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ANALYSIS OF THE RIM INERTIAL MEASURING
SYSTEM (RIMS)

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SUMMARY

Nonlinear equations of motion are derived for the Rim Inertial Measuring System (RIMS), which is mounted in a strapped-down configuration on a carrier vehicle. The RIMS can be used for measuring angular rates and linear accelerations and the equations derived can be easily interfaced with the dynamic model of a user-defined carrier vehicle. Two methods are presented for rate measurement, and the results of the nonlinear simulation are presented.

INTRODUCTION

The Rim Inertial Measuring System (RIMS) consists of very small annular momentum control devices (AMCD's), mounted in a strapped down configuration, which are used to measure angular rates and linear accelerations of a moving carrier vehicle such as an aircraft or a spacecraft. The concept of RIMS was introduced in reference 1, and that of AMCD was introduced in reference 2. An AMCD consists of a spinning rim suspended in a set of electromagnetic actuators (suspension stations). The rim is spun by a noncontacting electromagnetic spin motor. As described in reference 1, a minimum of two AMCD's will be required for measuring angular velocities about three orthogonal axes. However, a RIMS consisting of a single AMCD would be sufficient for the purpose of analyzing the RIMS performance.

In this report, nonlinear equations of motion are derived for the RIMS. The equations can be easily interfaced with a carrier model and need only the following user-supplied information to be complete: (1) nominal location of RIMS center relative to the carrier center of mass (c.m.), (2) carrier angular velocities and linear velocities in the carrier coordinates, and (3) carrier attitude and position in order to initialize the equations. The carrier angular and linear velocities must be obtained from the user's vehicle model. In this report, three actuator stations (fig. 1) are assumed. Each actuator station consists of an axial actuator and a radial actuator. It is also assumed that actuator forces do not affect the vehicle motion. Although infinite bandwidth actuators are assumed in the derivations, actuator models can be easily inserted for a more realistic simulation. Results of the computer simulation of the RIMS are presented.

SYMBOLS

A, B_f, B_{n_1}, B_{n_2}	coefficient matrices
C	measurement matrix
$C_p(\ )$	cross-product matrix of a vector
e_k	unit vector in k-direction (k = 1,2,3)
f_j	actuator force vector at station j
f_{r_j}, f_{z_j}	radial and axial actuator forces at station j
$f_{r_{cj}}, f_{z_{cj}}$	radial and axial actuator command forces at station j
f_{vz}	z-direction force acting on vehicle
f_z	axial actuator force vector
G	feedback gain matrix
g	gravitational acceleration vector
g_m	measured axial centering errors (3 × 1 vector)
g_{m_r}	measured axial centering error rates (3 × 1 vector)
g_z	z-axis gravitational acceleration
H	rim angular momentum about spin axis
H_r	rim angular momentum vector
I_a	transverse-axis rim inertia
I_r	rim inertia matrix
I_v	vehicle inertia matrix
K	Kalman gain matrix

K_{D_a}, K_{D_r}	rate gains
K_P, K_D	proportional and rate gain matrices
K_{P_a}, K_{P_r}	proportional gains
M_r	moment acting on rim
$M_{r_{ak}}$	kth component of moment acting on rim of unit A
$M_{r_{bk}}$	kth component of moment acting on rim of unit B
m_r	rim mass
m_v	vehicle mass
n_v	white noise torque acting on vehicle
r	rim radius
T	coefficient matrix defined in equation (29)
T_a	matrix consisting of the first two columns of T
T_j	transformation matrix (j = 1,2,3)
T_s	spin motor torque
T_v	torque acting on vehicle
t_j	jth column of T
V	covariance intensity matrix of n_v
v_a	actuator noise
v_r	inertial velocity of rim center of mass
w_g	proximity sensor noise
(X,Y,Z)	denotes coordinate frames; subscripts are described in the text
X	state vector
X_1	vector defined in equation (52)
(X_{r_v}, Y_{r_v})	X and Y components of ρ_{rv}
z_r	rim center-of-mass position in the inertial system

z_v	vehicle center-of-mass position in the inertial system
α_a, α_b	coefficient matrices
$\delta_{a_j}, \delta_{r_j}$	axial and radial centering errors at station j
$\delta_{a_{cj}}, \delta_{r_{cj}}$	axial and radial centering error commands at station j
$\delta_{a_{rcj}}, \delta_{r_{rcj}}$	axial and radial centering error rate commands at station j
ε	defined in equation (36)
ρ_{a_j}	position of the point on the rim closest to actuator station j (in a-system)
ρ_j	position of actuator station j in v-system
ρ_r	position of rim center-of-mass in inertial system
ρ_{r_v}	position of nominal rim center location in v-system
ρ_v	position of vehicle center-of-mass in inertial system
τ_s	spin motor torque vector
ϕ_{e_1}, ϕ_{e_2}	rotations (in order 1-2) for obtaining e-system from v-system
$\theta_{a_1}, \theta_{a_2}$	rotations (in order 1-2) for obtaining a-system from inertial system
θ_e	vector defined in equation (33)
θ_v	$= (\theta_{v_1}, \theta_{v_2})^T$
$\theta_{v_1}, \theta_{v_2}, \theta_{v_3}$	vehicle attitude angles
ψ_r	rim rotation angle relative to a-system
Ω	matrix defined in equation (35)
Ω_v	$= (\dot{\theta}_{v_1}, \dot{\theta}_{v_2})^T$
ω_o	rim spin speed relative to v-system
ω_a	absolute angular velocity of a-system
ω_{a_j}	components of ω_a (j = 1,2,3)
ω_r	absolute angular velocity of rim

ω_{r_j}	components of ω_r
ω_{sp}	$= (0,0,\omega_o)$
ω_{spin}	$= (0,0,\dot{\psi}_r)$
ω_v	absolute angular velocity of v-system
ω_v'	absolute angular velocity of v-system, expressed in e-system
ζ	$= (Y_{rv},-X_{rv})$

Superscripts

T	transpose of a matrix
$\cdot$	first derivative
$\cdot\cdot$	second derivative
$\hat{}$	estimated value of a variable

MATHEMATICAL MODEL DEVELOPMENT

Coordinate Systems

Let (X_i, Y_i, Z_i) be the inertial coordinate frame and let (X_a, Y_a, Z_a) denote a coordinate frame obtained by a rotation θ_{a_1} about the X_i-axis and θ_{a_2} about the new Y_i-axis. The origin of the a-system is the rim center of mass which is also the rim center. Let (X_{rb}, Y_{rb}, Z_{rb}) denote a coordinate system fixed to the rim. This system is obtained from the (X_a, Y_a, Z_a) system by a rotation ψ_r (rim roll angle) about the a-axis. Let (X_v, Y_v, Z_v) denote a coordinate system which is fixed to the vehicle and is centered at the vehicle center of mass. The v-system is obtained from the i-system by rotations θ_{v_1}, θ_{v_2}, θ_{v_3} in the order 1-2-3. Let (X_v', Y_v', Z_v') denote a vehicle-fixed coordinate system parallel to the v-system, centered at the nominal location of the rim c.m.

