A PRELIMINARY ATTEMPT TO INTERPRET THE POWER SPECTRUM OF THE SOLAR FIVE MINUTE OSCILLATIONS IN TERMS OF THE GLOBAL OSCILLATION MODEL.

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ABSTRACT

The observed power spectrum of the solar five minute oscillations is discussed from the viewpoint that the oscillations are excited by turbulent convection. The observations place significant constraints on the theory, and suggest constraints on the solar model structure.

I. INTRODUCTION

The solar five minute oscillations provide a sensitive probe of the properties of the solar atmosphere and upper convection zone, and a challenge to theories which attempt to explain the amplitude of the oscillations as a function of frequency and spatial wave number. This paper discusses some properties of the power spectrum of the oscillations, in the context of the specific excitation mechanism described by Goldreich and Keeley (1977). It also discusses the effect of the atmospheric temperature profile, and the mechanical boundary condition. In §III the observed power spectrum as a function of aperture size is used to suggest constraints on the theoretical surface velocity of individual normal modes having periods near five minutes. It has already been noted by Ulrich and Rhodes (1977), that the frequency spectrum is better represented by solar models with mixing length equal to two or three pressure scale heights. It is shown in §IV that the steep low frequency side of the peak in the power spectrum is also more readily explained if the mixing length is greater than one scale height. The high frequency end of the power spectrum is also discussed.

II. BRIEF DESCRIPTION OF OBSERVATIONAL DATA

The main observational data to be considered here are the shape of the power spectrum, and the dependence of the power density at a given frequency, on the horizontal scale observed. Examples of the data available are given by Fossat and Ricord (1975), and Fossat, Grec, and Slaughter (1977). For observations through circular apertures, the power spectrum shows a rather sharp peak very near 5 minute period, and the position and shape are rather insensitive to aperture diameter over the range from 22" up to several minutes of arc. One of the most striking features is the steep rise on the low frequency side of the peak; the power density increases by a factor of about 6.5 between about 7 minutes and 5 minutes. On the high frequency side, the drop-off is roughly half as steep. An additional piece of information is the fall-off in power density at the peak, as a function of aperture size. The data of Fossat and Ricord (1975) suggest a drop by a factor $\sim$2.5 in power density, when the aperture diameter goes from 22" to 60".

III. THE RELATIVE CONTRIBUTION OF DIFFERENT SPHERICAL HARMONIC MODES

a) Theoretical result of the averaging process.

Consider observations made through a circular aperture sufficiently small that the region viewed on the solar surface subtends a small solid angle at the center of the sun. Then the sphericity of the surface can be neglected and the vertical component of velocity can be written in the form

$$V(\theta, \phi, t) = \sum_{Lm} v_{Lm} \, Y_{Lm}(\theta, \phi) e^{-i\omega_{Lm} t},$$

in which θ and ϕ are spherical polar coordinate angles, Y_{Lm} is a spherical harmonic, and ω is the oscillation frequency. v_{Lm} is the velocity amplitude for a given Lm mode. The spatial averaging can be carried out most simply if the polar axis of the coordinates is chosen to be the line of sight. Let θ_0 be the angular radius of the disc as seen from the center of the sun. Then

$$V = \sum_{Lm} v_{Lm} e^{-i\omega_{Lm} t} \frac{1}{\Delta\Omega} \int_{\cos\theta_0}^{1} Y_{Lm} \, d\Omega$$

$$= \sum_{L} v_{Lo} e^{-i\omega_{Lo} t} \frac{2\pi}{\Delta\Omega} \left(\frac{2L+1}{4\pi}\right)^{\frac{1}{2}} \sin\theta_0 P_L^{-1}(\cos\theta_0),$$

where P_L^{-1} is an associated Legendre function. For $L\theta_0 \ll 1$, but $\theta_o \ll 1$

$$V = \sum_L v_{Lo} e^{-i\omega_{Lo} t} \left(\frac{2L+1}{4\pi}\right)^{\frac{1}{2}}$$

and for $L\theta_0 \gg 1$, but $\theta_0 \ll 1$,

$$V = \sum_L v_L \left(\frac{2}{\pi \theta_o^{1/2}}\right) \left(\frac{1}{L\theta_0}\right) \cos\left[(L+\frac{1}{2})\theta_0 - \frac{3\pi}{4}\right]$$

