EVIDENCE FOR THE EXISTENCE OF NONRADIAL SOLAR OSCILLATIONS: SOLAR ROTATION*

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I. INTRODUCTION

In the recent work of Hill and Caudell (1978) the solar origin of the observed long period oscillations in apparent diameter was demonstrated. One of the strongest pieces of evidence obtained at SCLERA[†] for this conclusion was the phase coherency of six oscillations over a two-week period in which seven days of equatorial diameter measurements in the 1973 solar oblateness data of Hill and Stebbins (1975) were analysed. This analysis further showed, through examination of the ratio of oscillatory power at two different scan amplitudes in the finite Fourier transform edge definition, the FFTD (see Hill, Stebbins, Oleson 1975 for discussion of diameter measurement technique), that the four higher frequency modes were likely to be p-modes while the two lower to be g-modes with large values of $\ell \approx 20\text{-}40$, where ℓ is the principal order number in the spherical harmonic expansion of the eigenfunction. The discovery that g-modes of large ℓ value are excited to an observable level allows one to place new constraints on solar models (Hill and Caudell 1978).

This paper extends this form of analysis through inclusion of an additional day of data, which falls between the first four and the last three days of the previous analysis. When placed on the phase diagrams, this subsequent addition confirms the original choice of phase solutions and strengthens the solar normal mode hypothesis. The power of the oscillations as a function of day for the p-modes are also analysed for periodicity. We observe that the power fluctuates for each of the peaks in a well defined manner, going nearly to zero at the minima. This implies the existence of beating between states of approximately equal amplitudes, an interpretation which in the light of the properties of the FFTD suggests that the beating is not between states of the different ℓ values. It further suggests that the beating is between rotational split states that differ only in m, the azimuthal order number, and is reminiscent of Wolff's (1977) work on rotation in DA white dwarfs. With this interpretation, mode identification of these oscillations is made, allowing inference of the eigenfunction weighted internal rotation rate of the sun (Hill 1978).

II. PHASE COHERENCY

The phase as defined by the Fourier transform for each peak in the power spectrum was computed and shifted to a common time of day. Since this is observationally determined to within a multiple of 2π, the solution was picked which gave the best fit for a constant phase shift per day (see Hill and Caudell 1978 for details). In Figure 1 is plotted the phase solutions for the equatorial observations as a function of date, with the new point on the 17th included. These are indeed unique solutions in that displacing the last three days up or down multiples of 2π increases significantly the χ^2 variant of the linear fit. To illustrate this proposition, the square root of χ^2 for a given fit to the original seven-days observations for each peak is given in Figure 2 plotted against the displacement in phase of the last three days as a unit. The standard deviation of an individual observation has been set arbitarily to unity. Note the parabolic nature of these curves and that it is unlikely that a phase shift of 2π has occurred between the four- and three-day sets of observations. This indicates that the oscillations observed at SCLERA are phase coherent over periods of two weeks, further proof of their solar origin.

III. TEMPORAL POWER VARIATIONS

The phase coherency of the oscillatory power over a two week period implies a relatively high level of stability, i.e. low damping. This fact allows one to examine the power of the oscillations as a function of day without such complicating factors that lead to a change in the mode during the interim. Variations in power will arise if beating occurs between different unresolved eigenfrequencies within a single observed peak (Wolff 1976). The beating pairs of modes may differ in m value as well as ℓ value when solar rotation is accounted for (Gough 1977). Due to the theoretically possible values of ℓ in the observed peaks and the variation in detection sensitivity with ℓ and m of the FFTD used in these diameter measurements, the temporal dependence of power can be valuable in distinguishing between the two cases.

In Figure 3 is plotted the equatorial oscillatory power against date for the four p-mode peaks. Through the data points have been fit a function of the form $A^2\cos^2(\Omega_b t + \phi)$ where amplitude A, beat frequency Ω_b and phase ϕ were adjusted to minimize the χ^2 variant. The resultant functions have been drawn in the figure. Here ν represents the frequency of the mode in milliHertz and Ω_b the best fit beat frequency in radians per day. Both quantities are listed in the first two columns of Table 1. Because of the sampling in the time on a daily basis there are higher aliased values of Ω_b which will give equally good fits.

