CORONAL HEATING BY STOCHASTIC MAGNETIC PUMPING

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ABSTRACT

Recent observational data cast serious doubt on the widely held view that the sun's corona is heated by traveling waves (acoustic or magnetohydrodynamic). It is here proposed that the energy responsible for heating the corona is derived from the free energy of the coronal magnetic field derived from motion of the "feet" of magnetic field lines in the photosphere. Stochastic motion of the feet of magnetic field lines leads, on the average, to a linear increase of magnetic free energy with time. This rate of energy input is calculated for a simple model of a single thin flux tube. The model appears to agree well with observational data if the magnetic flux originates in small regions of high magnetic field strength as proposed by Tarbell, Title and Schoolman. On combining this energy input with estimates of energy loss by radiation and of energy redistribution by thermal conduction, we obtain scaling laws for density and temperature in terms of length and coronal magnetic field strength.

I. INTRODUCTION

Following the general acceptance of the concept of a very hot corona in 1945 (Billings, 1966), Biermann (1948) and Schwarzschild (1948) independently suggested that the corona is heated by acoustic waves. However, recent observational data appear to make this hypothesis untenable.

Athay and White (1979), in their analysis of UV spectroscopic data obtained by means of the OSO-8 Spacecraft, argue that the acoustic wave flux in the chromosphere cannot exceed about 10^4 erg cm^{-2} s^{-1} whereas the radiation losses of the transition region and corona require an energy flux of $10^{5.7}$ erg cm^{-2} s^{-1}. Bruner (1980), from an independent analysis of OSO-8 UV spectroscopic data, argues that most of the wave motion in the transition region is in the form of standing waves (probably evanescent waves) rather than traveling waves, so that the net acoustic flux is at least three orders of magnitude lower than that needed to heat the corona.

The presence of a magnetic field complicates the discussion. It is possible that the change is minor, in that energy still propagetes as waves, but these are magneto-acoustic waves and/or Alfven waves (Osterbrock, 1961; Stein and Leibacher, 1974). If the waves are magnetoacoustic, we still have a problem in accounting for the required energy flux. If the waves are Alfven waves, then there is a problem in accounting for their dissipation (Stein and Leibacher, 1974) although Uchida and Kaburaki (1974) have argued that large-amplitude Alfven waves may convert to magneto-acoustic waves at coronal heights and thereby be dissipated.

On the other hand, the close association between magnetic field strength and coronal heating suggests that the magnetic field may play an active role, rather than a passive role, in the energy transport process. For instance, it is notable that the Rosner-Tucker-Vaiana model of hot coronal loops (Rosner, Tucker and Vaiana, 1978) seems to fit not only large-scale loop structures and active region loops, but also coronal loops produced by solar flares. Although this model deals only with the relationship between energy transport by heat conduction and by radiation, it does raise the question as to whether coronal loops are heated by some mechanism akin to that responsible for solar flares, which is believed to be the release of free magnetic energy by dissipative plasma processes (Sturrock, 1980). It is quite possible that the explosive nature of flares is due to the sudden rearrangement of magnetic field by an MHD instability: heating (and possibly acceleration) which occur during a flare may be due to dissipative processes which may also occur at a slower rate in the "quiet" solar atmosphere.

It appears that there are two requirements for the propagation of preflare energy from the photosphere into the corona: there must be a magnetic field, and the footpoints of the magnetic field must be moved in such a way that the configuration is raised in energy from the current-free "ground state" to the current-carrying "excited state" which, in the early stage of excitation, may with good approximation be taken to be a force-free magnetic-field configuration. These requirements can be met equally well in the "quiet" solar atmosphere: the photosphere is always permeated by magnetic field, and the photosphere is always in motion due to granulation, supergranulation and other motions. The aim of this article is to estimate the resulting flux of energy from the photosphere into the corona and to incorporate this assumption into the Rosner-Tucker-Vaiana model so as to obtain estimates of the density and temperature of the plasma in a coronal loop in terms of the basic parameters of the loop.

This possibility has recently been addressed also by Golub et al.

(1980). They begin with an analysis of observational data which indicates that there is a relationship between the plasma pressure in a coronal loop and the magnetic field strength in the loop, which they assert to be $p \propto B^{1.5}$. They estimate the rate of energy supply into a coronal loop in terms of the mean transverse (toroidal) magnetic field strength and the mean transverse (torsional) velocity at the photosphere, but without consideration of the cause-and-effect relationship of these two quantities or their stochastic nature. Analysis of the data contained in their Fig. 2(b) yields a pressure-magnetic-field-strength relationship of the form

$$p \propto B^b, \; b = 0.77 \pm 0.23. \qquad (1.1)$$

This is a better fit to the relationship obtained in this article ($b = 6/7$, see Section 5), than the value $b = 12/7$ found by Golub et al. (1980).

