NOTICE

THIS DOCUMENT HAS BEEN REPRODUCED FROM MICROFICHE. ALTHOUGH IT IS RECOGNIZED THAT CERTAIN PORTIONS ARE ILLEGIBLE, IT IS BEING RELEASED IN THE INTEREST OF MAKING AVAILABLE AS MUCH INFORMATION AS POSSIBLE
The Evolution of Supernova Remnants in Different Galactic Environments, and Its Effects on Supernova Statistics

M. Kafatos, S. Sofia, F. Bruhweiler
and T. R. Gull

MAY 1980

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland 20771
THE EVOLUTION OF SUPERNOVA REMNANTS IN DIFFERENT GALACTIC ENVIRONMENTS
AND ITS EFFECTS ON SUPERNOVA STATISTICS

Minas Kafatos\textsuperscript{+}, Sabatino Sofia\textsuperscript{\pm}, Frederick Bruhweiler\textsuperscript{*}
and Theodore R. Gull\textsuperscript{\ddagger}

\textsuperscript{+} George Mason University, Physics Department
\textsuperscript{\pm} Goddard Space Flight Center, Greenbelt, MD
\textsuperscript{*} Computer Sciences Corporation, Silver Spring, MD

Received 1980 February 5;
ABSTRACT

By examining the interaction between supernova (SN) ejecta and the various environments in which the explosive event might occur, we conclude that only a small fraction of the many SNs produce observable supernova remnants (SNRs). This fraction, which is found to depend weakly upon the lower mass limit of the SN progenitors, and more strongly on the specific characteristics of the associated interstellar medium, decreases from approximately 15 percent near the galactic center to 10 percent at Rgal \(\sim 10\) kpc and drops nearly to zero for Rgal \(>15\) kpc. Generally, whether a SNR is detectable is determined by the density of the ambient interstellar medium in which it is embedded. We find that SNRs are only detectable above some critical density \((n \sim 0.1 \text{ cm}^{-3})\). The presence of large, low density cavities around stellar associations due to the combined effects of stellar winds and supernova shells strongly suggests that a large portion of the detectable SNRs must have runaway stars as their progenitors. These results explain the differences between the substantially larger SN rates in the Galaxy derived both from pulsar statistics and from observations of SN events in external galaxies, when compared to the substantially smaller SN rates derived from galactic SNR statistics. These results also explain the very large number of SNRs observed towards the galactic center in comparison to few SNRs found in the anti-center direction.

Subject headings: nebulae; supernovae remnants – stars; stellar statistics
I. INTRODUCTION

The evolution of a SNR expanding within the "typical" interstellar medium (i.e., number density $n \sim 1 \text{ cm}^{-3}$, temperature $T \sim 10^2 - 10^4 \text{ K}$) has been studied with a great deal of detail and sophistication (for a recent review see Chevalier, 1977). In particular, detailed integrations of the basic shock equations have been performed (Chevalier 1974; Mansfield and Salpeter, 1974), and the complex phenomena which appear during the transition from the adiabatic to the isothermal phase have been addressed (Chevalier, 1975; Chevalier and Theys, 1975; Woodward, 1976; McCray et al., 1975, etc.).

Recognizing the inhomogeneous nature of the interstellar medium (ISM) considerable work has been carried out to study the effects of inhomogeneities in the structure and evolution of the SN shock waves. For example, McKee and Cowie (1975), Sgro (1975), and Woodward (1976) have investigated the interaction of supernova shock waves with interstellar clouds, and McKee and Ostriker (1977) have examined the effects of SN explosions on a cloudy interstellar medium.

In all the above work, the basic shock is always assumed to propagate within a typical (as defined earlier) interstellar medium, and until quite recently, no attempt had been made to study the evolution of a SN shock wave expanding into media representing the various possible environments of supernova progenitors within the Galaxy. For example, it is well known that most (if not all) stars are born in groups (clusters or associations). The precursor of the stellar association is a dense molecular cloud. The first supernova from the stars in the group is set off near, perhaps inside dense, cold gaseous media. The evolution and long term detectability of such an event (recently studied by Wheeler et al., 1980; and Shull, 1980) is obviously very different from that of the canonical supernova remnant that propagating in the typical interstellar medium.
Supernovae propagating in dense molecular clouds will be very rare, however, since the first one may disrupt one of the smaller clouds (Wheeler et al., 1980), whereas for the more massive clouds, the combined effect of stellar winds and the earliest supernovae (Bruhweiler et al., 1980) creates an expanding, hot, low density cavity, within which subsequent supernova shells will expand for times up to tens of thousands of years. It is this scenario, in fact, which is by far the most common encountered by supernova shells. As we shall discuss in § V, however, since most massive stars occur in binary systems, when the primary member of the system becomes supernova, it may impart a large velocity to the secondary star by means of the slingshot effect (Blaaauw, 1964). Many of these stars will in fact overtake the supershell and remain within the confines of the galactic disk until they become supernovae. These stars are the ones that produce the typical supernova remnants so extensively discussed in the scientific literature.

In this paper, we will study the evolution of supernova shells into these very dissimilar media. Because of the dramatic effects of the pre-supernova environment on the evolution of the remnant, we have confined our theoretical discussion on the simplest possible description. Thus, following Spitzer (1978), we characterize the supernova shock by three phases, namely:

(a) initial free expansion of the supernova material,
(b) intermediate adiabatic (or Sedov) expansion, and
(c) late isothermal expansion.
Although the structure of the pre-supernova environment can, in general, be fairly complex, it is usually made up by a combination of dense clouds, diffuse, warm medium, and rarefied, hot cavities. To that extent, we shall study the evolution of supernova remnants into these three types of media, which should span conditions for nearly all real cases. Thus, we will consider the remnant evolution into

(1) a dense \((n \sim 10^4 \, \text{cm}^{-3})\), cold \((T \lesssim 10^2 \, \text{K})\) molecular cloud

(2) the typical interstellar medium \((n \sim 1 \, \text{cm}^{-3}, T \sim 10^2 - 10^4 \, \text{K})\), and

(3) a bubble or supernova cavity \((n \sim 10^{-2} \, \text{cm}^{-3}, T \sim 5 \times 10^5 \, \text{K})\).

II. TYPES OF SUPERNOVA REMNANTS

Before describing each particular type of SNR let us introduce the relations which allow us to compute the several SN phases for each particular environment. As stated earlier, we shall follow the formalism of Spitzer (1978), and Gorenstein and Tucker (1976).