Equations of Motion of the Rim

The equations of motion are similar to those derived in reference 3 for the AMCD.

Rotational equations. - Let ω_a denote the absolute angular velocity of the a-system, and ω_{spin} the angular velocity of the rim with respect to the a-system. Then the absolute angular velocity of the rim expressed in the a-system is given by

$$\omega_r = \omega_a + \omega_{spin} \tag{1}$$

where

$$\omega_{spin} = (0,0,\dot{\psi}_r) \tag{2}$$

Rim angular momentum is given by

$$H_r = I_r\omega_r = I_r(\omega_a + \omega_{spin}) \tag{3}$$

$$\dot{H}_r + \omega_a \times H_r = M_r \tag{4}$$

where M_r is the total momentum acting on the rim. Therefore,

$$\dot{\omega}_r = -I_r^{-1}\left[C_p(\omega_a)I_r\omega_r - \sum_{j=1}^{3} \rho_{a_j} \times f_j - \tau_s\right] \tag{5}$$

where $C_p(\ )$ denotes the cross-product matrix of a vector, ρ_{a_j} denotes the position of the point on the rim closest to actuator j, f_j is the force at actuator j in the a-system and τ_s denotes the spin motor torque vector.

Since the a-system is obtained by rotations θ_{a_1} and θ_{a_2} about the X_i and Y_a axes,

$$\begin{bmatrix} \omega_{a_1} \\ \omega_{a_2} \\ \omega_{a_3} \end{bmatrix} = \begin{bmatrix} \cos\theta_{a_2} & 0 & 0 \\ 0 & 1 & 0 \\ \sin\theta_{a_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_{a_1} \\ \dot{\theta}_{a_2} \\ \dot{\theta}_{a_3} \end{bmatrix} \tag{6}$$

Therefore,

$$\begin{bmatrix} \dot{\Theta}_{a_1} \\ \dot{\Theta}_{a_2} \end{bmatrix} = \begin{bmatrix} \sec \Theta_{a_2} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \omega_{a_1} \\ \omega_{a_2} \end{bmatrix} \tag{7}$$

Since

$$\omega_{r_3} = \omega_{a_3} + \omega_{spin} = \omega_{a_3} + \dot{\psi}_r \tag{8}$$

substituting for ω_a from equation (6) gives

$$\dot{\psi}_r = -\sin \Theta_{a_2} \dot{\Theta}_{a_1} + \omega_{r_3} \tag{9}$$

Assuming that the magnetic actuator forces act only axially and radially, there is no torque generated by the actuator forces about the Z_a-axis. Assuming further that the spin motor generates a torque T_s only about the Z_a-axis, τ_s in equation (5) is given by

$$\tau_s = (0, 0, T_s)^T \tag{10}$$

(The spin-motor torque is included here for completeness, and is not used in the simulation described later.) Thus, ω_r is obtained by solving the differential equation (5); the rim spin speed $\dot{\psi}_r$ is then given by equation (9).

Translation equations. - Let v_r denote the inertial velocity of the rim center of mass expressed in the a-system. Then

$$m_r\left(\dot{v}_r + C_p(\omega_a)\ v_r\right) = \sum_{j=1}^{3} f_j + m_r g \tag{11}$$

where m_r is the rim mass and $m_r g$ is the gravitational force (in a-system).

Let ρ_r denote the position of the rim c.m. in the i-system. Then

$$\dot{\rho}_r = T_1^T \left(\Theta_{a_1}\right) T_2^T\left(\Theta_{a_2}\right) v_r \tag{12}$$

where T_k denotes the transformation matrix corresponding to rotation about axis k (k = X,Y,Z).

Expressions for the Actuator Gaps

The vehicle on which the RIMS would be installed could be a spacecraft, aircraft, etc. Let ρ_v denote the position of the vehicle c.m. in the i-system, and ω_v its absolute angular velocity in the v-system.

It is necessary to obtain expressions for the magnetic actuator gaps. An actuator gap or centering error is defined as the distance between the point midway between the actuator pole faces and the point on the rim (assumed to be a perfect circle of zero thickness for gap calculation) which is closest to it. Thus the exact expression for an actuator gap will involve (1) deriving an expression for the rim plane, (2) obtaining the point P_r on the plane through which the vector normal to the plane passes, such that this normal vector also passes through the point S located midway between the actuator pole faces, and (3) obtaining the point Q on the rim which lies on the line joining the rim center to P. The distance between the points S and Q then gives the actuator gap. Although the expression obtained in this manner will be exact, it is also quite cumbersome to evaluate. However, since the rim motion within the actuator pole faces is small, it is advisable to obtain an approximate expression for the actuator gaps. An approximate expression (in inertial coordinates) for the gap at actuator j is

$$\begin{aligned}\delta_j = {} & \rho_r - \rho_v - T_1^T\left(\Theta_{v_1}\right) T_2^T\left(\Theta_{v_2}\right)T_3^T\left(\Theta_{v_3}\right)\rho_{r_v} \\ & + \left\{T_1^T\left(\Theta_{a_1}\right)T_2^T\left(\Theta_{a_2}\right) - T_1^T\left(\Theta_{v_1}\right) T_2^T\left(\Theta_{v_2}\right)\right\} T_3^T\left(\Theta_{v_3}\right) \rho_j\end{aligned} \tag{13}$$

where ρ_j denotes the location of actuator j in the v'-system, and ρ_{r_v} denotes the nominal position of the rim c.m. in the v-system. In the vehicle coordinates, the gap at actuator j is given by

$$\delta_{v_j} = T_3\left(\Theta_{v_3}\right)T_2\left(\Theta_{v_2}\right)\ T_1\left(\Theta_{v_1}\right)\delta_j \tag{14}$$