II. TWISTED FLUX TUBE

We wish to consider the response of a coronal flux tube to photospheric motion. Since the free energy available for heating the plasma is due to currents, we consider in particular a twisting motion such as would result from a rotation of either or both ends of the flux tube at the photosphere. For simplicity, we consider a tube which is thin compared with its length. We also simplify the calculation by considering a tube for which the central field line is a straight line and the tube has cylindrical symmetry. As indicated in Section I, we assume that the plasma density is sufficiently low that the field is approximately in a force-free state.

It will be seen that the model we are considering is very close to that of Gold and Hoyle (1960) except that we are allowing for slow variation of the field with respect to the z coordinate of cylindrical coordinates (z,r,ϕ). On the other hand, we will simplify the model by considering only the lowest order significant terms in polynomial expressions of quantities in terms of r.

With these restrictions, we find that the solution of the equations

$$\vec{\nabla}.\vec{B} = 0 \tag{2.1}$$

and

$$\vec{B}\times(\vec{\nabla}\times\vec{B}) = 0 \tag{2.2}$$

may be expressed, to lowest significant order in r, as

$$\left.\begin{aligned} B_z(z,r) &= B_o(z) - \left[\frac{1}{4} B_o''(z) + b^2 B_o(z)\right] r^2, \\ B_r(z,r) &= -\frac{1}{2} B_o'(z)\, r, \\ B_\phi(z,r) &= b\, B_o(z)\, r. \end{aligned}\right\} \tag{2.3}$$

We see that each field line is rotated about the axis by an angle $\Delta\chi$ given by

$$\Delta \chi = b L, \tag{2.4}$$

when L (cm) is the length of the tube.

If the radius of the flux tube is R(z), the magnetic flux passing through the tube is given by

$$\Phi = \pi B_o R^2 - \frac{\pi}{8} B_o'' R^4 - \frac{\pi}{2} b^2 B_o R^4, \tag{2.5}$$

and the magnetic pressure at the surface of the tube is given by

$$p = \frac{1}{8\pi} B_o^2 + \frac{1}{8\pi} \left\{ \frac{1}{4} (B_o')^2 - \frac{1}{2} B_o B_o'' - b^2 B_o^2 \right\} R^2. \tag{2.6}$$

Let us now consider a flux tube, which is initially untwisted so that b = 0, immersed in a plasma so that the magnetic pressure at the surface of flux tube is balanced by an equal gas pressure outside the tube. Now suppose that the tube is twisted so that $b \neq 0$, but we require that the flux Φ is unchanged and that the magnetic pressure p at r = R is unchanged, since it is still balanced by the same external gas pressure. Then we must expect that $B_o(z)$ and R(z) will both change. It turns out, however, that to lowest order (quadratic) in b, R(z) is unchanged. On the other hand, $B_o(z)$ is found to increase quadratically with b according to

$$\left(\partial^2 B_o / \partial b^2 \right)_{b = 0} = B_o R^2 . \tag{2.7}$$

The "free energy" ΔW of the flux tube is the magnetic energy of the twisted tube less the magnetic energy of the untwisted tube. To lowest order in b, this is found to be

$$\Delta W = \frac{b^2}{16} \int dz \, B_o^2 R^4 , \tag{2.8}$$

which shows that the free energy is distributed uniformly along the length of the twisted flux tube. By using equations (2.4) and (2.5), we may re-express (2.8) as

$$\Delta W = \frac{\Phi^2(\Delta\chi)^2}{16\pi^2 L} . \tag{2.9}$$

III. STOCHASTIC MOTION

We now consider the twisting of a flux tube due to stochastic motions at the photosphere. The general surface motion may be divided into two components, according to the equation

$$\vec{v} = -\vec{\nabla}\Psi + \vec{\nabla}\times(A\vec{n}) \qquad (3.1)$$

where $\vec{n}$ is the unit vector normal to the surface. The first term on the right-hand side denotes the "curl-free" component and the second term denotes the "vortical" component.

We need to consider the transfer of energy from the photosphere into the magnetic field due to a random (horizontal) velocity field in the photosphere. A general treatment will be published at a later date. At this time, we undertake only a simplified analysis.

We assume that the velocity field is statistically homogenous and isotopic, and that half of the energy in the velocity field is in curl-free motion and half in vortical motion. Only the latter component contributes to twisting of magnetic field lines. We therefore consider an elementary contribution to the twisting by considering the rotation of one end of the flux tube where it meets the photosphere. Another contribution to the stored energy will come from the other end of the flux tube.