In the early phase, (a),

a shock wave will travel just ahead of the ejected shell, with a velocity \(V_s\). The shock will heat matter to a temperature

\[
T_s = \frac{3m_H}{4\mu} \frac{V_s^2}{16k} \gtrsim 1.3 \times 10^3 V_s^2
\]

(Landau and Lifshitz, 1959), where \(m_H\) is the mass of the hydrogen atom, \(\mu\) is the mean molecular weight and \(k\) is the Boltzmann constant. This phase ends when the swept-up interstellar material equals the mass of the ejecta, i.e., when

\[
R_s = \left(\frac{3M_{ej}}{4\pi \rho}\right)^{1/3}
\]

where \(M_{ej}\) is the mass of the ejecta and \(\rho\) the density of the ambient interstellar medium. The elapsed time, \(t\), between the supernova event and the end of phase (a) is given by

\[
t = \frac{R_s}{V_s}
\]
Phase (b), the intermediate non-radiative expansion phase, can be computed by means of the Sedov solution. The temperature immediately behind the shock is given by

\[ T_2 = \frac{3 \mu M V_s^2}{16 \kappa} = \frac{0.06 \mu}{k} \frac{E}{\rho S} \]  

where \( E \) is the kinetic energy of the ejecta;

\[ R_S = \left( \frac{2.0 \rho}{\rho} \right)^{1/5} t^{2/5} \text{ cm} \]  

and

\[ V_S = \left( \frac{3.0 \rho}{3 \rho} \right)^{1/2} \left( \frac{1}{R_S} \right)^{3/2} \text{ cm s}^{-1} \]  

and \( t \) can again be obtained from (3).

Phase (b) ends when \( T \) falls below \( \sim 10^6 \text{K} \), for \( n \sim 1 \text{ cm}^{-3} \) since radiative cooling then becomes important, which brings the onset of phase (c), the late isothermal expansion. This phase can be represented by the snowplow model, where conservation of momentum applies. Here the shell velocity is given by

\[ V_S = \frac{3}{4} \frac{M_1 V_1}{\rho_1} \left( \frac{1}{R_S} \right)^3 \]  

where \( M_1 \) and \( V_1 \) are, respectively, the shell mass and velocity at the end of phase (b). At this phase, most of the swept-up material is in a cool, dense shell although there may be some thermal X-radiation from the hot low density gas interior to the shell. This low density interior gas has a long cooling time.

We now discuss the three example SNRs and how differently they appear in the several expansion phases. In all of our calculations we shall assume that during the SN event, 4 \( M_\odot \) of the stellar material are ejected with a velocity of 5000 km sec\(^{-1}\), and thus, the total kinetic energy of the ejecta is \( 10^{51} \text{ erg} \). These parameters are a realistic representation of a type II SN which would be expected to occur in an OB association.
A. A SN in a Dense Molecular Cloud

While molecular clouds found in nature have a large range of size, mass and density, a typical molecular cloud can be approximated as a sphere with uniform density $n_{H_2} \approx 10^4 \text{cm}^{-3}$ having a diameter $D_C \approx 5 \text{ pc}$ (i.e., or cloud mass $M_C \approx 3.5 \times 10^4 M_\odot$), and with a temperature $T < 10^2$ (Burton, 1976).

Wheeler et al., (1980) and Shull (1980) have modeled the effects of a SN exploding within a molecular cloud. The characteristics of the SNR at the end of each phase using their results are summarized in Table 1.

We will subsequently call these SNRs the molecular cloud SNRs.

Because of the very high density, the phases occur rapidly for the molecular cloud SNRs. Wheeler et al. (1980) indeed suggest that the adiabatic phase may not exist if $n_{H_2} > 10^5 \text{cm}^{-3}$. Whether this happens depends on $n$ as well as on $M_{ej}$. The molecular cloud SNR represents the conditions within a young molecular cloud when the first, most massive stars become SNe. These conditions would be very short-lived as the SNRs would push the molecular cloud away from the remaining massive stars within the association and cause rapid cloud fragmentation (Elmegreen, 1979). Consequently, this picture will apply to a relatively small minority of the SNe which occur in the Galaxy. Moreover, at no point of its evolution does the resulting shell remotely resemble the familiar observed supernova remnant (SNR). In particular, during the earlier stages of evolution, optical and x-ray observations are useless as a means of detection because the cloud is optically thick to those wavelengths. If the SN were to produce a gamma ray pulsar as the stellar remnant, it would be observable. However, since only a minority of the pulsars are known to emit gamma rays, this is not an effective means to search for SNe within dense clouds. The SNR should be detectable by means of the infrared emission from the heated grains inside and outside of the cloud (Wheeler et al., 1980; Shull 1980; Silk and Burke, 1974). Even this technique is not fool-proof, though, since it may be very difficult to differentiate the cloud-embedded source of infrared emission.
as a SNR rather than a recently formed OB association.

B. A SN Within a Hot, Rarefied Cavity

Most SNs which occur in the older OB associations will first expand into the hot highly-evacuated volume produced by the combined effects of stellar winds and earlier SNs (Bruhweiler et al., 1980). The ejecta expand freely until enough gas is encountered to form a shock. The ejecta do not encounter significant gas until very late in the evolution of the SNR. Indeed, for a few thousands of years the SNR is hot, dilute gas expanding without bound. Such a gas is very difficult to observe (the question of detectability will be addressed in § III.

As an illustration, we compute the evolution of a SNR contained within a superbubble with radius $R \approx 105$ pc. From the model calculations of Bruhweiler et al., such a shell would exist around a typical OB association after a few million years. The characteristics of the SNR within a superbubble are summarized in Table I at the end of each phase. We shall call these SNRs the hot cavity SNRs.

C. A SN Surrounded by the Undisturbed ISM

In some instances, an intermediate mass star may become a SN outside molecular clouds and also outside the hot cavities surrounding OB associations. The resulting SNR which interacts with the previously undisturbed interstellar medium of the galactic disk is the canonical SNR that has been described in § I. The characteristics of the canonical SNR are summarized in Table I.

The canonical SNR has several intriguing differences in properties when compared to the other two types of SNRs. The total evolutionary lifetime is substantially longer than the lifetimes of either the molecular cloud SNR or the hot cavity SNR. The kinetics of a molecular cloud SNR are quickly transferred to the very massive molecular cloud and the expanding shell stalls within $9 \times 10^4$ years for $n \approx 10^4$ cm$^{-3}$. The hot cavity SNR, on the other hand, expands rapidly
until it encounters the outer, slowly moving supershell of neutral gas (as observed by Heiles, 1979). By then the SN shock is highly diluted, and consequently the ejecta are quickly decelerated. The slow accretion of material by the classical SNR in the undisturbed ISM extends the lifetime by more than twenty-fold compared to either alternate example.
III. OBSERVABLE SNRs

We discuss here the type of interstellar environment required to produce an observable SNR. By "observable" we mean an SNR which may be seen in a) visible light, b) radio waves, or c) X-rays.

a) Visible light SNRs

The majority of visible light SNRs are believed to be in the Sedov (adiabatic) phase. Exceptions may be very "old" SNRs like the Monoceros Loop, although McKee and Cowie (1975) have suggested that even those are in the Sedov phase, or very "young" SNRs like Cas A and the Crab Nebula. A SNR can be easily detected at visual wavelengths if the emission measure, \( EM = n_e^2 L \), exceeds 50 cm\(^{-6}\) pc where \( n_e \) is the electron density in the SN shell and \( L \) is the shell thickness. Careful observations will aid in detecting an SNR with \( EM \gtrsim 20 - 50 \text{ cm}^{-6} \text{ pc} \) for \( T \sim 10^4 \text{ K} \) but very special, tedious techniques are needed to detect a SNR with \( EM \gtrsim 5 \text{ cm}^{-6} \text{ pc} \) (for example a large Fabry-Perot etalon, etc.).