The axial centering error at actuator j is

$$\delta_{a_j} = e_3{}^T\ \delta_{v_j},\ j = 1,\ 2,\ 3 \tag{15}$$

where e_k denotes a 3×1 vector with all zeros except a "1" in the kth position. The radial centering errors are obtained by appropriate rotations about the Z-axis:

$$\delta_{r_1} = e_1^T\ T_3(60^o)\ \delta_{v1} \tag{16}$$

$$\delta_{r_2} = e_1^T\ T_3(-60^o)\ \delta_{v_2} \tag{17}$$

$$\delta_{r_3} = -e_1{}^T\ \delta_{v_3} \tag{18}$$

Radial centering errors are defined as being positive outwards and

$$T_3(\phi) = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{19}$$

The position of actuator j in the a-system

$$\rho_{a_j} = T_3{}^T\left(\Theta_{v_3}\right)\rho_j \tag{20}$$

Let f_{r_i} and f_{z_i} (i = 1, 2, 3) denote the radial and axial actuator forces generated at station i. Forces f_i at station i (in the v-system) are given by

$$f_1 = \begin{bmatrix} f_{r_1} \cos(60^\circ) \\ f_{r_1} \sin(60^\circ) \\ f_{z_1} \end{bmatrix} \tag{21}$$

$$f_2 = \begin{bmatrix} f_{r_2} \cos(60^\circ) \\ f_{r_2} \sin(60^\circ) \\ f_{z_2} \end{bmatrix} \tag{22}$$

$$f_3 = \begin{bmatrix} -f_{r_3} \\ 0 \\ f_{z_3} \end{bmatrix} \tag{23}$$

Since the forces f_i are in the v-system, they must be transformed into the a-system before their use in equation (5).

The control laws for the magnetic actuators are assumed to be proportional plus derivative type. The radial and axial command forces at actuator i are given by

$$f_{r_{ci}} = K_{P_r}\left(\delta_{r_i} - \delta_{r_{ci}}\right) + K_{D_r}\left(\dot{\delta}_{r_i} - \delta_{r_{rci}}\right) \tag{24}$$

$$f_{z_{ci}} = K_{P_a}\left(\delta_{a_i} - \delta_{a_{ci}}\right) + K_{D_a}\left(\dot{\delta}_{a_i} - \delta_{a_{rci}}\right) \tag{25}$$

where $\delta_{r_{ci}}$ and $\delta_{a_{ci}}$ denote the radial and axial centering error commands, and $\delta_{r_{rci}}$, $\delta_{a_{rci}}$ denote the centering error rate commands at the ith actuator. These commands are normally zero. K_{Pj}, K_{Dj} (j = a, r for axial and raidal) are the proportional and derivative gains.

ANGULAR RATE MEASUREMENT SCHEMES

In order to gain insight into the angular rate measurements, it is convenient to examine the linearized equations of motion of the carrier vehicle/RIMS. The linearization is performed about nonrotating carrier axes.

Linearized Equations of Motion

The linearized equations of motion of the rim are given by:

$$I_a \ddot{\Theta}_{a_1} + H \dot{\Theta}_{a_2} = t_1^T f_z \tag{26}$$

$$I_a \ddot{\Theta}_{a_2} - H \dot{\Theta}_{a_1} = t_2^T f_z \tag{27}$$

$$m_r \ddot{z}_r = t_3^T f_z + m_r g_z \tag{28}$$

where I_a is the transverse rim inertia, z_r the Z-axis rim c.m. position, $m_r g_z$ the Z-axis gravitational force, f_z the 3 × 1 axial force vector, H is the rim angular momentum (about spin axis) and t_i is the ith column of matrix T:

$$T = \begin{bmatrix} \sqrt{3}r/2 & -r/2 & 1 \\ -\sqrt{3}r/2 & -r/2 & 1 \\ 0 & r & 1 \end{bmatrix} \tag{29}$$

where r represents the rim radius.

Linearized two-axis vehicle equations are

$$\begin{bmatrix} \ddot{\Theta}_{v_1} \\ \\ \ddot{\Theta}_{v_2} \end{bmatrix} = I_v^{-1}\left(T_v + n_v\right) \tag{30}$$

where I_v and T_v denote the vehicle inertia matrix and the torque acting on the vehicle and n_v is a disturbance input torque which is assumed to be a zero-mean white noise with covariance intensity matrix V. It is assumed that RIMS actuator forces have a negligible effect on the vehicle. The vehicle c.m. position z_v (Z-axis) is given by

$$m_v \ddot{z}_v = f_{v_z} + m_v g_z \tag{31}$$

where f_{v_z} is the (Z-axis) force acting on the vehicle. Let X_{r_v} and Y_{r_v} denote the X and Y components of ρ_{r_v}, the nominal position of the rim c.m. in the v-system, and let $\zeta = (Y_{r_v}, -X_{r_v})^T$.

Dividing equations (26) and (27) by I_a and subtracting equation (30) from the result, we have

$$\ddot{\Theta}_e + \Omega\dot{\Theta}_e + \Omega\dot{\Theta}_v = \frac{1}{I_a} T_a{}^T f_z - I_v^{-1}\left(T_v + n_v\right) \tag{32}$$

where Θ_v denotes $\left(\Theta_{v_1}, \Theta_{v_2}\right)^T$ and

$$\Theta_e = \begin{bmatrix} \Theta_{e_1} \\ \\ \Theta_{e_2} \end{bmatrix} = \begin{bmatrix} \Theta_{a_1} - \Theta_{v_1} \\ \\ \Theta_{a_2} - \Theta_{v_2} \end{bmatrix} \tag{33}$$

$$T_a = \begin{bmatrix} t_1 & t_2 \end{bmatrix} \tag{34}$$

$$\Omega = \begin{bmatrix} 0 & H/I_a \\ -H/I_a & 0 \end{bmatrix} \tag{35}$$

Let

$$\varepsilon = z_r - \left(z_v + \zeta^T \Theta_v\right) \tag{36}$$

Dividing equation (28) by m_a, equation (31) by m_v, subtracting the latter from the former and using equations (36) and (30) yield

$$\ddot{\varepsilon} = \frac{1}{m_r} t_3{}^T f - \frac{1}{m_v} f_{v_z} - \zeta^T I_v{}^{-1}\left(T_v + n_v\right) \tag{37}$$

The measured axial centering errors at the three actuator stations are given by:

$$g_m = T_a \Theta_e + t_3 \varepsilon + w_g \tag{38}$$

where w_g represents the position sensor noise. The axial actuator forces are given by:

$$f_z = K_p g_m + K_D g_{m_r} \tag{39}$$

where K_p and K_D denote the 3 × 3 position and rate gain matrices and g_{m_r} denotes the measured (or derived) axial actuator centering error rate vector.

