NASA
National Aeronautics and
Space Administration

NASA CR-160192

Lyndon B. Johnson Space Center
Houston, Texas 77058

JSC-14722

APR 13 1979

PRELIMINARY MATHEMATICAL SPECIFICATIONS
FOR
AREA TARGETS AND SPACE VOLUMES

Job Order 92-011

Project Number 1281

Prepared By

Computer Sciences Corporation

for

Mission Planning and Analysis Division

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
LYNDON B. JOHNSON SPACE CENTER
HOUSTON, TEXAS

March 1979

CSC-0229

JSC FORM 2026A (REV AUG 78)
NASA-JSC

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
JSC- 14722	N/A	N/A

4. Title and Subtitle	5. Report Date
PRELIMINARY MATHEMATICAL SPECIFICATIONS FOR AREA TARGETS AND SPACE VOLUMES	March 1979
	6. Performing Organization Code: CSC/M30
7. Author(s): M. Bishop	8. Performing Organization Report No.: CSC-0229
	10. Work Unit No.: N/A
9. Performing Organization Name and Address: Computer Sciences Corporation, 1300 Bay Area Boulevard, Houston, Texas 77058	11. Contract or Grant No.: NAS 9-15700
	13. Type of Report and Period Covered: Mathematical Specifications
12. Sponsoring Agency Name and Address: NASA, Lyndon B. Johnson Space Center, Houston, Texas 77058	14. Sponsoring Agency Code: FM2

15. Supplementary Notes

This document was generated for Mr. A. A. Menchaca of the Flight Planning Branch.

16. Abstract

This document provides the preliminary level B/C mathematical specifications for the Mission Control Center (MCC) area targets and space volumes processor.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Area targets Space volumes	N/A

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified		

*For sale by the National Technical Information Service, Springfield, Virginia 22161

JSC-14722

PRELIMINARY MATHEMATICAL SPECIFICATIONS FOR AREA TARGETS AND SPACE VOLUMES

Job Order 92-011

Project Number 1281

Prepared By

M. Bishop

Approved By

O. Dial, Manager
Mission Analysis Section

K. Nickerson, Manager
Mission Planning Software Department

Prepared By

Computer Sciences Corporation

for

Mission Planning and Analysis Division

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
LYNDON B. JOHNSON SPACE CENTER
HOUSTON, TEXAS

March 1979

CSC-0229

CONTENTS

FIGURES

ACRONYMS AND SYMBOLS

AOS	Acquisition-of-Signal
ATSVP	Area Targets and Space Volumes Processor
LOS	Loss-of-Signal
MCC	Mission Control Center
M50	Mean-of-1950
RNP	Rotation, Nutation, and Precession
S/C	Spacecraft
TOE	True-Of-Epoch

Symbols:

a_F	"Adjustment factor" for computing altitude above the Fischer ellipsoid
$\vec{C}$	Vector along the centerline of a polyhedron
d	Perpendicular distance to the side of a polyhedron
F	Flattening coefficient
h	Altitude
n	Number of sides
$\hat{N}$	Unit outward normal vector
r_c	Radius of the Earth-referenced circle
R_{em}	Mean equatorial radius
$\vec{R}_{sc}$	Spacecraft position vector
$[RNP]_{M50}^{TOE}$	RNP matrix from the mean-of-1950 coordinate system to the true-of-epoch system
t	Time
t_e	Epoch time corresponding to the RNP matrix

α	Right ascension
δ	Declination
ϕ	Geodetic latitude
ϕ_c	Geocentric latitude
λ	Longitude
$\vec{\rho}$	Slant range vector
ω_e	Earth rotation rate

Subscripts:

i	Denotes the i^{th} vertex or side of a polygon or polyhedron. Also used to denote the i^{th} occurence of an event
B	Denotes a vector measured from the lower boundary (i.e., base) of a polyhedron
T	Denotes a vector measured from the upper boundary (i.e., top) of a polyhedron

Superscripts:

G	Denotes a vector in the rotating geocentric coordinate system
TOE	Denotes a vector in the true-of-epoch coordinate system
blank	Denotes a vector in the mean-of-1950 coordinate system
^	Denotes a unit vector

1.0 INTRODUCTION

This document provides the level B/C mathematical specifications for the Area Targets and Space Volumes Processor (ATSVP). Pursuant to the requirements of reference 1, this processor is designed to compute the acquisition-of-signal (AOS) and loss-of-signal (LOS) times for the following:

a. Area targets

   (1) Earth-referenced circles which are specified by a latitude, longitude, altitude, and radius.

   (2) Celestial circles which are specified by a right ascension, declination, and angular radius.

   (3) Earth-referenced polygons which are an arbitrary Earth-fixed figure having up to five sides with the "corner points" defined by latitude, longitude, and altitude.

   (4) Celestial polygons which are an arbitrary, inertially fixed figure having up to five sides with the corner points defined by right ascension and declination on the celestial sphere.

b. Space volumes

   (1) Earth-referenced space volumes which are an arbitrary, Earth-fixed, five-sided polyhedron. These volumes are defined by a lower-limit polygon at an altitude, h_1, and the projection of this polygon to an altitude, h_2. The corner points of the polygon will be defined by latitudes and longitudes and will rotate with the Earth.

   (2) Celestial-fixed space volumes which are an arbitrary, inertially fixed, five-sided polyhedron. These volumes are defined by a lower-limit polygon at an altitude, h_1, and the projection of this polygon to an altitude, h_2. The corner points of the polygon are defined by right ascension and declination on the celestial sphere.

The AOS and LOS times for these targets are defined (ref. 1) as follows:

a. Ground circles and polygons

   AOS - the time corresponding to the first subsatellite point to be just inside the area.

   LOS - the time corresponding to the last subsatellite point just prior to exiting the area.

b. Celestial circles and polygons

   AOS - the time corresponding to the first zenith point to lie just inside the area.

   LOS - the time corresponding to the last zenith point just prior to exiting the area.

c. Ground-fixed and celestial-fixed space volumes

   AOS - the time at which the spacecraft (S/C) is just entering the volume.

   LOS - the time just prior to the S/C exiting the volume.

Six data tables will contain the information necessary to completely describe the area targets and space volumes. These tables (ref. 1) are as follows:

a. Ground targets table containing 10 targets in 1 block of data.
b. Celestial circles table containing 10 targets in 1 block of data.
c. Ground polygons table containing 20 targets in 2 blocks of data.
d. Celestial polygons table containing 10 targets in 1 block of data.
e. Ground-fixed space volumes table containing 10 targets in 1 block of data.
f. Celestial-fixed volumes table containing 10 targets in 1 block of data.

Section 2 of this report presents the mathematical equations necessary to determine whether the S/C lies within the area target or space volume. Section 3 outlines the process required to determine the AOS and LOS times.

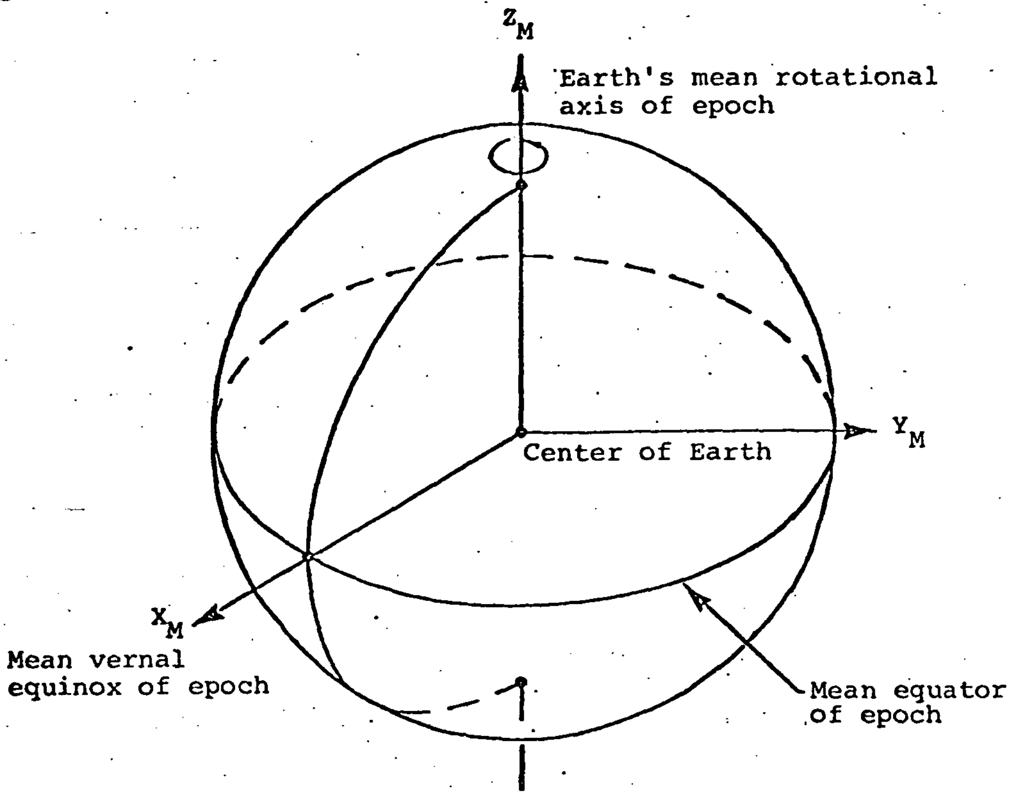
2.0 MATHEMATICAL EQUATIONS

The following subsections present the mathematical equations necessary to determine whether the S/C lies within each of the area targets and space volumes presented in section 1. Two reference coordinate systems will be used. The inertial Aries-mean-of-1950 (M50) coordinate systems (fig. 2-1) will be the reference system when dealing with area targets and space volumes which remain inertially fixed. The rotating geocentric coordinate system (fig. 2-2) will be used when dealing with area targets and space volumes which rotate with the Earth.

Each of the following six subsections is further subdivided into three topics:

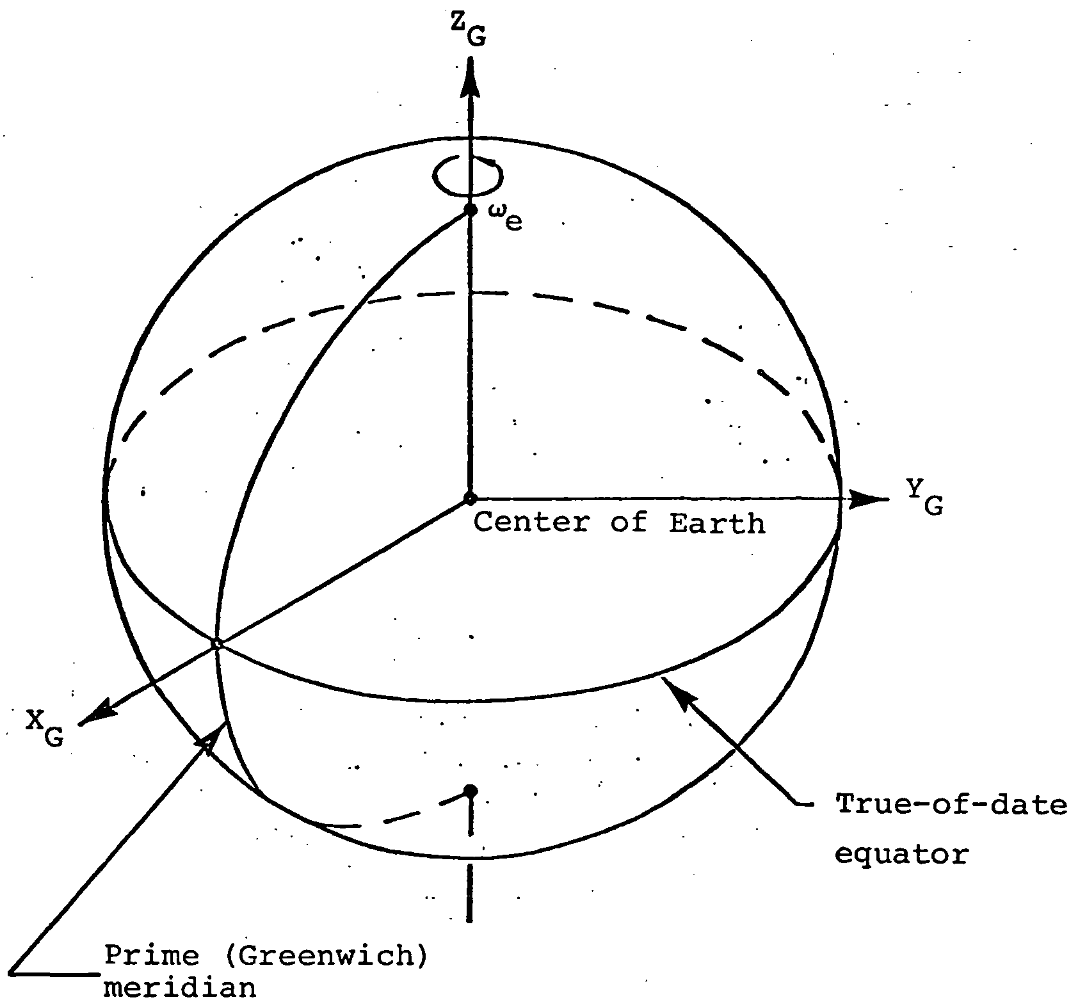
a. Procedure - a brief description of the steps to be performed.
b. Equations - a statement of the input parameter requirements and development of the mathematical equations.
c. Assumptions and limitations - description of any simplifying assumptions and/or mathematical restrictions.

For convenience, section 2.7 summarizes the equations for all of the area targets and space volumes.



NAME:	Aries-mean-of-1950, Cartesian, coordinate system.
ORIGIN:	The center of the Earth.
ORIENTATION:	The epoch is the beginning of Besselian year 1950 or Julian ephemeris date 2433282.423357.
	The X_M-Y_M plane is the mean Earth's equator of epoch.
	The X_M axis is directed towards the mean vernal equinox of epoch.
	The Z_M axis is directed along the Earth's mean rotational axis of epoch and is positive north.
	The Y_M axis completes a right-handed system.
CHARACTERISTICS:	Inertial, right-handed, Cartesian system.

Figure 2-1.- Aries-mean-of-1950, coordinate system.



NAME:	Geocentric Coordinate system
ORIGIN:	Center of the Earth
ORIENTATION:	X_G - Y_G plane is the Earth's true-of-date equator
	X_G passes through the Greenwich meridian
	Z_G is along the Earth's rotational axis
	Y_G completes the right-handed system
CHARACTERISTICS:	Rotating, right-handed, Earth-fixed

Figure 2-2.- Rotating geocentric coordinate system.

2.1 EARTH-REFERENCED CIRCLES

Earth-referenced circles are defined to be circular ground target areas whose centers are defined by geodetic latitude, longitude, and altitude (fig. 2-3). The S/C lies within this ground target area if its subsatellite point lies within the perimeter of the circular area.

2.1.1 Procedure

The following procedure will be used to determine whether the S/C lies within the ground target area:

a. The geodetic coordinates of the ground target area will be transformed to the geocentric system.

b. The S/C position vector will be transformed from the M50 system to the geocentric system.

c. A test will be performed to determine whether the S/C subsatellite point lies within the perimeter of the ground target area.