The emission measure can be expressed as (assuming, roughly, an average path length of \( R_s/12 \) in the Sedov phase)

\[
EM = \frac{4}{3} R_s n_o^2
\]  

(7)

where \( n_o \) is the ambient ISM density and \( R_s \) is the radius of the shock front. Equation (7) is actually an over-estimate because the density drops off rapidly behind the shock front (Spitzer 1978; Chevalier, 1974).

The EM observability criterion (\( EM \gtrsim 50 \text{ cm}^{-6} \text{ pc} \)) is a function of time. For the Sedov phase, we find

\[
n_o \gtrsim 1.8 F_{51}^{-1/9} t_4^{-2/9} \text{ cm}^{-3}
\]  

(6)

where \( EM \gtrsim 50 \text{ cm}^{-6} \text{ pc} \). Taking \( F_{51} = 1 \), \( t_4 = 3 \) (see part C of Table 1) we find that \( n_o \gtrsim 1.4 \text{ cm}^{-3} \) is required. Even for the SNR with \( EM \gtrsim 5 \text{ cm}^{-6} \text{ pc} \), we need \( n_o \gtrsim 0.2 \text{ cm}^{-3} \).

For an SNR in the isothermal (cooling) phase, the observability estimates are different. Large compressions can take place in this case (Bek, 1972b; McCray, Stein and Raitt, 1976). McCray, Stein and Raitt (1976)
find that compression ratios as high as $\approx 70$ may be reached if the ambient magnetic field is weak or as low as $\approx 20$ when the magnetic field cannot be ignored ($B > 3 \times 10^{-6} G$). The appropriate pathlengths is the characteristic cooling length (McCray et al. 1975). For a $100 \text{ km s}^{-1}$ shock with no ambient magnetic field (the most favorable case) the emission measure would exceed $50 \text{ cm}^{-6} \text{ pc}$ if $n_o \gtrsim 0.8 \text{ cm}^{-3}$. Correspondingly, for $\text{EM} \lesssim 5 \text{ cm}^{-6} \text{ pc}$ \nolinebreak $n_o \gtrsim 0.08 \text{ cm}^{-3}$. Since the ambient densities are lower, it is more favorable to observe an optical SNR during its isothermal phase than during its adiabatic phase. For an "easy" detection, it seems that the shock must collide with a dense cloud (locally, the intercloud medium has a density of $\approx 0.15 \text{ cm}^{-3}$, Falgarone and Lequeux, 1973).

The filling factor of clouds is quite small ($f \approx 1-10\%$ see McCray and Snow 1979). Moreover, the number of clouds drops off rapidly with height above the galactic plane (the cloud scale height is probably $\approx 1/2$ of the scale height of the diffuse intercloud medium; see Falgarone and Lequeux, 1973). Hence, few optically observable SNR's are expected at large heights above the galactic plane. The Cygnus Loop may be observable because it has collided with a neutral cloud or clouds. These clouds cannot be the same type of neutral clouds that produce $\text{H I}$ absorption profiles, as the $\text{H I}$ absorption clouds are found (Radhakrishnan et al., 1972) within 300 pc of the galactic plane.

b) Radio observable SNRs

Radio SNRs are much more numerous than either optical or X-ray remnants. About 130 radio SNRs have been observed in the galaxy (Clark and Caswell 1976), whereas only 30 of the radio SNRs are detected by any optical emission (van den Bergh, 1976). It is easier to detect a SNR at radio wavelengths both because lower ambient densities are necessary for radio detection and because dust does not absorb radio photons. It is usually assumed that
van der Laan's (1962) theory applies to the older SNRs. However, the
statistical investigation by Clark and Caswell (1976) confirmed that for
the majority of SNRs the Sedov solution properly describes the value of the
diameter D with time. Clark and Caswell derived an average \( \langle E/n_0 \rangle \sim 5 \times 10^{51} \)
ergs cm\(^3\) where \( E \) is the initial supernova energy and \( n_0 \) is again the ambient
density. If \( E = 10^{51} \) erg, the averaged \( n_0 \) implied by their analysis is 0.2 cm\(^{-3}\).

More recently, Caswell and Leche (1979) refined the \( E - D \) relation to include a z-
dependence. (\( E \) is the radio surface brightness.) The derived scale height of radio
SNRs is 200 pc. The implied frequency of SN producing radio SNRs within the Galaxy
if \( f \sim 1/80 \) yr\(^{-1}\) The lower values of the ambient density implied by the radio
observations confirm that SNRs are more easily detected in the radio.

There is also information on the galactic distribution of SNRs. Ylovaisky
and Lequeux (1972) find that the distribution of radio SNRs closely follows
the radial distribution of the non-thermal background radio emission. At
\( \sim 1 \) kpc from the galactic center the radio SNRs are three times as abundant
as at 10 kpc. Beyond 15 kpc there are very few radio SNRs.

(c) X-ray observable SNRs

Very few X-ray SNRs have been observed (for a recent review of the
X-ray SNRs see Clark and Culhane, 1976) perhaps being due in part to the
limited sensitivity of complete X-ray surveys. The \( E/n_0 \) value derived for
the X-ray SNRs support the average value obtained from radio observations
(Gorenstein, Harraden and Tucker, 1974; Clark and Caswell 1976) although the
two studies are vastly different. The thermal X-ray flux in the 1-100 keV
region of the spectrum for SNRs is given by Gorenstein, Harraden and Tucker
as \( L_X \sim 5 \times 10^{33} n_0^2 R_{pc}^3 \) erg s\(^{-1}\). Using the above expression for \( L_X \) and the
X-ray measured for Pup A, the Cygnus Loop and the Veil SNR, we conclude than an
X-ray SNR is not observable if the ambient density is approximately less
than \( \sim 0.1 \) cm\(^{-3}\). On the basis of recent HEAO - A (''Einstein'') surveys
for X-ray emission of radio and optical SNRs (Knox Long, private communication), initial ambient densities below 0.05 cm$^{-3}$ would not produce detectable X-ray fluxes. A value of the critical density of 0.1 cm$^{-3}$ is required to observe an X-ray remnant is consistent with the "Einstein" results. From the above discussion we conclude that if the ambient interstellar medium density $n_o$ exceeds a critical value $n_c$, the SNR is observable. It would most probably be observed as a radio SNR, but if $n_o$ is appreciably larger and the SNR is not too distant, it may be an optical or an X-ray observable SNR. We adopt the value $n_c = 0.1$ cm$^{-3}$ with the awareness that this is a realistic estimate really for the radio SNRs. In any case, we find that the study which follows would not change appreciable if $n_c$ were to change by a factor of 2 to 3. For optical SNRs in the Sedov phase, we find $n_c \sim 1.0$ cm$^{-3}$ but this value is probably unimportant since it is much easier to detect the radio SNR.