Estimation Schemes

Scheme I - direct inversion. - The actuator gains K_p and K_D are selected in such a manner that the centering errors are quickly corrected and returned to zero in steady state. The closed-loop bandwidth of the centering error loop is usually much higher than that of the vehicle motion: that is, the actuator centering errors quickly attain steady state. Thus, substituting $\ddot{\Theta}_e = \dot{\Theta}_e = 0$ in equation (32) and solving for $\dot{\Theta}_v$ yield an estimate of the two-axis vehicle angular rates:

$$\dot{\Theta}_v \simeq \Omega^{-1} \left[\frac{1}{I_a} T_a{}^T f_z - I_v{}^{-1} \left(T_v + n_v\right)\right] \tag{40}$$

The second term in the brackets denotes an error term which is a characteristic of rate gyros. This error term will be assumed negligible in this estimation scheme. The first term can be simplified by using equation (39) and making the centering error rate g_{m_r} equal to 0. Thus an estimate of vehicle angular rates based on position sensor outputs is given by

$$\hat{\dot{\Theta}}_v = \frac{1}{I_a} \Omega^{-1} T_a{}^T K_p g_m \tag{41a}$$

The same analysis was presented in reference 1 using the frequency-domain approach. From equations (31) and (37), the linear acceleration (Z-direction) is given by

$$\ddot{z}_v \simeq \frac{1}{m_r} t_3{}^T f_z + g_z$$

or

$$\ddot{z}_v \simeq \frac{1}{m_r} t_3{}^T K_p g_m + g_z \tag{41b}$$

Although these rate estimates are based on the linear analysis, it is shown below that they are also valid for the general nonlinear equations when the vehicle is rotating about all three axes.

Nonlinear analysis. - Let (X_e, Y_e, Z_e) denote a coordinate system obtained from the v-system by Euler angle rotations ϕ_{e_1} and ϕ_{e_2} in the order 1, 2. The origin of this coordinate system is fixed to the rim center and translates with it. Thus the (X_e, Y_e) plane defines the rim plane. Let ω_e denote the angular velocity of the e-system relative to the vehicle, expressed in the e-system. Let ω_{sp} be the angular velocity of the rim relative to the e-system. Then $\omega_{sp} = (0, 0, \omega_o)^T$ where ω_o = rim spin speed (relative to the vehicle). The absolute angular velocity of the rim, expressed in the e-system, is

$$\omega_r = \omega_v' + \omega_e + \omega_{sp} \tag{42}$$

where ω_v' = absolute angular velocity of the vehicle expressed in the e-system. The rim rotational equation is given by [from eq. (4)]:

$$I_r \left(\dot{\omega}_v' + \dot{\omega}_e + \dot{\omega}_{sp}\right) + C_p \left(\omega_v' + \omega_e\right) I_r \left(\omega_v' + \omega_e + \omega_{sp}\right) = M_r \tag{43}$$

In steady state (of the magnetic actuator loops), the e-system does not move relative to the vehicle, i.e., $\omega_e = \dot{\omega}_e = 0$. Also, spin rate is assumed to be constant with respect to the vehicle. Therefore, in steady state,

$$I_r \left(\dot{\omega}_v'\right) + C_p \left(\omega_v'\right) I_r \left(\omega_v' + \omega_{sp}\right) = M_r \tag{44}$$

The second term on the left-hand side is

$$\begin{bmatrix} 0 & -\omega_{v_3}' & \omega_{v_2}' \\ \omega_{v_3}' & 0 & -\omega_{v_1}' \\ -\omega_{v_2}' & \omega_{v_1}' & 0 \end{bmatrix} \begin{bmatrix} I_a \omega_{v_1}' \\ I_a \omega_{v_2}' \\ 2I_a \left(\omega_{v_3}' + \omega_o\right) \end{bmatrix} \tag{45}$$

where I_a denotes the transverse-axis rim inertia. Therefore, in steady state,

$$\begin{bmatrix} I_a\left(\omega'_{V_3} + 2\omega_o\right)\omega'_{V_2} \\ -I_a\left(\omega'_{V_3} + 2\omega_o\right)\omega'_{V_1} \\ 0 \end{bmatrix} = M_r - I_r\dot{\omega}'_V \tag{46}$$

The acceleration term $I_r\dot{\omega}'_V$ represents an error term which is present in all rate gyros. Ignoring this error term,

$$I_a \begin{bmatrix} 2\omega_o\omega'_{V_2} + \omega'_{V_2}\omega'_{V_3} \\ -2\omega_o\omega'_{V_1} - \omega'_{V_3}\omega'_{V_1} \end{bmatrix} = \begin{bmatrix} M_{r_{a_1}} \\ M_{r_{a_2}} \end{bmatrix} \tag{47}$$

where the right-hand side represents the steady-state torque exerted by the magnetic actuators. The torque can be expressed as a moment-arm matrix times axial and radial force vector, which, in steady state, can be expressed as a proportional gain matrix times the axial and radial centering error vector. Thus M_{r_a} can be expressed as

$$M_{r_a} = \left[\alpha_a\right] \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \tag{48}$$

where α_a is a 2 × 6 matrix.

