THE BARNES-EVANS COLOR-SURFACE BRIGHTNESS RELATION: A PRELIMINARY THEORETICAL INTERPRETATION

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ABSTRACT

Model atmosphere calculations from the published literature are used to assess the claim of Barnes, Evans, and their collaborators that their empirically derived relation between V-R and surface brightness is independent of a variety of stellar parameters, including surface gravity. This relationship is being used in a variety of applications, including the determination of the distances of Cepheid variables using a method based on the Baade-Wesselink method. The principal conclusion here is that the use of a main-sequence relation between V-R color and surface brightness in determining radii of giant ($\log g = 2$) stars is subject to systematic errors that are smaller than 10 % in the determination of a radius or distance for temperatures cooler than 12,000 K. The application of a main-sequence relation to white-dwarf stars is also considered; the error in white-dwarf radii determined from a main sequence color-surface brightness relation is again roughly 10 %.

I. INTRODUCTION

Stellar diameters derived from lunar occultations and Johnson UBVRI photometry have been used to develop an empirically calibrated relation between the surface brightness of a star and its V-R color (Barnes and Evans 1976; Barnes, Evans, and Parsons 1976; Barnes, Evans, and Moffett 1978). This relation, often called the Barnes-Evans relation, can be used for a variety of purposes. Since comparable techniques generally require observations having considerably higher spectral resolution, the Barnes-Evans relation, if it is indeed widely applicable, presents considerable promise for a variety of astrophysical problems. Yet this relation is empirically derived using measurements of nearby stars, mostly main sequence stars, and many of its most interesting applications (including those relevant to the variable star field) involve the extension of this relation to other types of objects. How valid is this extension? The range of applicability of a relation of the Barnes-Evans type is the central focus of this paper.

The surface brightness parameter used in the Barnes-Evans relation is

$$F_V = 4.2207 - 0.1\ V_0 - 0.5 \log \phi', \qquad (1)$$

where V_0 is the apparent visual magnitude corrected for interstellar extinction and ϕ' the stellar angular diameter in milliseconds of arc. Barnes, Evans, and their colleagues argue empirically that the relation between F_V and the Johnson (1966) V-R color is a single-valued one. This relation has been used to measure the radii of nearby stars (Lacy (1977b) and of white-dwarf stars (Moffett, Barnes, and Evans 1978). It has been used to determine distances of eclipsing binaries (Lacy 1977a), of Cepheid variables (Barnes et al. 1977) and of a nova (Barnes 1976). It has also been applied tentatively to solar flares, and a variety of other applications are contemplated (Evans 1978). A theoretical assessment is timely.

II. THEORETICAL FRAMEWORK

A disadvantage of dealing with broad-band photometric systems from a theoretical viewpoint is that broad-band photometry does not isolate some easily calculable part of the spectrum, but rather measures the integrated intensity of a broad spectral region containing both continuum emission and also the contamination of spectral lines. It is this feature of broad-band photometry that makes it extremely economical in terms of telescope time. A thorough evaluation of the applicability of the Barnes-Evans F_v versus V-R relation would require a consideration of all the lines that could affect these bands at all temperatures and gravities. There are considerable difficulties involved in computing synthetic Johnson colors, and they are only beginning to be solved for the UBV colors (Buser and Kurucz 1978). The ambition of this investigation is far more modest. Here I ask how badly changes in gravity and chemical composition affect the continuum fluxes at the central wavelengths of the V and R bands. Since at the temperatures considered here, line blanketing is minimal, changes in the continuum fluxes should provide a reasonable idea of changes in the V-R colors.

To recast the color-surface brightness relation into theoretically tractable terms, use the following relations:

$$\phi' = 206,264,806.2\ (2R/D), \tag{2}$$

$$V_o - 0.04 = -2.5 \log\ (f_v/3.52 \times 10^{-20}), \text{ and} \tag{3}$$

$$f_v = 4\pi\ (R/D)^2\ H_v, \tag{4}$$

where R is the stellar radius, D its distance, V_o the Johnson V magnitude, ϕ' is in milliseconds of arc, f_v is the stellar flux at the Earth in the central wavelength of the V band and H_v is the Eddington flux at the stellar surface at the central wavelength of the V band in ergs cm^{-2} sec^{-1} $ster^{-1}$ Hz^{-1}. Note that H is normalized so that $\int H_\nu d\nu = \sigma T_{eff}^4/4\pi$. For the purposes of this paper, the effective wavelengths are presumed constant at 0.55 microns (V) and 0.7 microns (R). Variations in effective wavelength with temperature are presumed to be second-order effects when considering the change in H_v versus V-R when gravity or chemical composition is altered. Equation (3) takes the V magnitude of Vega as 0.04 and takes the visual flux at 5556 A as 3.52×10^{-20} ergs cm^{-2} sec^{-1} Hz^{-1}, averaging the results of Hayes and Latham (1975), the corrected results of Oke and Schild (1970), and the recent calibration of Tug et al. (1977). Uncertainties in the transformation of monochromatic magnitudes to V-magnitudes undoubtedly exist and will affect the calibration of the zero point to some small degree; for present purposes they are not material. Equations (1) through (4) can be combined to yield the following definition of the surface brightness parameter F_v:

$$F_v = 5.047 \pm 0.002 + 0.25\, H_v \,, \tag{5}$$

where the uncertainty in the constant comes from a presumed 2 % uncertainty in the absolute flux calibration.

For present purposes a continuum color is defined:

$$v'-r' = -2.5 \log (H_v/H_r), \tag{6}$$

using the effective wavelengths noted above. Ideally this color would be equal to the Johnson V-R color plus an additive constant. Blanketing and changes in effective wavelength with temperature cause reality to fall short of the ideal. But in narrow color regions, one can presume that a change in the H_v versus v'-r' relation is paralleled by a change in the broadband color index V-R and the broadband flux F_V. It may be possible to calibrate the V-R index in absolute terms in the future (Buser 1978; Buser and Kurucz 1978).

III. RESULTS

Figure 1 illustrates the central results of this paper, Here the monochromatic Eddington fluxes H_v are plotted against the monochromatic v'-r' color index. These fluxes and colors are taken from a variety of models: those of Kurucz, Peytremann, and Avrett (1974) for main sequence (log g = 4.5) and giant (log g = 2) models with T_{eff} higher than 8,000 K, or v'-r' = -0.118; those of Carbon and Gingerich (1969) for main sequence (here log g = 4 because more models were available) and giant (log g = 2) models for T_{eff} between 4,000 and 8,000 K; and mine (Shipman 1972, 1977; McGraw and Shipman 1978) for the white-dwarf models (log g = 8). Since white-dwarf stars are either essentially pure H or essentially pure He, models for both compositions are shown. Also shown is the black body relation for a gray atmosphere where $H_v = B_v/4$ given the normalization of H.

The sets of models chosen for Figure 1 are worth noting. All models, except the He-rich white-dwarf models, include line blanketing to some extent. The Carbon-Gingerich models are fairly old, and do not use the most modern, elaborate treatment of line blanketing. However, published grids of models which cover the T_{eff} range that the Carbon-Gingerich models are used for here (Peytremann 1974, Bell _et al._ 1975) do not directly provide tables of emergent fluxes. Gustafsson _et al_. (1975) show that their models, which do include a state-of-the-art, elaborate statistical treatment of line blanketing, have temperature gradients quite similar to those of Carbon and Gingerich (1969) that are used here. Under the circumstances, the use of the Carbon-Gingerich models is probably justified, particularly since no models cooler than T_{eff} = 4000 K, where blanketing by TiO bands can greatly affect integrated fluxes in the V and R bands, are considered here. A test of the accuracy of the Carbon-Gingerich models is provided by the T_{eff} range between 8,000 K and 10,000 K, where the Carbon-Gingerich grid overlaps the Kurucz, Peytremann, and Avrett (1974) grid (which includes a fairly elaborate treatment of line blanketing). Differences between the H_v versus v'-r' relations provided by the two sets of models are less than 2 % in H_v.

