansen (1975) have shown that there exists a period-luminosity relationship for nonradial pulsations on white dwarfs. Measuring the highly stable pulsations of stars like R548 over a baseline of 50-100 years will allow a significant determination of the rate of change of the period and thus give a measurement of the cooling time of the star. A more immediate result might be to set limits on the core composition of the variables by measuring the baseline over which $\dot{P}$ does not appear to change. For example, $\dot{P}$ for an iron white dwarf is more than a factor of two greater than for a carbon core white dwarf.

Observers are beginning to find that nonradial pulsations are ubiquitous. In addition to the ZZ Ceti variables they are observed in the β CMa and B stars (Smith 1977). There is evidence that they occur in δ Scuti stars (Millis 1973), they have been suggested as the pulsation modes of white dwarfs in cataclysmic variables (Chanmugam 1972, Warner and Robinson 1972), and of course they occur in the sun (cf.

Rhodes, Ulrich and Simon 1977). Because of their short
periods and the ease with which they can be observed, the
ZZ Ceti stars are probably the most extensively observed
class of nonradially pulsating stars. When the theory of
pulsation on these stars can explain the observations, the
ZZ Ceti stars will be a laboratory of linear and nonlinear
nonradial pulsations, from which investigations into the
pulsational instabilities in other stars may be firmly
launched.

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Discussion

Aizenman: You mentioned that the m = 0 mode was missing. Would you explain this further?

McGraw: Well, I'll try. When we see patterns of what we call equally spaced frequencies in the power spectrum, we generally assume an ℓ = 2 mode and that what we are seeing is m = $\pm$ 2, $\pm$ 1, but in two cases that I can recall the m = 0 mode would be missing in that interpretation. I have no idea why that kind of selection would occur. In the case of ZZ Ceti, the detailed analysis of its light curve showed a high stability, and an attempt was made at mode identification, in which case the m = 2 mode was picked. But we can't really give a justification why the m = 0,1 modes aren't present as well.

A. Cox: I want to ask a technical question. Where are your side bands? Why don't you have side bands on all these? Aliases?

McGraw: You do have to contend with them, but the amplitude is very low. We have long data streams to begin with, and we "window" the data. I think what you are driving at is whether we are seeing significant peaks.

A. Cox: I was just wondering why you don't have those wiggles on the side?

McGraw: They are very low amplitude, compared to the amplitudes that we are measuring.