2.1.2 Equations

The following parameters are required:

ϕ and λ - geodetic latitude and longitude, respectively, of the center of the Earth-referenced circle (fig. 2-4)

h - altitude of the Earth-referenced circle, measured with respect to the Fischer ellipsoid of 1960 (fig. 2-4)

r_c - radius of the Earth-referenced circle (fig. 2-3)

$\vec{R}_{sc}$ - S/C position vector in the M50 coordinate system (fig. 2-3)

t - time corresponding to $\vec{R}_{sc}$

$[RNP]_{M50}^{TOE}$ - Rotation, nutation, and precession (RNP) matrix which is used to transform vectors from the M50 coordinate system to the true-of-epoch (TOE) coordinate system

t_e - epoch time corresponding to the RNP matrix

R_{em} - mean equatorial radius for the Fischer ellipsoid of 1960

F - flattening coefficient for the Fischer ellipsoid of 1960

ω_e - Earth rotation rate

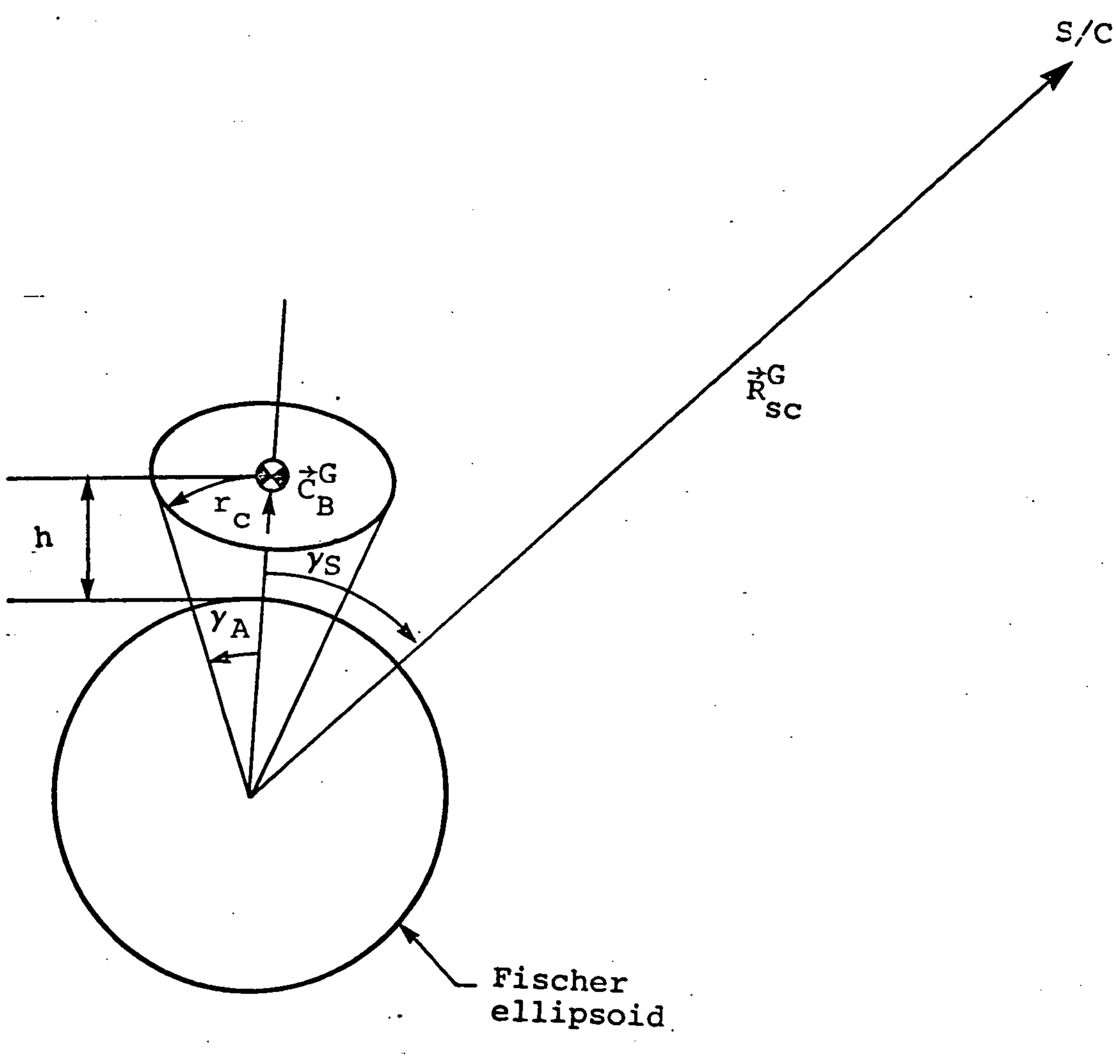
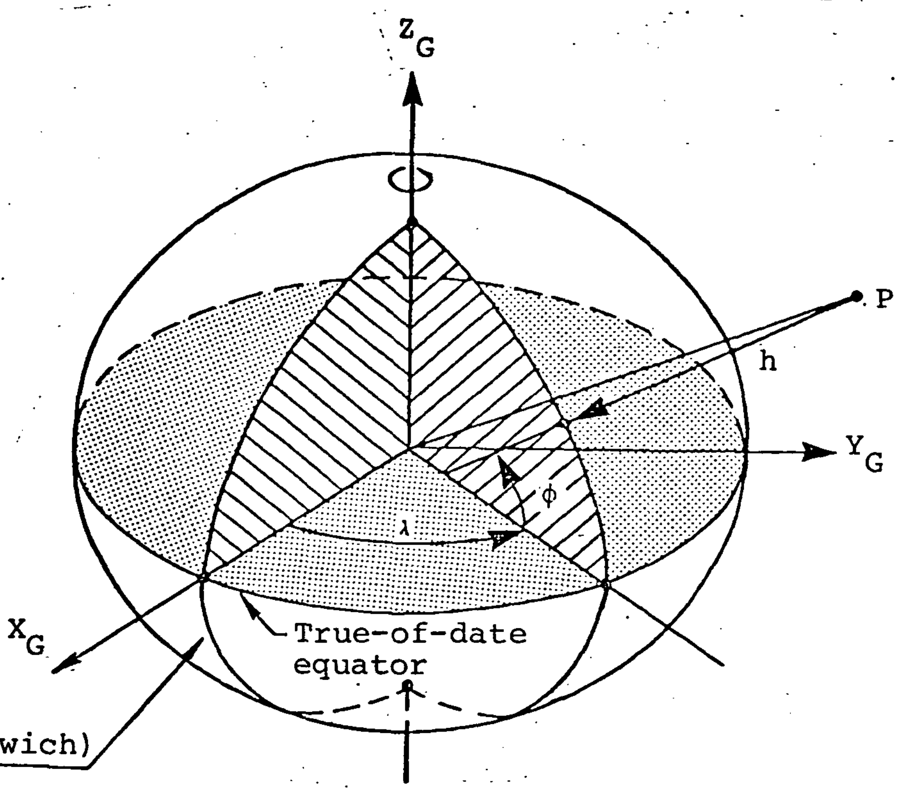


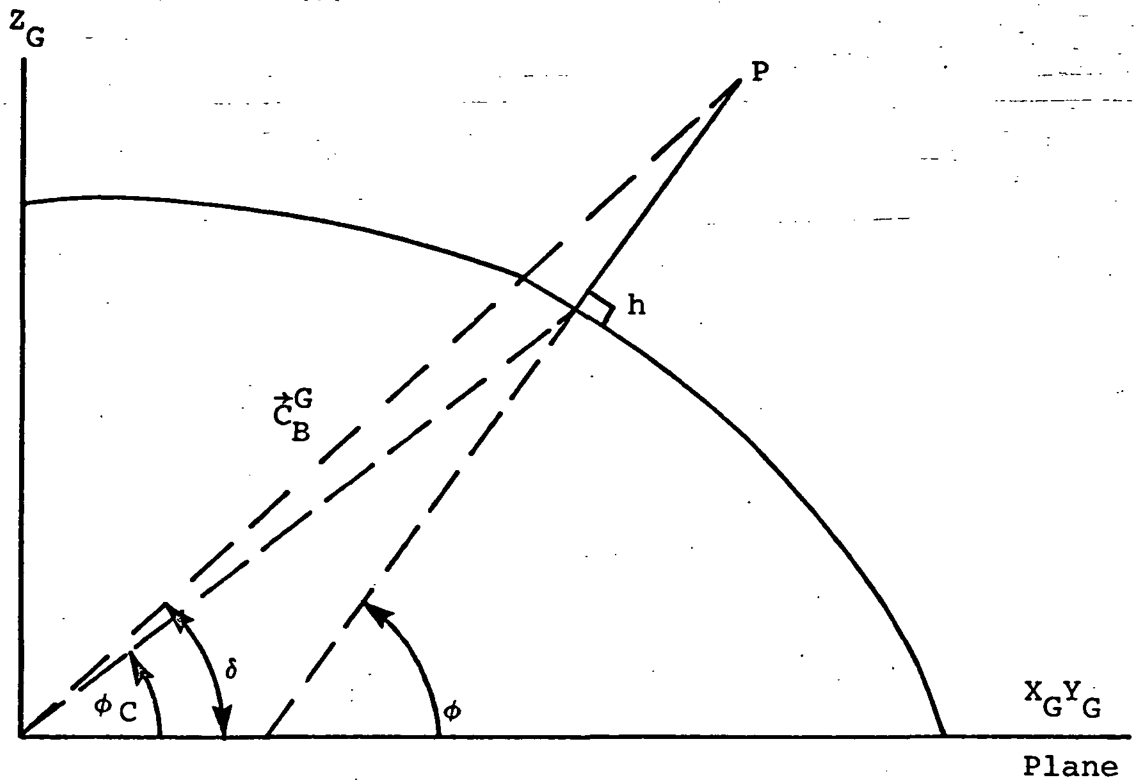
Figure 2-3.- Earth-referenced circle.



NAME:	Geodetic coordinate system.
ORIGIN:	This system consists of a set of parameters rather than a coordinate system; therefore, no origin is specified.
ORIENTATION:	This system of parameters is based on an ellipsoidal model of the Earth (e.g., the Fischer ellipse of 1960). For any point of interest we define a line, known as the geodetic local vertical, which is perpendicular to the ellipsoid and which contains the point of interest. h, geodetic altitude, is the distance from the point of interest to the reference ellipsoid, measured along the geodetic local vertical, and is positive for points outside the ellipsoid. λ is the longitude measured in the plane of the Earth's true equator from the prime (Greenwich) meridian to the local meridian, measured positive eastward. ϕ is the geodetic latitude, measured in the plane of the local meridian from the Earth's true equator to the geodetic local vertical, measured positive north from the equator. **NOTE:** A detailed explanation of declination, geodetic latitude, and geocentric latitude is provided on figure 2-4(b)
CHARACTERISTICS:	Rotating polar coordinate parameters. Only position vectors are expressed in this coordinate system. Velocity vectors should be expressed in the Aries-mean-of-1950, or the Aries true-of-date, polar for inertial or quasi-inertial representations, respectively. The Fischer ellipsoid model should be used with this system.

(a) Basic definitions.

Figure 2-4.- Geodetic coordinate system



NAME: Geodetic coordinate system of point P.

DEFINITIONS: h is the altitude of point P. Measured perpendicular from the surface of the referenced ellipsoid.

ϕ is the geodetic latitude of point P.

ϕ_C is the geocentric latitude of point P.

δ is the angle between radius vector and equatorial plane (declination).

λ is the longitude of point P. Angle (+ east) between plane of the figure and the plane formed by the Greenwich meridian.

(b) Detailed explanation.

Figure 2-4.- Concluded.

The first step is to transform the ground target area from the geodetic coordinate system to the geocentric system. The vector from the center of the Earth to the center of the ground target area can be expressed in the rotating geocentric coordinate system by

$$\vec{C}_B^G = \begin{Bmatrix} \left(h + a_F\right) \cos \lambda \cos \phi \\ \left(h + a_F\right) \sin \lambda \cos \phi \\ \left[h + (1 - F)^2 a_F\right] \sin \phi \end{Bmatrix} \tag{2-1}$$

where

$$a_F = \frac{R_{em}}{\sqrt{\cos^2\phi + (1 - F)^2 \sin^2\phi}} \tag{2-2}$$

The second step is to transform the S/C position vector from the M50 system to the geocentric system. This is accomplished by

a. Transforming the vector from the M50 system to the TOE system using the RNP matrix.
b. Transforming the resultant vector from the TOE system to the geocentric system.

The S/C position vector in the TOE system is given by

$$\vec{R}_{sc}^{TOE} = [RNP]_{M50}^{TOE} \vec{R}_{sc} \tag{2-3}$$

The S/C position vector in the geocentric system (fig. 2-5) is given by

$$\vec{R}_{sc}^{G} = \begin{Bmatrix} \cos \Delta\lambda & \sin \Delta\lambda & 0 \\ -\sin \Delta\lambda & \cos \Delta\lambda & 0 \\ 0 & 0 & 1 \end{Bmatrix} \vec{R}_{sc}^{TOE} \tag{2-4}$$

where

$$\Delta\lambda = \omega_e \left(t - t_e\right) \tag{2-5}$$

The final step is to determine if the S/C subsatellite point lies within the ground target area (fig. 2-3). The angle from $\vec{C}_B^G$ to the perimeter of the circular ground target area, γ_A, is given (in degrees) by

$$\gamma_A = \left\{ \frac{r_c}{\left|\vec{C}_B^G\right|} \right\} \frac{180}{\pi} \tag{2-6}$$

The angle from $\vec{C}_B^G$ to the S/C, γ_s, is given by

$$\gamma_s = \cos^{-1} \left\{ \frac{\vec{R}_{sc}^G \cdot \vec{C}_B^G}{\left|\vec{R}_{sc}^G\right| \left|\vec{C}_B^G\right|} \right\} \tag{2-7}$$

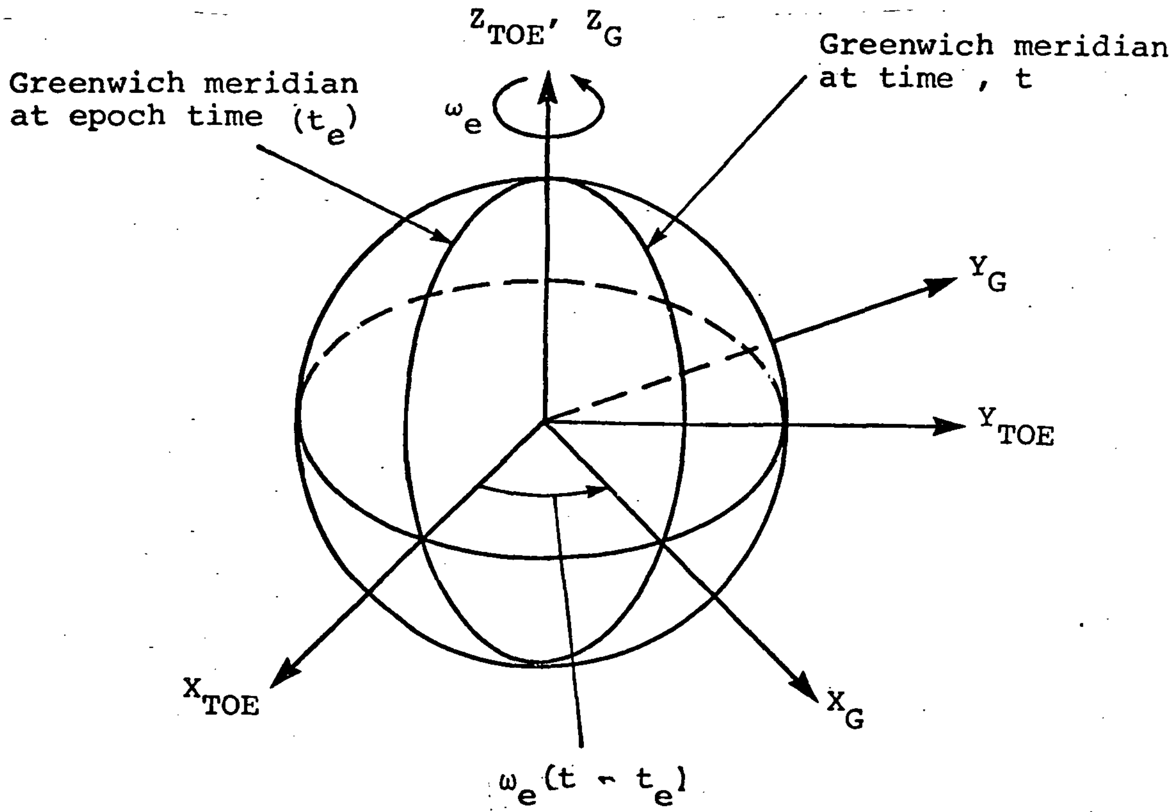


Figure 2-5.- Relationship between true-of-epoch and rotating geocentric coordinate systems.