There are several major conclusions to be drawn from the power plots. First, the cosine squared function of a single beat frequency does indeed fit the observations quite well. This indicates that perhaps only one pair of states dominates the beating in each peak. Second, the power at the minimum goes nearly to zero, often by a factor of 100 below the maximum value. Only if the amplitudes of the two constituent states are equal to within 20% will this be possible. If the beating states were of different ℓ value, this observational result requires that their amplitudes vary with ℓ just as the variation of the FFTD sensitivity with ℓ (see Hill 1978 for discussion of ℓ dependence of the sensitivity of the FFTD). This must be true for each of the observed peaks. However, such an assumption regarding the ℓ dependence of the amplitudes is not required if the beats occur between rotationally split modes. Since the FFTD favors the mode of highest ℓ and $|m|$, the beating in a given peak of the power spectrum will be dominated by the two modes with $m = \pm\ell$. This appears to be the most likely phenomenon to occur in light of the observations to date.

In the following we present an interpretation based on the case where modes of nonzero ℓ value are excited and the beating pattern is dominated by the associated $m = \pm\ell$ states. Beats between states with $|m| < \ell$ only add lower beat frequencies commensurate with those for the $|m| = \ell$ states. This will allow extraction of information about the internal rotation of the sun.

IV. ROTATIONAL SPLITTING AND INTERNAL ROTATION

The hydrogen atom, when given an axis of symmetry through the imposition of an external magnetic field, splits the otherwise energy degenerate m states and produces a multiplet of states, $2\ell + 1$ in number for each ℓ value. As in the hydrogen atom, when the sun is given an axis of symmetry by rotation, a degenerate set of states with a given ℓ value will split its eigenfrequency into a $2\ell + 1$ multiplet (Gough 1977, Ledoux and Walraven 1958). The eigenfunction in the presence of rotation has the form,

$$\vec{\xi}_{k\ell m}(\vec{r},t) = \vec{\xi}_{k\ell m}(\vec{r}) \exp\left\{i(2\pi\nu_{k\ell}t + m\beta_{k\ell}\overline{\Omega})\right\} \tag{1}$$

where

$$\beta_{k\ell}\overline{\Omega} = \frac{\int_0^{R_\odot} \rho\Omega\left\{\left(\xi_{k\ell} + \eta_{k\ell}\right)^2 + [\ell(\ell+1) - 2]\eta_{k\ell}^2\right\}r^2 dr}{\int_0^{R_\odot} \rho\left\{\xi_{k\ell}^2 + \ell(\ell+1)\eta_{k\ell}^2\right\}r^2 dr} , \tag{2}$$

Ω is the local rotational frequency, ρ the mass density, r the radius of the mass element, and the remaining functions are defined as components of the displacement vector for a normal mode of a nonrotating star written as

$$\left\{ \xi_{k\ell}(r)P_\ell^m(\cos\theta)e^{im\phi} \quad , \quad \eta_{k\ell}(r)\frac{d}{d\theta}P_\ell^m(\cos\theta)e^{im\phi} \quad , \right.$$

$$\left. im\ \eta_{k\ell}(r)\frac{1}{\sin\theta}P_\ell^m(\cos\theta)e^{im\phi} \right\} \quad . \tag{3}$$

The angles θ and ϕ are the standard spherical coordinates, $P_\ell^m(x)$ the associated Legendre polynomial and k is the radial order related to the number of radial nodes in the eigenfunction. Clearly $\beta_{k\ell}\bar{\Omega}$ is the eigenfunction weighted mean of the internal rotational frequency. Identification of the modes contributing to the four p-mode oscillations as well as the spatial filtering for SCLERA type diameter measurements must be clarified before $\beta_{k\ell}\bar{\Omega}$ can be determined.