If the vorticity at the center of the flux tube where it meets the photosphere is ω,

$$v_\phi = \omega r \qquad (3.2)$$

and the contribution to the twisting due to rotation at one end only is given by

$$\Delta\chi = \int_0^t \omega(t')dt'. \qquad (3.3)$$

This leads to the rate of change of energy given by

$$\frac{dW}{dt} = \frac{\Phi^2}{16\pi^2 L} \left\langle \frac{(\Delta\chi)^2}{\Delta t} \right\rangle . \qquad (3.4)$$

We see from equation (3.3) that

$$\left\langle (\Delta\chi)^2 \right\rangle = \int_t^{t+\Delta t} dt' \int_t^{t+\Delta t} dt'' \left\langle \omega(t') \, \omega(t'') \right\rangle . \qquad (3.5)$$

This becomes

$$\left\langle (\Delta\chi^2) \right\rangle \approx \Delta t \int_{-\infty}^{\infty} d\tau \; R(\tau) \qquad (3.6)$$

if

$$R(\tau) = \left\langle \omega(t)\omega(t+\tau) \right\rangle \qquad (3.7)$$

and Δt is large compared with the range of τ over which $R(\tau)$ is significant.

Hence if the "correlation time" τ_c is defined by

$$\int_0^{\infty} d\tau \; R(\tau) = R(0)\tau_c \qquad (3.8)$$

and if $\Delta t \gg \tau_c$, equation (3.6) becomes

$$\left\langle \frac{(\Delta\chi)^2}{\Delta t} \right\rangle = 2 \, R(0)\tau_c = 2 \left\langle \omega^2 \right\rangle \tau_c . \qquad (3.9)$$

If, considering only one end of the flux tube, the radius of the flux tube is $R_\star$ and the field strength is $B_\star$ at the photosphere, then

$$\Phi \left\langle \frac{(\Delta\chi)^2}{\Delta t} \right\rangle = 2\pi B_\star R_\star^2 \left\langle \omega^2 \right\rangle \tau_c . \qquad (3.10)$$

The combination $\langle \omega^2 \rangle R_\star^2$ is the mean square vortical velocity at the circumference of the flux tube, which is twice the mean square vortical component of the velocity over the flux tube. Hence it is the same as the mean square velocity $\langle v^2 \rangle$, including both vortical and curl-free components.

If we now consider both ends of the flux tube and assume that the field strength is the same at both ends, we find that the rate of increase of energy in the flux tube is given by

$$\frac{dW}{dt} = \frac{\Phi B_*}{4\pi L} \langle v^2 \rangle \tau_c . \tag{3.11}$$

Now suppose that R_c is the mean radius of the flux tube in the corona defined by

$$V = \pi R_c^2 L \tag{3.12}$$

where V is the volume of the flux tube. Suppose also that the mean field strength B_c is defined by

$$\Phi = \pi R_c^2 B_c . \tag{3.13}$$

Then if ε_I (erg cm^{-3} s^{-1}) is the mean rate of energy input into the flux tube, defined by

$$\frac{dW}{dt} = \varepsilon_I V, \tag{3.14}$$

we see that

$$\varepsilon_I = K B_c L^{-2} \tag{3.15}$$

where

$$K = \frac{1}{4\pi} B_* \langle v^2 \rangle \tau_c . \tag{3.16}$$

IV. SCALING LAWS FOR CORONAL LOOPS

We now consider a coronal loop and attempt to obtain scaling laws for the coronal density and temperature by investigating the energy balance. We de... by n_o (cm^{-3}) and T_o (K) the density and temperature at the top of the loop. For simplicity, we consider only loops which are sufficiently small that they are approximately isobaric. Then, if the mean density and temperature are n_c and T_c,

$$n_o T_o = n_c T_c . \tag{4.1}$$

We follow Rosner, Tucker and Vaiana (1978) in approximating the radiation energy-loss function ε_R ($erg\ cm^{-3}\ s^{-1}$) by

$$\varepsilon_R = \Gamma n_c^2 T_c^{-1/2} \tag{4.2}$$

wherein $\Gamma \approx 10^{-18.8}$.

If energy is deposited primarily near the top of a loop, it would be carried to the lower regions of the loop by heat conduction. If we denote by ε_H ($erg\ cm^{-3}\ s^{-1}$) the rate at which energy is extracted from the upper regions of the loop by heat conduction for transfer to the lower regions of the loop, then

$$\varepsilon_H = \Theta T_o^{7/2} L^{-2}. \tag{4.3}$$

Noting that the "temperature scale height" is of order 1/2 L, and that the thermal conduction coefficient for a fully ionized plasma (Spitzer, 1962) is approximately $10^{-6} T^{5/2}$, we see that $\theta \approx 10^{-5.4}$.