The S/C lies within the ground target area if

$$\gamma_s \leq \gamma_A \tag{2-8}$$

2.1.3 Assumptions and Limitations

The following assumptions are implicit in the equations presented in section 2.1.2:

a. The S/C geocentric subsatellite point is used to compute entry into the ground target area.

b. The effects of polar nutation and precession from time t_e to time t can be neglected.

2.2 CELESTIAL CIRCLES

Celestial circles are defined to be inertially fixed circular areas which extend from the center of the Earth to infinity (fig. 2-6). The center of this area target is defined by right ascension and declination on the celestial sphere. The criterion for a S/C to lie within this area is for the S/C zenith point to lie within the perimeter of the celestial circle.

2.2.1 Procedure

The following procedure will be used to determine if the S/C lies within the celestial circle:

a. The unit vector along the centerline of the celestial circle will be computed in the M50 coordinate system.

b. The dot product between this vector and the S/C position vector will be formed to determine if the S/C lies within the celestial circle.

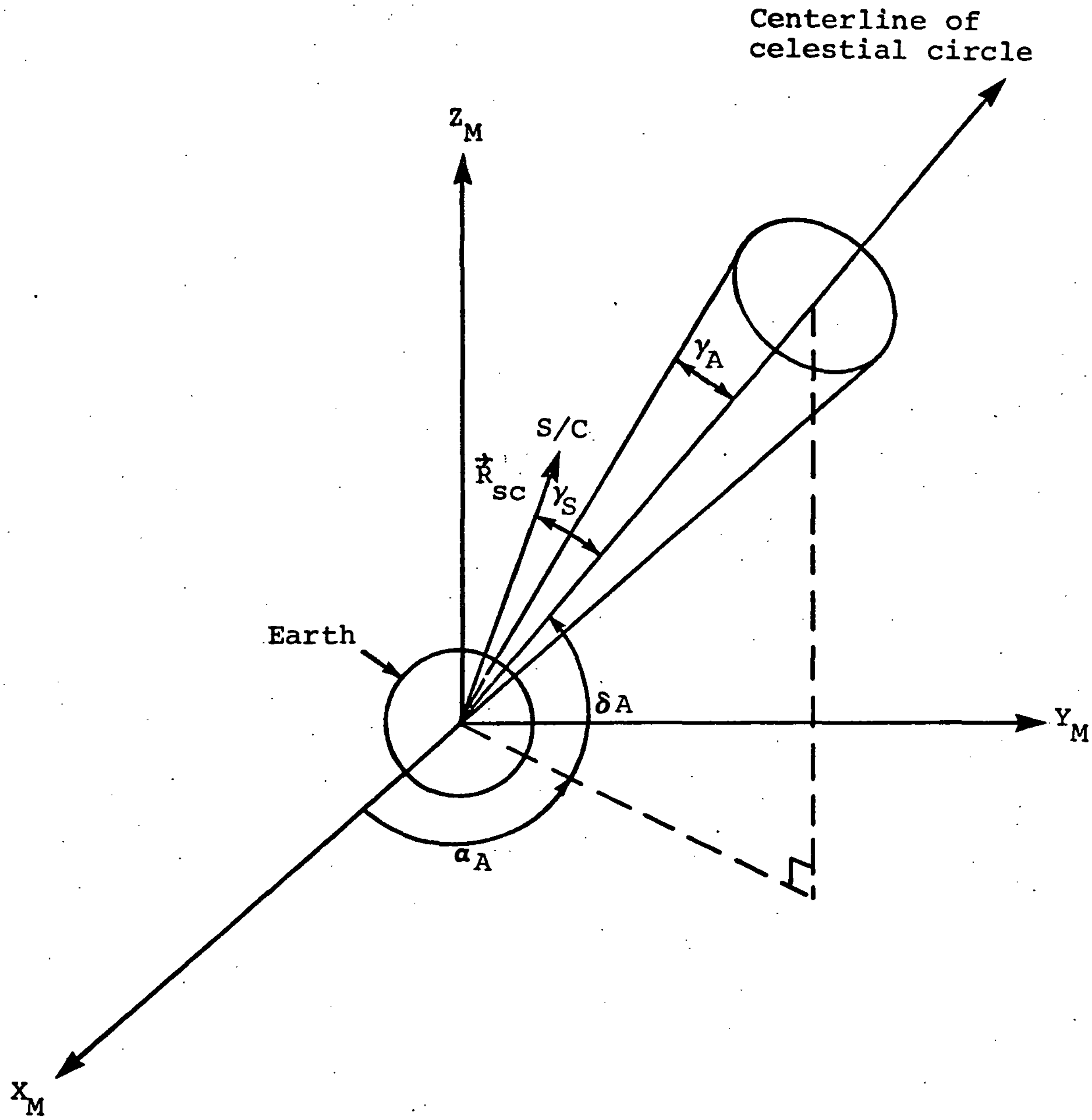


Figure 2-6.- Containment test for Celestial Circles.

2.2.2 Equations

The following parameters are required:

α_A and δ_A - the right ascension and declination, respectively, of the centerline of the celestial circle expressed in the M50 coordinate system (fig. 2-6)

γ_A - the celestial circle angular radius (fig. 2-6)

$\vec{R}_{sc}$ - S/C position vector in the M50 coordinate system

The unit vector from the center of the Earth to the center of the celestial circle, $\hat{C}_B$, is given by

$$\hat{C}_B = \begin{Bmatrix} \cos \alpha_A \cos \delta_A \\ \sin \alpha_A \cos \delta_A \\ \sin \delta_A \end{Bmatrix} \tag{2-9}$$

The angle between this vector and the S/C, γ_s, is

$$\gamma_s = \cos^{-1} \left\{ \hat{C}_B \cdot \frac{\vec{R}_{sc}}{|\vec{R}_{sc}|} \right\} \tag{2-10}$$

The S/C lies within the celestial circle if

$$\gamma_s \leq \gamma_A \tag{2-11}$$

2.2.3 Assumptions and Limitations

None.

2.3 EARTH-REFERENCED POLYGON

The Earth-referenced polygon is defined to be an arbitrary planar figure having up to five sides (fig. 2-7). This figure is fixed with respect to the rotating Earth. The corner points (i.e., vertices) of this polygon are defined by geodetic latitude, longitude, and altitude. The basic criterion for penetration into this ground target area is to ensure that:

a. The S/C lies within the proper hemisphere.
b. The S/C subsatellite point lies within the perimeter of the ground target area.

2.3.1 Procedure

The basic procedure for this ground area target is similar to the procedure presented in section 2.1.1. It consists of

a. Transforming the geodetic coordinates of each polygon vertex to the geocentric coordinate system.
b. Transforming the S/C position vector from the M50 system to the geocentric coordinate system.
c. Testing to ensure that the S/C lies in the proper hemisphere. If this test is failed, then no further computations are required.
d. Assuming step c. is passed, tests will be made to determine if the S/C is interior to all planes defining the sides of the polygon.

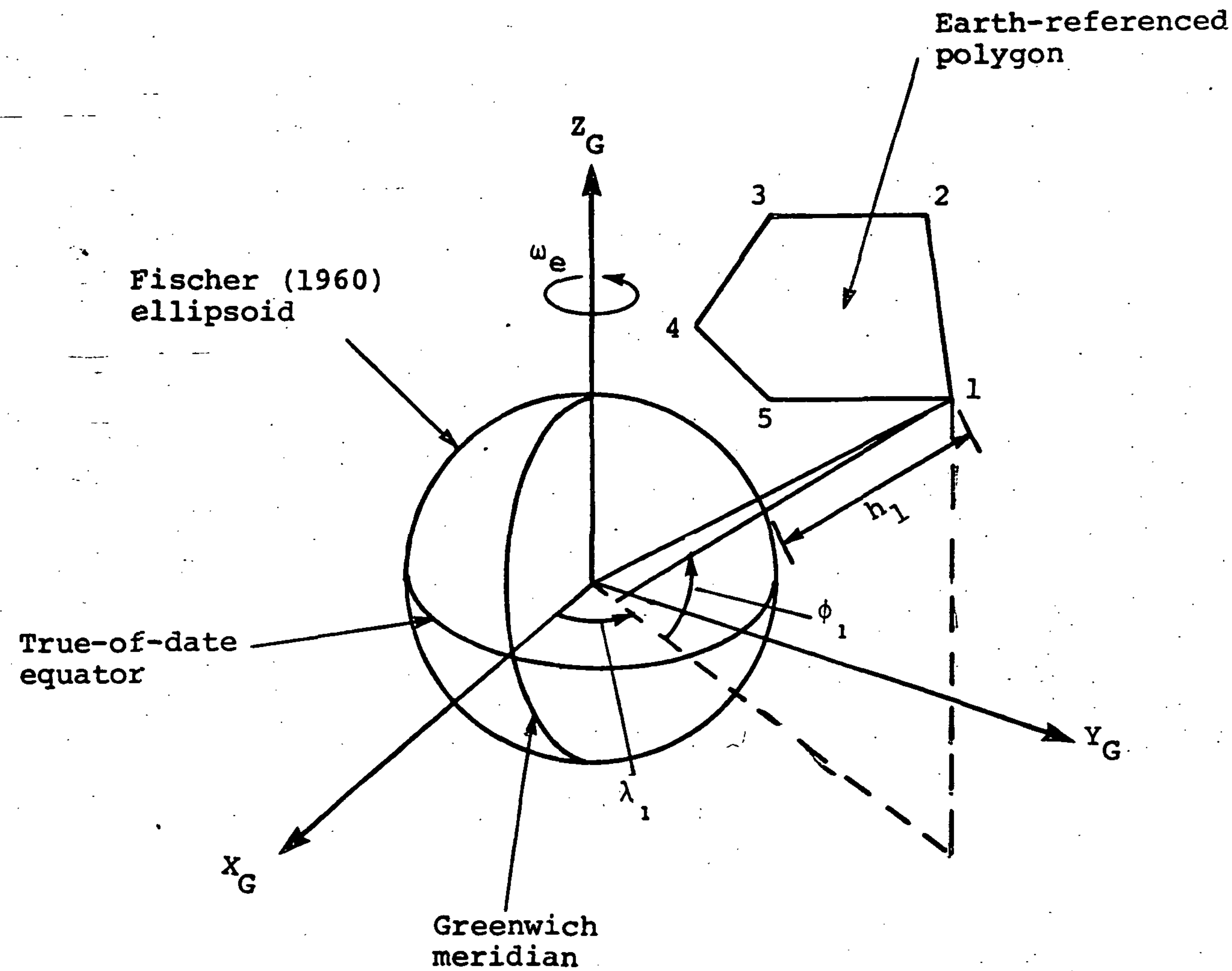


Figure 2-7.- Earth-referenced polygon.

2.3.2 Equations

The following parameters are required:

- n - number of sides
- ϕ_i and λ_i - geodetic latitude and longitude, respectively, of each vertex (fig. 2-7)
- h_i - geodetic altitude of each vertex (fig. 2-7)
- $\vec{R}_{sc}$ - S/C position vector in the M50 coordinate system
- t - time corresponding to $\vec{R}_{sc}$
- $[RNP]_{M50}^{TOE}$ - RNP matrix to transform from M50 to TOE
- t_e - epoch time corresponding to the RNP matrix
- R_{em} - mean equatorial radius for the Fischer ellipsoid of 1960
- F - flattening coefficient for the Fischer ellipsoid of 1960
- ω_e - Earth rotation rate

The first step is to transform the vectors defining each vertex of the polygon from the geodetic system to the rotating geocentric system. These vectors are given by

$$\vec{R}_i^G = \begin{Bmatrix} \left(h_i + a_F\right) \cos \lambda_i \cos \phi_i \\ \left(h_i + a_F\right) \sin \lambda_i \cos \phi_i \\ \left[h_i + (1 - F)^2 a_F\right] \sin \phi_i \end{Bmatrix} \quad i = 1,2,3 \ldots n \qquad (2\text{-}12)$$

where

a_F is defined by equation 2-2 with ϕ_i replacing ϕ.

The S/C position vector in the geocentric coordinate system, $\vec{R}_{sc}^G$, is then obtained by using equations 2-3 through 2-5.

The next step in the procedure is to determine if the S/C lies in the proper hemisphere. Figure 2-8 illustrates the geometry. All

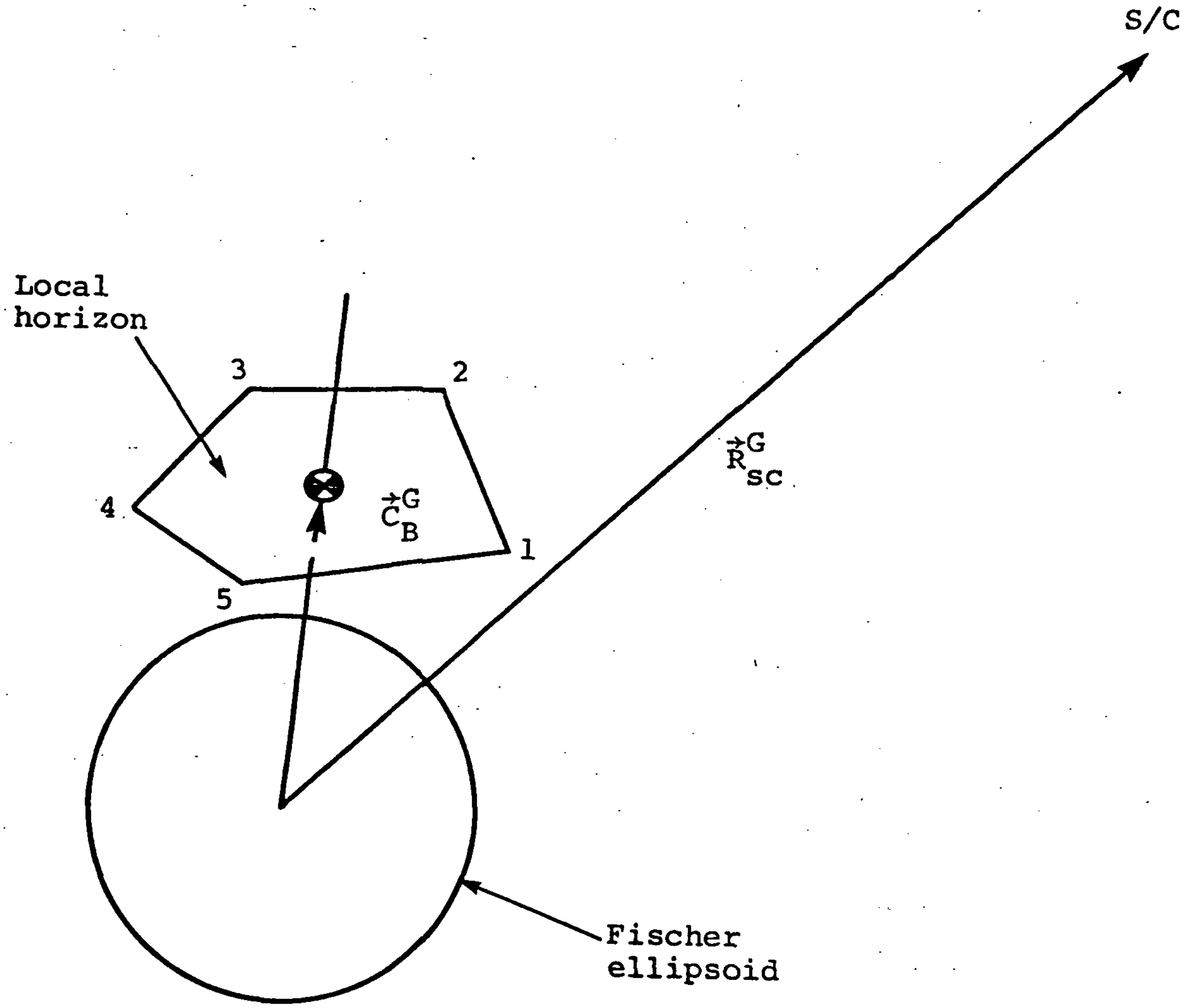


Figure 2-8.- Horizon test for Earth-referenced polygons.

