Optimal Periodic Binary Codes of Lengths 28 to 64

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Computer searches were performed to find repeated binary phase coded waveforms with optimal periodic autocorrelation functions. The best results for lengths 28 to 64 are given. These codes have extensive applications in radar and communications.

I. Introduction

Repeated binary phase coded waveforms with 100 percent duty factors form an important class of signals utilized extensively in radar and communications (Ref. 1).

Some codes with good periodic autocorrelation functions are known. For lengths of 27 and less, the best codes are fairly well known. For lengths of $2^n - 1$, maximal length P-N codes have optimal autocorrelation functions. However, for most lengths there is no practical algorithm for obtaining the best code (Ref. 2).

This article gives codes of lengths 28 to 64. Some of these codes are "optimal," others merely the best the authors have been able to find so far. These codes often represent a marked improvement over what has been reported previously in the available literature. Codes of these lengths are not merely useful in themselves; they can also be combined to give good codes of longer lengths. For instance, our best result for length 52 was obtained by copying the optimal code of length 13 four times and inverting one of the copies.

For codes of lengths above about 40, it is not practical to search exhaustively for the best code (Ref. 3). Smaller searches must be done. In this study, initial guesses were used and then modified until no further modification produced a superior code. Hopefully, methods for determining initial guesses and for modifying these will be improved as experience is gained in this field.

A periodic binary code can have several good features. In this study, we look at just two:

(1) The peak sidelobe in the autocorrelation function is small.

(2) The sum of the squares of the sidelobes in the autocorrelation function is small.

II. Examples

Consider a code of length 7:

+ + + + - + -

To get the elements of the autocorrelation function:

+ + + + - + -	is the original code.
- + + + + - +	is the code shifted one position.
- + + + - - -	is the arithmetic product for each position.
-1	is the sum of these products; it is the first sidelobe element of the autocorrelation function.

Shifting by 2:

+ + + + - + -

+ - + + + + -

+ - + + - + + +3 is the sum.

Shifting by 3:

+ + + + - + -

+ - + + + + -

- + - + - + - -1 is the next element of the autocorrelation function.

Shifts by 4, 5, 6 are equivalent to those of 3, 2, and 1.

The first element of the autocorrelation function is the main lobe. It corresponds to shifting by zero positions. The other elements are the sidelobes (the main lobe is not a sidelobe).

Thus the autocorrelation function of (+ + + + - + -)

is (7 -1 3 -1 -1 3 -1).

Here P = peak sidelobe magnitude = 3

S = sum of squares of sidelobes = 22

Two simple relationships manifest themselves:

(1) Each element of the autocorrelation function when taken modulo 4, is equal to the length of the code modulo 4.

(2) The sum of the elements of the autocorrelation function equals the square of the sum of the elements of the code (Ref. 4).

An "optimal" code: + + + - - + - would have autocorrelation function

$$7, -1, -1, -1, -1, -1, -1$$

where

P = 1

S = 6

For length 8, the "optimal" code is

+ + + + - + - -

with autocorrelation function

$$8, 0, 0, 0, -4, 0, 0, 0$$

Here P = 4

S = 16.

Two other results are used in determining the optimality of codes:

(1) No code of length greater than 4 has P = 0.

(2) No code has length 1 modulo 4, length greater than 13, and P = 1 (Ref. 5).

III. Results

Table 2 shows the best codes for each lengths 28 to 64. Table 1, showing the results for lengths 3 to 27, is included for completeness.

Length gives the length of the code.

$\Sigma(\text{Sidelobes})^2$ gives the lowest sum of the squares of the sidelobes discovered for any code of that length. When a code which has a lower peak sidelobe is found which has a higher than optimal $\Sigma(\text{Sidelobes})^2$, both codes are given.

When the value for the peak sidelobe or sum of squares is in parentheses, the authors feel that a better but as yet undiscovered code probably exists. When the value is underlined, a better value might exist. In all other cases, the value can be proved to be optimal.

The codes are written in hex notation. The first bit is always a +. For example, the code for 29 is given in hex as 14A7C111. In binary, this would be 0001 0100 1010 0111 1100 0001 0001 0001. By removing the leading zeroes, we get the code:

+ - + - - + - + - - + + + + + - - - - - + - - - + - - - +

In some cases a code from Table 2 was found in an earlier work. In these cases, the reference number is given.

References

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Table 1. Optimal codes of length less than 28

Length	Peak Sidelobe	Σ (Sidelobes)^2	Code (hex)
3	1	2	4
4	0	0	E
5	1	4	1D
6	2	20	25
7	1	6	4B
8	4	16	CB
9	3	24	1F4
10	2	36	350
11	1	10	716
12	4	16	941
13	1	12	1F35
14	2	52	36A3
15	1	14	647A
16	4	48	FAC4
17	3	64	19A3D
18	2	68	31EDD
19	1	18	7A86C
20	4	64	F6E8E
21	3	52	117BCE
22	2	84	3D1231
23	1	22	6650FA
24	4	32	DC20D4
25	3	72	18B082E
26	2	100	2C1AEB1
27	3	74	5A3C444

Table 2. Best results for codes of length 28 to 64

Length	Peak Sidelobe	Σ (Sidelobes)^2	Code (hex)	Ref.
28	4	80	B30FDD4	
29	3	92	14A7C111	
30	2	116	33927FAB	(6)
31	1	30	4B3E3750	(6)
32	4	80	89445BC1	(7)
33	3	64	18A5C2401	
34	2	132	24D1F7112	(8)
35	1	34	71F721592	(6)
36	4	64	C6859AE80	
37	3	84	1BD623E316	
38	2	148	3D69144620	(8)
39	(5)	(118)	7C744B905E	
39	3	(134)	5CC00AD278	
40	4	80	918547E90C	
41	3	104	1F0D19DF14A	(8)
42	2	164	33A970D33F4	(8)
43	1	42	653BE2E00D6	(6)
44	4	(144)	A042EA0F334	
45	3	(124)	17473C9B1AD0	
46	2	180	3B9BA0712495	
47	1	46	7BCAE4D82C20	(6)
48	4	112	CBF089223A51	
49	3	(192)	120AF28D1C5E0	
50	2	196	2E92B0050EE1C	
51	(5)	(226)	60B957CC485B0	
52	4	192	F9AFCD7E6A0CA	
53	(5)	(228)	12030BA906D987	
53	3	(260)	196EB81901D769	(6)
54	(6)	(276)	30EA0DB237B100	
55	(5)	(230)	74E705812DC456	(6)
56	4	(272)	DEC4518357C968	
57	(5)	(248)	166EA046116D4F0	
58	(6)	(292)	2C985A631F53A00	
59	1	58	5D49DE7C1846D44	(6)
60	4	224	FA32C756BD9E480	(6)
61	(5)	(268)	18F5981E02FBDBA4	
61	3	(300)	1B89F34A052CF91D	(6)
62	2	244	225746DC22583D20	
63	1	62	4314F4725BB357E0	(6)
64	(8)	(352)	A804EA630D727C2C	
64	4	(384)	EC10845E8B3CB0AC	