MIRA VARIABLES: AN INFORMAL REVIEW

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The Mira variables can be either fascinating or frustrating -- depending on whether one is content to watch them go through their changes or whether one insists on understanding them. Virtually every observable property of the Miras, including each detail of their extraordinarily complex spectra, is strongly time-dependent. Most of the changes are cyclic with a period equal to that of the light variation. It is well known, however, that the lengths of individual light cycles often differ noticeably from the star's mean period, the differences typically amounting to several percent. And if you observe Miras -- no matter what kind of observation you make -- your work is never done, because <u>none</u> of their observable properties repeats exactly from cycle to cycle.

The structure of Mira variables can perhaps best be described as <u>loose</u>. They are enormous, distended stars, and it is clear that many different atmospheric layers contribute to the spectra (and photometric colors) that we observe. As we shall see, these layers can have greatly differing temperatures, and the cyclical temperature variations of the various layers are to some extent independent of one another. Here no doubt is the source of many of the apparent inconsistencies in the observational data, as well as the phase lags between light curves in different colors. But when speaking of "layers" in the atmosphere, we should remember that they merge into one another, and that layers that are spectroscopically distinct by virtue of their vertical motions may in fact be momentarily at the same height in the atmosphere.

I sometimes find it helpful to think of Miras as jellyfish. As they move through the water, their general oscillatory motion can be expected to continue, but it is impossible to predict all the details of their changes in shape and appearance; if you tweek a jellyfish on one side, you don't know if, when, and with what amplitude the disturbance will reach the other. If you think of Miras this way, you will stop worrying about their failure to repeat exactly in their variations.

No one has ever called Miras a theoretician's delight. Certainly they have not been very useful in testing theories of stellar pulsation. There is just too much going on -- it's hard to know which observable properties are even relevant to pulsation. In fact the question has sometimes been raised as to whether the Miras are pulsating at all. Merrill (1955) wrote that "the evidence for volume pulsation is so meagre that skepticism is warranted", and Wallerstein (1977) has proposed that the apparent radius changes in Miras are caused not by the outward movement of the gas but simply by changes in the atmospheric opacity. Recent results from infrared spectroscopy (Hinkle 1978) have clarified the picture considerably, basically by allowing us to look more deeply into the atmosphere, and I think that the last doubts that Mira variables are pulsating have finally been laid to rest.

A scheme has recently been devised by Cahn and Wyatt (1978) by which one can estimate the masses and luminosities (and hence ages) of individual Miras from two readily observable quantities, the mean period and the mean spectral type at maximum. This scheme is based on the assumption that Miras are pulsating stars, more specifically that they are pulsating in the first overtone. Since we have few opportunities of determine masses and luminosities of individual Miras directly, it will be difficult to decide observationally whether this picture is correct. At this stage the question is not whether the relations of Cahn and Wyatt will eventually need recalibrating, but whether they exist at all. However, the fact that Cahn and Wyatt were able to construct a self-consistent picture suggests that they may be on the right track, and that it may indeed be possible to understand the gross observable properties of individual Miras, such as surface temperature and mean period, in terms of their masses and evolutionary states.

In describing to you the observed behavior of Miras, I will attempt to select those particular observations that have a bearing on the question of pulsation -- although, as I have already indicated, it is not always obvious which observations these are. The discussion will be centered around the sizes of these stars, or more particularly the evidence for changes in size. No mention will be made of their absolute magnitudes, ages, chemical compositions, population types, galactic distribution, statistical properties, or other matters not directly related to pulsation. Rather, we will be concerned with the variations of individual stars, and with some spectroscopic peculiarities that are related to the enormous sizes of their atmospheres.

As soon as we try to talk about the sizes of Mira variables, we run into a serious problem. The reason that we can see so many different atmospheric layers at the same time is that the continuous opacity -- between the spectral lines and bands -- is extremely low. A Mira is about as translucent as a jellyfish: you can practically see right through it. Some regions contributing to the spectrum are much farther from the center of the star than others, and this is all within the region we call the "photosphere". In order to speak of the size of a Mira, we must specify the wavelength precisely. Furthermore, there may be no real discontinuity between the photosphere and the circumstellar shell that contributes zero-volt absorption lines and infrared emission, since presumably the shell consists of material which has drifted away from the star, and which may receive new contributions with every light cycle. In this respect, Miras are worse than jellyfish. Although the radius of a jellyfish is both time-dependent and angle-dependent, at least there is a membrane to show us where the jellyfish ends and the ocean begins. The size of a jellyfish is certainly difficult to measure, but at least the creature has a size.

The methods that can be used to determine the sizes of Miras fall into three classes:

(1) Direct. Angular diameters of Miras have been measured directly with Michelson interferometers, by observations of lunar occultations, and by speckle interferometry. The distance must be known to calculate the ab-

solute size, but useful information about <u>changes</u> in size can be obtained directly from the observed changes in angular diameter.

(2) <u>Photometric</u>. If you know the luminosity L (or equivalently the absolute bolometric magnitude M_{bol}) and the effective temperature T_e, you can calculate the size of the emitting surface area and hence the stellar radius R, since $L = 4\pi R^2 \sigma T_e^4$. If you don't know the distance and have only the apparent bolometric magnitude m_{bol}, you can still calculate the change in radius from the observed changes in T_e and m_{bol}.

(3) <u>Spectroscopic</u>. Measurements of radial velocities of absorption and emission lines give information about vertical motions in the atmosphere. Integration of the radial velocity curve then gives the distance moved by the gas producing the lines.

All of these methods are clear-cut in concept, and all of them are known to "work" in the sense that they give reasonable results for the sizes of other kinds of stars. But all of them get into trouble when they are applied to the Miras, and in general the results from the three methods are in poor agreement.

For example, Figure 1 shows a famous illustration from the paper by Pettit and Nicholson (1933). From the variations in bolometric magnitude (obtained from the observed radiometric magnitudes, with crude corrections for absorption by the earth's atmosphere) and temperature (from a <u>very</u> broad-band color index, which compares the radiation shortward and longward of 1.3 μ), they computed the variation in the angular diameter of Mira (o Cet). Differentiation of the diameter curve then gave the "radiometric radial velocity curve", which is compared to the radial velocity curve measured spectroscopically by Joy (1926) for the mean absorption spectrum of Mira. The two curves have similar amplitude but are badly out of phase -- in fact, they are nearly mirror images of one another. Similar comparisons for three other Miras have recently been published by Wallerstein (1977); the higher quality of the data used in his analysis did not make the discrepancy go away.

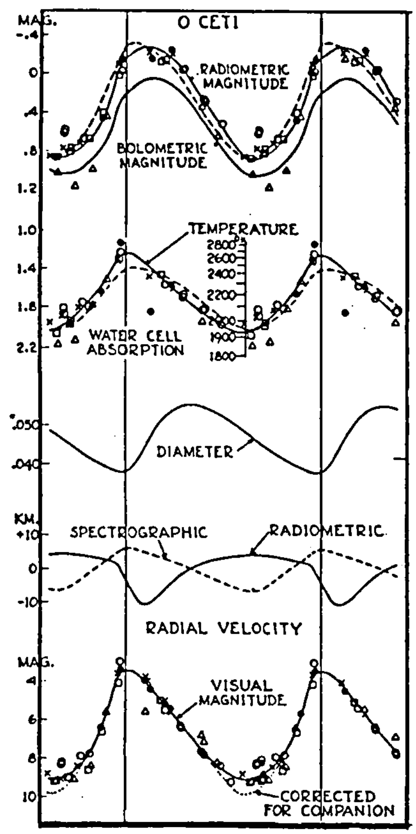

Fig. 1 - The variations of o Cet (Mira), according to Pettit and Nicholson (1933). Note that the color temperature variations are in phase with the visual light curve but not the bolometric curve. The diameter curve is computed from the temperature and bolometric magnitude. Its derivative, the radiometric radial velocity curve, disagrees with the radial velocities determined spectroscopically.

What is the problem? It's not that the observations of Miras are so difficult. All three methods (including Michelson interferometry) had been applied to Miras by the 1920's, and in general the measurements by each method are reproducible to sufficient accuracy. Also, impressive advances in each of the three areas of observation have been made within the past decade, and yet the discrepancies persist. The problem, it seems to me, is simply that when we observe a Mira by any of these methods, <u>we don't know what we're looking at</u>.

For most of the remainder of my talk, I will give examples of the use of each method, focusing attention on the problems of interpretation. I think it will be clear why observers can derive a lot of pleasure from the study of Miras, and why theoreticians, for the most part, can not.

THE DIRECT APPROACH

The basic problem with the direct approach to measuring the sizes of Miras is that it really is not very direct. We don't measure the size of an image with a ruler. Rather, we observe some optical phenomenon that is related to the size of the star -- the visibility of interference fringes at different mirror separations, or the degree of degradation of the diffraction pattern as the star disappears behind the moon, or the character of the speckles in the seeing disk. In each case, a model for the distribution of light in the true stellar image must be assumed before the observed quantity can be related to image size.

How much limb darkening (or brightening) is there? Is the star round? Does it have spots on it? If the answer assumed for any of these questions is wrong, so is the angular diameter that we get. Nevertheless, the results obtained by these techniques have certainly been instructive. The main problem with using these results is that there haven't been enough of them.

The angular diameter of Mira was measured with a Michelson interferometer in January 1925 by Pease at Mount Wilson (see Kuiper 1938). Unfortunately, the method could be employed only when the variable was at maximum light.

Diameter measurements of Miras by lunar occultation observations are a rather recent innovation, since a time resolution of a few milliseconds is needed to resolve the diffraction pattern. A few years ago Nather and Wild (1973) succeeded in observing an occultation of R Leo at V = 8 on the declining branch. Its phase was estimated to be 0.27, so that it should have been very nearly at its maximum diameter if the diameter curve of Pettit and Nicholson (Figure 1) is valid. The light curve of this event is shown in Figure 2. Anyone who has seen occultation data for just about any other star will recognize that R Leo is enormous. The diffraction pattern is completely smeared out. From the slope of the decline, Nather and Wild computed a uniform-disk diameter of 0.067 arcsec.