vectors in this figure are with respect to the geocentric coordinate system. The centroid of the ground target area, $\vec{C}_B^G$, is defined as follows

$$\vec{C}_B^G = \frac{\sum_{i=1}^{n} \vec{R}_i^G}{n} \tag{2-13}$$

The S/C lies in the proper hemisphere if

$$\vec{C}_B^G \cdot \vec{R}_{sc}^G \geq 0 \tag{2-14}$$

Assuming equation 2-14 is satisfied, the final step is to determine if the S/C lies interior to all planes defined by the sides of polygon. Figure 2-9 illustrates the geometry. The unit normal vectors to each side of the polygon are given by

$$\hat{N}_i^G = \frac{\vec{R}_{i+1}^G \times \vec{R}_i^G}{\left|\vec{R}_{i+1}^G \times \vec{R}_i^G\right|} \qquad i = 1,2,3,\ldots n-1 \tag{2-15a}$$

$$\hat{N}_n^G = \frac{\vec{R}_1^G \times \vec{R}_n^G}{\left|\vec{R}_1^G \times \vec{R}_n^G\right|} \tag{2-15b}$$

The perpendicular distance from $\vec{C}_B^G$ to each polygon side is

$$d_i = \hat{N}_i^G \cdot \left(\vec{R}_i^G - \vec{C}_B^G\right) \quad i = 1,2,3,\ldots n \tag{2-16}$$

The S/C must lie interior to the i^{th} plane if

$$\vec{N}_i^G \cdot \vec{\rho}_B^G \leq 0 \tag{2-17a}$$

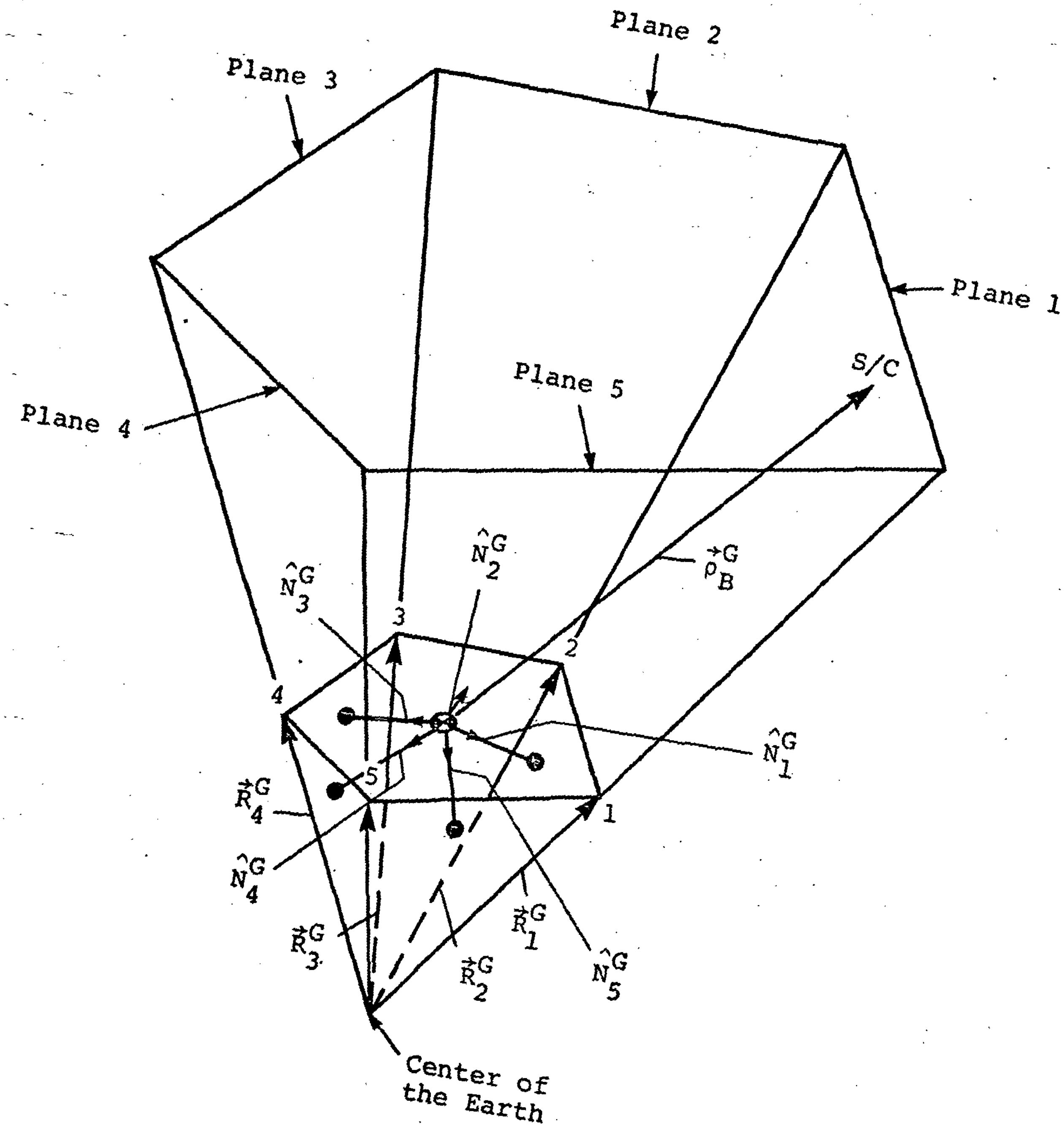


Figure 2-9.- Containment test for Earth-referenced polygons.

where

$$\vec{\rho}_B^{\,G} = \vec{R}_{sc}^{\,G} - \vec{C}_B^{\,G} \tag{2-17b}$$

Assuming equation 2-17a is failed, the S/C may lie between $\vec{C}_B^{\,G}$ and the i^{th} plane if the following condition is met.

$$\hat{N}_i^G \cdot \vec{\rho}_B^{\,G} \leq d_i \tag{2-18}$$

All of the d_i's will be positive if the vertices of the polygon are ordered counterclockwise (as viewed from the top of the polygon). Thus, equation 2-18 provides a single necessary and sufficient test to determine if the S/C lies interior to each plane. This test is performed for all sides of the polygon. If the test is failed for any side, the S/C subsatellite point does not lie within the perimeter of the polygon.

2.3.3 Assumptions and Limitations

The following assumptions and limitations are implicit in the equations presented in section 2.3.2:

a. The S/C geocentric subsatellite point is used to determine entry into the ground target area.
b. The effects of polar nutation and precession from time t_e to t can be neglected.
c. The vertices of the polygon are specified in a counterclockwise order as viewed from the top.
d. The polygon is convex; i.e., the interior angles between the sides defining the vertices are less than 180 degrees.*
e. The angular separation between two consecutive vertices is sufficient to permit a nonsingular cross product (eq. 2-15).

*Concave polygons could be accommodated by subdividing them into two or more convex polygons. The appendix discusses this procedure.

2.4 CELESTIAL POLYGONS

Celestial polygons are defined to be arbitrary figures having up to five sides with the corner points (i.e., vertices) defined by right ascension and declination on the celestial sphere. The criterion for penetration into this area is to ensure that the S/C zenith point lies within the confines of the polygon. Figure 2-10 illustrates the celestial polygon. It is noted that this polygon also represents a variable area polyhedron which remains inertially fixed.

2.4.1 Procedure

The basic procedure for this target area is similar to the procedure presented in section 2.3.1, i.e.,:

a. The unit vector along the "centroid" of the polyhedron will be computed in the M50 coordinate system.

b. A test will be made to ensure that the S/C lies above the apex of the polyhedron. If this test is failed, then no further computations are required.

c. Assuming step b. is passed, tests will be made to determine if the S/C is interior to all planes defining the sides of the polyhedron.

2.4.2 Equations

The following parameters are required:

n - number of sides

α_i and δ_i - right ascension and declination, respectively, of each vertex in the M50 coordinate system ($i = 1,2,3,\ldots n$)

$\vec{R}_{sc}$ - S/C position vector in the M50 coordinate system

The first step is to determine the unit vector along the centroid of the polyhedron. The unit vectors from the center of the Earth to each vertex in the M50 coordinate system are given by

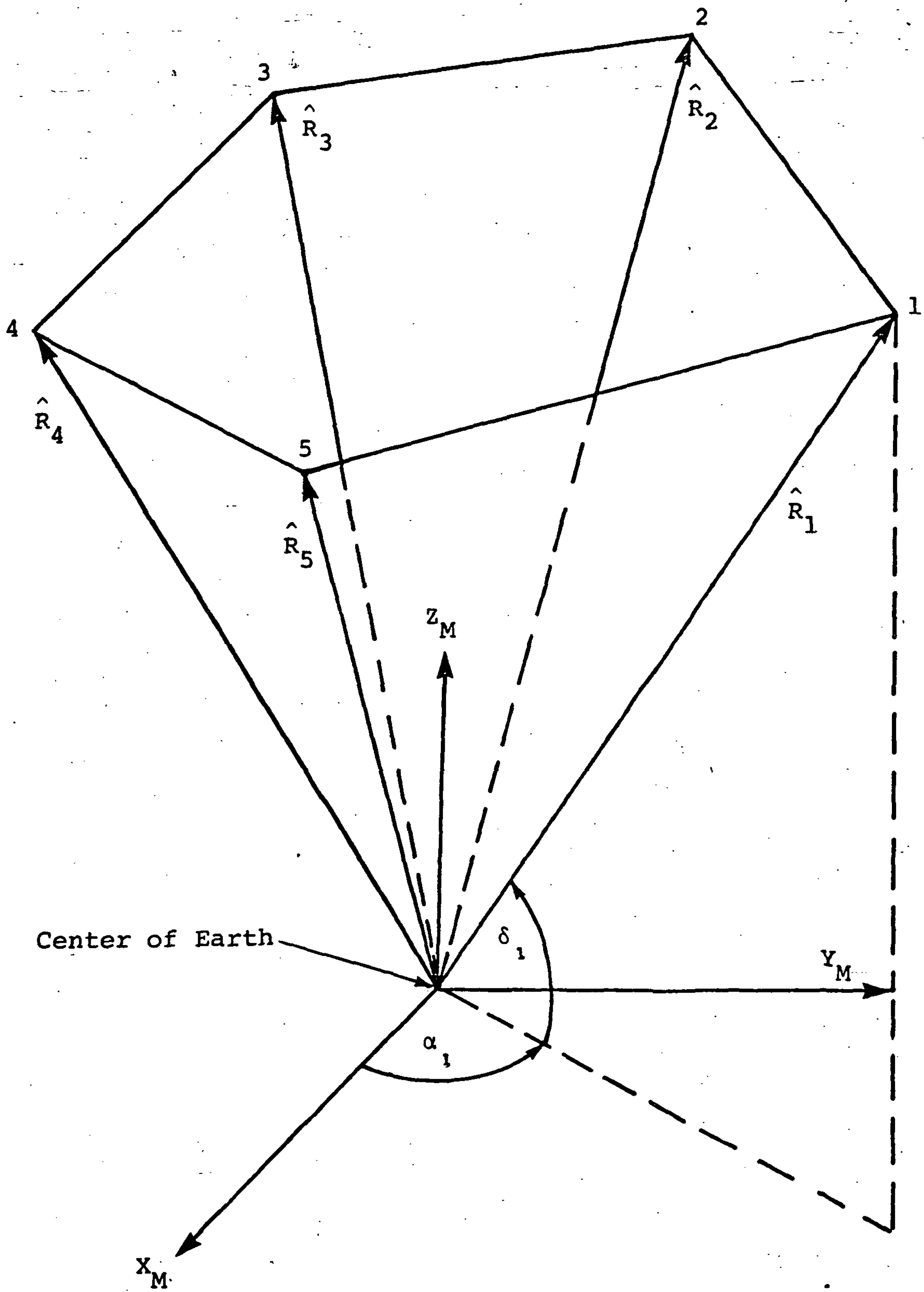


Figure 2-10.- Celestial polygon.

$$\hat{R}_i = \begin{Bmatrix} \cos \alpha_i \cos \delta_i \\ \sin \alpha_i \cos \delta_i \\ \sin \delta_i \end{Bmatrix} \quad i = 1,2,3,\ldots n \tag{2-19}$$

The unit vector along the centroid of the polyhedron is

$$\hat{C} = \frac{\vec{C}}{|\vec{C}|} \tag{2-20}$$

where

$$\vec{C} = \frac{\sum_{i=1}^{n} \hat{R}_i}{n} \tag{2-21}$$

The next step is to determine if the S/C lies above the apex of the polyhedron. The S/C lies above the apex if

$$\hat{C} \circ \vec{R}_{SC} \geq 0 \tag{2-22}$$

Assuming equation 2-22 is satisfied, the final step is to determine if the S/C lies interior to all planes defined by the sides of the polyhedron. Figure 2-11 illustrates the geometry. The slant range from $\hat{C}$ to the S/C is

$$\vec{\rho}_B = \vec{R}_{SC} - \hat{C} \tag{2-23}$$

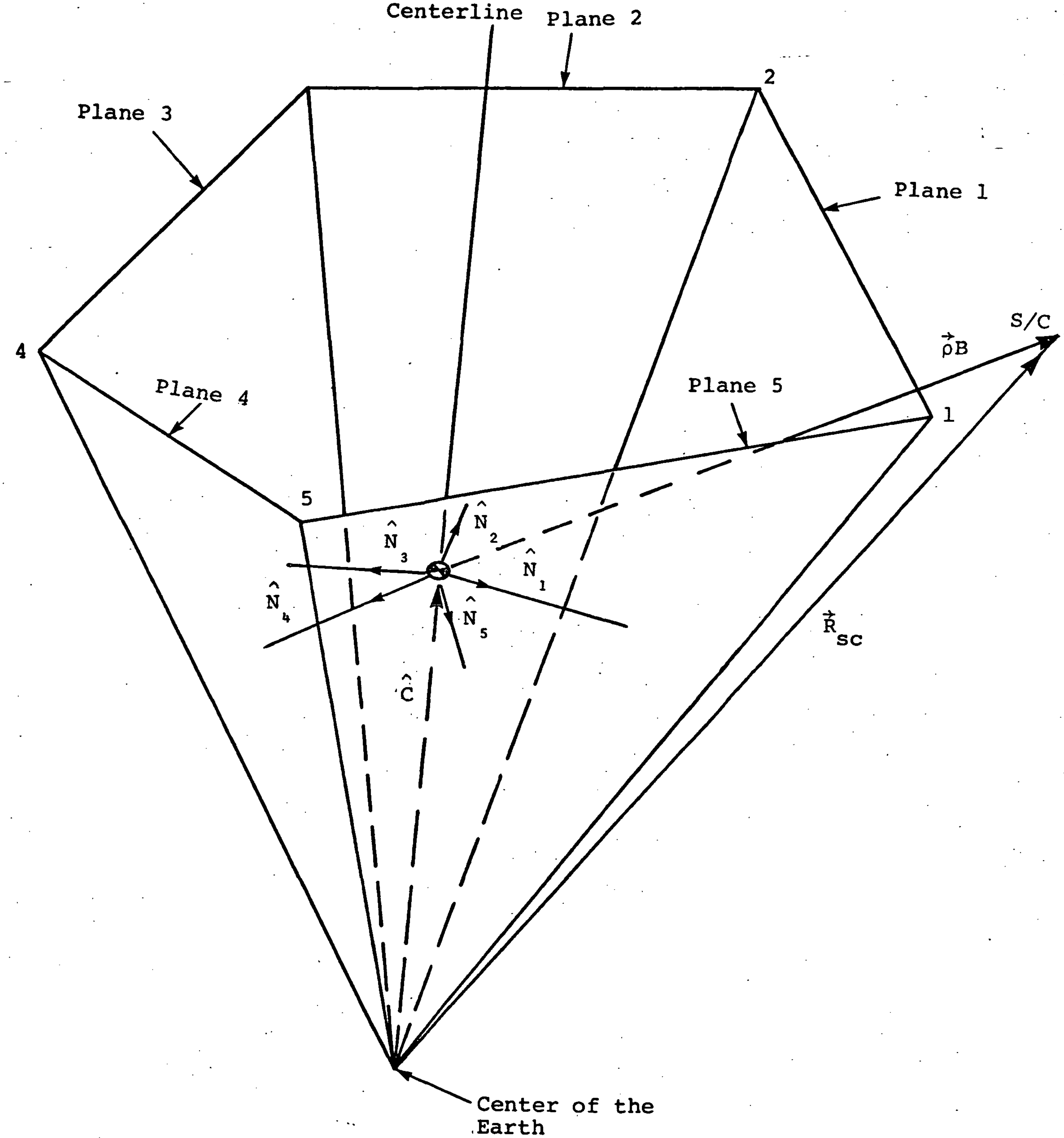


Figure 2-11.- Containment test for celestial polygons.