If the vehicle is not rotating about the yaw-axis, $\omega'_{V_3} = 0$ and equation (47) can be solved for ω'_{V_1} and ω'_{V_2}, the solution being basically the same as equation (41a). However, if $\omega'_{V_3} \neq 0$, it is necessary to use at least one more unit for complete rate measurement. The equations for the second unit (unit B), which consists of a rim suspended in the Y_V - Z_V plane, can be similarly derived:

$$I_a \begin{bmatrix} 2\omega_o \omega'_{v_3} + \omega'_{v_3}\omega'_{v_1} \\ \\ -2\omega_o \omega'_{v_2} - \omega'_{v_1}\omega'_{v_2} \end{bmatrix} = \begin{bmatrix} M_{r_{b_1}} \\ \\ M_{r_{b_2}} \end{bmatrix} \tag{49}$$

In steady state,

$$M_{r_b} = \begin{bmatrix} \alpha_b \end{bmatrix} \begin{bmatrix} \delta_a \\ \\ \delta_r \end{bmatrix} \tag{50}$$

Equations (47) and (49) represent four algebraic equations in three unknowns: ω'_{v_1}, ω'_{v_2}, and ω'_{v_3}. From these equations:

$$2\omega_o\left(\omega'_{v_3} - \omega'_{v_1}\right) = \frac{1}{I_a}\left(M_{r_{b_1}} + M_{r_{a_2}}\right)$$

or

$$\omega'_{v_3} = \omega'_{v_1} + \frac{1}{2I_a\omega_o}\left(M_{r_{b_1}} + M_{r_{a_2}}\right) \tag{51}$$

Substituting for ω'_{v_3} from equation (51) into the top equation of the vector equation (49), a quadratic equation in ω'_{v_1} is obtained. The two roots of the equation can be computed. In order to determine the root which represents ω'_{v_1}, the first root is substituted in equation (51), which gives ω'_{v_3}. Using these values of ω'_{v_1} and ω'_{v_3}, ω'_{v_2} is computed from the top part of equation (47), and also from the bottom part of equation (49). If these two estimates of ω'_{v_2} are equal (within some tolerance), the root chosen was the correct representation of ω'_{v_1}. If not, the other root must represent ω'_{v_1}. Three-axis rate information can be obtained from two orthogonal units in this manner. Through algebraic manipulations, it should be possible to further simplify the solution.

Scheme II. - This scheme is based on an application of the Kalman-Bucy filter. Returning to the linearized equations and denoting $\Omega_v = (\dot{\theta}_{v_1}\ \dot{\theta}_{v_2})^T$

$$X_1 = \begin{bmatrix} \Theta_e \\ \varepsilon \\ \Omega_v \end{bmatrix}_{5 \times 1} \tag{52}$$

Equations (30), (32) and (37) can be expressed in the form:

$$\dot{X} = AX + B_f f_z + B_{n_1} \left(T_v + n_v\right) + B_{n_2} f_{vz} + B_f v_a \tag{53}$$

where X is the 10×1 state vector consisting of X_1 and $\dot{X}_1$, B_f, B_{n_1}, B_{n_2} are appropriately dimensioned input matrices, and v_a is the actuator noise vector. For the purpose of estimator design, T_v is assumed to be zero, and f_{vz}, n_v and v_a are the white-noise inputs acting on the system. The Kalman-Bucy filter for this system is given by:

$$\dot{\hat{X}} = A\hat{X} + B_f f_z + K \left(g_m - C\hat{X}\right) \tag{54}$$

where C is obtained from equations (38) and (52). The actuator force vector is synthesized as

$$f_z = G\hat{X} \tag{55}$$

where G is the gain matrix obtained from equations (38), (39) and (52). The steady-state filter has constant gain matrix K, and is more suitable for practical implementation than the nonsteady-state version. The filter generates estimates of X, which also include estimates of Ω_v. The sensor and actuator noise covariance matrices are needed for the design. The vehicle input torque covariance matrix (V) can be used as a design parameter for the Kalman-Bucy filter. Further analysis is necessary in order to evaluate the performance of this scheme using the nonlinear equations for a vehicle rotating about all three axes.

NUMERICAL RESULTS

In order to complete the analysis of RIMS, a computer program was written for its simulation using the nonlinear equations developed [eqs. (1) - (25)]. A single AMCD was used in this simulation. The rim mass was 453.6 gm (1 lb), the rim radius was 5.715 cm (2.25 in.), rim spin speed was 1,000 rpm, and the rim center was nominally located at the vehicle c.m. With zero initial conditions, a torque T_{v_1} about the X_v axis was used to force the vehicle for 1 sec and to obtain a rate of 0.286 rad/sec at the end of 1 sec. The system was unforced for $1 < t \leq 4$ sec, and a torque $-T_{v_1}$ was applied for $4 \leq t \leq 5$ sec, after which time the torque was again zero. Figures 2 to 5 show the actual and measured vehicle rate, the actual and measured vehicle attitude angle (Θ_{v_1}), the rate measurement error, and the axial and radial centering errors at actuator station 1. The actuator-loop damping ratio used was 2 (at zero spin speed) in all the computer runs. In figures 2 to 5, the radial and axial actuator control-loop bandwidth ω_{c1} used was 30 rad/sec. Figures 6 to 8 show the RIMS performance for ω_{c1} = 60 rad/sec. The measurement error as well as the actuator centering error is much smaller. The actual and measured rate for ω_{c1} = 100 rad/sec is shown in figure 9. As the actuator control loops are made tighter, the RIMS performance improves. However, since the actuator gaps (centering errors) also get smaller with higher ω_{c1}, more accurate proximity sensors are required if the implementation uses gap measurements rather than force measurements. If the rim momentum is increased, higher ω_{c1} has to be used for getting satisfactory performance. This too results in small gaps and will need more accurate proximity sensors. Therefore, from the viewpoint of implementation, it would be better to use force measurements [in eq. (40)]. In that case it would be advisable to use higher momentum and higher ω_{c1}, since the actuator forces to be measured would be large, resulting in a more sensitive measurement scheme. It may also be advisable to use an integrator in the actuator control loop. This would result in relatively easy measurement of force (by measuring the integrator output), and also maintain the actuator centering errors near zero. The effect of the latter would be beneficial in maintaining the actuators in the linear region. Further investigation is needed in this area in order to evaluate the dynamic response of RIMS using integrators.

In summary, the numerical results presented indicate that, with properly selected rim momentum and ω_{c1}, the RIMS can provide highly accurate rate measurements.

CONCLUSIONS

Nonlinear equations of motion have been derived for the RIMS using one AMCD. The equations can be easily interfaced with a model of a carrier vehicle. The complexity of the equations can be easily increased by making additions such as the magnetic actuator models. Two methods have been presented for the measurement of vehicle angular velocities. The first method is obtained from nonlinear equations, while the second method uses linearized equations and the Kalman-Bucy filter. Results of the nonlinear simulation using the first method indicate that RIMS can offer an attractive and accurate inertial measurement system. Further analyses will be required in order to completely evaluate the second measurement method, and also to evaluate the performance of both methods in a stochastic environment.