The theoretical eigenfrequency spectrum of the sun is rich in states of differing ℓ value, many of which lie close together in frequency (Iben 1976 and Wolff 1978a). Due to the low resolution of an eight hour data run, several of these states may form the constituents of a single observed peak in power (Hill 1978, §2.2.3). The detection sensitivity to a particular ℓ value for the diameter measurements made at SCLERA depends on the dimensions of the aperture and the latitude of the measurement. Hill (1978) calculated the detection sensitivity for oscillations as a function of ℓ value at the equator using the solutions to the wave equation given by Hill, Rosenwald and Caudell (1978). It was found in the region between $\ell = 0$ and 12 the relative detection sensitivity increases with the ℓ value by a factor of nearly 4 in power. Out of this analysis also came the result that for a given ℓ value, the relative sensitivity to m reaches a pronounced maximum when $m = \pm\ell$ and is a minimum for the $m = 0$ case. Combining these two theoretical results we conclude that although several different eigenmodes may be contributing to a given oscillation, the largest ℓ value will be detected the easiest. If rotationally split, the $m = \pm\ell$ states will produce the strongest detectable effect.

Consulting the eigenfrequency spectrum associated with a standard solar model (Iben 1976, Wolff 1978a), a tentative mode identification can be made for the modes that dominate the observed power spectrum. This is accomplished by picking the theoretical frequencies which fall within the observed peaks. The k value and the maximum ℓ values associated with each observational peak are listed in columns 3 and 4 of Table 1. Applying the results of the previous paragraph, we

conclude that beating must be occurring between the plus and minus m states for these largest ℓ values. Column 5 of the table is the result of dividing the appropriate alias of the beat frequency by these maximum ℓ values to give the smoothest variation in rotation rate; column 6 is the inverse of column 5 in days where the standard deviation has been estimated to be one day. There is also another solution giving a smoother rotation curve. For this solution, the rotation rate increases more rapidly than the one given in the table.

The last column in Table 1 gives estimates of the depth to which the p-mode has substantial amplitude (Wolff 1978b, Figure 2). Inside this point, the amplitude exponentially decreases to the center, thus weighting the rotational velocity curve exterior to this point. These numbers illustrate the tenet that the various beat frequencies contain the requisite information to extract the depth dependence of solar rotation.

The correlation between columns 6 and 7 in Table 1 appears to be statistically significant. Based on this analysis the interior of the sun appears to be rotating at a somewhat larger rate over the surface, a result not inconsistent with the recent work on the five-minute oscillation (Ulrich, Rhodes, and Deubner 1978) and Gough's (1977) interpretation of 12.2 day periodicities reported in the Princeton solar oblateness observations (Dicke 1976).

V. CONCLUSION

The addition of a new data point on the phase plots of Hill and Caudell (1978) confirms the coherent properties of the observed oscillations. The two large ℓ g-mode oscillations identified in the previous work may be candidates for the slowly rotating mode-locked structures of Wolff (1976). For the four low frequency p-modes, periodic nature is observed in the daily power levels, varying with periods of several days. This is attributed to beating between rotationally split m states for a given ℓ value, an effect similar to that suggested for the DA white dwarfs (Wolff 1977). In any case, nonradial modes are a major contribution to the observed solar oscillations; the nonradial character of the observed modes allows the depth dependence of the internal solar rotation to be investigated.

[†]SCLERA is an acronym for the Santa Catalina Laboratory for Experimental Relatively by Astrometry and is a research facility jointly operated by the University of Arizona and Wesleyan University.