Equations 2-15 through 2-18 (with $\vec{R}_i^G$, $\vec{C}_B^G$, and $\vec{\rho}_B^G$ replaced by $\hat{R}_i$, $\hat{C}$, and $\vec{\rho}_B$, respectively) are then used to determine if the S/C is interior to all polyhedron planes.

2.4.3 Assumptions and Limitations

The following assumptions and limitations are implicit in the equations presented in section 2.4.2:

a. The vertices of the polygon are specified in a counterclockwise order as viewed from the top.
b. The polygon is convex; i.e., the interior angles between the sides defining the vertices are less than 180 degrees.*
c. The angular separation between two consecutive vertices is sufficient to permit a nonsingular cross product (eq. 2-15).

* Concave polygons could be accommodated by subdividing them into two or more convex polygons as discussed in the appendix.

2.5 EARTH-REFERENCED SPACE VOLUMES

The Earth-referenced space volumes are arbitrary five-sided figures which are defined by a lower-limit polygon at an altitude, h_1, and the projection of this polygon to an altitude, h_2. The vertices of this polygon are defined by geodetic latitude and longitude and rotate with the Earth. Figure 2-12 illustrates this type of space volume. As shown, the planar cross sectional area of this polyhedron remains constant with respect to altitude.

2.5.1 Procedure

The following procedure will be used to determine whether the S/C lies within the space volume:

a. The geodetic parameters defining each vertex of the lower boundary will be transformed to geocentric position vectors.

b. The centroid vectors to the lower and upper boundaries will be computed in the geocentric coordinate system.

c. The S/C position vector will be transformed from the M50 system to the geocentric system.

d. Tests will be performed to ensure that the S/C lies above the lower boundary and below the upper boundary. If either of these tests is failed, then no further computations are required.

e. Assuming step d. is passed, tests will be performed to determine whether the S/C is interior to all planes defining the sides of the polyhedron.

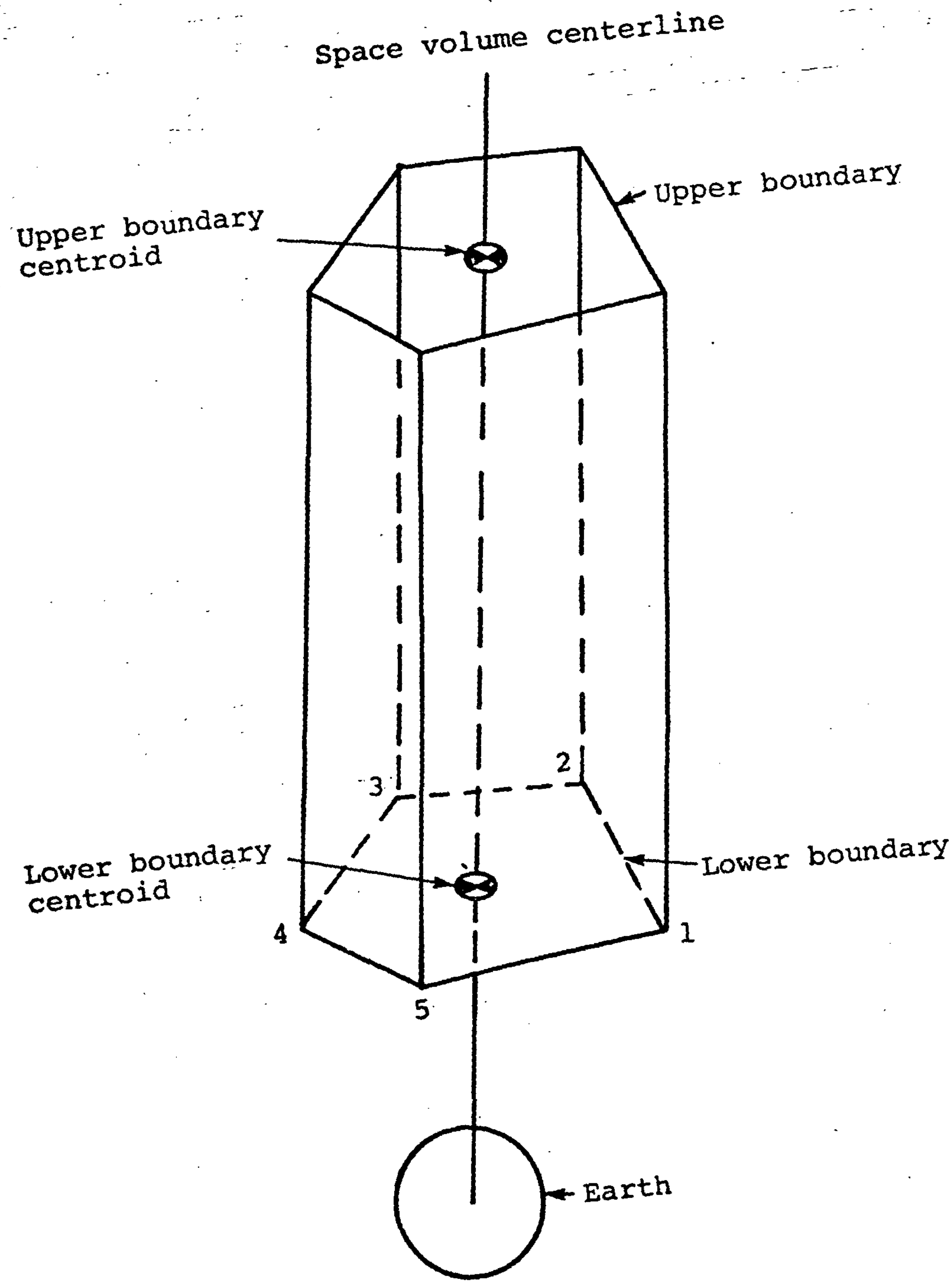


Figure 2-12.- Constant area polyhedron.

2.5.2 Equations

The following parameters are required:

- ϕ_i and λ_i - geodetic latitude and longitude, respectively, of each vertex of the lower boundary ($i = 1,2,3,...,5$)
- h_1 - geodetic altitude of the lower boundary
- h_2 - geodetic altitude of the upper boundary
- $\vec{R}_{sc}$ - S/C position vector in the M50 coordinate system
- t - time corresponding to $\vec{R}_{sc}$
- $[RNP]_{M50}^{TOE}$ - RNP matrix to transform from M50 to TOE
- t_e - epoch time corresponding to the RNP matrix
- R_{em} - mean equatorial radius for the Fischer ellipsoid of 1960
- F - flattening coefficient for the Fischer ellipsoid of 1960
- ω_e - Earth rotation rate

The first step is to transform the vectors defining each vertex of the lower boundary to the geocentric coordinate system. Equation 2-12 (with $h_i = 0$) provides the necessary transformation.

The centroid of these vectors is given by

$$\vec{C}^G = \frac{\sum_{i=1}^{n} \vec{R}_i^G}{n} \qquad (2\text{-}24)$$

The geocentric vectors to the centroids of the lower and upper boundaries (fig. 2-13) are given by

$$\vec{C}_B^G = \vec{C}^G + h_1 \frac{\vec{C}^G}{|\vec{C}^G|} \qquad (2\text{-}25)$$

$$\vec{C}_T^G = \vec{C}^G + h_2 \frac{\vec{C}^G}{|\vec{C}^G|} \qquad (2\text{-}26)$$

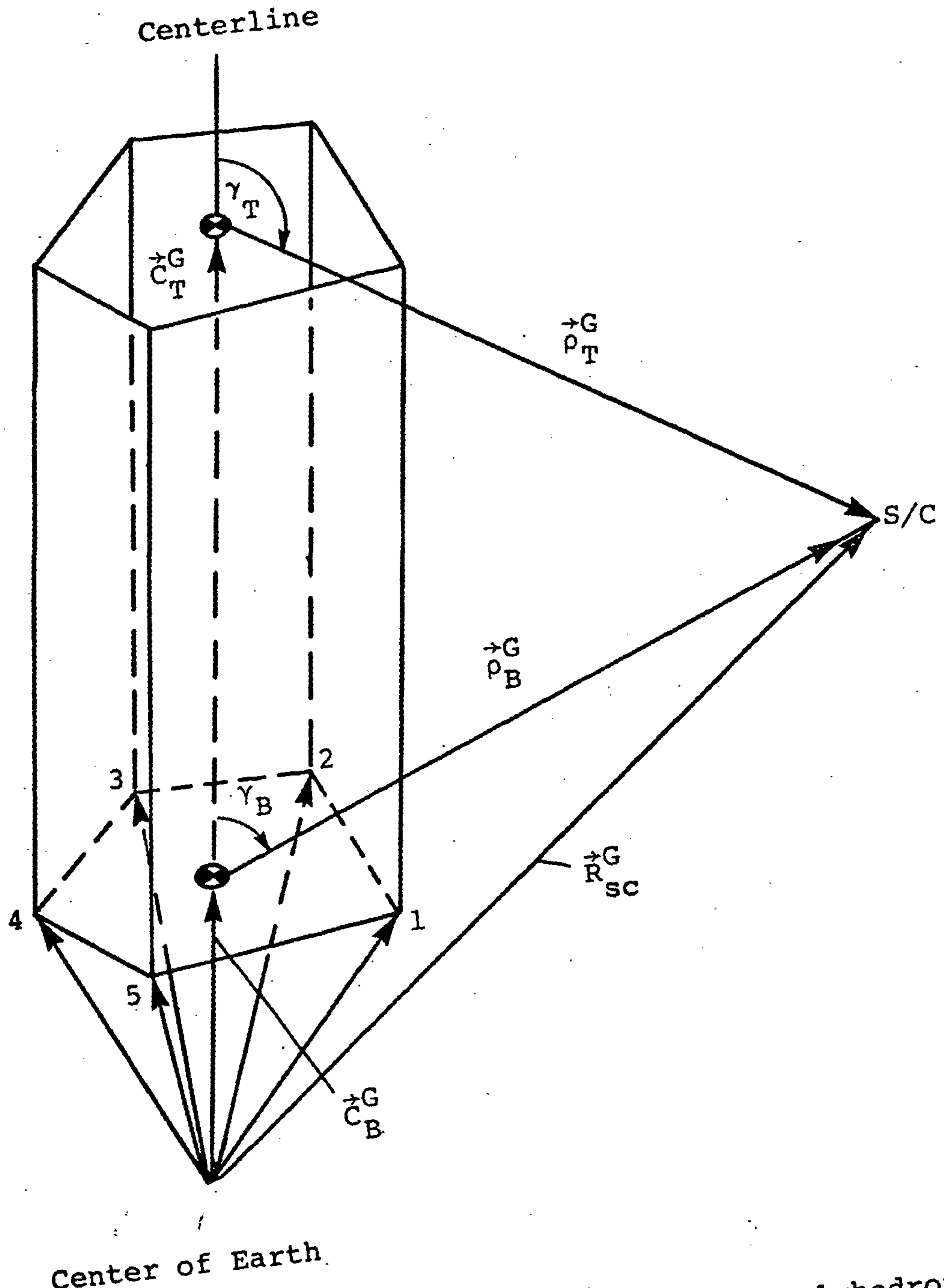


Figure 2-13.- Boundary tests for constant area polyhedrons.

Next, the S/C position vector in the geocentric coordinate system, $\vec{R}^G_{sc}$, is computed using equations 2-3 through 2-5.

The fourth step in the procedure is to ensure that the S/C lies between the upper and lower boundaries. Figure 2-13 illustrates the geometry. All vectors in this figure are with respect to the geocentric system. The slant range from $\vec{C}^G_B$ to the S/C, $\vec{\rho}^G_B$, is given by

$$\vec{\rho}^G_B = \vec{R}^G_{sc} - \vec{C}^G_B \tag{2-27}$$

The angle, γ_B, between $\vec{C}^G_B$ and $\vec{\rho}^G_B$ is computed by

$$\gamma_B = \cos^{-1}\left\{\frac{\vec{\rho}^G_B \cdot \vec{C}^G_B}{\left|\vec{\rho}^G_B\right| \left|\vec{C}^G_B\right|}\right\} \tag{2-28}$$

The S/C lies above the lower boundary if

$$\gamma_B \leq 90^{\circ} \tag{2-29}$$

Similarly, for the upper boundary

$$\vec{\rho}^G_T = \vec{R}^G_{sc} - \vec{C}^G_T \tag{2-30}$$

$$\gamma_T = \cos^{-1}\left\{\frac{\vec{\rho}^G_T \cdot \vec{C}^G_T}{\left|\vec{\rho}^G_T\right| \left|\vec{C}^G_T\right|}\right\} \tag{2-31}$$

The S/C lies below the upper boundary if

$$\gamma_T \geq 90^\circ \tag{2-32}$$

Assuming that both equations 2-29 and 2-32 are satisfied, the final step is to determine if the S/C lies interior to the side planes of the polyhedron. Figure 2-14 illustrates the geometry. The unit normal vectors from $\vec{C}_B^G$ to each side of the polygon are given by

$$\hat{N}_i^G = \frac{\vec{C}_B^G \times \left(\vec{R}_i^G - \vec{R}_{i+1}^G\right)}{\left|\vec{C}_B^G \times \left(\vec{R}_i^G - \vec{R}_{i+1}^G\right)\right|} \quad i = 1,2,3,4 \tag{2-33a}$$

$$\hat{N}_5^G = \frac{\vec{C}_B^G \times \left(\vec{R}_5^G - \vec{R}_1^G\right)}{\left|\vec{C}_B^G \times \left(\vec{R}_5^G - \vec{R}_1^G\right)\right|} \tag{2-33b}$$

Equations 2-16 and 2-18 are then used to determine if the S/C is contained within the polyhedron.

2.5.3 Assumptions and Limitations

The following assumptions and limitations are implicit in the equations presented in section 2.5.2:

a. The altitude of the lower boundary, h_l, is measured with respect to the centroid of the vertices which lie on the Fischer ellipsoid.

b. The effects of polar nutation and precession from time t_e to t can be neglected.

c. The vertices are specified in a counterclockwise order as viewed from the top.