To simplify our analysis, we assume that all SNRs located within a medium with ambient density exceeding $n_c$ will be observable, but all SNRs located where $n_o < n_c$ are not observable. For example, SNRs with diameters approaching the cloud scale height would be expected to be brighter on their edge nearest to the plane (Clark and Stephenson, 1977; Caswell and Lerche, 1979) and therefore, would be observable even though a portion of the SNR is located in a medium with density less than $n_c$. The largest observable SNRs have diameters $\lesssim 50$ pc which is less than the cloud scale height, so our results will not be significantly changed by this effect.
The average expansion rate for OB associations is $\sim 5$ km s$^{-1}$. Since
the radius of a typical supercavity at the Sun's distance from the galactic
center is 250 pc, it takes about $5 \times 10^7$ years for an association member to
overtake the supershell. During this time all stars more massive than $7 M_\odot$
will have completed their evolution, so that if a lower mass limit of $8 M_\odot$
for stars that produce SNe is adopted, we find that very few stars are
capable of escaping the cavity before they become supernovae (the numbers
are higher for closer distances to the center—see below—but still small).
If the mass limit of stars producing SNe is extended down to $4 M_\odot$ then
the majority of SN explosions that produce SNRs in the solar neighborhood
would originate in $4-8 M_\odot$. A significant fraction of the progenitors
would escape the low density cavity before exploding. As there is an
uncertainty of the mass limit of stars which produce SNe we will consider
two limits: $8 M_\odot$ and $4 M_\odot$. In part (A) below, we use $8 M_\odot$ while in part
(B), where we also include the effects of the gravitational field from the
galactic disk, we adopt $4 M_\odot$ as the lower mass limit. By these two examples,
we represent the upper limits and lower limits to supernova occurrences
within OB associations.

Early type stars have a high incidence of binaries. A survey of
early B stars (Abt and Levy, 1978) shows that about half of these stars are
multiple systems.

Blauw (1954) suggests that when a SN occurs in such a binary system
the companion can become a runaway star. In his classical work, 19
runaways are identified out of which the latest spectral type is B3 with
an assigned mass of $\sim 10 M_\odot$. A later study of 304 O stars (Cruz et al., 1974)
concludes that at least 20 percent of all O stars are runaway stars. Using
the above evidence, and assuming that all binaries produce runaways, it follows that one-third of all SNs are from runaway progenitors. It is the runaway stars that have an opportunity to escape the superbubbles and in turn produce observable SNRs.

However, not all runaway progenitors produce observable SNRs. In order to estimate the fraction that produce observable SNRs, we need to estimate the following:

1) the initial mass function (IMF),
2) the total evolutionary lifetime, $T$, for stars with different masses,
3) the average peculiar velocity for runaway stars, $V_p$, which when used with the total evolutionary times will allow us to estimate the total distance, $d$, that a runaway star would travel from the OB association, and
4) the effective critical scale height, $H_c$, for observable SNRs.

We now discuss each of these variables that will influence the estimate of observable SNRs.

The IMF for the massive stars is uncertain at best, especially for the O stars. Ostriker et al., (1974) used the observational data of Richstone and Davidson (1972) to derive an IMF for O stars. Their mass function predicts significantly more massive O stars than that predicted by an extrapolation of the IMF deduced for the mid to late-B stars by McCuskey (1966). Theoretical evolutionary calculations, when compared with eclipsing binary data (Stothers, 1972), indicate a different mass versus spectral type relationship than used by Ostriker et al. Bruhweiler (1980) has re-analyzed the Richstone and Davidson (1972) data and has determined the masses for MK standards based upon the work of Stothers (1972). Both the data of Richstone and Davidson and of McCuskey (1966) can be represented by an IMF which relates the total number of stars as:
We now estimate the distances, \( d \), that a runaway star travels from an OB association before it becomes a supernova:

\[
d = v_p (\tau_2 - \tau_1) = v_p \Delta \tau_{21},
\]

(10)

where \( \tau_1 \) and \( \tau_2 \) are the total evolutionary times of the primary and the secondary in a binary system and \( \Delta \tau_{21} \) is the difference. The average mass of the primary, \( M_1 \), can be expressed in terms of the mass of the secondary, \( M_2 \), by the relation:

\[
\left( \frac{M_1}{M_2} \right)^{-2.25} = 1
\]

(11)

The higher mass limit for the IMF is not critical. In equation (11) we assume that the primary has a random mass distribution described by the IMF in equation (9). In Table 2, we present the resultant mass (which is a mean value) of the primary, \( M_1 \), computed from equation (11) and the timescale, \( \Delta \tau_{21} = \tau_2 - \tau_1 \), which enters the expression in (10).

**FOOTNOTE**

*In our discussion we have ignored the effects of mass loss and mass exchange in the evolution of O and B stars. These processes affect \( \Delta \tau_{12} \), which in turn affects the fraction of runaway stars that escape from the supercavity. Mass loss is expected to lengthen (by about 10%) the evolutionary lifetimes of the more massive stars (Chiosi, Nasi and Sreenivasan, 1978). However, the domain of extensive mass loss is limited to O or earlier stars (Snow and Morton, 1976), i.e., stars with initial masses \( \geq 17 M_\odot \). From our calculations, these stars become supernovae either inside the supercavity or outside the galactic plane. Thus, the proposed increase in evolutionary timescales has a
negligible effect.

A potentially more significant effect is due to the mass distribution of the binaries. For the sake of simplicity, we have assumed a random distribution of mass ratios among binaries. This assumption gives $M_2/M_1 \geq 0.73$ for masses of interest in this work. On the other hand, if close binaries are formed as bifurcation products, then the mass ratio should be of order unity.

Observationally determined values for this ratio range from 0.35 (Stone, 1979) to about 1 (Heintze, 1973). In view of these uncertainties, our assumption of $M_2/M_1 \gtrsim 0.73$ is reasonable. If the true ratio were smaller, $\Delta \tau_{12}$ would be larger than our tabulated values, whereas the bifurcation hypothesis leads to smaller $\Delta \tau_{12}$ than our tabulated values. In any event, the effects are not overwhelming, and our procedure is justifiable at present. A final uncertainty is introduced by our having ignored the effect of mass transfer in computing $\Delta \tau_{12}$. Due to the speeding up of the evolution beyond core H-exhaustion stage, the effects of mass dumping by the primary is expected to be minimal when compared to the shortening of the evolutionary timescale of the secondary. This would lead to generally smaller $\Delta \tau_{12}$. Since in order to produce a runaway the mass ejected must exceed the mass of the companion, mass transfer could modify the statistics of close binaries.