Table 1

ν(mHz)	$\Omega_b\left(\frac{rad}{day}\right)$	k	ℓ	$\beta_{k\ell}\overline{\Omega}\left(\frac{rad}{day}\right)$	T_{rot}(days)	$r_{eff}/R_\odot$
0.606	0.57	1	10	0.258	24.4 ± 1.0	.77
0.539	1.15	1	8	0.249	25.2 ± 1.0	.75
0.463	1.41	1	6	0.288	21.8 ± 1.0	.72
0.414	1.05	1	4	0.263	23.9 ± 1.0	.64

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Wolff, C. L. 1977, Ap. J., 216, 784.

Wolff, C. L. 1978a, see §2.2.3 of Hill 1978.

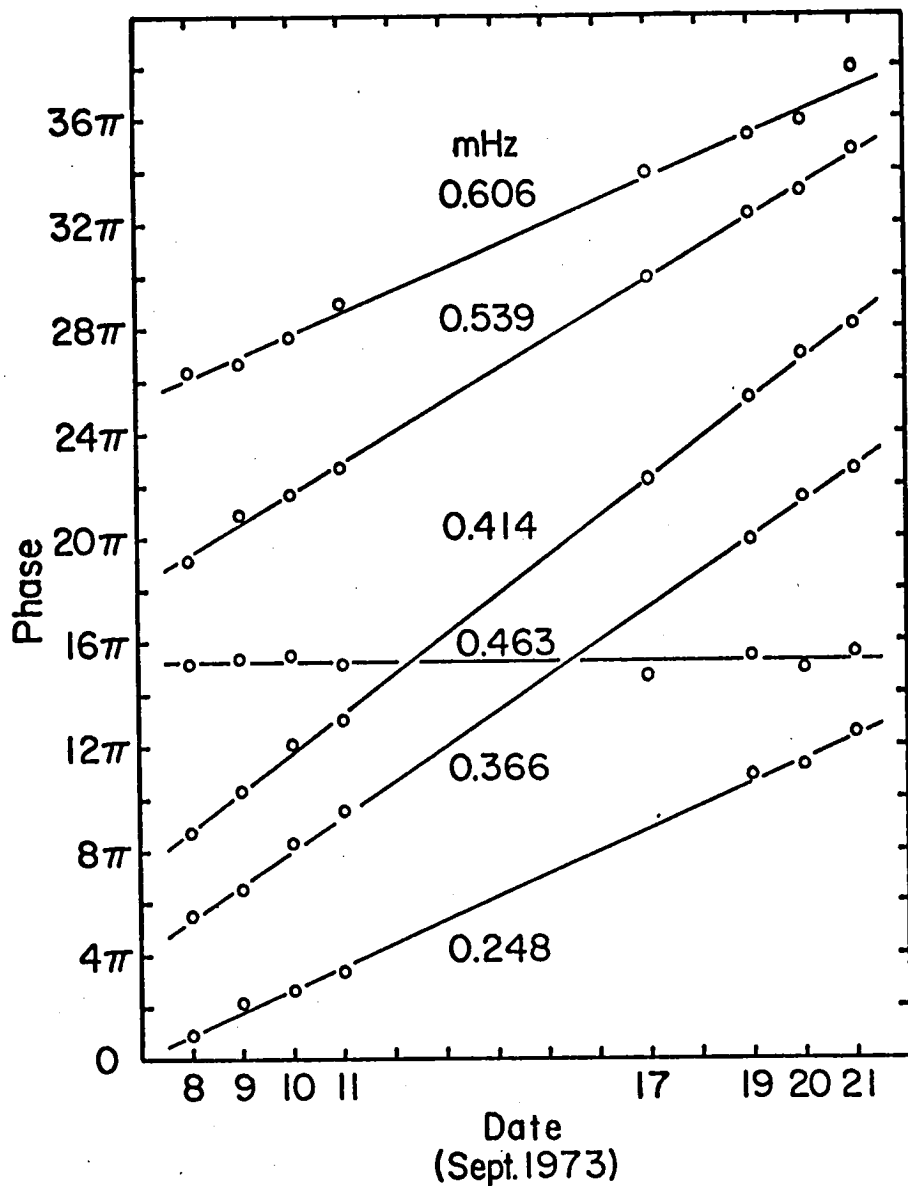Wolff, C. L. 1978b, submitted for publication.