For stars as large as this, the occultation technique runs into a

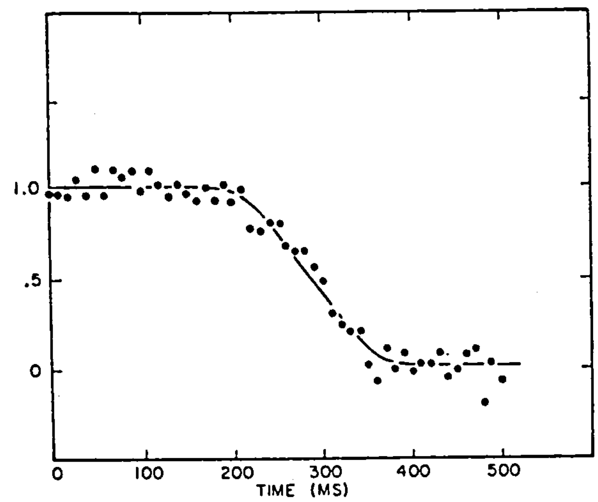

Fig. 2 - Light curve of the occultation of R Leo on 19 May 1972, as measured by Nather and Wild (1973). The computed angular diameter is 0.067 arcsec.

snag, as Nather and Wild point out. Since there is no diffraction pattern, there is no information as to the slope of the lunar limb at the point of contact. One rock could spoil the result. Fortunately this problem can be overcome by planning simultaneous observations at different observatories. It does not arise if the star is, say, one-quarter to one-tenth the size of R Leo; there are many Miras in this range of angular size, but of course they are correspondingly fainter, and photon noise becomes a problem.

Occultation observations of Miras are now being pursued vigorously by Ridgway and his colleagues at Kitt Peak (Ridgway, Wells, and Joyce 1977). Most of their measurements are being made at 2 μ in the infrared, so that daytime observations are possible. Since it is impossible to control the motion of the moon, this method will never produce a true diameter curve for any single star. However, even a simple pair of observations of the same Mira could be useful in indicating whether the spectroscopic or the photometric diameter curve tends to be confirmed by the direct method. Occultation measurements of U Ori were recorded by Ridgway, Wells, and Joyce in two different lunations, and they indicate that the diameter is several percent larger at phase 0.36 than at phase 0.99. This result appears to confirm the photometric diameter curve, but unfortunately no conclusion can be drawn. The two observations were made with different filters, one in the continuum and one in an H_2O band, and it is quite possible that the measured change in diameter has more to do with wavelength dependence than with time dependence.

Exciting results obtained by speckle interferometry have recently been published by Labeyrie, Koechlin, Bonneau, Blazit, and Foy (1977). R Leo and o Cet were found to be twice as large at wavelengths affected by strong TiO bands as they are at continuum wavelengths. Information about changes in diameter with phase is still very limited, but the indications are that changes with wavelength at a given phase are much more dramatic than changes with phase at a given wavelength. The acquisition of further speckle data, especially if timed to cover a substantial portion of the light cycle of a single Mira, would be enormously valuable.

THE PHOTOMETRIC APPROACH

The formula $L = 4\pi R^2 \sigma T_e^4$ is straightforward enough, but the Miras are not. One problem is that the formula assumes the stars are round -- not shaped like jellyfish. We now examine the problems associated with determining L and T_e.

Observationally, it takes a lot of work to determine L, or even the apparent quantity m_{bol}. But at least m_{bol} -- the total radiation from the star reaching the top of our atmosphere -- is a well-defined quantity. Since the time of Pettit and Nicholson (1933), much more sophisticated methods have been applied to the determination of m_{bol}, including the use of Stratoscope scans to interpolate between the various infrared magnitudes measured from the ground (Smak 1966). A few observations of Miras have also been made from high-altitude aircraft (Strecker, Erickson, and Witteborn 1978). I am currently collaborating with J. Smak on the determination of m_{bol} for a large set of Miras from extensive wide- and narrow-band photometry. My impression is that if you are willing to do the work and are careful about the photometric calibrations, you can determine m_{bol} to an accuracy of a few percent. Recent work on this problem has not changed the character of the bolometric light curves derived by Pettit and Nicholson. In other words, I don't think errors in the determination of m_{bol} can be responsible for the discrepancies mentioned above.

Light curves measured at carefully-chosen continuum points in the infrared can give a good approximation to the bolometric light curve. Lockwood and Wing (1971) have published light curves for 25 Miras in I(104), measured photoelectrically with a narrow bandpass at 10400 Å, and they were found to have the same amplitudes and phasing as the bolometric light curves of Pettit and Nicholson. Since I(104) is very much easier to measure than m_{bol}, it is nice to know that it provides essentially the same information.

The I(104) light curves have proved quite interesting, especially since each measurement of magnitude has been part of a set of narrow-band photometry which also gives the spectral type (from the strengths of TiO and VO bands) and the near-infrared continuum color. When I started this work in 1965, my hope was that the I(104) curve of any Mira would repeat so well from cycle to cycle that its characteristics could be established once and for all. Several Miras were followed through two or three cycles to test this idea, and the first results seemed promising. Figure 3 shows the visual and I(104) curves for W Peg in two successive cycles. The V curves, measured photoelectrically with a UBV photometer, show typical cycle-to-cycle differences: one maximum is 0.3 mag brighter than the other, and it occurred ahead of schedule; the slopes on the declining branches are also different. On the other hand, the I(104) magnitudes followed the same curve in both cycles. I would like to be able to tell you that the differences in the visual maxima were caused by differences in blanketing of the visual region by TiO, but the fact of the matter is that W Peg attained the same spectral type, M7.0, at both maxima.

These observations of W Peg, from 1965 and 1966, were made with a spectrum scanner. Since 1969 I have been using a set of eight interference filters to obtain similar information. In addition, Lockwood (1972) has published extensive photometry of Miras on a five-color system in the same near-infrared spectral region. These three systems have enough in common that it has been possible to work out the transformations between them (Lockwood and Wing 1971; Wing and Lockwood 1973); in particular, all three measure I(104).

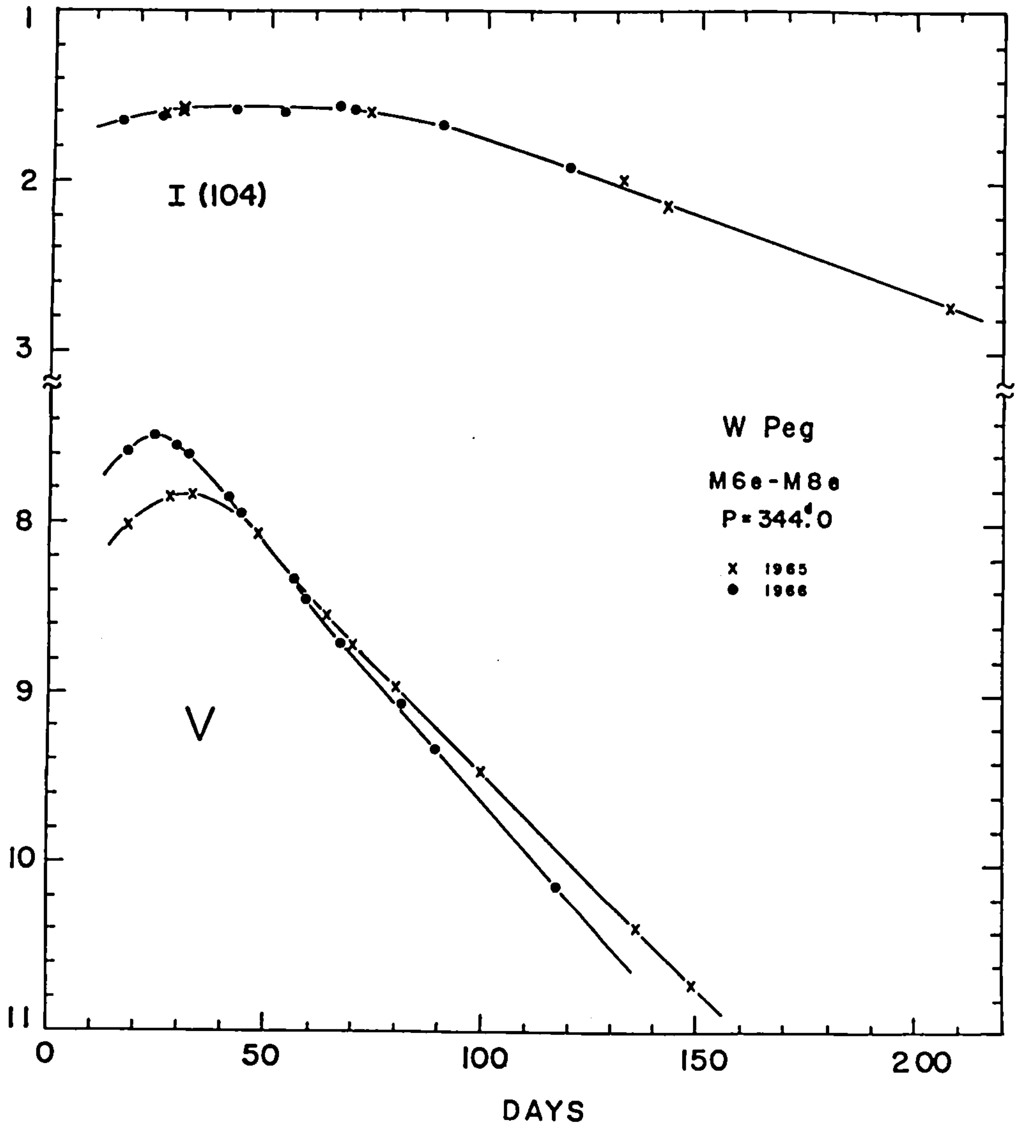

Fig. 3 - Photoelectric light curves in V and I(104) for W Peg, a typical Mira variable, during the accessible portions of two consecutive cycles. In this case the differences in the visual curves are not reflected in the I(104) curve. Note also that the infrared maximum occurs well after the visual maximum. From Wing (1967).