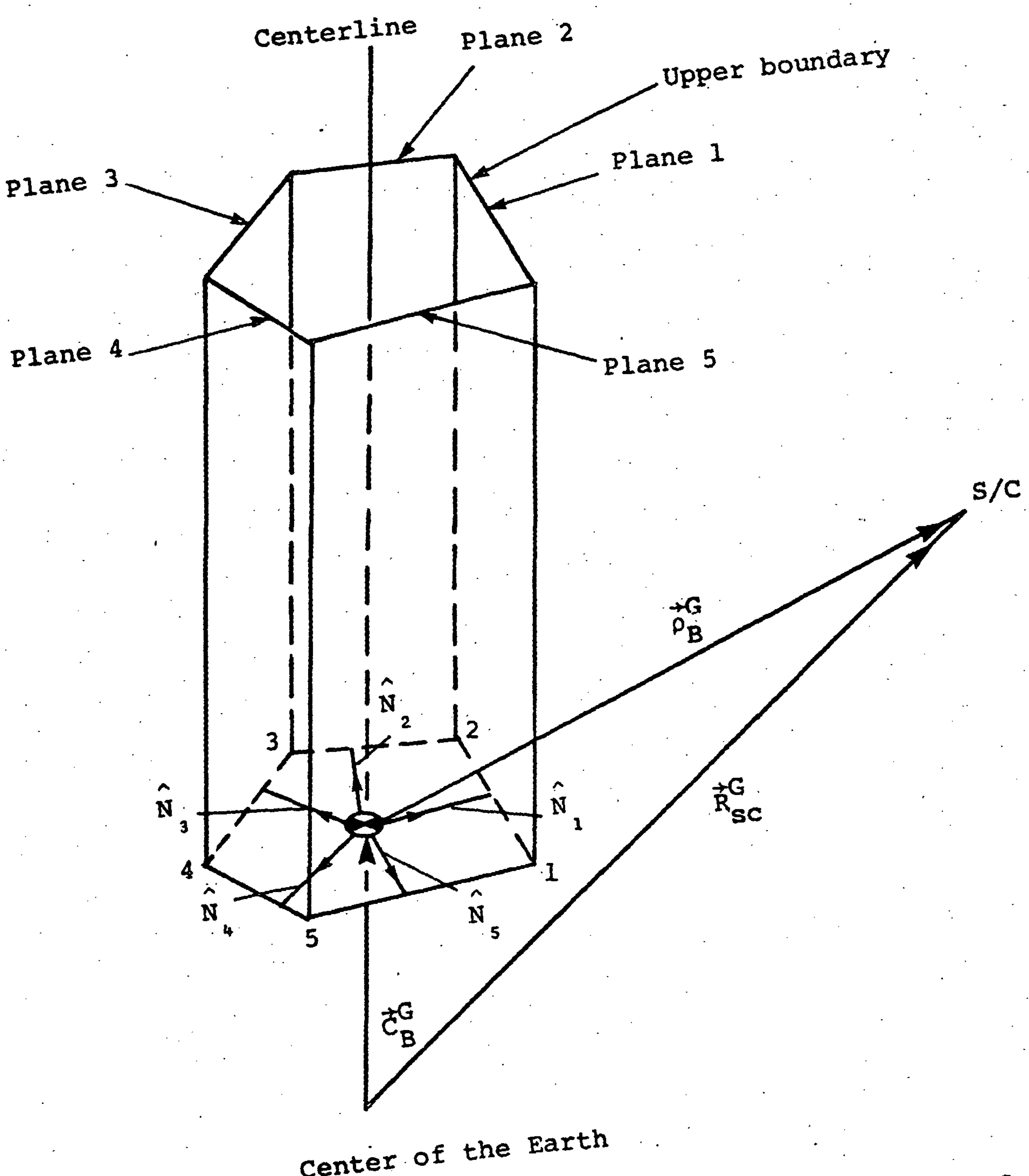


Figure 2-14.- Containment test for constant area polyhedrons.

d. The planar area of the polyhedron is convex; i.e., the interior angles between the sides defining the vertices are less than 180 degrees.*

e. The local horizon for determining if the S/C is between the upper and lower boundaries is perpendicular to the centroid vector.

f. The distance between two consecutive vertices is sufficient to permit a nonsingular cross product (eq. 2-33).

---

*Concave polyhedrons could be accommodated by subdividing them into two or more convex portions as discussed in the appendix.

2.6 CELESTIAL-FIXED SPACE VOLUMES

The celestial-fixed space volumes are arbitrary five-sided figures which are defined by a lower-limit polygon at an altitude, h_1, and a projection of this polygon to an altitude, h_2. The vertices of the polygon are defined by right ascension and declination and are inertially fixed. Figure 2-12 can also be used to illustrate this type of space volume. As mentioned previously, the planar cross sectional area of this polyhedron remains constant with respect to altitude.

2.6.1 Procedure

The basic procedure for this space volume is very similar to the procedure presented in section 2.5.1. It consists of:

a. Computing the centroid vectors to the lower and upper boundaries in the M50 coordinate system.

b. Testing to ensure that the S/C lies above the lower boundary and below the upper boundary. If either of these tests is failed, then no further computations are required.

c. Assuming step b. is passed, further tests will be performed to determine if the S/C is interior to all planes defining the sides of the polyhedron.

2.6.2 Equations

The following parameters are required:

α_i and δ_i - right ascension and declination, respectively, of each vertex of the lower boundary in the M50 coordinate system (i = 1,2,3,...5)

h_1 - altitude of the lower boundary

h_2 - altitude of the upper boundary

R_{em} - mean equatorial radius

The unit vector to the centroid of the polyhedron, $\hat{C}$, is computed via equations 2-19 through 2-21. The vectors to the centroids of the lower and upper boundaries ($\vec{C}_B$ and $\vec{C}_T$, respectively) are given by

$$\vec{C}_B = \left(R_{em} + h_1\right) \hat{C} \tag{2-34}$$

$$\vec{C}_T = \left(R_{em} + h_2\right) \hat{C} \tag{2-35}$$

Equations 2-27 through 2-32 (with variables $\vec{R}^G_{sc}$, $\vec{C}^G_B$, and $\vec{C}^G_T$ replaced by $\vec{R}_{sc}$, $\vec{C}_B$, and $\vec{C}_T$, respectively) are then used to ensure that the S/C lies between the upper and lower boundaries.

Assuming equations 2-29 and 2-32 are both satisfied, equation 2-33 (with $\vec{C}^G_B$ and $\vec{R}^G_i$ replaced by $\vec{C}_B$ and $\hat{R}_i$, respectively) is used to define the unit normal vectors to each side of the polyhedron. Finally, equations 2-16 and 2-18 are used to determine if the S/C lies within the space volume.

2.6.3 Assumptions and Limitations

The following assumptions and limitations are implicit in the equations presented in section 2.6.2:

a. The altitudes of the lower and upper boundaries are measured with respect to the mean equatorial radius.
b. The vertices are specified in a counterclockwise order as viewed from the top.
c. The planar area of the polyhedron is convex; i.e., the interior angles between the sides defining the vertices are less than 180 degrees.*
d. The distance between two consecutive vertices is sufficient to permit a nonsingular cross product (eq. 2-33).

*Concave polyhedrons could be accommodated by subdividing them into two or more convex portions as discussed in the appendix.

2.7 SUMMARY OF EQUATIONS

This section summarizes all of the equations presented in the previous sections for the various area targets and space volumes. The order of presentation and equation numbers correspond to the computation sequence discussed in the text.

2.7.1 Earth-Referenced Circles

$$\vec{C}_B^G = \begin{Bmatrix} \left(h + a_F\right) \cos \lambda \cos \phi \\ \left(h + a_F\right) \sin \lambda \cos \phi \\ \left[h + (1 - F)^2 a_F\right] \sin \phi \end{Bmatrix} \tag{2-1}$$

where

$$a_F = \frac{R_{em}}{\sqrt{\cos^2\phi + (1 - F)^2 \sin^2\phi}} \tag{2-2}$$

$$\vec{R}_{sc}^{TOE} = [RNP]_{M50}^{TOE} \vec{R}_{sc} \tag{2-3}$$

$$\vec{R}_{sc}^G = \begin{bmatrix} \cos \Delta\lambda & \sin \Delta\lambda & 0 \\ -\sin \Delta\lambda & \cos \Delta\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} \vec{R}_{sc}^{TOE} \tag{2-4}$$

where

$$\Delta\lambda = \omega_e \left(t - t_e\right) \tag{2-5}$$

$$\gamma_A = \left\{\frac{r_c}{\left|\vec{C}_B^G\right|}\right\} \frac{180}{\pi} \tag{2-6}$$

$$\gamma_s = \cos^{-1}\left\{\frac{\vec{R}_{sc}^G \cdot \vec{C}_B^G}{\left|\vec{R}_{sc}^G\right| \ \left|\vec{C}_B^G\right|}\right\} \tag{2-7}$$

$$\gamma_s \leq \gamma_A \tag{2-8}$$

2.7.2 Celestial Circles

$$\hat{C}_B = \begin{Bmatrix} \cos\alpha_A \cos\delta_A \\ \sin\alpha_A \cos\delta_A \\ \sin\delta_A \end{Bmatrix} \tag{2-9}$$

$$\gamma_s = \cos^{-1}\left\{\hat{C}_B \cdot \frac{\vec{R}_{sc}}{\left|\vec{R}_{sc}\right|}\right\} \tag{2-10}$$

$$\gamma_s \leq \gamma_A \tag{2-11}$$

2.7.3 Earth-Referenced Polygons

$$\vec{R}_i^G = \begin{Bmatrix} \left(h_i + a_F\right) \cos \lambda_i \cos \phi_i \\ \left(h_i + a_F\right) \sin \lambda_i \cos \phi_i \\ \left[h_i + (1-F)^2 a_F\right] \sin \phi_i \end{Bmatrix} \quad i = 1,2,3,\ldots n \quad (2\text{-}12)$$

where

$$a_F = \frac{R_{em}}{\sqrt{\cos^2 \phi_i + (1-F)^2 \sin^2 \phi_i}} \quad (2\text{-}2)$$

$$\vec{R}_{sc}^{TOE} = [RNP]_{M50}^{TOE} \, \vec{R}_{sc} \quad (2\text{-}3)$$

$$\vec{R}_{sc}^G = \begin{bmatrix} \cos \Delta\lambda & \sin \Delta\lambda & 0 \\ -\sin \Delta\lambda & \sin \Delta\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} \vec{R}_{sc}^{TOE} \quad (2\text{-}4)$$

where

$$\Delta \lambda = \omega_e \left(t - t_e\right) \quad (2\text{-}5)$$

$$\vec{C}_B^G = \frac{\sum_{i=1}^{n} \vec{R}_i^G}{n} \quad (2\text{-}13)$$

$$\vec{C}_B^G \cdot \vec{R}_{sc}^G \geq 0 \tag{2-14}$$

$$\hat{N}_i^G = \frac{\vec{R}_{i+1}^G \times \vec{R}_i^G}{\left|\vec{R}_{i+1}^G \times \vec{R}_i^G\right|} \qquad i = 1,2,3,\ldots n-1 \tag{2-15a}$$

$$\hat{N}_n^G = \frac{\vec{R}_1^G \times \vec{R}_n^G}{\left|\vec{R}_1^G \times \vec{R}_n^G\right|} \tag{2-15b}$$

$$d_i = \vec{N}_i^G \cdot \left(\vec{R}_i^G - \vec{C}_B^G\right) \quad i = 1,2,3,\ldots n \tag{2-16}$$

$$\vec{\rho}_B^G = \vec{R}_{sc}^G - \vec{C}_B^G \tag{2-17b}$$

$$\hat{N}_i^G \cdot \vec{\rho}_B^G \leq d_i \qquad i = 1,2,3,\ldots n \tag{2-18}$$

2.7.4 Celestial Polygons

$$\hat{R}_i = \begin{Bmatrix} \cos \alpha_i & \cos \delta_i \\ \sin \alpha_i & \cos \delta_i \\ & \sin \delta_i \end{Bmatrix} \quad i = 1,2,3,\ldots n \qquad (2\text{-}19)$$

$$\hat{C} = \frac{\vec{C}}{|\vec{C}|} \qquad (2\text{-}20)$$

where

$$\vec{C} = \frac{\sum_{i=1}^{n} \hat{R}_i}{n} \qquad (2\text{-}21)$$

$$\hat{C} \cdot \vec{R}_{sc} \geq 0 \qquad (2\text{-}22)$$

$$\vec{\rho}_B = \vec{R}_{sc} - \hat{C} \qquad (2\text{-}23)$$

$$\hat{N}_i = \frac{\hat{R}_{i+1} \times \hat{R}_i}{\left|\hat{R}_{i+1} \times \hat{R}_i\right|} \quad i = 1,2,3,\ldots n\text{-}1 \qquad (2\text{-}15a)$$

$$\hat{N}_n = \frac{\hat{R}_1 \times \hat{R}_n}{\left|\hat{R}_1 \times \hat{R}_n\right|} \qquad (2\text{-}15b)$$

$$d_i = \hat{N}_i \cdot \left(\hat{R}_i - \hat{C}\right) \quad i = 1,2,3,\ldots n \qquad (2\text{-}16)$$

$$\hat{N}_i \cdot \vec{\rho}_B \leq d_i \quad i = 1,2,3,\ldots n \qquad (2\text{-}18)$$

2.7.5 Earth-Referenced Space Volumes

$$R_i^G = \begin{Bmatrix} a_F \cos\lambda_i \cos\phi_i \\ a_F \sin\lambda_i \cos\phi_i \\ (1-F)^2 a_F \sin\phi_i \end{Bmatrix} \qquad ; \; i = 1,2,3,\ldots 5 \qquad (2\text{-}12)$$

where

$$a_F = \frac{R_{em}}{\sqrt{\cos^2\phi_i + (1-F)^2 \sin^2\phi_i}} \qquad (2\text{-}2)$$

$$\vec{C}^G = \frac{\sum_{i=1}^{n} \vec{R}_i^G}{n} \qquad (2\text{-}24)$$

$$\vec{C}_B^G = \vec{C}^G + h_1 \frac{\vec{C}^G}{|\vec{C}^G|} \qquad (2\text{-}25)$$

$$\vec{C}_T^G = \vec{C}^G + h_2 \frac{\vec{C}^G}{|\vec{C}^G|} \qquad (2\text{-}26)$$

$$\vec{R}_{sc}^{TOE} = [RNP]_{M50}^{TOE} \vec{R}_{sc} \qquad (2\text{-}3)$$

$$\vec{R}_{sc}^G = \begin{bmatrix} \cos\Delta\lambda & \sin\Delta\lambda & 0 \\ -\sin\Delta\lambda & \cos\Delta\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} \vec{R}_{sc}^{TOE} \qquad (2\text{-}4)$$

where

$$\Delta\lambda = \omega_e \left(t - t_e\right) \tag{2-5}$$

$$\vec{\rho}_B^{\,G} = \vec{R}_{sc}^{\,G} - \vec{C}_B^{\,G} \tag{2-27}$$

$$\gamma_B = \cos^{-1}\left\{\frac{\vec{\rho}_B^{\,G} \cdot \vec{C}_B^{\,G}}{\left|\vec{\rho}_B^{\,G}\right| \; \left|\vec{C}_B^{\,G}\right|}\right\} \tag{2-28}$$

$$\gamma_B \leq 90^\circ \tag{2-29}$$

$$\vec{\rho}_T^{\,G} = \vec{R}_{sc}^{\,G} - \vec{C}_T^{\,G} \tag{2-30}$$

$$\gamma_T = \cos^{-1}\left\{\frac{\vec{\rho}_T^{\,G} \cdot \vec{C}_T^{\,G}}{\left|\vec{\rho}_T^{\,G}\right| \; \left|\vec{C}_T^{\,G}\right|}\right\} \tag{2-31}$$

$$\gamma_T \geq 90^\circ \tag{2-32}$$

$$\hat{N}_i^G = \frac{\vec{C}_B^{\,G} \times \left(\vec{R}_i^{\,G} - \vec{R}_{i+1}^{\,G}\right)}{\left|\vec{C}_B^{\,G} \times \left(\vec{R}_i^{\,G} - \vec{R}_{i+1}^{\,G}\right)\right|} \qquad i = 1,2,3,4 \tag{2-33a}$$

$$\hat{N}_5^G = \frac{\vec{C}_B^G \times \left(\vec{R}_5^G - \vec{R}_1^G\right)}{\left|\vec{C}_B^G \times \left(\vec{R}_5^G - \vec{R}_1^G\right)\right|} \tag{2-33b}$$

$$d_i = \hat{N}_i^G \cdot \left(\vec{R}_i^G - \vec{C}_B^G\right) \qquad i = 1,2,3,\ldots 5 \tag{2-16}$$

$$\hat{N}_i^G \cdot \vec{\rho}_B^G \leq d_i \qquad i = 1,2,3,\ldots 5 \tag{2-18}$$