The effective critical scale height, $H_c$, for observable SNRs is determined by the scale height of the gas. In Figure 1 we show the structure of the supercavity produced at three different distances from the galactic center, $R_{\text{gal}} = 5, 10,$ and $20$ kpc. These are structures based upon the model we calculated (Bruhweiler et al., 1980). We assumed an exponential density distribution

$$n = n_0 \exp \left(-\frac{z}{H}\right)$$  \hspace{1cm} (12)

with $H = 70, 150, 500$ pc and $n_0 = 3, 1, 0.1 \text{ cm}^{-3}$ for $R_{\text{gal}} = 5, 10, 20$ kpc, respectively. The assumed densities are appropriate for the H I medium as determined in Paul, Cassé and Cesarsky (1976) and the scale height is
from Kerr (1969). Exponential distributions were found by Celnik, Fohlfs and Braunsfurth (1979) for large distances away from the galactic plane. They give detailed formulae for the scale heights, $H$, although the values of $n_0$ are harder to determine from their work. Assuming that 5 percent of the total mass of the galaxy is in gaseous form, we find – using the work of Celnik, Rohlfs and Braunsfurth – that $n_0 \sim 6, 0.7$ and $4 \times 10^{-3}$ cm$^{-3}$ for $R_{\text{Gal}} = 5, 10$ and 20 kpc respectively. Bohlin et al. (1978) on the other hand, find that $n_0 \sim 0.9$ for the H I medium and $\sim 1.2$ for the H I + H$_2$ medium (in the solar neighborhood).

Even though there are large uncertainties in the parameters of the ISM gas, the structure of the bubble–SN cavity is not affected very much. This is so because the radius of the bubble is only weakly proportional to the ambient density ($\sim n_0^{-1/5}$) and the radius of the SN produced shell is $\propto n_0^{-1/3}$ near the plane. For the case where the uncertainties are large ($R_{\text{Gal}} = 20$ kpc) we find that even the higher density $n_0 = 0.1$ cm$^{-3}$ produces such large supercavities that essentially no runaway stars escape. Therefore, this result would still be true if we choose $n_0$ to be smaller.

A. Supernova Statistics Derived with 8 M$_\odot$ Lower Limit for SN Progenitors, and Ignoring Gravitational Effects of the Disk.

We now compute the percentage of all SN progenitors that produce observable SNRs. In this case we assume 8 M$_\odot$ to be the lower limit for SN progenitors, and ignore the gravitational effects of the disk. The gravitational force of the disk tends to restore gas to the galactic plane and in the $\tau$-direction decreases the size of the supercavities. Hence, the percentages found in this case will be a lower limit to the actual percentage.
In Figure 2 we present the fraction of runaway stars in mass intervals $\Delta M = 1 \, M_\odot$ that escape the supercavity to produce observable SNRs. The results change for different ratios of $H_c/V_p$; hence, we show the curves for a range of $H_c/V_p$. For example, if we assume $V_p = 50 \, \text{km s}^{-1}$ and we want to appropriate curve for $R_{\text{Gal}} = 5 \, \text{kpc}$, from Figure 1, $H_c = 240 \, \text{pc}$ and therefore $H_c/V_p = 4.8$. Similarly, for $R_{\text{Gal}} = 10 \, \text{kpc}$ $H_c = 350 \, \text{pc}$ and $H_c/V_p = 7$.

We now illustrate the application of this figure by presenting numerical estimates for the fraction of stars in a $1 \, M_\odot$ mass interval that escape the supercavity and produce an observable remnant. We chose $n = n_0 e^{-z/H}$ as in paper 1 at the two galactic distances $R_{\text{Gal}} = 5$ and 10 kpc. The critical density, $n_c = 0.1 \, \text{cm}^{-3}$, makes the total fraction of SN producing observable remnants at $R_{\text{Gal}} = 20 \, \text{kpc}$ equal to zero. Decreasing the critical density to $0.05 \, \text{cm}^{-3}$ would predict a total fraction at $R_{\text{Gal}} = 20 \, \text{kpc}$ of less than 1 percent.

Table 3 presents the lower limit on the percentage of SNs that produce observable SNRs. In Column 1, rows 2-10, we increment the mass range in bins of $1 \, M_\odot$. However, row 1 has a bin of 17-70 $M_\odot$. In column 2, we present the fraction of all SN progenitors that are within the mass bin. In columns 3 and 4, the fraction of all SN progenitors, which are within the mass bin, is given for those that produce observable SNRs at 5 and 10 kpc. Column 5, which would list the fraction of SN progenitors that produce SNRs at $R_{\text{Gal}} = 20 \, \text{kpc}$, is empty to emphasize that no SNRs would be produced in the ambient interstellar gas at 20 kpc. In row 11 we add the incremental percentages to find the total percentage of runaway SN progenitors that produce observable SNRs. To this total, we must add in row 12 the few SN progenitors in the low mass range that survive long enough to escape the supercavity even at the association expansion velocity of 5 km s$^{-1}$. By comparison, we have assumed that the runaway progenitors have $V_p = 50 \, \text{km s}^{-1}$. We see that no slow moving stars escape the supercavity at $R_{\text{Gal}} = 10 \, \text{kpc}$ while only 4.3 percent of the SN
progenitors are slow moving stars which escape the supercavity of \( R_{Gal} = 5 \) kpc.

Even if a SN explodes outside the supercavity formed by the parent association, such a SN might not be in the ambient interstellar gas. Rather the SN may find itself in another supercavity and hence, it would not form a visible SNR. The fraction of the ISM occupied by these supercavities is hard to estimate. The O VI gas (McCray and Snow, 1979) has a filling factor estimated to be twenty percent whereas the hot gas responsible for the X-ray background (Kraushaar 1977) has a filling factor estimated to be 50 percent. We find that about 30 percent of the ISM is occupied by these superbubbles produced by OB associations. However, this is likely to be a lower limit since we do not include in our model the (older) B associations. With 30 percent of the ISM assumed to be in the hot phase, we find the percentages listed in line B. Note that we have decreased the percentage of the runaway progenitors by 1/3 and not the percentage of the slow-moving progenitors, as the latter will be just beyond the superbubble and would be very unlikely within another supercavity.

B. Supernova Statistics Derived with 4 \( M_\odot \) Lower Limit for the SN Progenitors, and Including Gravitational Effects of the Disk.

The runaway progenitors and the gas are subjected to a gravitational restoring force towards the disk which we have ignored thus far. Close to the galactic plane the gravitational force law can be approximated by \( \ddot{z} = -kz \) with the resultant motion being that of an undamped harmonic oscillator.