The theoretical calculations give mean square velocities for each normal mode; these are assumed to add incoherently. Thus the formulae above yield

$$V^2 = \sum_L v_L^2 \left(\frac{2L+1}{4\pi}\right), \quad L\theta_0 \ll 1, \text{ and}$$

$$V^2 = \sum_L v_L^2 \frac{4}{\pi^2 \theta_0} \frac{1}{(L\theta_0)^2} \cos^2\left[(L+\frac{1}{2})\theta_0 - \frac{3\pi}{4}\right], \quad L\theta_0 \gg 1.$$

To compare with observations made with a finite bandwidth $\Delta\omega$, the sum over L's is taken for all modes having frequencies within that band. If $\Delta\omega$ is not too small, there will be one or more modes with no radial nodes, one or more with one radial node, one or more with two radial nodes, etc., contributing. Within each such group the L value will vary over a range depending on the bandwidth $\Delta\omega$, and the L values involved in different groups will be quite different except when L itself is small or the bandwidth is wide. From the calculated relation between frequency and L for modes with a fixed number of radial nodes, $\frac{\Delta L}{L} \approx 2 \frac{\Delta\omega}{\omega}$. Thus the number of modes expected in $\Delta\omega$ is $\sim 2 L \frac{\Delta\omega}{\omega}$, provided this number is greater than unity;

if the bandwidth is very small some groups may not contribute at all. In this case, the bandwidth of the individual normal modes may be important. If within each group it is assumed that V_L^2 is the same, the power can then be written as

$$V^2 = \frac{\Delta\omega}{\omega} \sum_L v_L{}^2 \left(\frac{2L+1}{4\pi}\right) (2L), \quad L\theta_0 \ll 1$$

$$V^2 = \frac{\Delta\omega}{\omega} \sum_L v_L{}^2 \, \frac{4}{\pi^2\theta_0} \, \frac{1}{(L\theta_0)^2} \, (2L)\cos^2\left[(L+\tfrac{1}{2})\theta_0 - \tfrac{3\pi}{4}\right], \quad L\theta_0 \gg 1 .$$

where the sum now is over the central L values of each group. The power per unit frequency interval follows directly from these formulae.

b) Trial distributions of $v_L{}^2$

It is instructive to plug in some trial distributions for $v_L{}^2$ as a function of L, to see whether any clues to the actual distribution of mode energies in the sun can be obtained. The L values for the groups depend on ω; for definiteness, a frequency $\omega = 2 \times 10^{-2}$ sec^{-1} was chosen. Approximate L values for groups near this frequency are as follows, for a model with mixing-length equal to one pressure scale height: 1004, 620, 386, 270, 195, 150, 120, 98, 80, 68, 56, 50, 43, 36, 30, 25, 19. Relative power densities were calculated corresponding to the following three cases: Case 1) $v_L{}^2 = 1$ for all L. Case 2) $v_L{}^2 = L$. Case 3) $v_L{}^2 = L^{-1}$. The calculations were simplified by omitting the $\cos^2$ factor, and by using the asymptotic form for small $L\theta_0$ up to the point where it intersected the form for large $L\theta_0$. This occurred at $L\theta_0 \approx 2\pi^{-\frac{1}{3}}$. The results are shown for six different aperture sizes, in table 2. The most important point to note is that in all three cases, the ratio of power densities at 22" and 60" is greater than the observed ratio of 2.5 noted earlier. Although these calculations are extremely crude, they suggest that there may be problems if v_L^2 is constant or an increasing function of L. In fact, current calculations (Keeley 1977) suggest that v_L^2 increases with L for periods longer than ~3.5 minutes, if turbulence provides the only damping mechanism. This suggests that it will be worthwhile to repeat the calculations presented in table 1, using an accurate representation of the Legendre function. Results similar to those of the crude calculation may present a severe challenge to the turbulent excitation theory, and a significant constraint on any theory which predicts amplitudes of individual normal modes. If, on the other hand, the observations at 22" suffer from seeing or guiding effects which reduce the power observed at high spatial wave number, then the results may be compatible.

IV. THE SHAPE OF THE POWER SPECTRUM

Since the observations suggest that the shape is relatively independent of aperture size, it is convenient to discuss the power spectrum corresponding to fixed values of L. Results for L = 100, 200, and 300, all of which contribute to the power at periods as long as about 10 minutes, are considered in detail below. The discussion naturally divides into consideration of the energy to which an individual mode is excited, and the shape of the eigenfunction for the velocity amplitude. Of course, these are not totally independent, but this separation will be useful.