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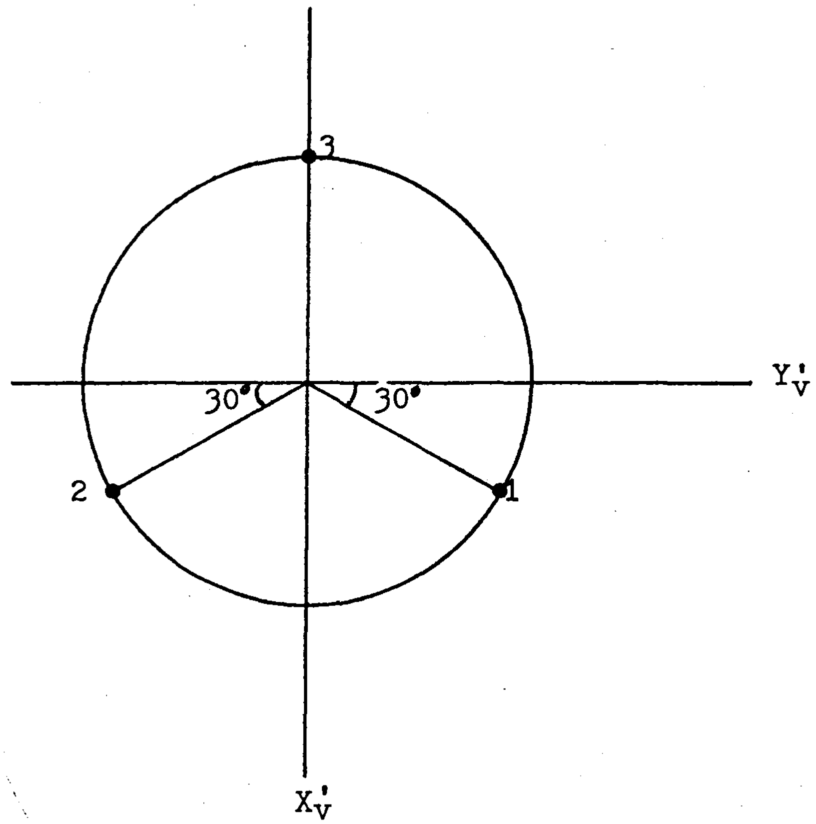

Figure 1. RIMS actuator station locations.

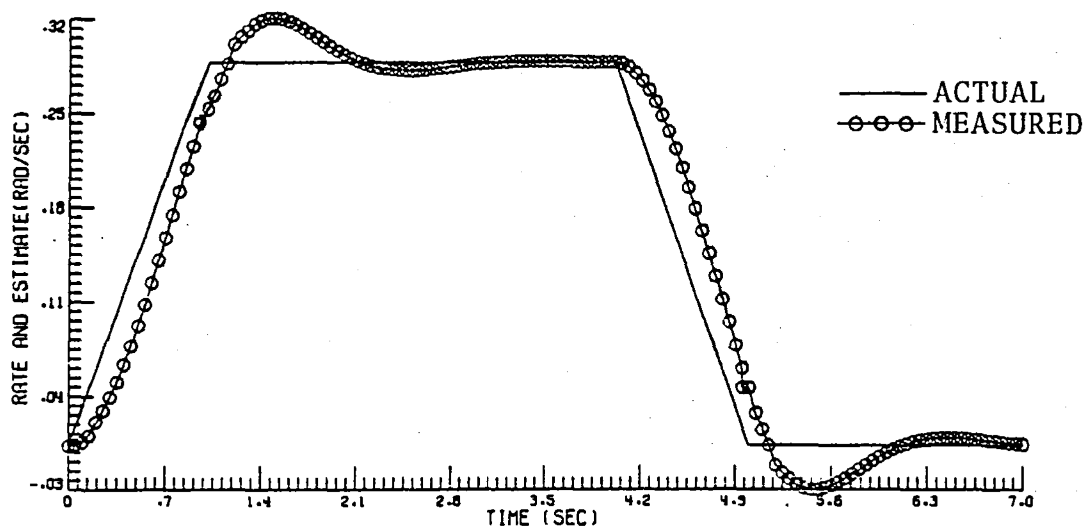

Figure 2. Actual and measured rate, ω_{c1} = 30 rad/sec.

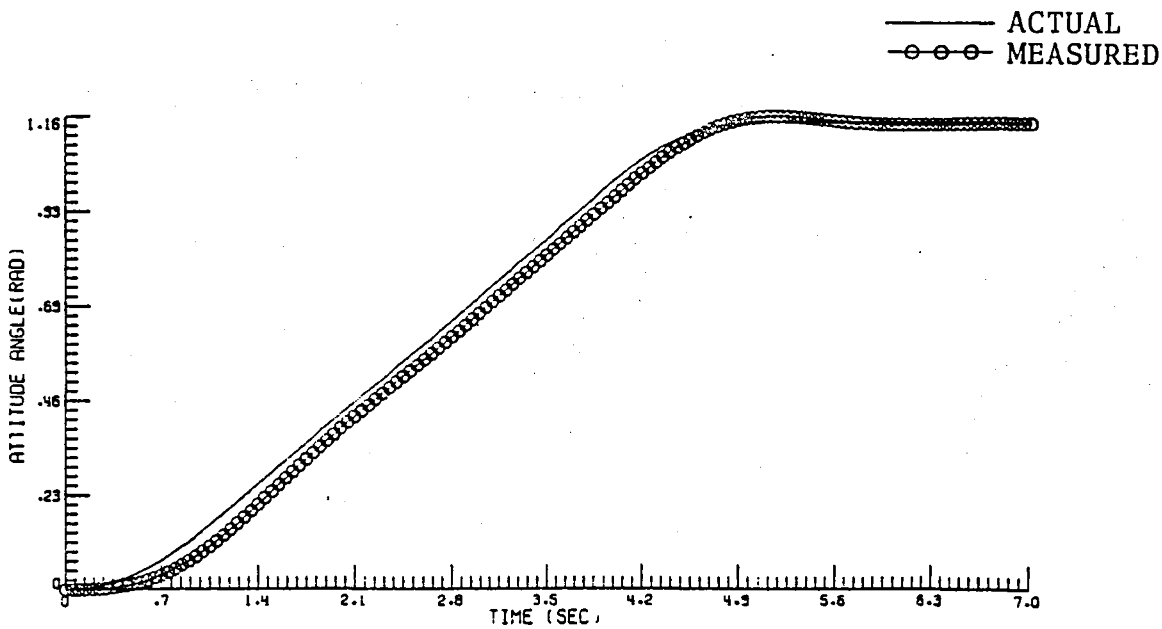

Figure 3. Actual and measured vehicle attitude angle, ω_{c1} = 30 rad/sec.