We now calculate the critical angle, \( \theta_H \), at which runaway progenitors would produce SNe at a height \( H_c \) above the plane for a selected mass range.
as represented by $\Delta \tau_{21} = \tau_2 - \tau_1$. In terms of the z-direction gravitational force, we can express $\dot{z}$ and $H_c$:

$$\dot{z} = v_p \sin \theta_H = A \sqrt{k}$$

and

$$H_c = A \sin (\sqrt{k} \Delta \tau_{21})$$

We can solve for $\theta_H$ as:

$$\sin \theta_H = \frac{H \sqrt{k}}{v_p} \left\{ \sin (\sqrt{k} \Delta \tau_{21}) \right\}^{-1}$$

Within the mass range represented by $\Delta \tau_{21}$, all runaway progenitors that are ejected from the galactic plane at ejection angles less than $\theta_H$ will produce SNs at heights less than $H_c$ above the galactic plane. The fraction of runaway progenitors that produce observable SNRs for a given mass range $\Delta M_2$ is then:

$$f_{\text{SNR}} = \frac{2\pi \int_{-\Delta \tau_{21}}^{\Delta \tau_{21}} \left( v_p \Delta \tau_{21}(\Delta M_2) \right)^2 \cos \theta \, d\theta}{4\pi \Delta \tau_{21}(\Delta M_2)^2} = \sin \theta_H$$

The fraction $f_{\text{SNR}}$ can be evaluated by substitution of equation (16) into equation (15).

We are aware that large uncertainties are inherent in the gravitational force law for the Galaxy. Although errors in the force could be quite large at high latitudinal distances, we are mostly concerned with z-distances less than 300 pc. The $\ddot{z} = -kz$ approximation has estimated errors of less than 20 percent (see paper 1) in the solar neighborhood ($R_{\text{Gal}} \approx 10$ kpc). However, at 5 and 20 kpc the force law is much more uncertain.

Based upon the curves published by Schmidt (1956), we adopted in Bruhweiler et al. (1980) linearized force laws out to 300 pc in z with the values of $k$ being $6.06 \times 10^{-15}$, $2.58 \times 10^{-15}$ and $3.65 \times 10^{-16}$ sec$^{-2}$ for
\( R_{\text{Gal}} = 5, 10 \) and \( 20 \) kpc respectively.

Two other uncertainties are the values for \( H_c \) and \( V_p \). The derived values for \( H_c \) depend upon the assumed values of \( n_0 \) and the assumed density law (Equation 12). The average velocity, \( V_p \), for runaway progenitors is also very uncertain. Several runaway stars are known to have \( V_p \approx 100 \) km s\(^{-1}\) (Blauw, 1964; Stone 1979). The average runaway velocity we use is based on the velocities of 19 runaways presented by Blauw. Various selection effects are known that strongly suggest the lower velocity runaways may not be recognizable as such. While a reasonable value for \( V_p \) may be on the order of \( 50 \) km s\(^{-1}\) (or lower), we feel constrained to express \( H_c \) and \( V_p \) in terms of one variable, namely, their ratio, \( H_c / V_p \).

We now show in Figures 3 and 4 curves similar to those in Figure 2, but with the gravitational restoring force included. Within the mass intervals, \( \Delta M = 1 \) \( M_\odot \), we plot the fraction of runaway progenitors that escape the supercavity and produce observable SNRs. Figure 3 is for \( R_{\text{Gal}} = 5 \) kpc and Figure 4 is for \( R_{\text{Gal}} = 10 \) kpc. We present the results for the ratios of \( H_c / V_p = 1.0 \) and \( 2.0 \).

The percentages of SNs that would produce observable SNRs are summarized in Table 4. As in part (A) we assume that \( 1/3 \) of all SNs are from runaway stars and that \( 1/3 \) of the interstellar medium is occupied by supercavities. The percentages listed in Table 4 are for an evolved supercavity with the internal SN from low-mass progenitors being at large distances from the shell (labeled in Figure 1 as SN2). The SNR from the more massive stars have either dissipated or have been overtaken by the expanding supercavity around the old association. If stars appreciably less than \( 8 \) \( M_\odot \) produce SNs, then it is possible that these older supercavities would be ringed with SNRs from the slow moving progenitors. For example, if the expansion velocity were \( 5 \) km s\(^{-1}\) for the association, then \( 5 \) \( M_\odot \) stars would diffuse \( 500 \) pc, which
in the solar neighborhood is twice the shell radius, along the plane, by the
time they become SNs.

We have also computed similar models for young supercavities where only
stars with spectral type BO or earlier have become SN (the radius is shown
in Figure 1 as "SN1"). These younger supercavities would be surrounded by
SNRs from the relatively more massive progenitors \( M > 11 \, M_\odot \). A possible
example of such a system may be the Gum Nebula. It is a roughly spherical
cavity with a 125 pc radius. Two known SNRs in close proximity are Vela X-1
and Puppis A.

The computed estimates of SNs producing SNRs for the relative younger
and older supercavities are very similar. The total percentages, summarized
in Table 4, are 29.8 percent and 23.1 percent for \( R_{\text{Gal}} = 5 \) kpc and 10 kpc
respectively. About one half of the SNRs are from the slow-moving (5 km s\(^{-1}\)
stars.

We also computed the percentages of SNs creating observable SNRs at
\( R_{\text{Gal}} = 20 \) kpc and find that less than one percent of the SNs would yield
observable SNRs.
V. DISCUSSION AND CONCLUSIONS

Several important results are derivable from Tables 3 and 4. The presence of supercavities drastically changes the mass distribution of SN progenitors that produce the classically detectable SNRs.

A greater percentage of higher-mass stars escape the supercavity at small $R_{\text{Gal}}$ primarily because the sizes of the supercavities are smaller toward the galactic center. The model prediction that more SN produce detectable SNRs at small $R_{\text{Gal}}$ is in qualitative agreement with the observed distribution (van der Bergh, 1978; Clark and Caswell, 1976; Ilovaisky and Lequeux, 1972). Had we chosen $n_0$ to be 6 cm$^{-3}$ — which may be more appropriate for the dense inner arms—we would have predicted 2.2 times more SNRs at 5 kpc than at 10 kpc. Due to the uncertainties involved we consider this to be satisfactory agreement with the observations. This also implies that our assumption that the rate of SN outbursts throughout the galactic plane is nearly uniform may be close to reality. The distribution of observable SNRs is only determined by the presence and size of the supercavities and by the fraction of "runaway" SN progenitors that escape before becoming SNs.

The exact fraction of SNs that produce observable SNRs cannot be estimated accurately with the present data. Upon a variety of assumptions and parameters hopefully encompassing the real situations, we find that this fraction ranges from approximately 10 percent to 35 percent depending on the $R_{\text{Gal}}$, with 5 percent variations in either direction being reasonable.