2.7.6 Celestial-Fixed Space Volumes

$$\hat{R}_i = \begin{Bmatrix} \cos\alpha_i & \cos\delta_i \\ \sin\alpha_i & \cos\delta_i \\ & \sin\delta_i \end{Bmatrix} \qquad i = 1,2,3,\ldots n \tag{2-19}$$

$$\hat{C} = \frac{\vec{C}}{|\vec{C}|} \tag{2-20}$$

where

$$\vec{C} = \frac{\sum_{i=1}^{5} \hat{R}_i}{5} \tag{2-21}$$

$$\vec{C}_B = \left(R_{em} + h_1\right) \hat{C} \tag{2-34}$$

$$\vec{C}_T = \left(R_{em} + h_2\right) \hat{C} \tag{2-35}$$

$$\vec{\rho}_B = \vec{R}_{sc} - \vec{C}_B \tag{2-27}$$

$$\gamma_B = \cos^{-1} \left\{ \frac{\vec{\rho}_B \cdot \vec{C}_B}{\left|\vec{\rho}_B\right| \left|\vec{C}_B\right|} \right\} \tag{2-28}$$

$$\gamma_B \leq 90^\circ \tag{2-29}$$

$$\vec{\rho}_T = \vec{R}_{sc} - \vec{C}_T \tag{2-30}$$

$$\gamma_T = \cos^{-1} \left\{ \frac{\vec{\rho}_T \cdot \vec{C}_T}{\left|\vec{\rho}_T\right| \left|\vec{C}_T\right|} \right\} \tag{2-31}$$

$$\gamma_T \geq 90^\circ \tag{2-32}$$

$$\hat{N}_i = \frac{\vec{C}_B \times \left(\hat{R}_i - \hat{R}_{i+1}\right)}{\left|\vec{C}_B \times \left(\hat{R}_i - \hat{R}_{i+1}\right)\right|} \quad i = 1,2,3,4 \tag{2-33a}$$

$$\hat{N}_5 = \frac{\vec{C}_B \times \left(\hat{R}_5 - \hat{R}_1\right)}{\left|\vec{C}_B \times \left(\hat{R}_5 - \hat{R}_1\right)\right|} \tag{2-33b}$$

$$d_i = \hat{N}_i \cdot \left(\hat{R}_i - \vec{C}_B\right) \quad i = 1,2,3,\ldots 5 \tag{2-16}$$

$$\hat{N}_i \cdot \vec{\rho}_B \leq d_i \quad i = 1,2,3,\ldots 5 \tag{2-18}$$

3.0 LOGIC FOR COMPUTING AOS AND LOS TIMES

Section 2.0 presented the equations necessary to determine whether the S/C lies within the various area targets or space volumes. This section outlines the method by which these equations will be used to compute the AOS and LOS times. The basic procedure consists of performing a sequential time search between user specified start and end times. Tests will be performed at each time point to determine whether the S/C lies within the area target or space volume. The AOS times are defined to be the time point at which the S/C first enters the area target or space volume. If the S/C lies within the area target or space volume at the beginning of the search, the first AOS time will be set equal to the start time. LOS times are defined to be the last time point prior to the S/C exiting the area target or space volume. If the S/C lies within the area target or space volume at the end of the time search, then the last LOS time will be set equal to the end time. It should be noted that the time increment used to perform the sequential time search will limit the accuracy and resolution of the AOS and LOS times (e.g., if the time increment is 1 minute, this implies that the AOS and LOS times will be determined to the nearest minute and that visibility periods of less than 1 minute may be skipped).

The area targets and space volumes defined in section 1.0 fall into two general categories:

a. Earth-fixed which include
   (1) Earth-referenced circles.
   (2) Earth-referenced polygons.
   (3) Earth-referenced space volumes.
b. Celestial-fixed which include
   (1) celestial circles.
   (2) celestial polygons.
   (3) celestial-fixed space volumes.

The logic for computing the AOS and LOS times for each of these categories is presented in the following subsections.

3.1 EARTH-FIXED AREA TARGETS AND SPACE VOLUMES

Figure 3-1 provides a functional flowchart for the Earth-fixed area targets and space volumes. The required inputs are

a. Target table (either the ground target table, ground polygon table, or ground-fixed space volume table).
b. Target ID.
c. Start and end times and time increment (t_{start}, t_{end} and Δt, respectively) for the sequential time search.
d. S/C ephemeris and ephemeris ID.
e. RNP matrix and associated epoch time.

A brief description of the process is provided below. The heading numbers correspond to the numbered blocks on figure 3-1.

1. The geodetic parameters defining the area target or space volume are obtained from the appropriate target table based upon the input target ID. Sections 2.1.2, 2.3.2, and 2.5.2 describe the specific parameters which are required.
2. The RNP matrix and its associated epoch time are obtained.
3. The geodetic coordinates of the area target or space volume are transformed to the rotating geocentric coordinate system (secs. 2.1.2, 2.3.2, and 2.5.2).
4. The current time, t, is initialized to the start time, t_{start}. The first AOS time, AOS_1, is initialized to zero. The AOS/LOS counter, i, is initialized to one.
5. A loop is established which will terminate when t exceeds the end time, t_{end}.
6. The S/C position vector (at time t in the M50 coordinate system) is obtained from the ephemeris file based upon the ephemeris ID.
7. The S/C position vector is transformed to the rotating geocentric coordinate system and computations are performed to determine whether the S/C lies within the area target or space volume (secs. 2.1.2, 2.3.2, 2.5.2, and the app.).

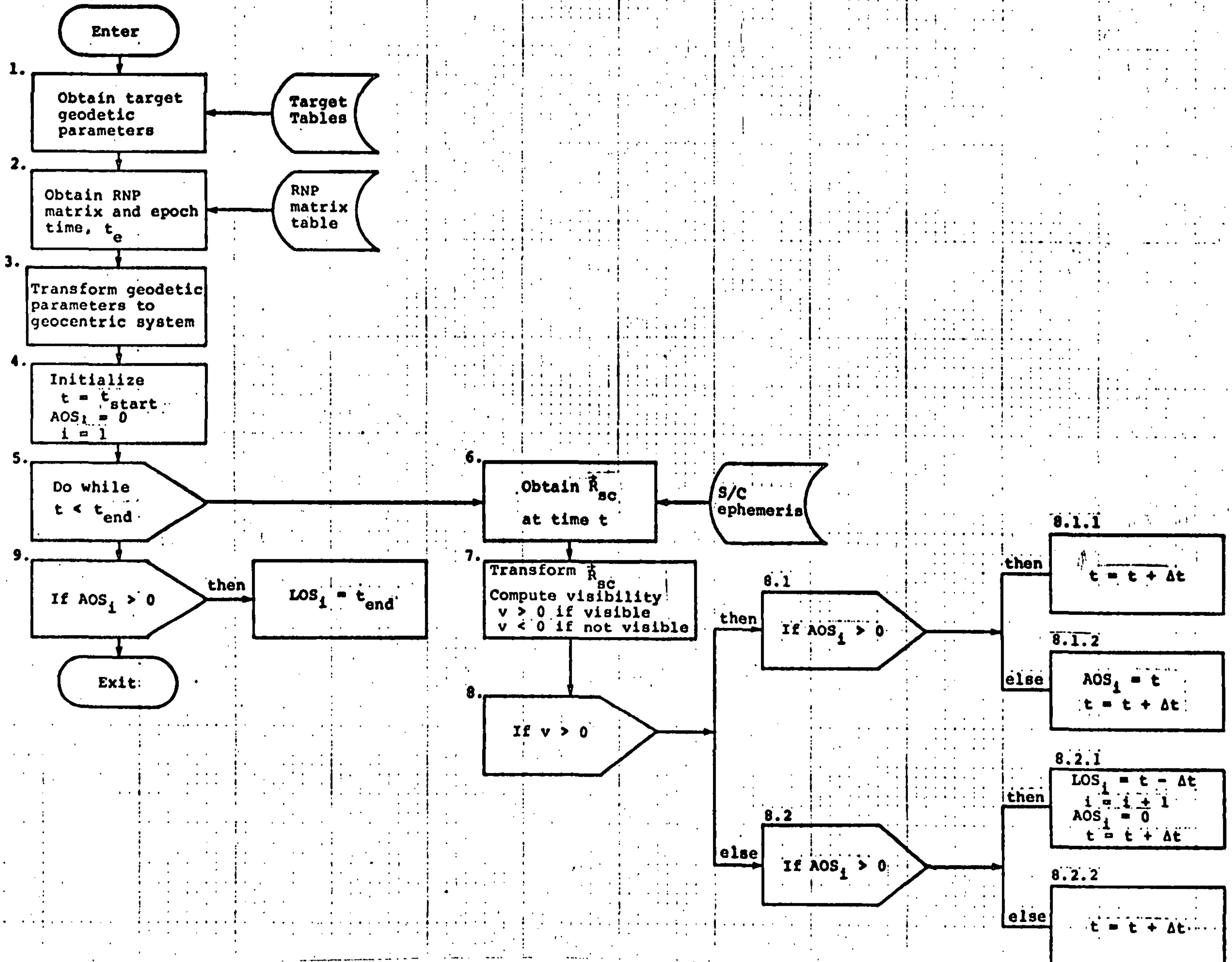


Figure 3-1.- Functional flow for Earth-fixed area targets and space volumes.

Based upon the results of these tests, the S/C visibility parameter, V, is set

> $V > 0$ if the S/C lies within the area target or space volume
>
> $V < 0$ if the S/C lies exterior to the area target or space volume

8. A test is performed on V.

   If $V > 0$

   8.1 a further test is performed on AOS_i to determine whether the S/C was also visible during one or more of the previous time steps.

   If $AOS_i > 0$

   8.1.1 then the S/C was visible during one or more of the previous time steps. Hence, no transition has occurred. The current time is incremented by Δt and the search continues

   If $AOS_i \leq 0$

   8.1.2 then a transition into the area target or space volume has occurred. The current AOS time, AOS_i, is set equal to the current time. The current time is incremented by Δt and the search continues.

   If $V < 0$

   8.2 a further test is performed on AOS_i to determine whether the S/C was visible during the previous time step.

   If $AOS_i > 0$

   8.2.1 then the S/C was visible during the previous time step and a transition out of the area target or space volume has occurred. The current LOS time, LOS_i, is set equal to the time of the previous time step $(t - \Delta t)$. The AOS/LOS counter, i, is incremented by one. The next AOS time is initialized to zero. The current time is incremented by Δt and the search continues.

8.2.2 the S/C was not visible during the preceding time step. Thus no transition has occurred. The current time is incremented by Δt and the search continues.

9. At the completion of the sequential time search, a test is made to determine if the S/C was visible at the end time. If the test is true, the last LOS time is set equal to t_{end}.

3.2 CELESTIAL-FIXED AREA TARGETS AND SPACE VOLUMES

Figure 3-2 illustrates the computational logic for the celestial-fixed area targets and space volumes. This logic is similar to the approach presented in section 3.1. The required inputs are

a. Target table (either the celestial circles table, celestial polygons table, or celestial-fixed space volumes table).
b. Target ID.
c. Start and end times and time increment (t_{start}, t_{end}, and Δt, respectively).
d. S/C ephemeris and ephemeris ID.

A brief description of the process is provided below. The heading numbers correspond to the numbered blocks on figure 3-2.

1. The parameters defining the celestial-fixed area target or space volume are obtained from the appropriate target table based upon the input target ID. Sections 2.2.2, 2.4.2, and 2.6.2 describe the specific parameters which are required.
2. The current time, t, is initialized to the start time, t_{start}. The first AOS time, AOS_1, is initialized to zero. The AOS/LOS counter, i, is initialized to one.
3. A loop is established which will terminate when t exceeds the end time, t_{end}.
4. The S/C position vector (at time t in the M50 coordinate system) is obtained from the ephemeris file based upon ephemeris ID.

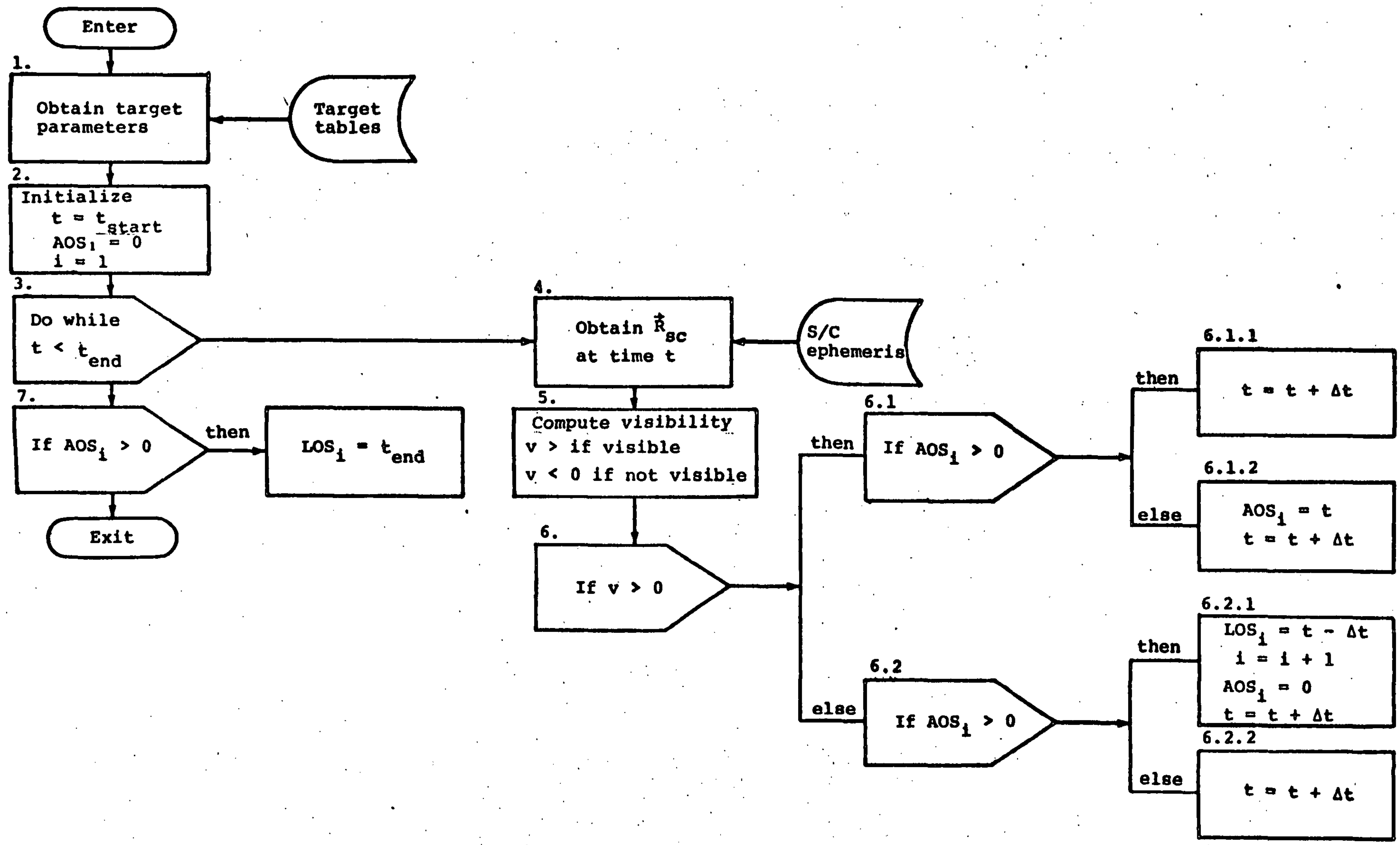


Figure 3-2.- Functional flow for celestial-fixed area targets and space volumes.