When Lockwood and I combined our data to form I(104) light curves for several Miras over a number of cycles, it became clear that cycle-to-cycle differences <u>do</u> occur in the I(104) curves quite commonly, and that the nice behavior shown by W Peg in Figure 3 is the exception rather than the rule. Several of these I(104) light curves are shown in Figure 4, where different

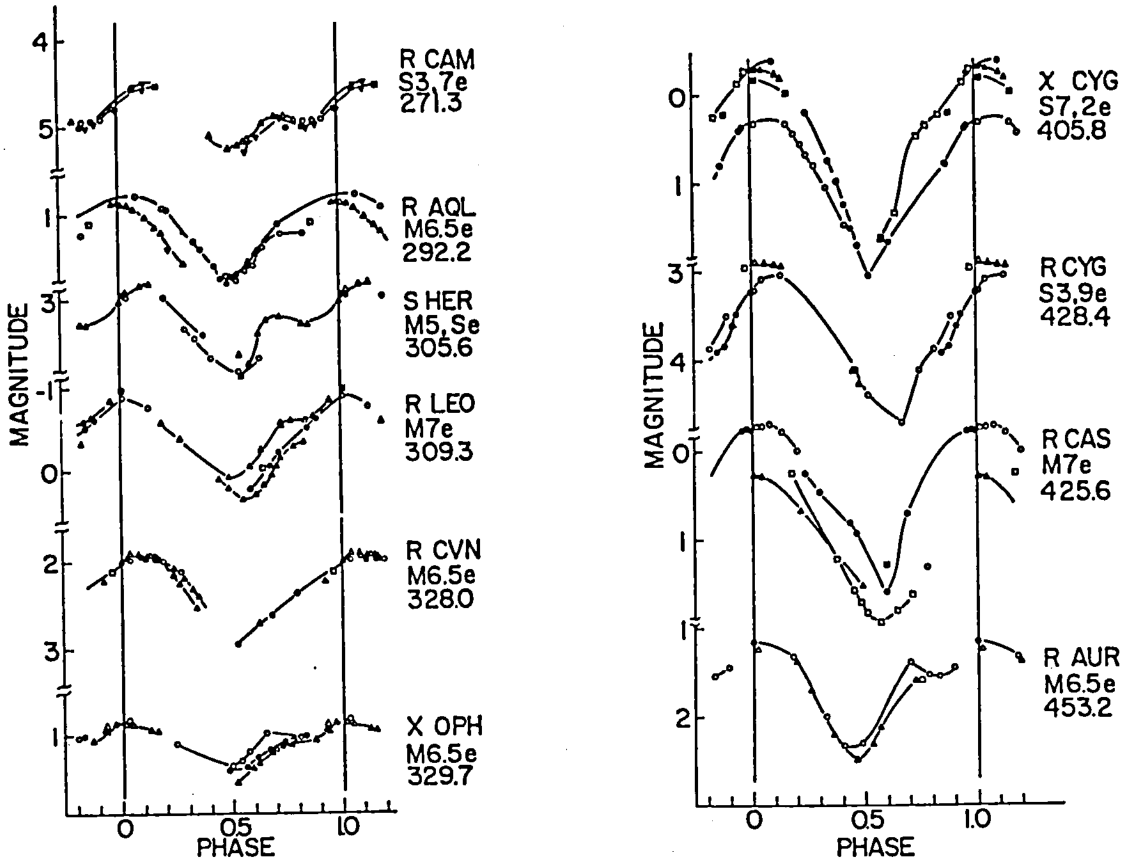

Fig. 4 - Light curves in I(104), an infrared continuum point, for Miras of relatively short period (left) and long period (right). Below the name of each star is its mean spectral type at maximum light (Keenan 1966) and the mean period used in calculating the phases. From Lockwood and Wing (1971).

symbols have been used to distinguish the different cycles. The stars of relatively short period usually repeat fairly well, while the large-amplitude, 400-day Miras show more substantial cycle-to-cycle differences. From an analysis of these differences in terms of the simultaneously measured spectral types and color temperatures, we were forced to conclude that cycle-to-cycle differences affect the bolometric curves as well.

Several of the stars in Figure 4 show humps on the rising branch, at about phase 0.7. Similar humps have long been known to occur in the visual light curves of certain Miras, and it was not known whether they are caused by superficial changes in spectroscopic features or by more basic changes in the continuum radiation. Now we see that the humps are present in the continuum radiation; blanketing changes do not affect the I(104) magnitude, and in any case the spectral types were observed to remain constant, at their latest value, throughout the interval from phase 0.6 to 0.8, whether or not a hump occurred in the light curve. The TiO and VO bands used for spectral classification are evidently formed very far from the layer emitting the continuum; these molecules do not start to dissociate until a month or two after the photospheric temperature has started to rise.

The infrared data show that humps on the rising branch are quite common: most stars observed in two or more cycles show a hump in at least one cycle. On the other hand, few stars seem to have humps in every cycle.

It is difficult to avoid the conclusion that humps also occur in the bolometric light curve, whenever they occur in I(104). If we know the shape of the bolometric curve, we can use the color temperatures measured in the infrared continuum, along with the usual formula, to inquire how the radius changes when a hump occurs. Interestingly, the color temperatures are observed to increase smoothly and monotonically, from minimum to visual maximum, no matter whether a hump occurs in the light curve or not; there is never a hump in the temperature curve. Thus the leveling-off or decrease in luminosity following a hump must be the result of a rapid decrease in radius prior to maximum light. Lockwood and I suggested that the occurrence (or not) of a hump of the rising branch is simply the result of the interplay between rising temperature and decreasing radius during this part of the cycle.

This brings us to the question of what temperature is really appropriate to use in the formula $L = 4\pi R^2 \sigma T_e^4$. The fundamental problem with applying this formula to the Mira variables, it seems to me, is that the effective temperature T_e is _defined_ by this formula and has meaning _only_ if we can attach a meaning to the radius R. Since the star has no membrane, we have to think of the radius as the distance from the center of the star at which the optical depth takes on some value, such as unity; as we have seen, the radius is then strongly wavelength-dependent and may vary by as much as a factor of two over the width of a strong spectral feature. Some kind of averaging is needed, but it is not clear what kind of mean opacity, or mean radius, corresponds to the "effective" temperature. There are innumerable ways of estimating the temperature of a Mira spectroscopically or photometrically, but different methods often give substantially different results, in part because they refer to different layers of the atmosphere which really do have different temperatures, and in part because most line ratios and photometric color indices are not pure indicators of temperature.

So what do we do? The usual response is to go ahead and use the formula anyway. That is, we determine m_{bol} as best we can, estimate T_e from a color index that we hope is representative (or worse, from the spectral type), and bravely plug them into the formula to compute the size. This gives us a number, but we really don't know how this number is related to the size of the star.

Applications of the photometric method to Mira variables do at least give internally consistent results, as exemplified by Pettit and Nicholson's diameter curve in the center of Figure 1. The size (or rather, this number) is smallest near the time of visual maximum, and it increases most rapidly between the times of visual maximum and bolometric maximum, which occurs one or two months later. Nearly all spectroscopic and photometric temperature indicators agree that the highest temperature occurs very close to the time of visual maximum; if the temperature really drops during the following month or so, a rapid increase in the size of the emitting region is needed to account for the increase in bolometric flux.

There are three further results from the narrow-band photometry of Miras which, although not clearly related to radius variations, do tell us a good deal about the structure and extent of their atmospheres: (1) the temperatures measured in the continuum are usually much higher than would be expected from the spectral type; (2) spectral types determined from different TiO bands are often grossly discordant; and (3) the variations in spectral type are only loosely coupled to the variations in color temperature. While each of these findings came as a surprise, I believe they are all manifestations of the same thing, namely the great stratification of these stars' atmospheres.

Color temperatures that are abnormally high for the spectral type do not always occur -- as a consequence of the loose coupling indicated in the third result above, the effect can go either way -- but most Miras have high temperatures for their spectral types at most phases. The effect is most conspicuous (and best established) in the early-type Miras near maximum light, when the near-infrared color temperatures are completely free from blanketing effects. For example, R Tri at its 1965 maximum attained a color temperature as high as that of a normal K4 giant, but its spectral type was never earlier than M3 (Spinrad and Wing 1969). I interpret this as meaning simply that the continuum and the absorption spectrum are formed very far apart, in regions of very different temperature. In other words, the atmosphere of a Mira is more stratified than that of a normal giant (Wing 1967).

A recent study of hydrodynamical phenomena in Mira variables (Willson and Hill 1979) lends credence to the conclusion that their atmospheres may be more distended than those of non-variable M giants of the same luminosity. There is simply not enough time for the atmosphere to recover from the effects of one shock wave before the next shock starts to propagate through it. The atmosphere is thus never in a "normal" state, and although the star has the energy output of a giant, the physical characteristics of its atmosphere, such as density and temperature structure, may more closely resemble those of a supergiant. Indeed, Mira variables seem spectroscopically to have the luminosities of supergiants, if their pressure-sensitive line ratios are interpreted in the usual way. For this reason, Keenan has always refrained from assigning luminosity classes to Mira variables (Keenan 1966; Keenan, Garrison, and Deutsch 1974). Unfortunately, not all investigators have exercised such restraint, and supergiant luminosity classifications have been published for several Miras, leading to possible confusion as to their actual luminosities.