The interstellar gas densities affect the above percentages somewhat. However, the fraction of SNs producing observable SNRs more than doubles when the lower mass cutoff decreases from 8 $M_\odot$ to 4 $M_\odot$. Moreover, if 4 $M_\odot$ stars are progenitors, the slowly moving stars which diffuse at the association...
velocity of expansion (5 km s\(^{-1}\)) would make an equal contribution to the total SNR percentage.

Since our model indicates that only one to three out of every ten SN produce an observable SNR, the disagreement between rates deduced from observable SNRs on the one hand and SN and pulsars on the other hand can be understood. Tammann (1974) suggests a mean interval between SN, \(\tau_{\text{SN}} \sim 30\) years. This leads to a SNR production rate of one every 100 to 300 yr. On the other hand pulsar statistics (Taylor and Manchester, 1978) imply \(\tau_{\text{SN}} \sim 10\) years. This leads to a SNR production rate of one every 30 to 100 years. Since the mean \(\tau_{\text{SNR}}\) derived by Caswell and Lerche (1979) is \(\sim 80\) years, there is a weak support for the higher SN rate. However, regardless of whether one SN occurs every 10 or 30 years, we can now understand how both these numbers are lower than the 80 year interval for the production of detectable SNR derived from radio data.

Recently, Higdon and Lingenfelter (1980), have proposed an alternative to our point of view. They propose that if a hot \((\lesssim 10^6\text{K})\), tenuous gas fills 90% of interstellar space, the observed number and surface brightness distribution of galactic remnants implies a SN rate of once every \(\sim 30\) yr. However, even though in this way the statistics may equally be reconciled, we feel that their filling factor is excessively large, although not inconsistent with the O VI data (Jenkins and Meloy 1974).

Our mechanism would operate in any case, further reducing the number of observable SNRs. Because of this, the 90% filling factor is not justified. The results of Tables 3 and 4 would indicate that filling factors as large as 70% could be tolerated but not appreciably higher than that.
We must emphasize that our discussion has been confined to type II supernovae. Type I supernovae, which probably originate from low-mass population II stars, share almost none of the considerations addressed in this paper, and they should be investigated separately. However, since some statistical studies of SN do not differentiate the SN types, they cannot be compared with our results without previously estimating the type I SN contribution. If one, however, examines the known SN of the last millennium, all of which left observable remnants, the question of differentiating between SN of type I and SN of type II becomes important.

We conclude that the supercavity concept—that of a hot, low-density gas around stellar associations created by stellar winds and SNs—provides a most effective scenario for understanding the general structure of the interstellar medium. Moreover, we believe that supercavities have already been detected observationally as the HI supershells (Heiles 1979) and as the X-ray superbubble in Cygnus (Cash et al. 1980). This provides a compelling argument that many SN do not occur in environments conducive to detectable SNRs.
Figure 1. Geometry of the supercavity structures at $R_{\text{Gal}} = 5$ kpc and 10 kpc as modeled by Bruhweiler, et al. (1980). The three stages of evolution are depicted. The curve labelled B is the limit of the supercavity formed by the 28 massive O stars having significant stellar winds. The curve labelled SN1 defines the size of the shell at the evolutionary time when all 28 massive O stars have become SNs. However, 180 more B stars have masses greater than $8 M_\odot$. These too become SNs and drive the supercavity to the SN2 size.
Figure 2. Fraction of runaway stars that escape the supercavity to produce observable SNRs. The various curves are for various assumptions of $H_c/V_p$ where $H_c$ is the critical z distance beyond which a SNR would not be detectable and $V_p$ is the runaway star velocity. Note that here the gravitation restoring force is not considered for the runaway star.
Figure 3. Fraction of runaway stars that escape the supercavity to produce observable SNRs. This is for $R_{\text{Gal}} = 5 \text{ kpc}$ and includes the gravitational restoring force.
Figure 4. Fraction of runaway stars that escape the supercavity to produce observable SNRs. This is for $R_{\text{Gal}} = 10$ kpc including the gravitational restoring force.
### TABLE 1

**A. The Molecular Cloud SNR (n = 10^4 cm^{-3}, T ≤ 10^2 K)**

<table>
<thead>
<tr>
<th>t (yr)</th>
<th>V_s (km s^{-1})</th>
<th>T_s (k)</th>
<th>R_s (pc)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5000</td>
<td>3.5x10^8</td>
<td>0</td>
<td>Explosion occurs</td>
</tr>
<tr>
<td>23</td>
<td>5000</td>
<td>3.5x10^8</td>
<td>0.12</td>
<td>Free expansion ends</td>
</tr>
<tr>
<td>112</td>
<td>1200</td>
<td>2x10^7</td>
<td>0.3</td>
<td>Adiabatic phase ends</td>
</tr>
<tr>
<td>9x10^4</td>
<td>5</td>
<td>2.5x10^3</td>
<td>1.9</td>
<td>Shell stalls</td>
</tr>
</tbody>
</table>

**B. The Hot Cavity SNR (n ~ 10^{-2} cm^{-3}, T ~ 5 x 10^5 K)**

<table>
<thead>
<tr>
<th>t (yr)</th>
<th>V_s (km s^{-1})</th>
<th>T_s (k)</th>
<th>R_s (pc)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5000</td>
<td>3.5x10^8</td>
<td>0</td>
<td>Explosion occurs</td>
</tr>
<tr>
<td>3.7x10^3</td>
<td>5000</td>
<td>3.5x10^8</td>
<td>19</td>
<td>Free expansion ends</td>
</tr>
<tr>
<td>1.4x10^5</td>
<td>300</td>
<td>1.3x10^6</td>
<td>105</td>
<td>SN shock encounters moving bubble shell and it quickly gets decelerated to the</td>
</tr>
</tbody>
</table>

**C. The Classical SNR (n ~ 1 cm^{-3}, T ~ 10^2 - 10^4 K)**

<table>
<thead>
<tr>
<th>t (yr)</th>
<th>V_s (km s^{-1})</th>
<th>T_s (k)</th>
<th>R_s (pc)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5000</td>
<td>3.5x10^8</td>
<td>0</td>
<td>Explosion occurs</td>
</tr>
<tr>
<td>6.2x10^2</td>
<td>5000</td>
<td>3.5x10^8</td>
<td>3.1</td>
<td>Free expansion ends</td>
</tr>
<tr>
<td>2.9x10^4</td>
<td>265</td>
<td>10^6</td>
<td>19.6</td>
<td>Adiabatic phase ends</td>
</tr>
<tr>
<td>3.6x10^6</td>
<td>5</td>
<td>2.5x10^3</td>
<td>73.6</td>
<td>Shell stalls</td>
</tr>
</tbody>
</table>

29
TABLE 2

Primary and Secondary Masses of the Binary System and the Evolutionary Timescale Differences