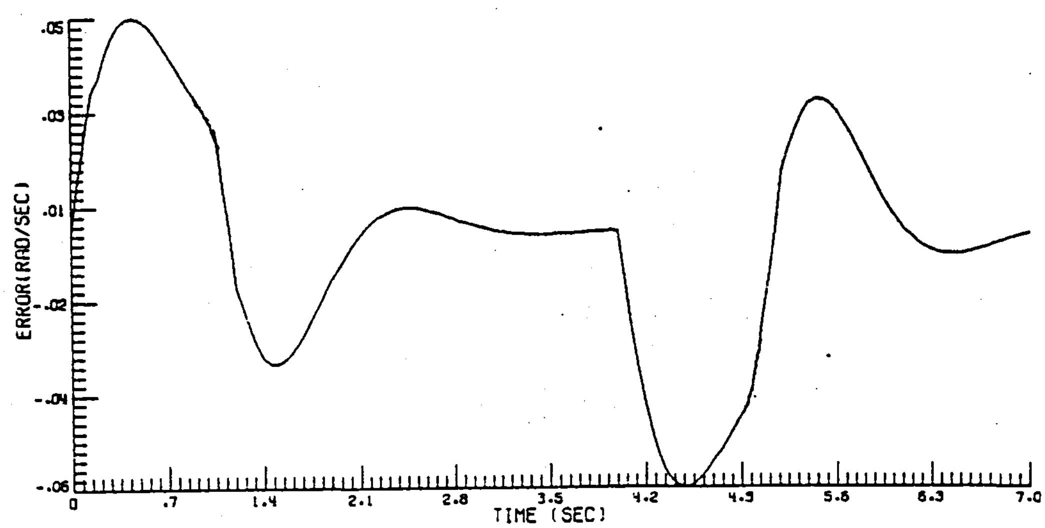

Figure 4. Rate measurement error, ω_{c1} = 30 rad/sec.

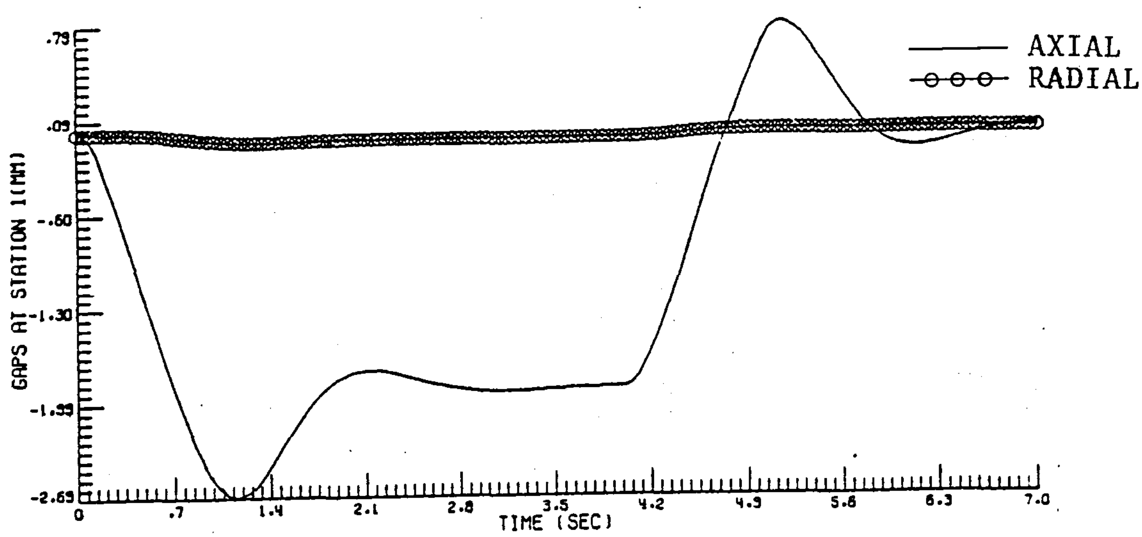

Figure 5. Axial and radial gaps at station 1, ω_{c1} = 30 rad/sec.

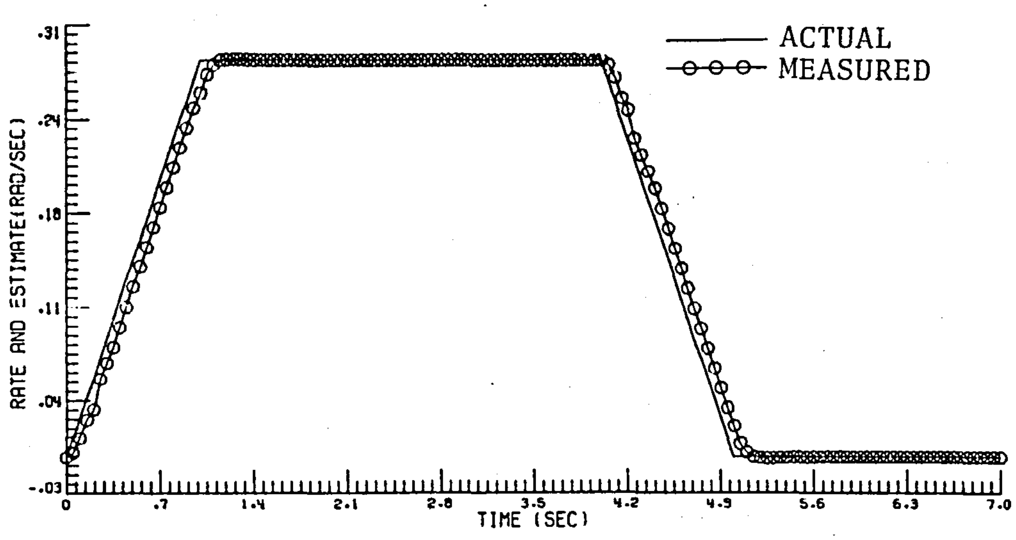

Figure 6. Actual and measured rate, ω_{c1} = 60 rad/sec.