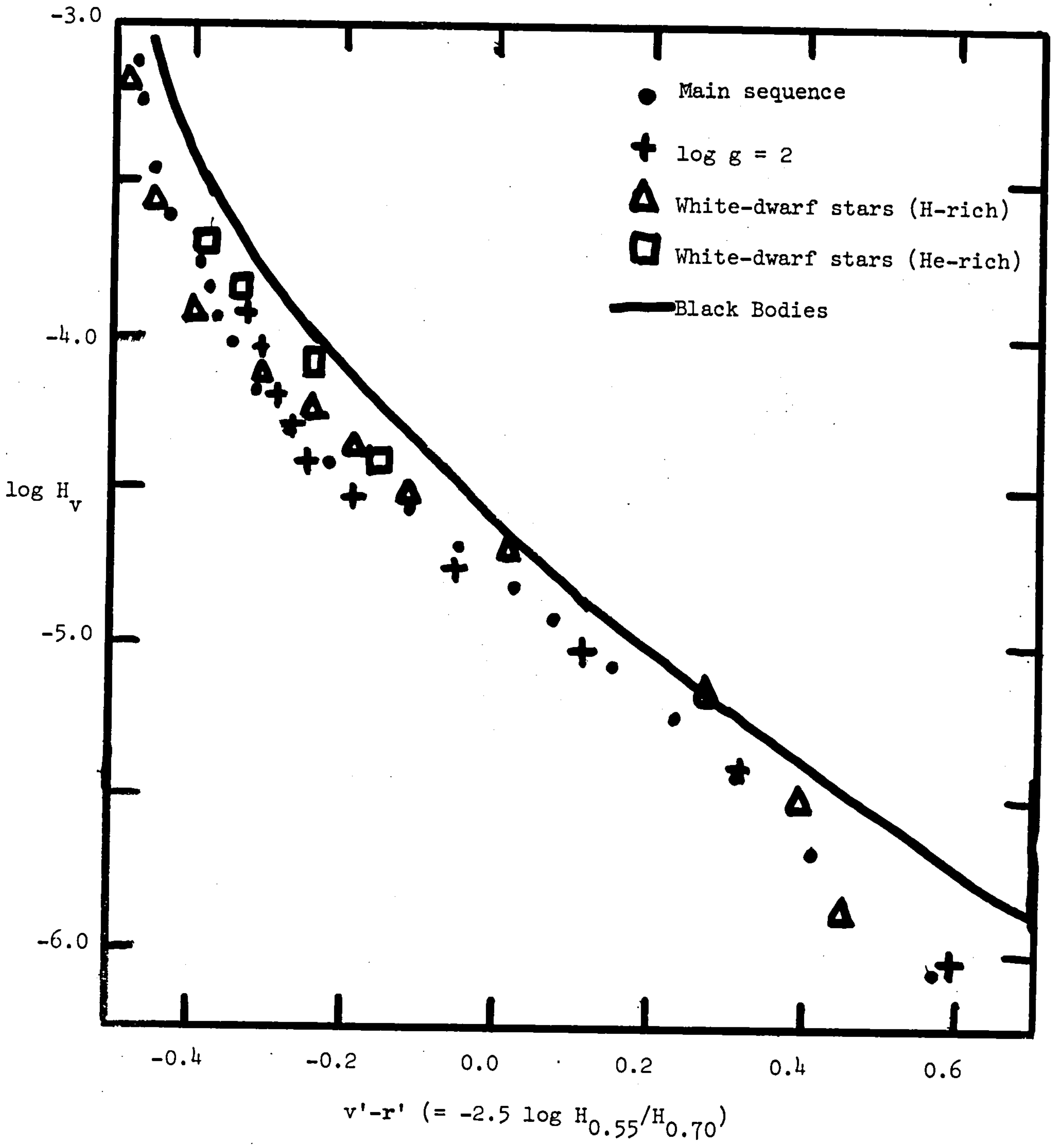

Figure 1. Color-Surface Brightness relations. The relation between Eddington flux in the visual band (effective wavelength 0.55 microns) and the monochromatic color index v'-r' is shown for a variety of model atmospheres and for black bodies. References to the specific model calculations is provided in the text.

It is worth examining the sensitivity of the conclusions presented here to the assumptions used, particularly to the simplified treatment of line blanketing used in the Carbon-Gingerich models. For most models, the observed blanketing coeffecients (defined as the fraction of the total flux removed by lines) in the V and R bands are 5 % or less, as tabulated by Carbon and Gingerich. Differential effects on the V and R colors should be smaller still. The effects are somewhat larger but still less than 20 % for the cooler T_{eff} = 5,000 and 4,000 K models. Carbon and Gingerich argue that the continuum energy distribution is relatively insensitive to the way that line blanketing is treated. Thus the approximate treatment of line blanketing will not affect the conclusions here for $T_{eff} > 6,000$ K, and will probably not affect the results at cooler temperatures. Stars cooler than 4,000 K were not included in this investigation; an examination of the M-dwarf models of Mould (1976) indicated that TiO blanketing could seriously affect the R colors, so that the behavior of v'-r' might differ from the behavior of the observed color V-R.