The preliminary results reported by Keeley (1977) showed that for models with mixing length equal to one or two pressure scale heights, the surface (velocity)^2 for $200 \leq L \leq 600$ had a peak near $\omega \approx 2 \times 10^{-2}$ (period ~5.25 minutes). The peak was steepest on the low frequency side, in general agreement with observations. However, it was noted at that time that the peak was not nearly sharp enough. These calculations include only turbulent dissipation in the calculation of the excitation energy, and the eigenfunctions were calculated for the adiabatic case.

a) The shape of the eigenfunctions

It is convenient to define an effective mass as the excitation energy required to produce a v^2 (averaged over the surface, and in time) of $(1\ cm/sec)^2$, at any particular depth in the atmosphere. This depends on the shape of the eigenfunction, but not on the actual excitation energy. The actual v^2 is obtained as the quotient of the excitation energy and the effective mass.

On the low frequency side of the peak, the increase in v^2 is due to an initial decrease in effective mass as the number of radial nodes in the eigenfunction increases. Physically, this occurs because the kinetic energy of the higher modes is more concentrated in the surface region, where the density is lower, and less energy is required to produce a given velocity. In the models discussed by Keeley, the excitation energy decreased with ω, but this effect was more than compensated by the decrease in effective mass, for $\omega < 2 \times 10^{-2}$. At higher frequencies (in a sequence with fixed L) the effective mass dropped off relatively slowly, and the net result was the high-frequency cut-off noted above.

The problem of insufficient steepness at low frequency can be approached from two directions. The first is to construct models in which the fall-off of effective mass is more rapid in the frequency range $\omega = 1.5 \times 10^{-2}$ to 2×10^{-2}, and the second is to find models in which the excitation energy decreases more slowly with ω, at least for periods greater than about 5 minutes. The latter problem is discussed in b) below. The ratio of effective masses over a given frequency range is conveniently expressed as $-\log(m(\omega_2)/m(\omega_1))/\log(\frac{\omega_2}{\omega_1})$. In table 2 this ratio is shown for solar models with three different values of the mixing length. The eigenfrequencies are not the same in these three cases, but $\omega_1 \approx 1.5 \times 10^{-2}$ and $\omega_2 \approx 2 \times 10^{-2}$ for the functions chosen. It is clear from the table that the model with largest mixing length is the most favorable, at all L values considered. If the excitation energies were roughly equal over this frequency range, the slope would be almost steep enough. Of course, the exact comparison with observations through a circular aperture requires that the results for various L values be combined as discussed in §III above. The results shown in table 2 are for models which have a fairly realistic atmosphere out to a temperature minimum of 4180°K.

Some preliminary calculations of nonadiabatic eigenfunctions have also been done. Including turbulent viscosity in the equations of motion does not have a significant effect on the shape of the eigenfunction, as reflected in the effective masses for the low frequency, low L modes studied so far. On the other hand, a fully nonadiabatic treatment of the radiative dissipation has a significant effect, apparently because the dissipation is very strongly localized near the top of the convection zone. For the cases studied, the result is to steepen the decrease in effective masses with increasing frequency, and thus to steepen the low-frequency side of the peak of the power spectrum. The nonadiabatic calculations can't at present be done correctly with a realistic solar atmosphere, since the radiation flux is not given simply in terms of the temperature gradient, as in the approximation usually used for stellar interiors. Also, the effects of convection are not included.

i) Effect of surface temperature and the mechanical boundary condition.

Models computed using the diffusion approximation all the way to the surface had surface temperatures of about 4880°. A more realistic value of T at the temperature minimum is about 4180° (Allen 1976). Adiabatic eigenfunctions and frequencies were computed for the diffusion models, and for models in which the empirical temperature profile was used at the surface. In addition, the models were computed with two different boundary conditions, one being that the Lagrangian pressure perturbation vanish at the surface, and the other that an outgoing wave existed (evanescent or propagating) with radial wave number determined as if the surface had an isothermal region attached to it at the boundary point of the model. The effect of a lower temperature at the surface is to make the waves more evanescent, and is expected to be most important at high frequency. At periods $\lesssim$ 5 minutes, it was found that with the $\delta^L P = 0$ boundary condition the low T model had lower effective masses (at optical depth 10^{-3}) and a slightly steeper decline in effective masses with increasing ω than the high surface temperature model. With the outgoing wave condition the situation was the opposite for both the magnitude of the effective mass, and the ratios of effective masses. In general, for either surface temperature, models with the outgoing wave condition had lower effective masses. The differences were largest at the highest frequency, where the waves were closest to being able to propagate. Significant changes in eigenfrequencies occurred only when the modes were close to propagating. Otherwise, the two atmosphere models and two boundary conditions gave nearly equal frequencies for the same physical oscillation modes.