5. Computations are performed to determine whether the S/C lies within the area target or space volume (secs. 2.2.2, 2.4.2, 2.6.2, and the app.). Based upon the results of these tests, the S/C *visibility parameter, V, is set*

$V > 0$ if the S/C lies within the area target or space volume

$V < 0$ if the S/C lies exterior to the area target or space volume

6. A test is performed on V.

If $V > 0$

6.1 a further test is performed on AOS_i to determine whether the S/C was also visible during one or more of the previous time steps.

If $AOS_i > 0$

6.1.1 then the S/C was visible during one or more of the previous time steps. Hence, no transition has occurred. The current time is incremented by Δt and the search continues

If $AOS_i \leq 0$

6.1.2 then a transition into the area target or space volume has occurred. The current AOS time, AOS_i, is set equal to the current time. The current time is incremented by Δt and the search continues.

If $V < 0$

6.2 a further test is performed on AOS_i to determine whether the S/C was visible during the previous step.

If $AOS_i > 0$

6.2.1 then the S/C was visible during the previous time step and a transition out of the area target or space volume has occurred. The current LOS time, LOS_i, is set equal to the time of the previous time step $(t - \Delta t)$. The AOS/LOS counter, i, is incremented by one. The next AOS time is initialized to zero. The current time is incremented by Δt and the search continues.

6.2.2 the S/C was not visible during the preceding time step. Thus, no transition has occurred. The current time is incremented by Δt and the search continues.

7. At the completion of the sequential time search, a test is made to determine whether the S/C was visible at the end time. If the test is true, the last LOS time is set equal to t_{end}.

APPENDIX

SUBDIVIDING CONCAVE POLYGONS

APPENDIX

SUBDIVIDING CONCAVE POLYGONS

This appendix discusses the procedure for subdividing complex concave polygons into two or more simpler convex polygons.[1] This subdivision process will permit the equations in section 2.0 of this report to be easily extended to accommodate any arbitrarily shaped polygon. For convenience, the equations and figures presented in this appendix will use a pentagon as an example.[2] However, this approach is easily extended to any n-sided polygon.

Figure A-1 presents three examples of concave pentagons. Figure A-1(a) illustrates a pentagon having one concave vertex. Figures A-1(b) and A-1(c) illustrate pentagons having two concave vertices. These pentagons can always be subdivided into triangles by selecting an "appropriate" vertex and connecting nonadjacent vertices (fig. A-2). The maximum number of triangles necessary to completely subdivide any arbitrarily shaped polygon is

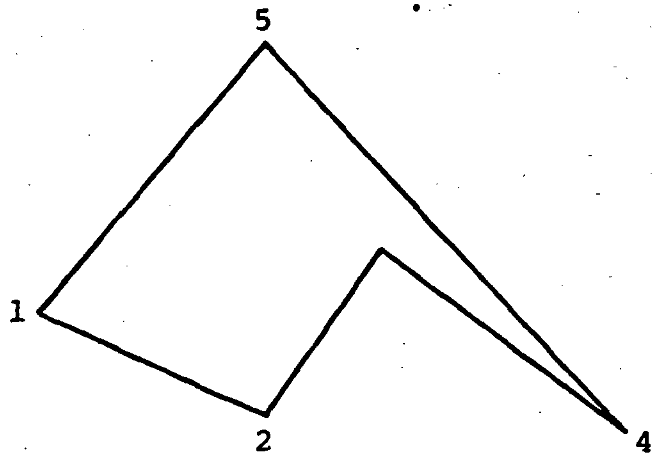
$$N_{seg_{max}} = n - 2 \tag{A-1}$$

where

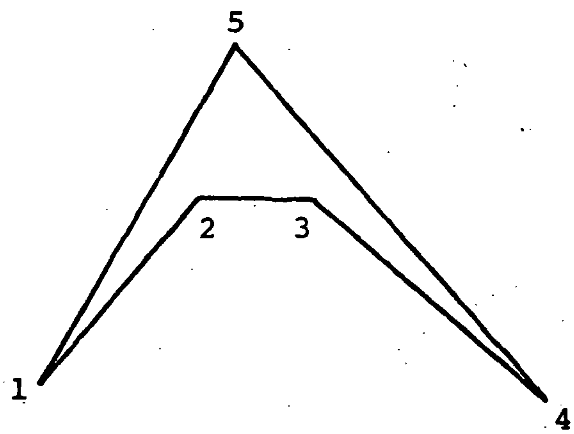
$$n = \text{number of sides}$$

[1]A concave polygon is defined to be a polygon which has one or more interior vertex angles exceeding 180 degrees. A convex polygon is defined to be a polygon which has all of its interior angles less than 180 degrees.

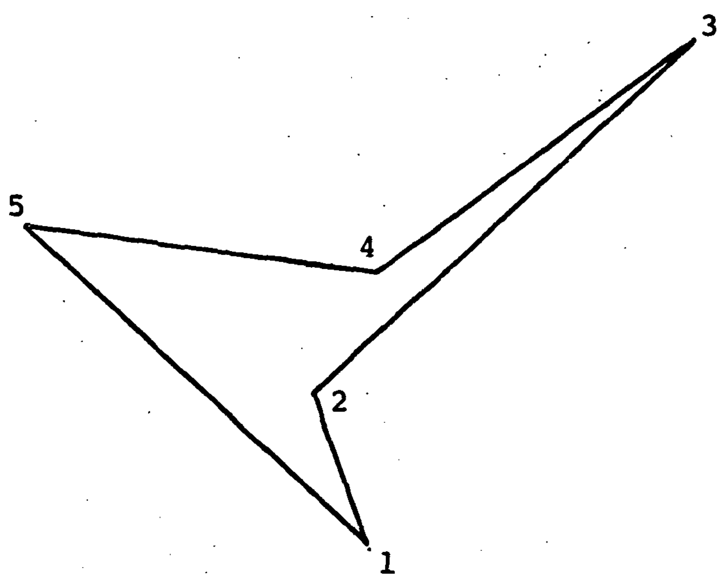
[2]This corresponds to the maximum number of sides specifically addressed in the requirements defined in reference 1.



(a) One concave vertex.

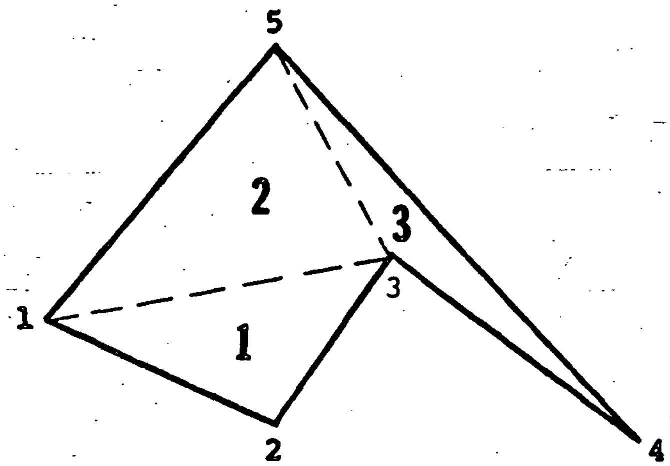


(b) Two adjacent concave vertices.

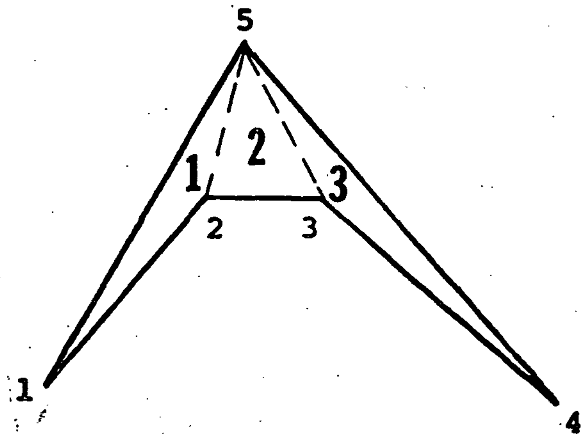


(c) Two nonadjacent concave vertices.

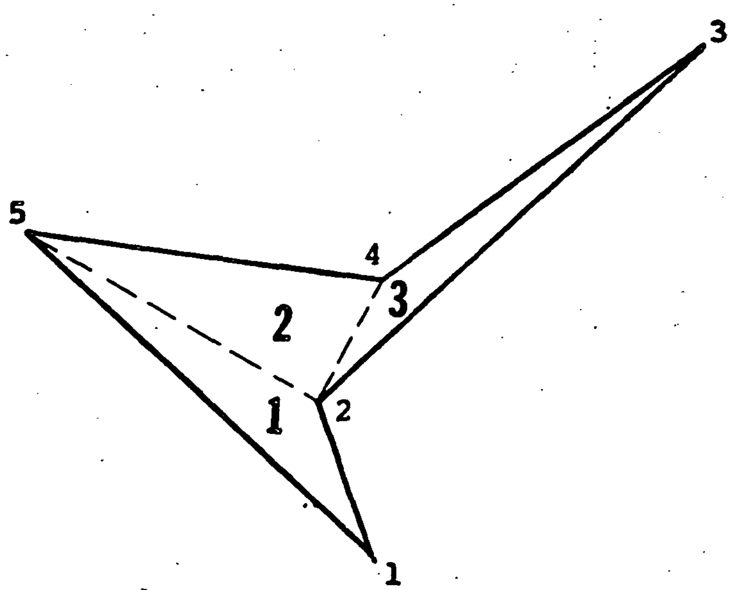
Figure A-1.- Examples of concave pentagons.



(a) One concave vertex.



(b) Two adjacent concave vertices.



(c) Two nonadjacent concave vertices.

Figure A-2.- Examples of subdividing concave pentagons.

Furthermore, the maximum number of interior angles exceeding 180 degrees can also be determined by noting that the sum of the interior angles of the polygon must be equal to the sum of the interior angles of all triangles into which it can be subdivided. Thus, the sum of the interior vertex angles for any arbitrarily shaped polygon is given by

$$\sum_{i=1}^{n} \gamma_i = (n - 2)\ 180 \tag{A-2}$$

where

γ_i = interior vertex angles of the polygon

Thus, the maximum number of interior vertex angles exceeding 180 degrees, γ^*, is given by

$$N(\gamma^*)_{max} = (n - 3) \tag{A-3}$$

This equation limits the maximum number of concave vertices for a pentagon to two. Figures A-1(b) and A-1(c) illustrate two examples. In figure A-1(b), the two concave vertices are adjacent to each other. In figure A-1(c) the two concave vertices are non-adjacent.

The selection of the "appropriate" vertex to begin the subdivision process is highly dependent upon the shape of the polygon and the number and relationship of the concave vertices. Also, it is not always necessary to subdivide the polygon into triangles. Figure A-3 illustrates another method for subdividing the polygon of figure A-1(a). In this case, the concave pentagon is subdivided into a four-sided convex polygon and one triangle. Furthermore, figure A-1 by no means exhausts all of the potential pentagon shapes which could be constructed.

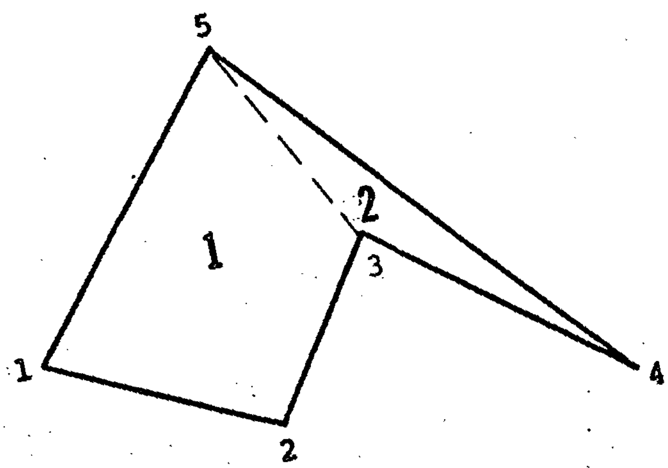


Figure A-3.- Alternate subdivision.

Since the shape of the area targets and space volumes will remain static during a mission, it is recommended that the subdivision process be performed manually.* There are two distinct advantages to this approach:

a. It eliminates the coding and execution of complex subdivision logic.
b. It can be performed once for each concave area target and space volume and does not have to be repeated each time AOS and LOS times are desired.

The treatment of concave polygons will place additional requirements on the targets tables other than those specifically mentioned in reference 1. In addition to the number of sides and coordinates for each vertex, the target tables must also contain the following for each polygon-shaped target

N_{seg} - number of segments into which the target is subdivided ($3 \geq N_{seg} \geq 1$ for polygons having five or less sides)

VO_i - integers defining the counterclockwise ordering of the vertices for each segment ($i = 1,2,...N_{seg}$)

The use of these additional parameters can best be illustrated by example. For figure A-3, this pentagon is subdivided into two segments. The first segment is a four-sided polygon defined by vertices 1, 2, 3 and 5. The second segment is a triangle defined by vertices 3, 4, and 5. The corresponding parameters for this pentagon would be

$$N_{seg} = 2$$

$$VO_1 = 1235 \text{ (or 2351 or 3512 or 5123)}$$

$$VO_2 = 345 \text{ (or 453 or 534)}$$

*This can easily be performed by plotting the vertex points on a Mercator projection.

Similarly, for figures A-2(b) and A-2(c):

a. Figure A-2(b)

$$N_{seg} = 3$$

$$VO_1 = 125 \text{ (or 251 or 512)}$$

$$VO_2 = 235 \text{ (or 352 or 523)}$$

$$VO_3 = 345 \text{ (or 453 or 534)}$$

b. Figure A-2(c)

$$N_{seg} = 3$$

$$VO_1 = 125 \text{ (or 251 or 512)}$$

$$VO_2 = 245 \text{ (or 452 or 524)}$$

$$VO_3 = 234 \text{ (or 342 or 423)}$$

For consistency, this approach can also be used for convex polygons. In this case, N_{seg} would be one and VO_1 would be set to the counterclockwise vertex sequence.

For computational purposes, the number of sides for each segment, n_i, can be extracted from the vertex ordering integer, VO_i, as follows

$$n_i = \text{highest values of } n_i \text{ where TRUNC} \left\{ \frac{VO_i}{10^{n_i - 1}} \right\} > 0 \qquad \text{(A-4)}$$

where

TRUNC implies integer truncation.

The vertex numbers, V_j, corresponding to each vertex of the i^{th} subpolygon can also be extracted from the vertex ordering integer as follows

$$V_1 = \text{TRUNC}\left\{\frac{VO_i}{10^{n_i-1}}\right\} \qquad \text{(A-5a)}$$

$$V_j = \text{TRUNC}\left\{\frac{VO_i - \sum_{\ell=1}^{j-1} V_\ell \, 10^{n_i-\ell}}{10^{n_i-j}}\right\} \; j = 2,3,\ldots n_i \qquad \text{(A-5b)}$$

The criterion for the S/C (or S/C subsatellite point) to be contained in the concave area target or space volume is that it must be contained in any one of the subpolygons. The equations presented in section 2.0 of this report can be used on a segment-by-segment basis to test for containment.

REFERENCE

1. Torres, F: Statement of Requirements (SR) for Area Targets and Space Volumes. CG5-78-342, December 13, 1978.

TECH PUBS
FINAL
ed. SL
date 3/1/79