The second of the results from narrow-band photometry mentioned above refers to the assignment of temperature classes. For the Miras, temperature classification becomes ambiguous as soon as we consider two different classification criteria -- even if they are just different bands of the same molecule. In Figure 5, we see that the relative strengths of the TiO bands measured by filters 1 and 3 on the eight-color system are not the same in the Mira as they are in the giant. For the giant we get the same spectral type from both TiO bands and from the continuum color, while for the Mira we get

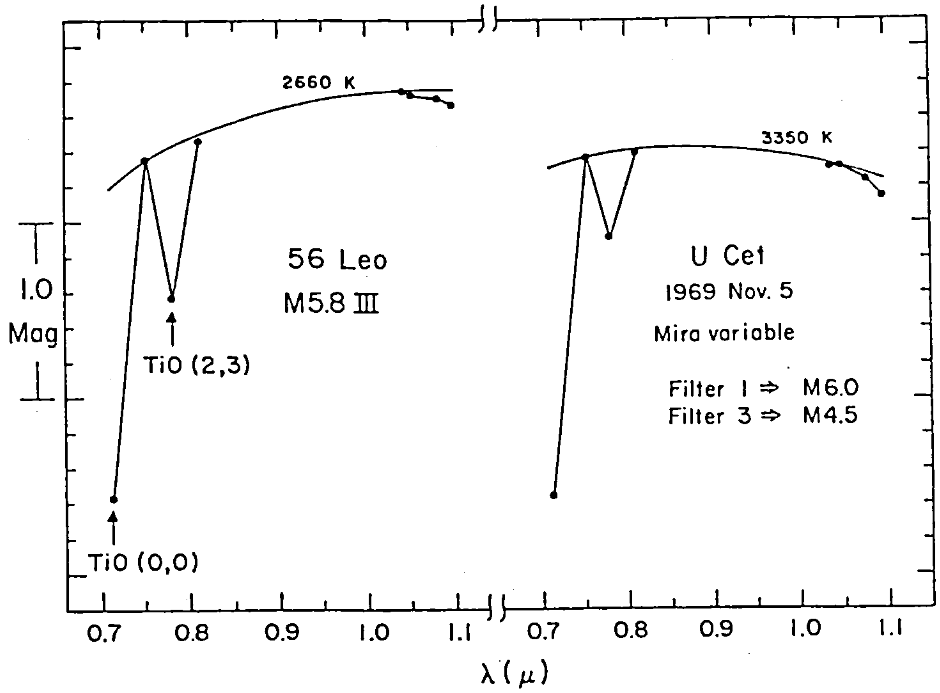

Fig. 5 - Eight-color photometry for 56 Leo, a normal, unreddened giant, and U Cet, a Mira variable. The two stars have nearly the same TiO strength at filter 1 but different TiO strengths at filter 3 and very different color temperatures. From Wing (1974).

M6 from the zero-volt TiO band, M4.5 from the excited TiO band, and M3 from the color. Clearly we must be careful in using spectral classifications of Miras; in particular, we should not use them to infer the temperature of the photosphere. At the same time, these results encourage me to hope that the infrared color temperature from the eight-color photometry may indeed be suitable to use in calculations of the radius, since it appears to refer to the same deep layer from which most of the total flux is emitted.

The loose coupling between color temperature and band strength is illustrated in Figure 6. The loops executed by Miras are really enormous -- the band strengths can differ by a factor of two or more between phases of the same color temperature. Because of this, bolometric corrections for Miras must be tabulated as two-dimensional functions of band strength and color, rather than as one-dimensional functions of spectral type as have always been used in the past. Another consequence of these loops is that they render the band-strength data virtually useless for abundance determinations.

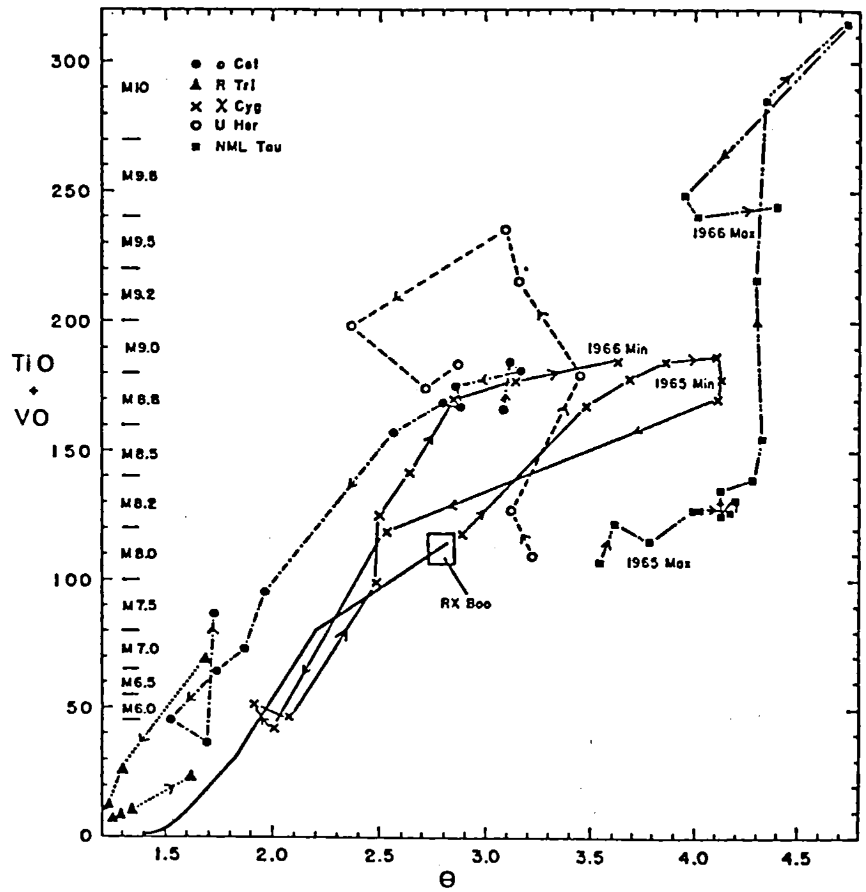

Fig. 6 - An index of molecular band strength is plotted against the reciprocal color temperature (5040/T), both measured in the near infrared with a scanner (Wing 1967). Normal giants define the heavy line ending in the box labeled RX Boo, whereas Mira variables execute large loops. See Spinrad and Wing (1969) for details.

These loops can be interpreted in the same way as the other phenomena we have discussed -- the band strengths and the continuum color refer to widely separated regions, the temperature variations in which are out of phase. This behavior could be modeled if all Miras showed loops that were at least qualitatively similar, but they're all different! In Figure 6, χ Cyg goes clockwise while U Her goes counter-clockwise. In fact, the loops shown by the same star in different cycles are not necessarily any more similar than the loops of two different stars.

Well, what do you expect of a jellyfish? Remember that the layers contributing to the spectra are near-perfect vacua separated by millions of miles, and you will be able to excuse their poorly-coordinated performance.

THE SPECTROSCOPIC APPROACH

The spectra of Miras are incredibly complex. They are dominated in the ultraviolet and blue regions by atomic absorption lines, in the visual and near infrared by bands of metallic oxide molecules, and in the infrared beyond 1.5 μ by innumerable lines from the rotation-vibration transitions of CO and H_2O. In addition to these absorption features, emission lines of various descriptions are present. Hydrogen lines of the Balmer, Paschen, and Brackett series are strong in emission during more than half the cycle, from just before maximum to approximately the time of minimum light; since absorption lines can be seen (and identified) within the broad emission profiles of the Balmer lines, it is clear that the hydrogen emission is produced in a deep layer of the atmosphere (Joy 1947). Some of the weaker emission lines are known to be produced by fluorescence, and although I will not discuss these particular lines further here, I should mention that the careful study of fluorescence mechanisms in Miras can provide important information about the structures and motions of their atmospheres (Wing 1964; Willson 1976). In fact, the very fact that fluorescence mechanisms are operative shows that these atmospheres are so rarefied that the populations of excited levels in atoms are governed by radiative processes rather than by collisions. Other metallic emission lines seem to be produced by recombination, some remain unidentified, and still other emission lines have been found to have molecular origins. Interesting reviews of the line spectra of Miras have been published by Merrill (1960) and Willson (1976).

Recent observations of Miras have revealed additional emission lines. Just six weeks ago, the first emission lines to be detected in the ultraviolet spectrum of a Mira variable below the atmospheric cut-off were recorded in R Leo with the IUE satellite (Wing and Carpenter 1978). At the other end of the spectrum, lines emitted by OH, H_2O, SiO, and CO have been detected in the microwave region.

Spectroscopic studies of the pulsational properties of Miras are based on the measurement of radial velocities. Unfortunately, an accurate measurement is not enough; there are three problems with which we must deal before the radial-velocity data can be converted into information about the expansion and contraction of the atmosphere. First, because of projection effects, the measured radial velocity does not tell us immediately the motion of the surface of the star; we must apply a very uncertain correction for geometry and limb darkening. Emission lines are particularly difficult to use, since they may be either limb-darkened or limb-brightened, depending on the depth of their formation. This problem has been discussed recently by Wallerstein (1977). A more important problem, also discussed by Wallerstein, is that it is not sufficient to know the absolute motion of the surface; we must also know the radial velocity of the center of mass of the star before we can tell whether the surface is moving up or down. Finally, when we find that different spectral features have different radial velocities, we must somehow decide which feature to use as an indicator of the photospheric velocity.

For many years, most of the radial-velocity work on Miras was done at Mount Wilson Observatory, mainly by Merrill and Joy. The spectroscopic radial-velocity curve shown in Figure 1 was taken from an early study of Mira by Joy (1926); 28 years later Joy (1954) published a second study of Mira, from which Figure 7 was taken. Quite generally, the emission lines show smaller radial velocities than absorption lines, i.e. the emitting regions are moving outward and/or the absorbing regions are falling inward. [The only <u>red</u>-shifted emission lines that have been identified in Miras are certain fluorescent lines that are excited by off-center coincidences (Wing 1964)]. Different emission lines, however, display very different behavior: note in particular the curves for Joy's "standard" metallic emission lines, the Fe II lines, and the hydrogen lines in panels (b) and (d) of Figure 7. The behavior of the absorption spectrum is also rather complex: Merrill, in several papers, reported that the radial velocities of absorption lines show a dependence upon excitation potential -- another indication of extreme stratification. In addition, a few instances of doubling of atomic lines have been reported, and evidence of incipient line doubling (fuzzy or irregular line profiles) is rather common.