b) The Excitation Energy

In the theory described by Goldreich and Keeley (1977), the expression for the excitation energy of a normal mode is given in the form of a quotient of two numbers. The denominator is the damping rate of the normal mode, and the numerator is a double integral, over depth in the model, and over eddy sizes at a given depth. An improved approximation to the inner integral has been used in the calculations described here.

i) Low frequency behaviour

In the discussion of the low frequency side of the peak in the power spectrum, it was noted that if the excitation energies over that frequency range were roughly equal, then the steep decline in effective mass for the high mixing-length models produced a slope much more in line with the observations. However, for all three mixing lengths tested, the excitation energy decreased significantly with increasing frequency. One way of equalizing the excitation energies for periods greater than about 5 minutes is to make all modes derive the main contribution to their excitation energy from a single set of eddies. The desired result will then be expected, provided the equipartition argument (Goldreich and Keeley 1977) is approximately valid. The result can be achieved, in fact, by a decrease in the correlation time for eddies in the outer part of the convection zone. If $\omega \tau_c < 1$ at $\omega \lesssim 2 \times 10^{-2}$ for the largest, most energetic eddies at a given depth, then all modes with $\omega < 2 \times 10^{-2}$ will tend towards energy equipartition with these eddies. If $\omega \tau_c > 1$ for the largest eddies, then the modes will tend to equipartition with a smaller, less energetic eddy having a correlation time satisfying $\omega \tau \approx 1$. In the present theory, the correlation time for the largest eddies is taken to be the mixing length divided by the convective velocity, and is scaled to smaller eddies by assuming a Kolmogoroff spectrum. Then the integral over eddy sizes depends on the correlation time assumed for the largest eddies in the form

$$\mathcal{J}(\omega) = \tau_c \int_0^1 \frac{dx}{X} X^{7.5} \exp\left(-\frac{1}{4}\omega^2\tau_c^2 X^2\right)$$

This is a slow function of ω for $\omega\tau_c \ll 1$, but drops like $\omega^{-7.5}$ for $\omega\tau_c \gg 1$. The changeover between the two types of behaviour is rather gradual; thus a change in τ_c which shifts the low frequency modes into the flat region results in a slower fall-off of the excitation energy, and therefore of the power spectrum, at high frequency.

The behaviour of excitation energy expected from the above discussion was verified by artificially increasing the convective velocity near the surface of the convection zone. A factor of less than two was sufficient to achieve the desired result. Thus it appears possible to reproduce, more or less, the steep rise in the power spectrum by a decrease in correlation time, in combination with a model with large mixing length. Of course, some of this gain is at the expense of the high-frequency fall-off.

ii) High frequency behaviour

The power spectrum as presently computed does not fall off very fast at high frequencies, at fixed L, except for L > 1000 or so (Keeley 1977). If such high L values do make substantial contribution to the power observed through typical apertures, they will improve the shape of the high frequency end substantially. However, it seems likely that radiative damping will play a significant role in decreasing the excitation energy for high-frequency modes. The only published nonadiabatic results on radiative damping are those of Ando and Osaki (1975, 1977). They find that the modes around 5 minutes are linearly unstable, but that modes near 3 minutes are stable. Strong damping sets in at somewhat lower frequency in their 1977 calculations, which include a chromosphere and corona. For the present models, the turbulent damping exceeds the radiative driving in all cases, but they are comparable for some modes. The result of a negative contribution to the damping would be an increase in the excitation energy of those modes; this could have a significant effect on the spectrum. For periods greater than about 7 minutes, the radiative growth or decay rate is relatively small compared to the turbulent decay rate. One important source of uncertainty is the calculation of the damping by the turbulent viscosity approximation. In addition, the calculations by Ando and Osaki do not include convection, and also do not calculate the radiative flux perturbation strictly correctly in the part of their model which employs an empirical $T(\tau)$ relation. Some preliminary nonadiabatic calculations using models with the high-temperature boundary, but no convective perturbations, find stability for all modes checked in the 3 to 10 minute range, without the effect of turbulent viscosity. A further source of uncertainty is the convective velocity profile (and magnitude) in the outer part of the convection zone, since this region contributes strongly to both the turbulent damping, and the total excitation.