Fig. 1. The phase as defined by the Fourier transform of the observed oscillations as a function of date. The phase for a given day and predetermined time of day is defined to within an additive multiple of 2π radians. For this plot the multiple has been previously choosen (Hill and Caudell 1978) to yield the best fit to the data not including the 17th of September. The results of the 17th have been added making no change in the previous fits. The frequency for each oscillatory period obtained from these observations is listed on the figure in mHz.
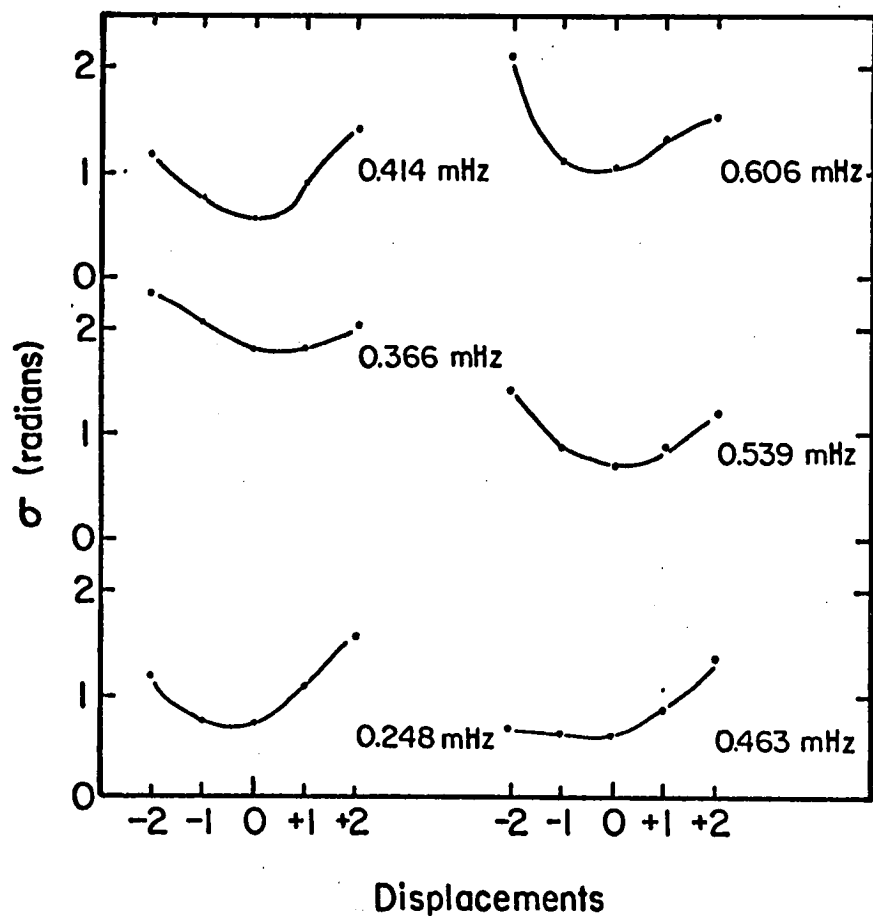
Fig. 2. The square root of the χ^2 variant of the linear fits in Fig. 1 where the last three days have been displaced up and down multiples of 2π. The standard deviations for the individual observations have been set to one. The zero of the horizontal scale is taken as the solutions plotted in Fig. 1.
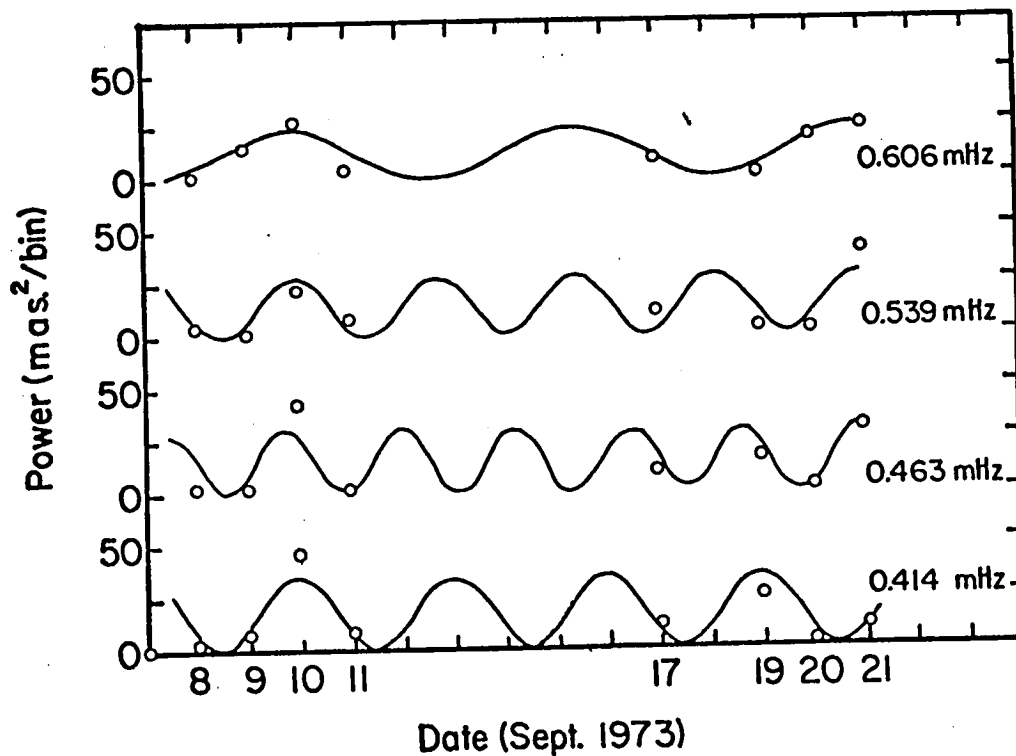
Fig. 3. The power level in units of milli-arcsec squared per frequency bin (0.03 - mHz) for the four p-mode oscillations as a function of date. The smooth curve is the least square fit to the points with the function $A^2 \cos^2(\Omega_b t + \phi)$.