Faced with the many different velocities that are present in the spectrum at any given phase, how can we decide which velocity best represents

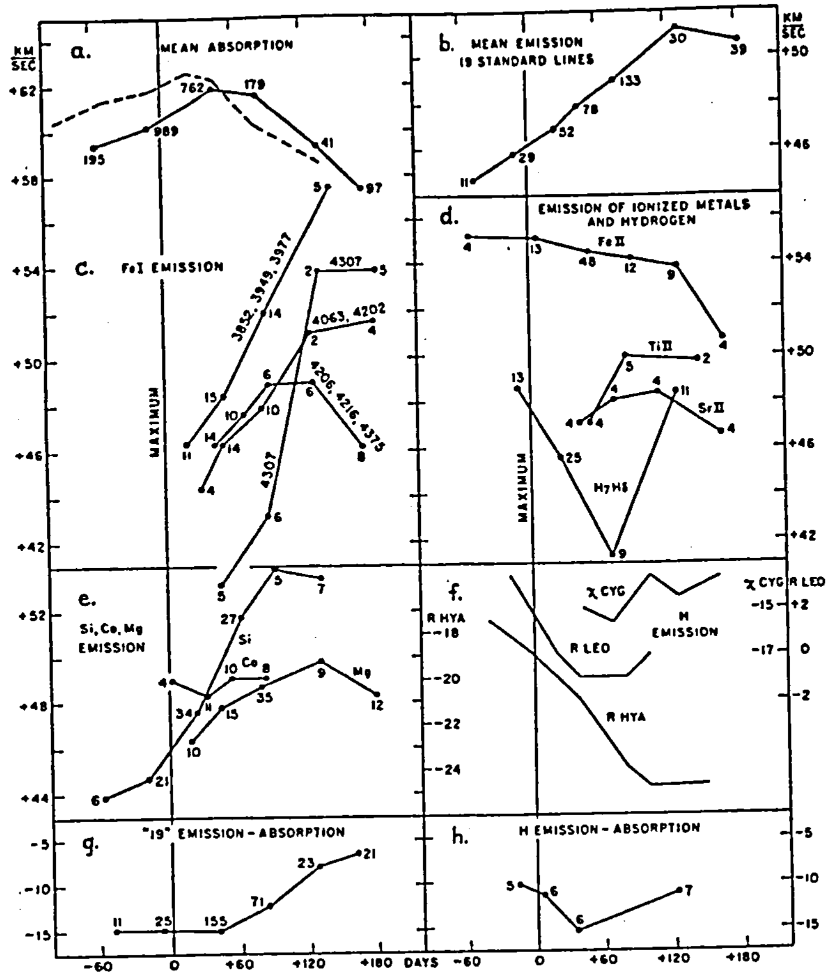

Fig. 7 - Radial velocities measured in the spectrum of Mira, from Joy (1954). Absorption and emission lines, all from the blue spectral region, have been grouped according to their behavior. Numbers next to the points indicate the number of measurements entering the mean.

the motion of the photosphere and which the motion of the center of mass? Many approaches to this question have been tried. Usually some kind of average of the atomic absorption-line velocities is taken to be the photospheric velocity. Common choices for the center-of-mass velocity are the mean absorption velocity at the time of maximum light, the velocity of Fe II emission lines (which appear to be formed in the chromosphere and show relatively little velocity variation with phase), and the velocities of certain molecular microwave emission lines (which are formed still farther out). Unfortunately, all these velocities are different.

For 7 stars with high-quality optical and microwave velocity data, Wallerstein (1975) prepared Figure 8 to illustrate the differences in the

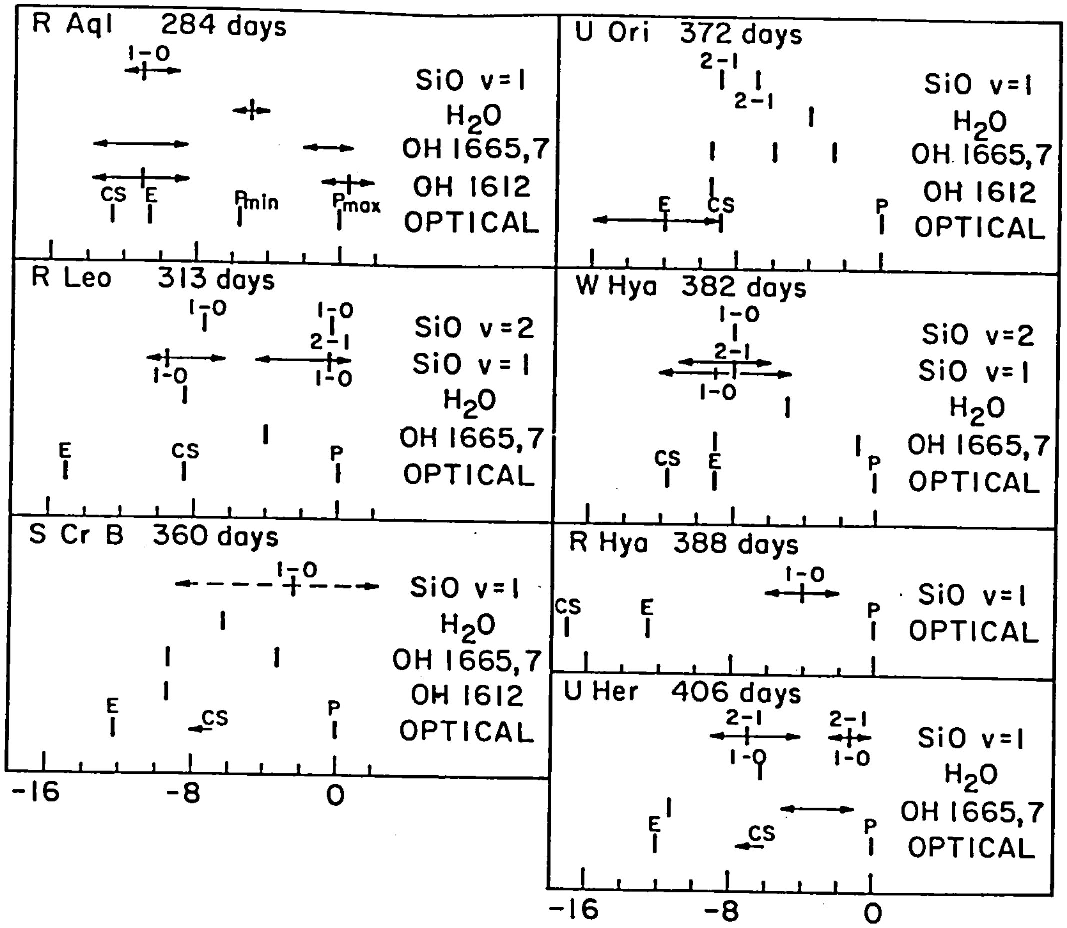

Fig. 8 - Radial velocities of various optical and radio lines plotted relative to the velocity obtained from high-excitation absorption lines (labeled P for "photospheric") for 7 well-studied Mira variables. Characteristic velocities of optical emission (E) and circumstellar (CS) lines are plotted as well as the velocities of radio lines of SiO, H_2O, and OH. From Wallerstein (1975).

various observed velocities which might be considered to represent the motion of the center of mass. Also included are the characteristic velocities of optical emission lines and circumstellar absorption components. The reference velocity in each case is that of the absorption spectrum and is labeled P for "photospheric". Because of the dependence of the absorption-line velocity upon excitation potential, Wallerstein used only high-excitation lines, which are formed in deeper layers than the low-excitation lines, in the determination of P. Even with this precaution, however, it is doubtful whether this velocity refers to a layer as deep as the true photosphere, and in fact the results from the infrared CO lines discussed below seem to show that it does not.

Since Figure 8 was originally drawn, thermal (as opposed to maser) SiO emission has been detected from several Miras. Since this emission must arise in a large, low-density region, it should indicate fairly directly the center-of-mass velocity of the star. According to Reid and Dickinson (1976), the circumstellar emitting region has a modest velocity of expansion (as inferred from the SiO line profile), and the center-of-mass velocity is smaller than the velocity derived from atomic absorption lines at bright phases, i.e. the gas producing the absorption is seen falling back in.

Once the necessary decisions have been made, the radial-velocity curve can be integrated to determine the distance moved by the gas producing the measured lines. The corresponding change in surface area can then be calculated and compared to that obtained by the photometric method. Whenever this exercise has been carried out, as for example by Pettit and Nicholson (see Figure 1) and more recently by Wallerstein (1977), it has revealed a disturbing discrepancy: the motion of the gas inferred from the photometry is simply not confirmed by the measured radial velocities. In fact, Wallerstein showed that a discrepancy exists no matter what value is assumed for the center-of-mass velocity, since part of the problem is that the amplitude of the radial-velocity variations -- at least for atomic lines in the blue -- is much too small to correspond to the photometric variations.

If, like Wallerstein, we choose to assume that the atomic absorption-line velocities are indicating the actual motion of the photosphere, then we are forced to look for an error in the interpretation of the photometry. Wallerstein's (1977) suggestion is that we have been fooled by an opacity effect: the apparent increase in size between the times of visual and bolometric maxima is not due to an actual outward movement of the gas but is simply the result of an increase in the atmospheric opacity as the temperature drops and molecules and grains form. However, this explanation has a fatal flaw. Although an increase in opacity can indeed cause an increase in the apparent size as measured directly (say by an interferometer of some kind), there is no way that it can produce an increase in size as "seen" by a photometer, since an increase in opacity cannot make the star become bolometrically brighter.

If, on the other hand, we assume that there is nothing basically wrong with the conventional interpretation of the photometry, we must conclude that the atomic absorption lines seen in the blue spectral region, even those of high excitation, do not arise in the photosphere, i.e. the layer producing most of the bolometric radiation seen by a photometer. This could be the case if the opacity is much greater in the blue than in the infrared. Evidence in favor of this conclusion has finally been produced by studies of high-resolution infrared spectra which show lines which _do_ have the radial velocities, and the large velocity amplitudes, that are expected for the motion of the photosphere.

The real break-through, it seems to me, came from investigations of line doubling, particularly Maehara's (1968) study of the doubling of atomic lines on near-infrared spectrograms of χ Cyg, an S-type Mira. Instances of line doubling in Miras had been reported earlier -- in the S star R And (Merrill and Greenstein 1958; Spinrad and Wing 1969) and in the carbon star R Lep (Phillips and Freedman 1969) -- but Maehara was the first to carry out a spectroscopic analysis of each set of lines separately and to establish that the blue component is produced in much hotter gas than the other. He therefore was able to construct a reasonable model involving a layer of shock-heated gas rising through a stratum of cooler gases.

Maehara also measured the velocities of the lines of TiO and CN on his near-infrared plates of χ Cyg. These were not doubled, but they didn't have the same velocity, either. The CN lines were found to be formed in the warm, rising layer, while the TiO lines were formed in the cooler layer. No wonder it has been hard to interpret the molecular band strengths of Miras in terms of a single-slab model!