V. SUMMARY OF RESULTS

The crude calculation of the power density near five minute period, as a function of aperture size, is not in agreement with observations; if a more accurate calculation establishes this discrepancy more firmly, then the observations may provide a powerful constraint on any theory of the excitation of the oscillations. In the present state of the theory, it seems possible to explain the steep low frequency side of the power spectrum peak. This requires, however, that the mass distribution of the sun be more like that of a model with a mixing length of three pressure scale heights, than one scale height. Important uncertainties in the damping due to radiative and convective energy transport preclude any strong statements about the high frequency end of the power spectrum; if there is linear driving comparable to the turbulent damping for periods near five minutes, this could have a significant effect on the sharpness and position of the peak.

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TABLE 1

Power density as a function of aperture size for three different assumptions about the velocities.

Case 1: $v_L^2 = 1$

Case 2: $v_L^2 = L$

Case 3: $v_L^2 = L^{-1}$

	Aperture diameter	<2"	5"	10"	22"	60"	120"
	θ_o	< .00135	.00325	.0065	.0143	.0390	.078
	L_c	> 1004	418	209	95	35	17
Case 1	Power density $L < L_c$	3.25×10^6	4.66×10^5	2.13×10^5	4.38×10^4	3.85×10^3	~0
	Power density $L > L_c$	0	3.87×10^5	1.65×10^5	6.83×10^4	1.33×10^4	3.02×10^3
	Total	3.25×10^6	8.53×10^5	3.78×10^5	1.12×10^5	1.72×10^4	3.02×10^3
Case 2	Power density $L < L_c$	2.69×10^9	1.84×10^8	2.97×10^7	2.63×10^6	1.01×10^5	~0
	Power density $L > L_c$	0	2.96×10^8	7.40×10^7	1.39×10^7	1.20×10^6	$\sim 1.82 \times 10^5$
	Total	2.69×10^9	4.80×10^8	1.03×10^8	1.65×10^7	1.30×10^6	$\sim 1.82 \times 10^5$
Case 3	Power density $L < L_c$	6.52×10^3	3.27×10^3	1.95×10^3	8.23×10^2	1.51×10^2	~0
	Power density $L > L_c$	0	5.32×10^2	4.45×10^2	4.68×10^2	2.29×10^2	8.7×10^1
	Total	6.52×10^3	3.80×10^3	2.40×10^3	1.29×10^3	3.80×10^2	8.7×10^1

TABLE 2

Logarithmic slope $\frac{\Delta \log(\text{effective mass})}{\Delta \log(\omega)}$ between $\omega \approx 1.5 \times 10^{-2}$ and $\omega \approx 2 \times 10^{-2}$, at three optical depths, for three values of mixing length/pressure scale height.

	Mixing length	1	2	3
L = 100	$\tau = 10^{-3}$	2.596	4.306	4.944
	$\tau = 10^{-2}$	2.087	3.929	4.563
	$\tau = 0.7$	1.213	3.072	3.688
L = 200	$\tau = 10^{-3}$	2.846	4.491	5.060
	$\tau = 10^{-2}$	2.426	4.150	4.719
	$\tau = 0.7$	1.561	3.378	3.949
L = 300	$\tau = 10^{-3}$	2.756	4.746	5.337
	$\tau = 10^{-2}$	2.323	4.419	5.011
	$\tau = 0.7$	1.517	3.676	4.271

Discussion

A. Cox: You have this high value of the mixing length -- three pressure scale heights -- so how deep in either mass or temperatures does the eigenfunction have any size? Does it go in very deep -- 2000°K? Half way into the Sun?

Keeley: Well, if you mean 0.1 of the surface amplitude, it goes in a very short distance -- a few percent of the radius. It doesn't go very deep into the convection zone. For a reasonably low ℓ value -- say $\ell = 10$, which I haven't shown -- then it goes a lot deeper. For $\ell = 1000$ it really stays way out at the surface.

Shipman: Presumably, it does go more than one mixing length, however.

Keeley: Yes, the scale height is around 10^7 - 10^8 cm, and it goes in farther than that.