<table>
<thead>
<tr>
<th>$M_1$ ($M_\odot$) *</th>
<th>$M_2$ ($M_\odot$) +</th>
<th>$\Delta \tau_{21}$ (yr)**</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.85</td>
<td>19</td>
<td>2.8x10^6</td>
</tr>
<tr>
<td>24.49</td>
<td>18</td>
<td>2.9x10^6</td>
</tr>
<tr>
<td>23.13</td>
<td>17</td>
<td>3.7x10^6</td>
</tr>
<tr>
<td>21.77</td>
<td>16</td>
<td>4.0x10^6</td>
</tr>
<tr>
<td>20.4</td>
<td>15</td>
<td>4.2x10^6</td>
</tr>
<tr>
<td>19.05</td>
<td>14</td>
<td>5.1x10^6</td>
</tr>
<tr>
<td>17.69</td>
<td>13</td>
<td>5.4x10^6</td>
</tr>
<tr>
<td>16.33</td>
<td>12</td>
<td>5.6x10^6</td>
</tr>
<tr>
<td>15.0</td>
<td>11</td>
<td>7.9x10^6</td>
</tr>
<tr>
<td>13.6</td>
<td>10</td>
<td>9.9x10^6</td>
</tr>
<tr>
<td>12.24</td>
<td>9</td>
<td>1.49x10^7</td>
</tr>
<tr>
<td>10.88</td>
<td>8</td>
<td>1.86x10^7</td>
</tr>
<tr>
<td>9.53</td>
<td>7</td>
<td>2.43x10^7</td>
</tr>
<tr>
<td>8.16</td>
<td>6</td>
<td>3.03x10^7</td>
</tr>
<tr>
<td>6.8</td>
<td>5</td>
<td>4.25x10^7</td>
</tr>
<tr>
<td>5.44</td>
<td>4</td>
<td>7.69x10^7</td>
</tr>
</tbody>
</table>

* Average mass of the primary for the tabulated mass of the runaway secondary.

+ Mass of the runaway secondary.

** Difference in the total evolutionary times, $\Delta \tau_{21} = \tau_2 - \tau_1$, where $\tau_1$ is the evolutionary time of the primary and $\tau_2$ is the total evolutionary time of the runaway secondary.
<table>
<thead>
<tr>
<th>SN Progenitor Mass (M☉)</th>
<th>Relative Fraction of Total SN Progenitors</th>
<th>Percentage of SNs Producing Observable SNRs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>R_{Gal} = 5kpc</td>
</tr>
<tr>
<td>17-70</td>
<td>17.9</td>
<td>0.0</td>
</tr>
<tr>
<td>17-16</td>
<td>2.7</td>
<td>0.9</td>
</tr>
<tr>
<td>16-15</td>
<td>3.3</td>
<td>1.1</td>
</tr>
<tr>
<td>15-14</td>
<td>4.1</td>
<td>1.4</td>
</tr>
<tr>
<td>14-13</td>
<td>5.2</td>
<td>1.6</td>
</tr>
<tr>
<td>13-12</td>
<td>6.7</td>
<td>1.8</td>
</tr>
<tr>
<td>12-11</td>
<td>8.7</td>
<td>2.0</td>
</tr>
<tr>
<td>11-10</td>
<td>11.7</td>
<td>2.0</td>
</tr>
<tr>
<td>10-9</td>
<td>16.3</td>
<td>2.1</td>
</tr>
<tr>
<td>9-8</td>
<td>23.4</td>
<td>2.1</td>
</tr>
<tr>
<td>Runaway Total</td>
<td>100.0%</td>
<td>15.0%</td>
</tr>
<tr>
<td>Additional Contribution by Slowly Moving Stars</td>
<td>4.3%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

**Total Allowing for 30% ISM as Supercavities**

14.3% 9.0% 0.0%

---

*The mass bin is only 13.0 to 13.5 M☉.*

**A density law n = n_o e^{-z/H} is assumed and with following values:**

<table>
<thead>
<tr>
<th>R_{Gal} (kpc)</th>
<th>n_o (cm⁻³)</th>
<th>H (pc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3</td>
<td>70</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>150</td>
</tr>
<tr>
<td>20</td>
<td>0.1</td>
<td>500</td>
</tr>
</tbody>
</table>

**These total percentages are computed by assuming that 1/3 of the galactic plane is occupied by supercavities, i.e. it is computed by multiplying the Runaway Total by 2/3, but all slow moving stars contribute.**

**Assumptions:** SM ☉ is the lower limit on SN progenitors.

The gravitational restoring force is negligible.

V_p = 50 km s⁻¹
<table>
<thead>
<tr>
<th>Mass Bin (M_☉)</th>
<th>Percentage of SN Producing Observable SNR</th>
<th>R_{Gal} = 5kpc</th>
<th>R_{Gal} = 10kpc</th>
</tr>
</thead>
<tbody>
<tr>
<td>19-18</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>18-17</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>17-16</td>
<td>0.12</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>16-15</td>
<td>0.23</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15-14</td>
<td>0.29</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14-13</td>
<td>0.36</td>
<td>0.36</td>
<td>0</td>
</tr>
<tr>
<td>13-12</td>
<td>0.46</td>
<td>0.46</td>
<td>0</td>
</tr>
<tr>
<td>12-11</td>
<td>0.61</td>
<td>0.61</td>
<td>0</td>
</tr>
<tr>
<td>11-10</td>
<td>0.72</td>
<td>0.67</td>
<td>0</td>
</tr>
<tr>
<td>10-9</td>
<td>1.13</td>
<td>0.72</td>
<td>0</td>
</tr>
<tr>
<td>9-8</td>
<td>1.63</td>
<td>0.89</td>
<td>0</td>
</tr>
<tr>
<td>8-7</td>
<td>2.45</td>
<td>1.35</td>
<td>0</td>
</tr>
<tr>
<td>7-6</td>
<td>3.85</td>
<td>2.67</td>
<td>0</td>
</tr>
<tr>
<td>6-5</td>
<td>6.8</td>
<td>6.8</td>
<td>0</td>
</tr>
<tr>
<td>5-4</td>
<td>12.39</td>
<td>7.22</td>
<td>0</td>
</tr>
</tbody>
</table>

| Runaway Total | 31.05% | 21.75 |
| Slowly Moving Stars | 9.13% | 8.6% |
| Total++       | 29.8%  | 23.10%|

*The same density law as in Table 3 is assumed.*

++This total is obtained by multiplying row 16 by 2/3, (i.e. we assume as before that 1/3 of all ISM is occupied by cavities) and adding it to row 17.
REFERENCES

Cruz-Gonzalez, C., Recillas-Cruz, E., Costero, R., Peimbert, M., Torres-Peimbert, S., 1974, Revista Mexicana de Astronomia y Astrofisica, 1, 211.
Elmegreen, B. C., 1979, preprint.


Kraushaar, W. L., 1977, Review lecture, 149th meeting, AAS.


McCuskey, S. W., 1966, Vistas in Astronomy, 7, 141.


Snow, T. P., 1979 (preprint)