With a two-component model, it is not difficult to see how the discrepancy between the photometric and spectroscopic results might be resolved. We must simply suppose that, during the maximum and post-maximum phases, most of the light comes from the deep, rising layer -- which brightens bolometrically as it swells and rises above the sources of continuous and molecular opacity in the cooler, in-falling layer -- while most of the absorption lines seen spectroscopically are produced in the in-falling layer.

It is only when double lines can be seen in the spectrum that we can measure the temperatures and motions of both layers and thus obtain the information we need to specify the parameters of a two-component model. Spectrograms of Miras in the blue region generally do not show double lines; evidently the opacity in the blue is too great to allow the deep layer to be seen. Rayleigh scattering, with its λ^{-4} dependence, is likely to contribute to the opacity in the blue, along with TiO bands and the overlapping wings of atomic lines. The advantage gained by Maehara (1968) in using the near-infrared and by Spinrad and Wing (1969) in using the one-micron region is considerable. It is also no mere coincidence that all reported instances of atomic line doubling have been found in S- and C-type Miras, which have lower atmospheric opacities than the much more common M-type Miras. [I do not count the doubled lines reported in o Cet by Adams (1941), since the displaced components that he saw were from an expanding circumstellar shell, rather than a deep layer of rising gas].

Much more complete information about the two-component model has come from the study of the infrared lines of carbon monoxide. The two-micron region where the first-overtone CO bands lie corresponds to the minimum opacity due to H^-, which in any case is seriously depleted at the cool temperatures of Miras because of the shortage of free electrons. Although the CO lines themselves are strong, they are not very densely packed, and it is possible to see down to the photosphere between these lines. Furthermore, the CO molecule is very stable and can exist at the high temperatures of the shock-heated photosphere. Hence it is possible to see photospheric CO absorption lines whenever the motion of the photosphere displaces them from the corresponding lines formed in the cool outer envelope.

Figures 9 and 10 show a section of the spectrum of χ Cyg in the region of the first-overtone CO bands on two different dates. They were obtained with a Fourier-transform spectrometer at the Kitt Peak National Observatory. Wavenumber increases from upper left to lower right. Most of the absorption features visible in these spectra are due to CO, although telluric H_2O lines are also present. For orientation I have labeled the (3,1) and (2,0) band heads of $C^{12}O^{16}$ on the third and fourth strips, respectively, of Figure 9.

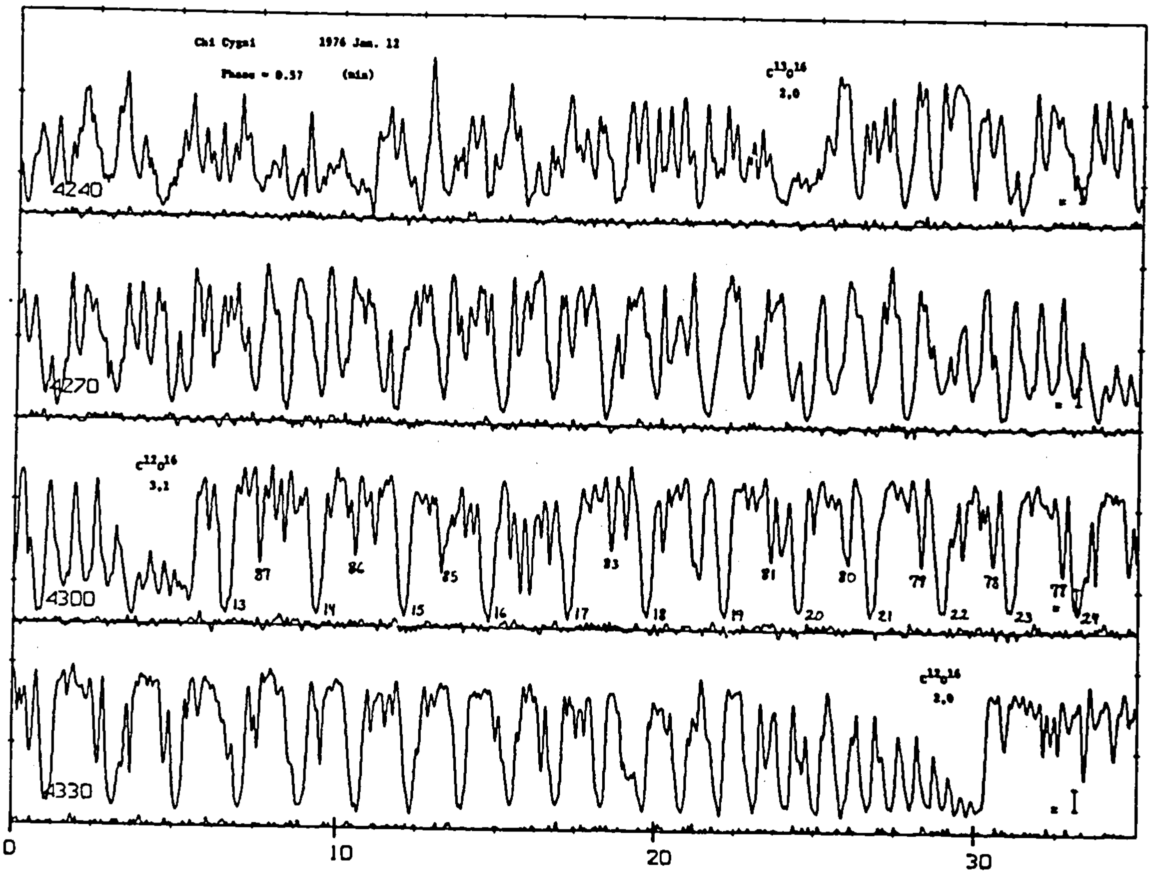

Fig. 9 - The spectrum of χ Cyg from 4240 to 4360 cm^{-1} (2.36 to 2.29 μ) on 1976 Jan. 12, when the star was at minimum light (phase 0.57). The (2,0) and (3,1) heads of $C^{12}O^{16}$ and the (2,0) head of $C^{13}O^{16}$ are labeled, as are the rotational quantum numbers of some of the lines of the (2,0) band (in the third strip). All CO features are sharp and single. From an unpublished Kitt Peak spectrum (courtesy D. N. B. Hall, S. T. Ridgway, and K. H. Hinkle).

The (4,2) head is in the left half of the first strip, where the spectrum is messy because of H_2O contamination, and the (2,0) head of the isotopic molecule $C^{13}O^{16}$ is clearly visible in the right half of the first strip. I have also labeled, on the third strip, the rotational quantum numbers of some of the R-branch lines of the (2,0) band of $C^{12}O^{16}$; the two sequences can, of course, be followed into the fourth strip, but they become blended together as they approach the band head, which occurs at about quantum number 50.

The spectrum shown in Figure 9 was taken at minimum light when the CO lines are sharp and single. The one shown in Figure 10 was obtained seven

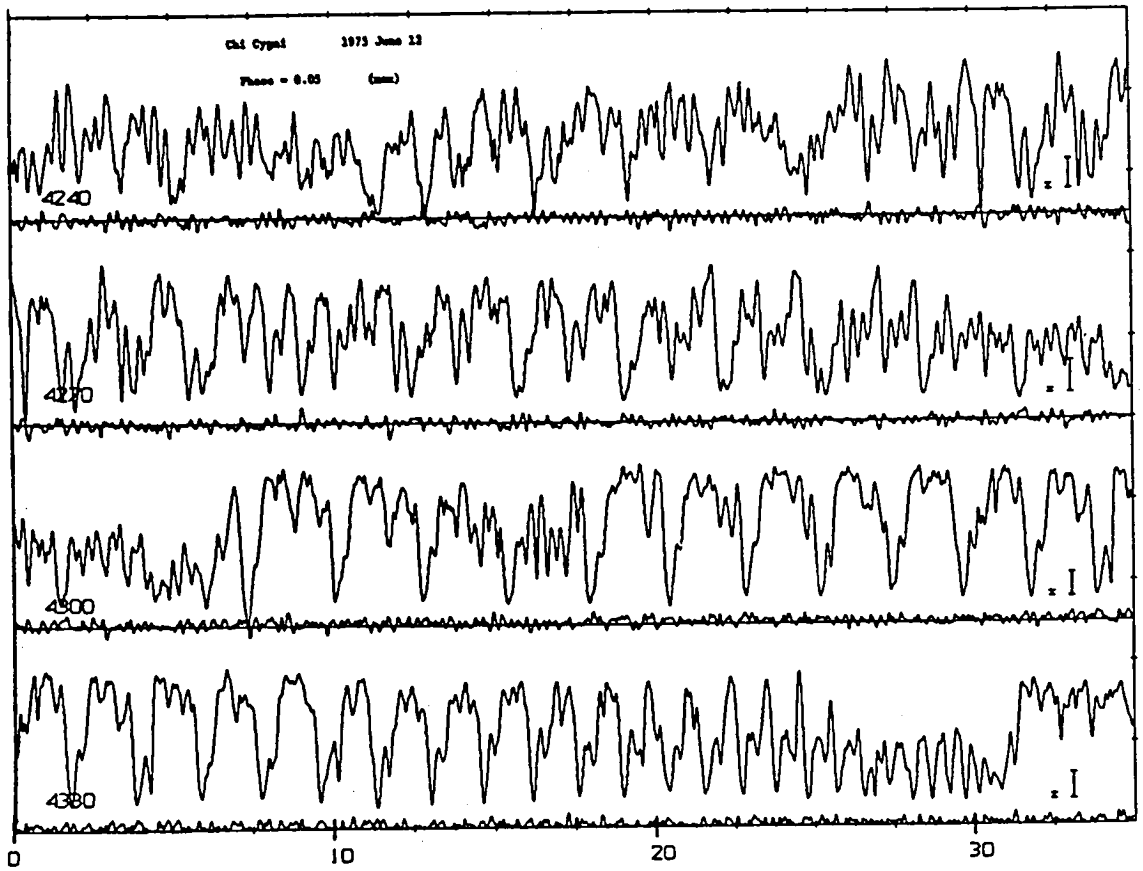

Fig. 10 - The same as Figure 9, except that the date was 1975 June 12, when χ Cyg was at maximum (phase 0.05). Here the CO features are double.

months earlier when the variable was at maximum, and we see at once that all the CO features, including the band heads, are doubled. The doubling is particularly obvious in the fourth strip. The weaker components, shifted to shorter wavelengths, can be identified with the deeper, warmer layer.