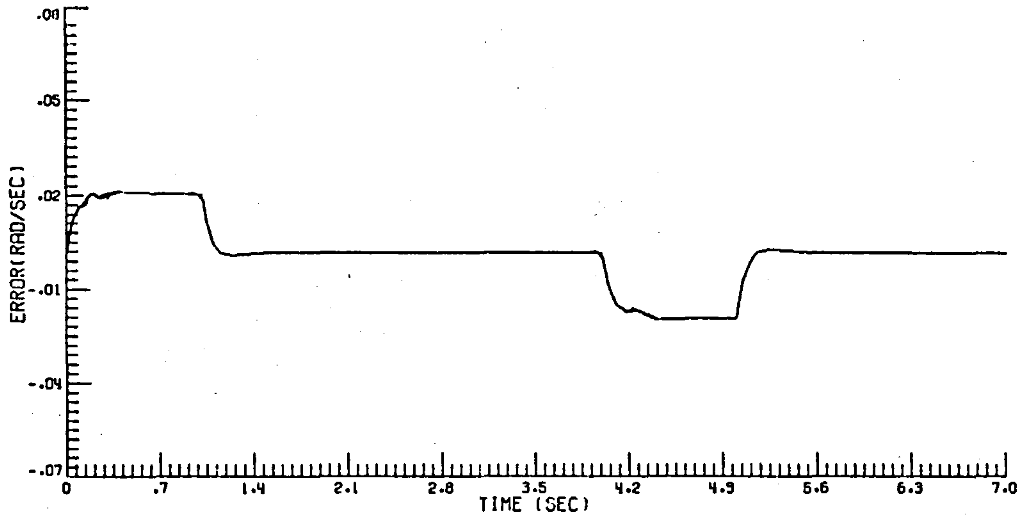

Figure 7. Rate measurement error, ω_{c1} = 60 rad/sec.

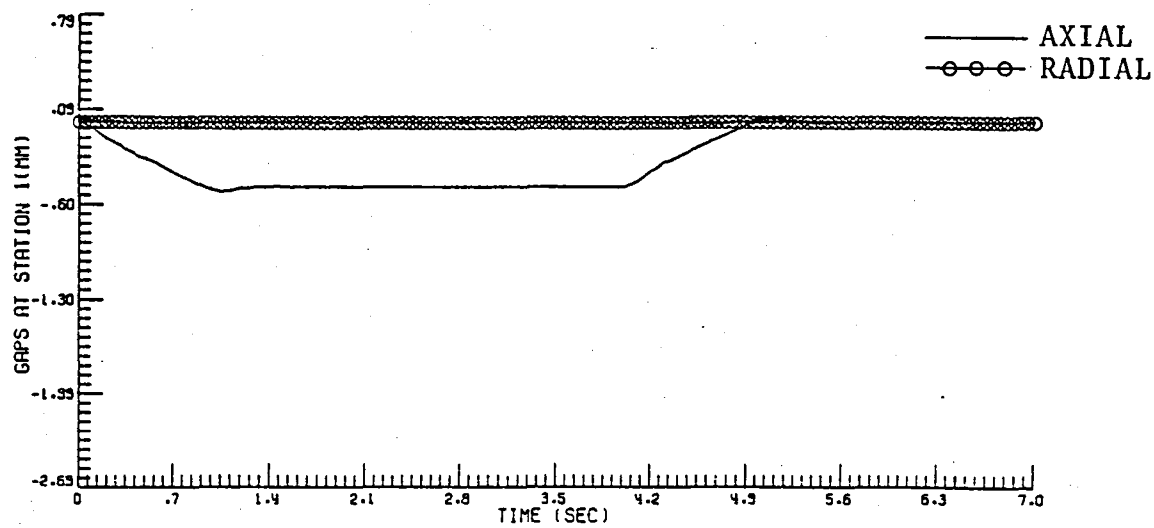

Figure 8. Axial and radial gaps at station 1, ω_{c1} = 60 rad/sec.

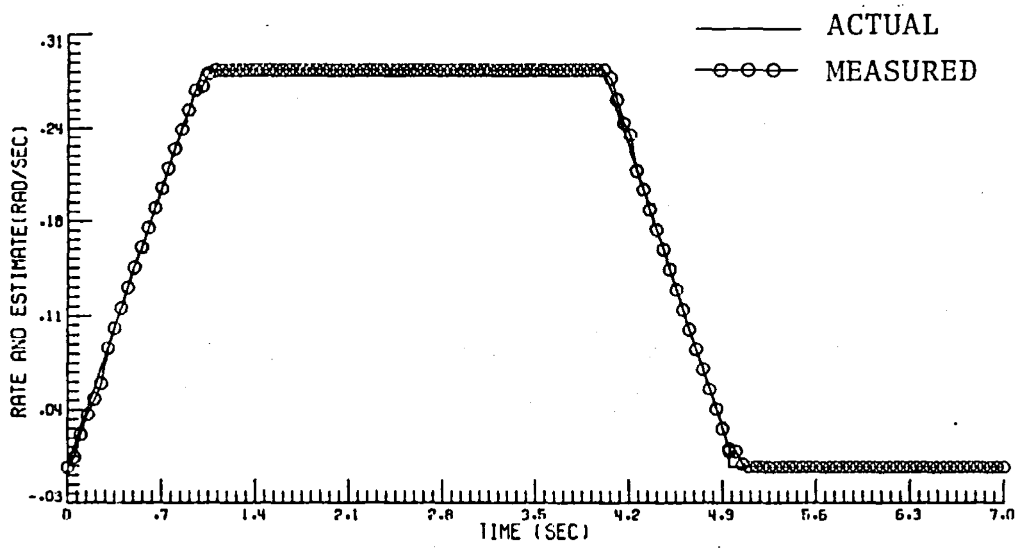

Figure 9. Actual and measured rate, ω_{c1} = 100 rad/sec.

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Langley Technical Monitor: Nelson J. Groom
Final Report

16. Abstract

Nonlinear equations of motion are derived for the Rim Inertial Measuring System (RIMS), which is mounted in a strapped-down configuration on a carrier vehicle. The RIMS can be used for measuring angular rates and linear accelerations and the equations derived can be easily interfaced with the dynamic model of a user-defined carrier vehicle. Two methods are presented for rate measurement, and the results of the nonlinear simulation are presented.

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