The results in Figure 1 show that H_v (or equivalently F_v, see equation 5) is quite tightly correlated with the v'-r' color. Figure 2 shows the difference between the value of H_v calculated from the models and the value derived from a piecewise-linear fit to the main sequence relation. This difference should parallel the difference between the empirical Barnes-Evans relation, derived for main sequence stars, and the true relation for giant stars or for white-dwarf stars. Cooler than v'-r' = -0.3, or T_{eff} = 12,000 K, the main-sequence and giant (log g = 2) relations coincide within 0.1 in log H_v or (equation 5) 0.025 in F_v. This correspondence agrees with the observations of Barnes _et al._ (1978). They

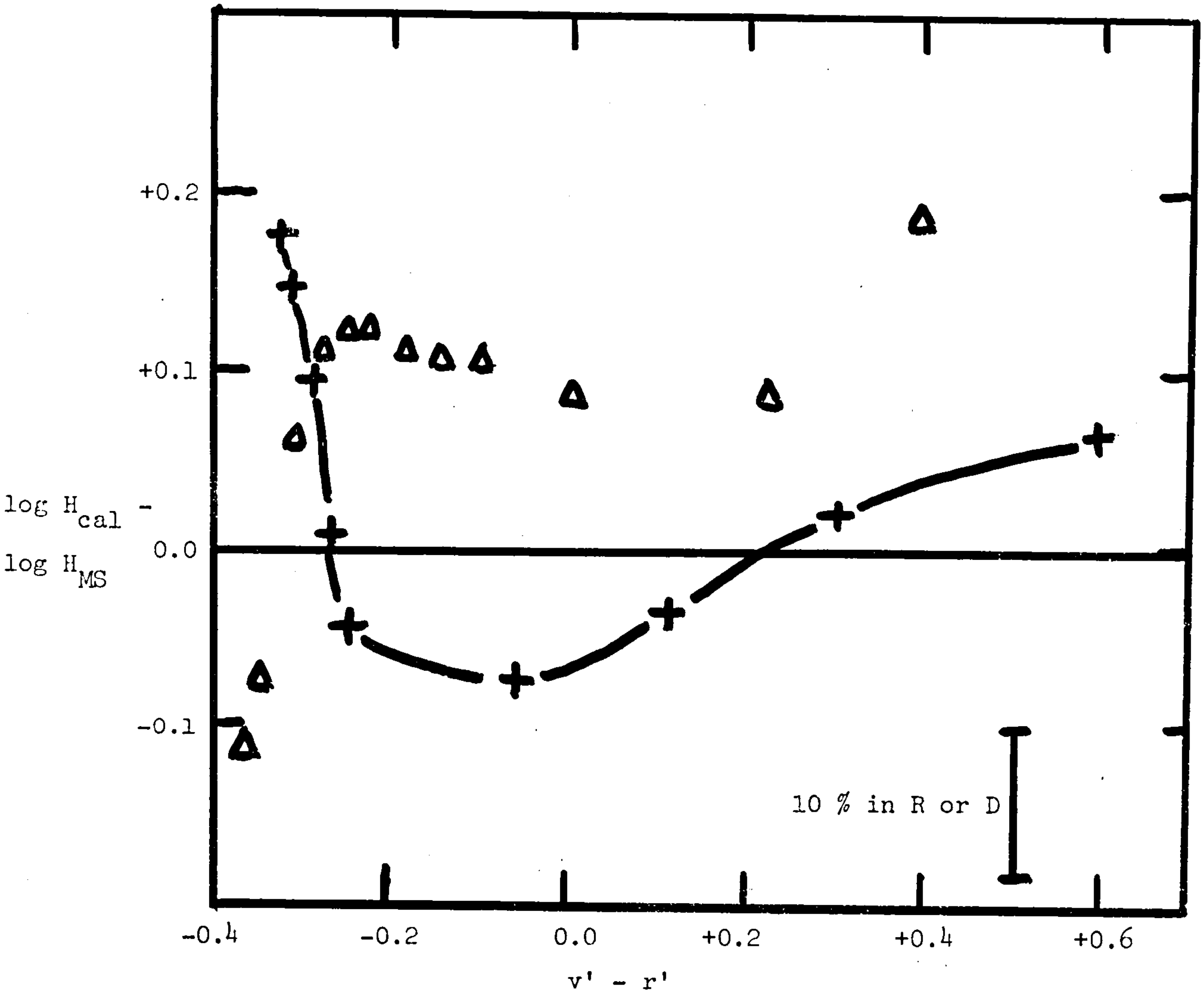

Figure 2. The difference between the visual flux calculated from the models and the flux determined from a main-sequence H_v vs. v'-r' relation. Plus signs and solid curve: log g = 2. Triangles: H-rich white-dwarf stars (log g = 8). The scale bar shows the change in H_v that corresponds to a 10 % change in a radius or distance measured using a surface-brightness method.