The comparison of Figures 9 and 10 points out two great advantages of the infrared CO bands in studies of line doubling, in addition to those already mentioned. First, since Miras have relatively small light amplitudes in the infrared, spectra of the same high quality can be obtained throughout the cycle. Second, because the doubling is shown by a large number of lines covering a substantial range in excitation, it is possible to measure the velocities and temperatures of both atmospheric layers with high precision.

The first report of the doubling of CO lines in a Mira variable was published by Maillard (1974), who described the spectrum of R Leo. Two extensive programs of infrared spectroscopy of Miras have also been undertaken: one the recently-completed Ph.D. dissertation of K. H. Hinkle, supervised by T. G. Barnes and D. L. Lambert, at the University of Texas, and the other an on-going project at Kitt Peak National Observatory, where D. N. B. Hall and S. T. Ridgway, who initiated it, have now been joined by Hinkle. The Texas results for R Leo, the star observed most extensively, have been published (Hinkle 1978 - CO and OH bands) or are in press (Hinkle and Barnes 1979 - H_2O bands). These papers are extremely illuminating, and I hope that the brief summary I will give here will inspire you to read them. Results from the Kitt Peak program have not yet been published, but since it uses even higher spectral resolution and involves extensive coverage of several stars, it may be expected to shed still more light on the Mira phenomenon.

From the positions and strengths of lines of both CO and OH measured between 1.6 and 2.5 μ in the spectrum of R Leo at nine distinct phases, Hinkle (1978) has been able to give a much clearer picture of the velocity and temperature structure and the manner in which these quantities vary through the cycle. Line doubling is most apparent just before maximum light, since at that time we begin to see the blue-shifted components from the deep part of the photosphere, which has just encountered the shock wave of a new cycle, while the infalling material from the previous cycle has not yet faded from visibility. In addition, Hinkle identified a third layer, a cool outer shell which produces only low-excitation lines and is falling slowly back toward the star; thus some of the infrared lines, at certain phases, were actually seen as triple.

These results are shown in Figure 11, in which velocity is plotted against phase. This figure is based on the first-overtone OH bands; similar results were obtained for the first- and second-overtone CO bands. For the purposes of the present discussion, two results stand out as being the most important. (1) The velocities of the warm, blue-shifted component near maximum light are algebraically much smaller than any previously seen in the absorption spectrum and are similar to those of the hydrogen emission lines;

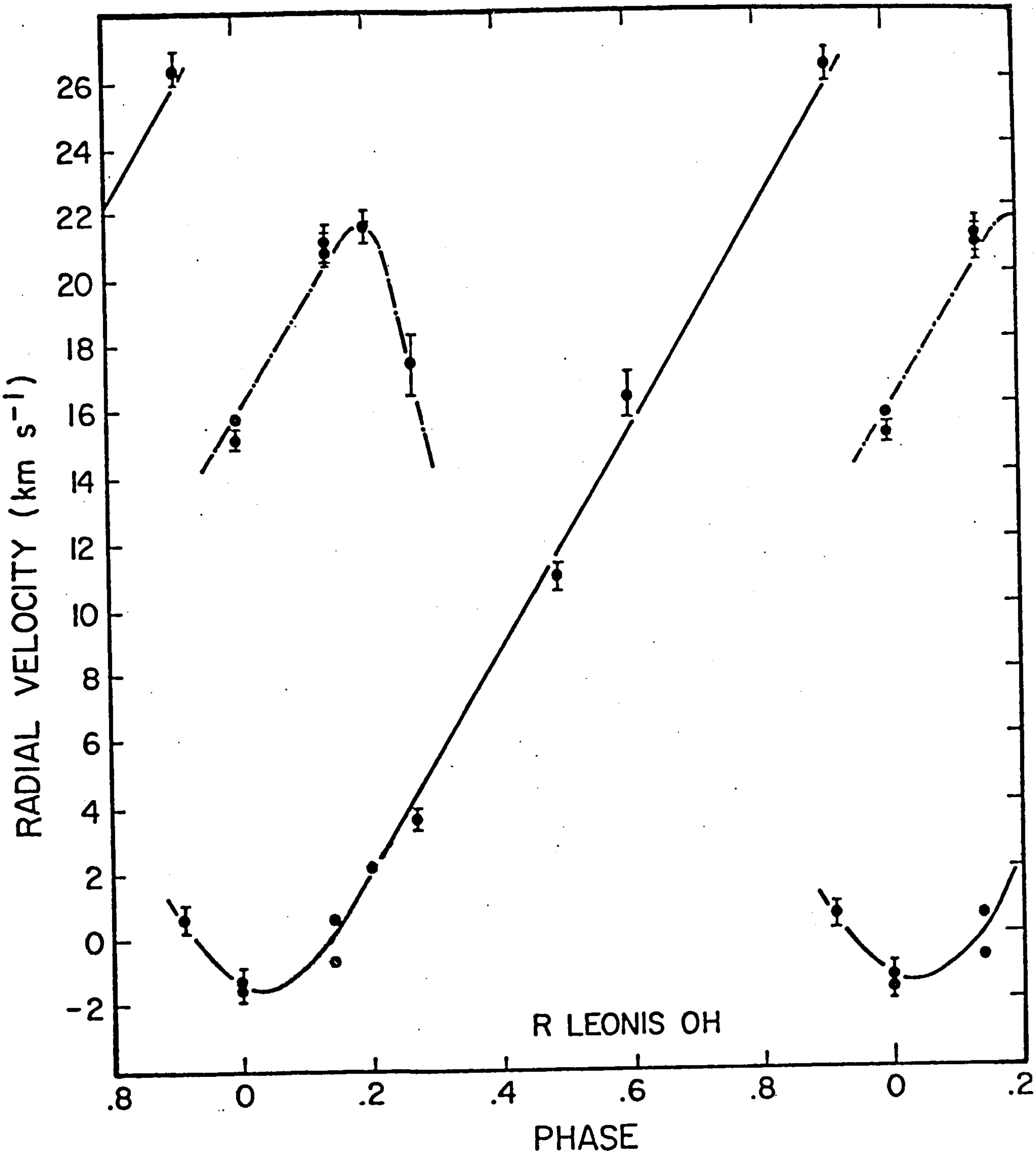

Fig. 11 - Velocity variations shown by OH lines in R Leo. Near maximum light (phase 0) the velocity is smallest as the gas rises rapidly; the motion can be followed for a full cycle as the gas decelerates and falls back in, and near phase 0.9 it is possible to see the photospheric layers from two successive cycles. In addition, the low-excitation OH lines show another component (dot-dash curve) which is identified with the cool circumstellar shell. From Hinkle (1978).

this is, in fact, the sort of velocity required for the expanding photosphere inferred from the photometry between the times of the visual and bolometric maxima. (2) The amplitude of the velocity variation, 27 km s^{-1}, is larger than any previously found and is, for the first time, consistent with the photometric results. The same gas can be seen throughout a complete cycle as it rises, decelerates, and falls back in.

Hinkle (1978) finds that a consistent pulsational model for R Leo can be derived if the center-of-mass velocity is 8 ± 1 km s^{-1}. This value is significantly smaller than most of the absorption-line velocities measured in the blue, which range from 7 to 15 km s^{-1} as a function of phase (Merrill 1946, 1952). This displacement of the mean absorption velocity from the center-of-mass velocity, which indicates that the absorbing material is falling in, is consistent with the suggestion of Reid and Dickinson (1976) discussed earlier.

Another molecule that produces a great many lines in the infrared spectra of Miras is H_2O, and its lines, too, are double throughout much of the cycle (Hinkle and Barnes 1979). The spectrum of H_2O is so complicated that the doubling of its lines might have gone unnoticed, were it not for the fact that the H_2O lines are sometimes sharp and single. The component that is always present can be identified with the cool circumstellar layer which also produces CO and OH lines of the same velocity. The other, which shows a greater range in velocity, comes from the photosphere. The behavior of the H_2O spectrum is thus similar to that of CO and OH, but there is an interesting difference. At maximum light, when the warm layer is moving rapidly outward and the doubling of the CO and OH lines is easily seen, the H_2O lines are single. Hinkle and Barnes offer a simple explanation for this difference: the H_2O molecules are dissociated at the high temperatures of the photosphere near maximum light, and they do not start to form in appreciable numbers until about 0.1 cycle later, when the photospheric temperature has dropped sufficiently.

Figure 12 illustrates the doubling of H_2O lines in R Leo. Nearly all the absorption in this spectral interval is due to stellar H_2O. In the upper spectrum, taken at maximum light, the lines are single and only the