Discussion

[Questions interjected during the paper]

Stellingwerf: What determines the slopes in Fig. 1?

Hill: For instance, if there were exactly 24 periods of oscillation in one day, the slope can, for example, be either 0, +1, +2, -1, or -2. But if the period comes out to be, say, 45.5 minutes -- that's 31.65 periods in a day, then it's the 0.65 that determines the slope.

A. Cox: What is the 0.463 mHz oscillation like in Fig. 1?

Hill: This one is quite close to being diurnal -- it's off by about 1 .

[After Talk]

J. Cox: One question about differential rotation -- if you imagine that the rotational angular frequency is constant on a cylinder, isn't the interpretation ambiguous because of the variation from latitude to latitude in the rotation rate, i.e. differential rotation?

Hill: But if the oscillation is coherent over a time long compared to the period of oscillation, then when one measures rotational splitting, one measures an average over the entire Sun. So it doesn't matter whether it's a longitudinal slice in cylinders or a slice in spherical surfaces. One doesn't measure the splitting locally; it's a weighted average. The equation that was written down by van Horn had Ω outside the integral because he had uniform rotation, but as soon as you have non-uniform rotation, Ω goes inside the integral and you get a weighted average of Ω over the entire volume.

Wesselink: Did I see that the rotational period was decreasing as you go down?

Hill: Yes, the period was decreasing as you went in.

Wesselink: Then it goes up again?

Hill: No, I think that's just scattering. The scatter was around one day in the period. There must be several interpretations, and what I've presented is just one interpretation. If you want to use it, you must conclude there's a trend there. There may also be more structure.