state that the scatter in the F_V versus V-R relation, a relation presumably very similar to the theoretical H_v versus v'-r' relation plotted in Figures 1 and 2, is 0.033; they attribute this to observational uncertainties, meaning that the cosmic scatter caused by gravity and composition changes is less than this. For the white-dwarf stars, the main-sequence relation is within 0.15 in log H_v of that given for the white-dwarf models for all but the coolest one (T_{eff} = 5,000 K) one plotted in Figure 2. Even the He-rich models fall within 0.2 in log H_v of the main-sequence line. The coincidence between the color-surface brightness relations for the two types of white-dwarf stars was noted by Shipman (1978) in connection with the monochromatic g - r color (this discovery provided the motivation for the present paper). In applications, the Barnes-Evans relation is generally used to determine stellar radii or distances, and a 10 % error in either of these quantities is shown in Figure 2. Note that there are some additional uncertainties when the slope of the Barnes-Evans relation is used to determine distances to variable stars; see the discussion below.

IV. PHYSICAL BASIS OF THE RELATION

A natural question to ask now is why? Why is v'-r', or equivalently V-R, such a good predictor of the visual flux H_v? Why does the Barnes-Evans relation work? This section provides some nonrigorous arguments directed towards a qualitative answer to this question.

Consider two model atmospheres with the same visual flux H_v. They are thus characterized by the same temperature at visual optical depth 2/3, where the flux in the V band is produced. Were these atmospheres isothermal --were the temperatures at $\tau_v = 2/3$ and $\tau_r = 2/3$ the same-- there would be no dependence of v'-r' on gravity, composition, or anything else, for it would be the wavelength (not depth) dependence of the source function which would drive the dependence of H_V on v'-r' color. In such a case, the H_V versus v'-r' color would follow the black body line. The relation almost follows the black body line (Figure 1), reflecting the fact that the v' and r' fluxes come from almost the same layer in the stellar atmosphere, but the coincidence is not exact.

The deviation of the H_V versus v'-r' relation from the black body line is caused by the fact that the v' and r' bands sample different layers in the stellar atmosphere. In all cases, the opacity in the v' band is less than the opacity in the r' band, and the stellar atmosphere is bluer than a black body with the same H_V (same temperature at visual optical depth = 2/3) would be. In cool (T_{eff} less than 8,000 K) stars, opacity in the 0.5 - 0.7 micron region comes from H^-. H^- is a very gray opacity source, with less than a 30 % change from 0.55 microns to 0.7 microns. Further, since both giant and dwarf stars have H^- as the principal opacity source in this wavelength region, the relative change in the opacity from v' to r' is the same in both types of stars, as well as in the white-dwarf stars. Thus we can understand the relative independence of the H_V versus v' -r' relation on surface gravity at cool temperatures. The V-R Johnson index is the only line-free index that samples two parts of the same continuum. A color index that samples two sides of a discontinuity (for example, U - V or V - I) would produce greater gravity effects, in all probability.

For the hotter stars, with temperatures exceeding 8,000 K and v'-r' colors bluer than -0.25, Figure 2 shows that the character of the H_V versus v'-r' relation changes drastically. This change can be attributed to a change in opacity source, since at these temperatures neutral H rather than H^- dominates in the v' and r' bands. Atomic H is less gray than H^-; the opacity doubles when going from the v' band to the r' band. The character of the difference between main sequence and giant stars changes. For white-dwarf stars, this transition occurs at slightly higher temperatures but similar values of the v'-r' color index (see Figure 2). At the extremely high temperatures, exceeding T_{eff} = 12,000 K (v'-r' = -0.30), the main sequence relation is no longer such a good predictor of H_v. However, at such high temperatures, the slope of the H_v versus v'-r' relation is such that this type of relation is less useful--observational uncertainties in the measured color indices translate into larger uncertainties in the surface brightness parameter H_v (or equivalently the parameter F_v).