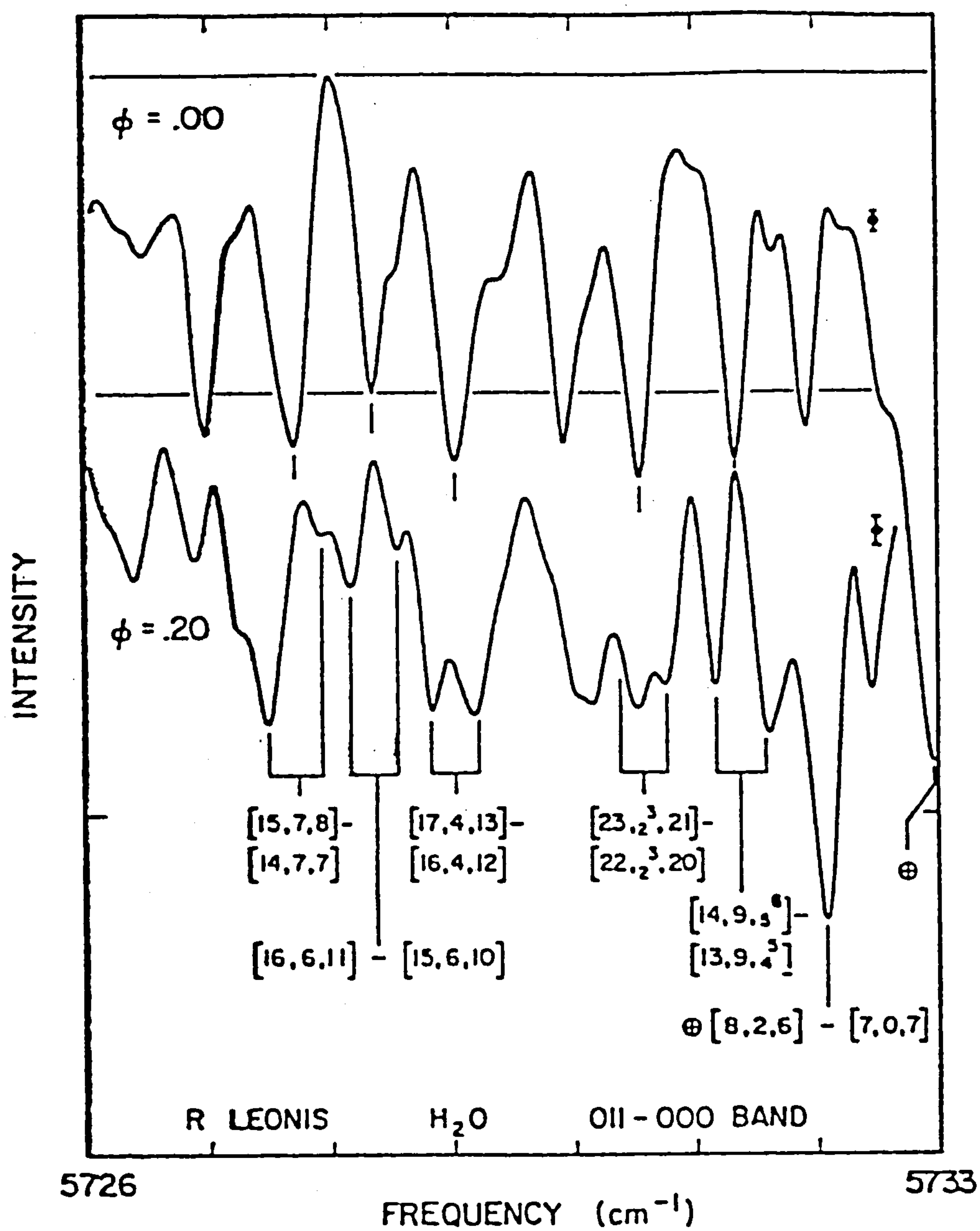

Fig. 12 - A section of the infrared spectrum of R Leo dominated by lines of H_2O. The lines are single at maximum light (upper spectrum) and double at phase 0.2 (lower spectrum). From Hinkle and Barnes (1979).

cool shell component is present. In the lower spectrum, taken 0.2 cycle later, the shell component of each line is shifted to the left as the material falls back into the star, and a second component from the rising photosphere appears, shifted to the right.

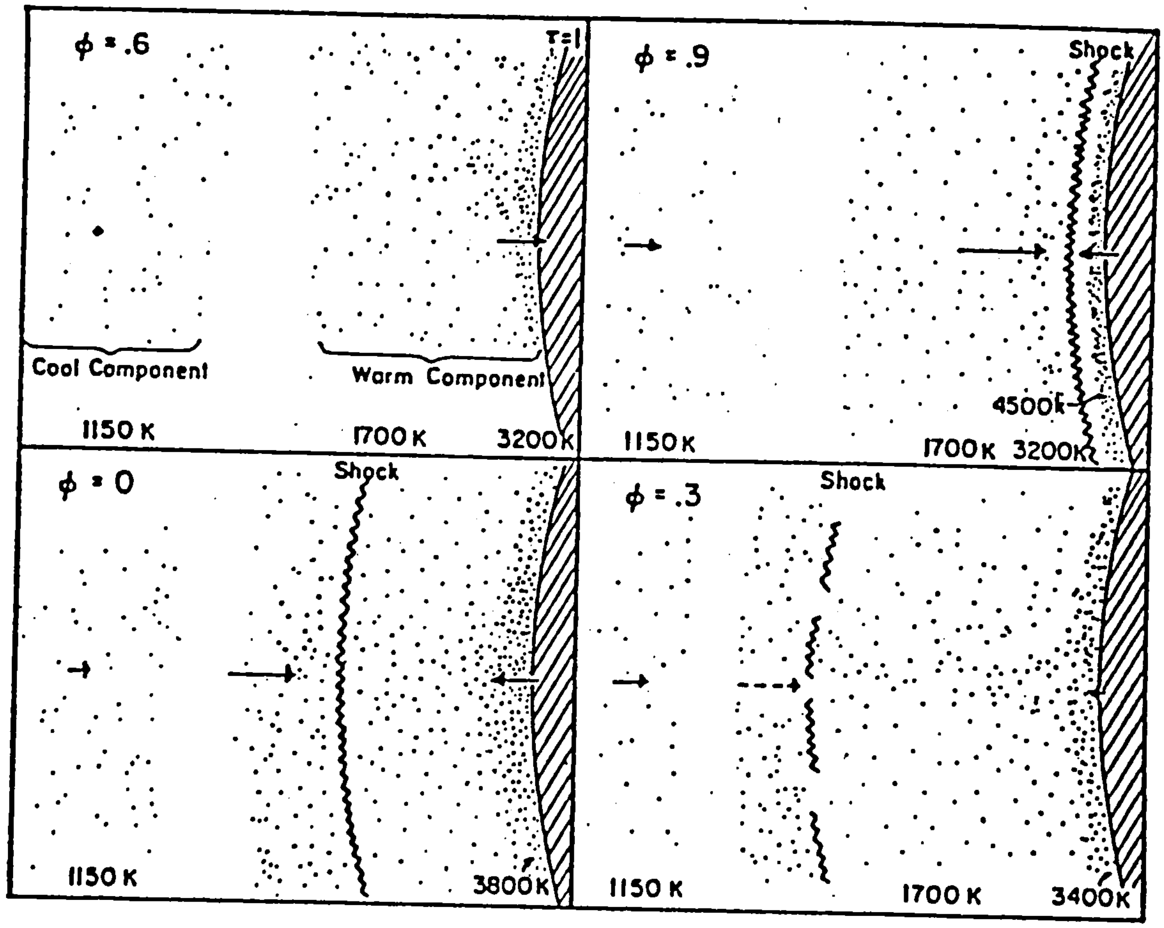

Fig. 13 - Schematic representation of the outer atmosphere of a Mira variable at phases 0.6, 0.9, 0.0, and 0.3, based on infrared spectroscopy of R Leo. The arrows represent gas velocities. From Hinkle and Barnes (1979).

Hinkle and Barnes (1979) have summarized in Figure 13 the picture of R Leo that they have put together from their infrared observations. At all phases a cool circumstellar shell is present, contributing low-excitation lines. The stellar photosphere pulsates and is at all times hotter than the temperatures conventionally ascribed to Miras -- a result anticipated by narrow-band photometry in the infrared (Wing 1967). Intermediate temperatures occur in the star's upper atmosphere, where most of the atomic and molecular absorption lines are formed and through which the shock wave propagates.

I find the recent results from infrared spectroscopy extremely encouraging. On the one hand they show that the spectroscopic approach can indeed give information about the pulsational properties of Miras. At the same time they show us how to construct a model which, at least in its broad outline, is consistent with the photometric as well as the spectroscopic observations.

CONCLUDING REMARKS

This review has touched upon a wide variety of topics; the common theme has been the great extent of the observable atmospheres of the Mira variables. We have considered various methods for measuring the sizes of these atmospheres, and more particularly the manner in which the size changes through the cycle. The results obtained by different methods have been compared, and the differences thus found have reminded us that observations of Miras are not always easy to interpret.

I have emphasized the problems of interpretation because it seems to me that these are not always given sufficient attention. I hope I have distinguished between the problems that are the star's fault (such as departures from spherical symmetry) and those which, dear Brutus, are our own (such as unwittingly combining results which refer to different parts of the star). While the Miras will always be difficult objects to treat, some of the problems that have baffled astronomers for decades have recently disappeared. In particular the famous discrepancy between the photometric and spectroscopic diameter curves turns out not to be a real discrepancy at all, since the two methods are not looking at the same gas. The discrepancies between molecular band strength and continuum color temperature can be accounted for in the same way.

A simple model for the atmospheric structure and motions of Miras, based on Hinkle's recent observations of the doubling of infrared molecular lines, has been described. This model, consisting of two atmospheric layers plus a circumstellar shell, has been remarkably successful in providing a physically plausible picture of the atmosphere which is consistent with the photometrically-measured magnitude and temperature variations as well as the spectroscopic data. However, it is of course much <u>too</u> simple to account for <u>all</u> the observations. For example, in Figure 13 the outer atmosphere is represented by a large region at the uniform temperature of 1700°K, whereas we have long known that temperature and velocity gradients must exist since the measured velocities of absorption lines show a dependence on excitation potential. In another paper at this conference, Pilachowski, Wallerstein, and Willson (1979) treat the absorption-line velocities as functions of excitation potential, ionization potential, wavelength, and line strength; their results for the outer atmosphere should now be combined with the broader picture given by studies of line doubling to obtain a more complete and realistic model for the atmosphere.

Much observational work on Miras remains to be done. In particular I would like to encourage work in three areas. First, it is important to find out how well the radial velocity variations of the infrared molecular lines repeat from cycle to cycle. In the work done to date it has been necessary to combine observations from different cycles, and this procedure generally has not been very successful with other kinds of observations of Miras. The current program of infrared spectroscopy at Kitt Peak should settle this point. Second, measurements of molecular band strengths and photospheric color temperatures should be made around the cycle by narrow-band photometry, but unlike previous measurements of this kind, they should be accompanied by high-resolution spectroscopy so that the region of formation of each spectral feature can be identified from its radial velocity. Such combined data could form the basis for a more detailed model of the atmosphere. Finally, direct diameter measurements through as much of the cycle as possible, by speckle interferometry in well-defined wavelength bands, are badly needed. Now that it is possible to derive diameter curves from photometry and spectroscopy that are at least qualitatively the same, we must be brave and ask whether the same result can be obtained by direct measurement.

I would like to thank Dr. Kenneth H. Hinkle for showing me his results prior to publication, and for providing the Kitt Peak spectra shown in Figures 9 and 10. I also thank Dr. George Wallerstein for helpful correspondence.

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