V. CONCLUSIONS

Figures 1 and 2, the centerpiece of this paper, indicate that the relation between surface brightness H_v (equivalent to the Barnes-Evans parameter F_v) and the v'-r' color is independent of surface gravity at the level of approximately 0.1 in log H_v for stars cooler than T_{eff} = 14,000 K, with errors not too much larger at higher temperatures where the slope of the relation makes v'-r' a less useful estimator of F_v. As long as differential blanketing effects are unimportant, and we believe that they are, the monochromatic v'-r' color should behave the same as the broadband V-R color, and therefore this claim can be made of the Barnes-Evans relation as well. This paper does not consider the problem of the cool stars (T_{eff} less than 4,000 K), where TiO blanketing effects could be important.

Consequently distance measurements of stars hotter than T_{eff} = 4,000 K, such as those made by Lacy (1977a, 1977b) should not be seriously affected by systematic errors. While there are some errors at the 10% level in measuring white-dwarf radii using a main-sequence relation of the Barnes-Evans type, these errors reverse sign at T_{eff} = 12,000 K and consequently measurements of the radii of a group of stars should be relatively free from systematic errors. Parallax errors are generally larger than 10 %, so that use of the Barnes-Evans relation should not introduce any significant additional errors in most applications. The value of this relation lies in its applicability with small telescopes; the broad bands of the V and R filters reduce the requirements of telescope size or observing time.

When the Barnes-Evans relation is used to determine the distances to Cepheid variables using an extension of the Baade-Wesselink method (Barnes _et al._ 1977), a problem arises because it is the slope of the relation that enters the distance-finding method. Is the slope of the H_V versus v'-r' relation the same for Cepheids (log g = 1.5 to 2) as it is for main-sequence stars? In the 6,000 K to 8,000 K range, the slope from model atmospheres changes from -0.445 at log g = 4 to -0.397 at log g = 2. Small uncertainties in the models could produce large changes in this conclusion, and these models are affected by convection, which produces uncertainties in the models (Relyea and Kurucz 1978). At the present time it is only possible to state that a change in slope of this magnitude cannot be ruled out, and that a change in slope greater than this might be possible but a significantly greater change in slope is unlikely. This possibility of a change in slope should be considered as a possible uncertainty in an evaluation of radii using the method of Barnes _et al._ (1977).

This investigation has some limitations. Perhaps the most severe is that it is restricted to continuum colors. In the temperature range being considered, line blanketing is a relatively minor effect in the V and R bands, and differential line blanketing (which would affect the V-R color through changes in line strengths with surface gravity) should be smaller still. Thus the extension of the present line of work to models including a more thorough treatment of line blanketing should probably not affect the principal conclusions of this paper. In addition, the present investigation does not include stars larger than log g = 2 and M-type stars.

Within these uncertainties, the principal conclusion of this paper is that a main-sequence relation between V-R and F_V such as that proposed by Barnes and Evans can be applied to stars with surface gravities between log g = 2 and log g = 8 and produce radii or distances with systematic errors of 10 % or less in most cases. Precise values of the errors as a function of T_{eff} and log g can be determined by examining Figures 1 and 2.

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Discussion

Smith: The previous question as to why the Barnes-Evans relationship works was actually answered, ironically, several years ago by Cayrel, who was interested in looking at the competing effects of backwarming and line blocking in model atmospheres when you went from normal composition to metal-poor stars. He came to the conclusion that (G-R) -- if I remember correctly -- on the Stebbins-Kron system was just the right color for these two effects to cancel out. And this also tends to explain why the (V-R) color works as a black-body indicator in this relationship.

Shipman: Good, I'm glad to get the history straight.

Wesselink: Have you any idea how much the effect of the bandwidth might be, if you change from a narrow band to a broad band?

Shipman: I don't have a good estimate of the uncertainty of the broad-band filters, but the bad estimate I have is that the absolute value of the line blocking in all of these filters is only around 10%. What I worry about is the differential line blocking between main sequence stars and giant stars. But even if that has not been done correctly in the Gingerich and Carbon models, I would say the error is probably 0.05 mag or less.

Nather: You stopped at 4000° going redward, but the relationship seems to work appreciably redward of that. Do you have any feeling as to why?

Shipman: I really don't know why. The reason I stopped there is that I took a look at some models for M dwarf stars, and there were TiO bands all over the place. So I can't argue that the continuum color resembles a broad-band color in that case.