If we now assume that

$$T_c = \delta T_o, \tag{4.4}$$

we may obtain a relationship between n_o and T_o by assuming that $\varepsilon_R \approx \varepsilon_H$. This relation is found to be

$$T_o = \Theta^{-1/4} \, \Gamma^{1/4} \, \delta^{-5/8} \, n_o^{1/2} \, L^{1/2}. \tag{4.5}$$

Since the mean rate of energy input must be balanced by the mean rate of energy output (by radiation), we obtain what should be a more reliable relationship by equating the expressions in equations (3.15) and (4.2). This leads to the relation

$$n_c^2 \, T_c^{-1/2} = \Gamma^{-1} \, K B_c \, L^{-2} \tag{4.6}$$

for n_c and T_c.

Alternatively, pursuing the assumption that $\varepsilon_R \approx \varepsilon_H$, we may set $\varepsilon_H \approx \varepsilon_I$ and so obtain, from equation (3.15) and (4.3), the expression

$$T_o = \Theta^{-2/7} \, K^{2/7} \, B_c^{2/7}. \tag{4.7}$$

V. DISCUSSION

Equation (4.5) has the same form as a relation obtained by Rosner, Tucker and Vaiana (1978). The more detailed model analysis of Vesecky, Antiochos and Underwood (1979) leads to a similar relationship with the numerical form

$$T_o = 10^{-3.25} n_o^{1/2} L^{1/2}. \qquad (5.1)$$

We find that this agrees with equation (4.5) if we adopt the proposed values of Θ and Γ and adopt $\delta \approx 10^{-0.15}$, i.e. $T_c/T_o \approx 0.7$, which seems not unreasonable.

Equation (4.7) is of special interest in that it leads to a relationship between T_o and B_c, independent of the length L. Although we have many magnetograph maps of the solar disk, unfortunately we do not know the magnetic field strength at the top of a coronal loop. Golub et al. (1979) have recently presented data for the coronal temperatures of three typical features: x-ray bright points, active regions, and large-scale structures. The data presented by Golub et al. (1979) suggests the approximate numerical relationship

$$T_o \approx 10^{5.9 \pm 0.1} B_c^{2/7}. \qquad (5.2)$$

In order for equation (4.7) to have this numerical form, we require that $K \approx 10^{15.2 \pm 0.4}$. We see from equation (3.16) that this requires the following combination of parameters at the photosphere:

$$B_* \langle v^2 \rangle \tau_c \approx 10^{16.3 \pm 0.4}. \qquad (5.3)$$

Tarbell, Title and Schoolman (1979) propose that most of the magnetic field in the solar atmosphere arises from small knots of intense field of strength $B_* \approx 1200$ and that the r.m.s. velocity field in the photosphere is about 10^5 cm s^{-1}. Hence our theory leads to the empirical relationship (5.2) if the correlation time for the photospheric velocity field has the value $\tau_c \approx 10^{3.3 \pm 0.4}$ s, i.e. in the range 10 - 80 m. The lower limit of this range is comparable with current estimates (8 m) of the mean lifetime of granules (Allen, 1973).

At this time, observational data concerning photospheric motions and the photospheric magnetic field, and observational data concerning magnetic-field strength at coronal levels, are insufficiently precise to provide a definitive check of the expression (4.7) for the coronal temperature in terms of the coronal magnetic field strength. In any case, relations derived in Sections IV and V depend on highly idealized assumptions concerning the structure and energy balance of coronal loops. The present estimates have been made simply to show that the heating mechanism proposed in this article is not obviously ruled out by observational data. In order to obtain more detailed consequences for comparison with observational data, we intend to incorporate the heating mechanism discussed in Section III into the model analyzed by Vesecky, Underwood and Antiochos (1979). In this way we shall avoid certain restrictive assumptions, such as constant pressure and the assumption that $\varepsilon_H \approx \varepsilon_R$, and we shall be able to include the important effect of magnetic-field geometry.

Nevertheless, the fact that the models of Vesecky, Antiochos and Underwood (1979) lead to a scaling law similar to that derived, on a simpler basis, by Rosner, Tucker and Vaiana (1978) give some cause for optimism that the scaling laws derived on the basis of the restrictive assumptions of the current article may also prove to have wider applicability than might initially be expected. Ignoring the numerical coefficients, we see from equations (4.6) and (4.7) that the density and temperature in a coronal loop will scale with the length of the loop and the coronal magnetic-field strength as

$$n_c \propto B_c^{4/7} L^{-1} \tag{5.4}$$

$$T_c \propto B_c^{2/7}, \tag{5.5}$$

so that

$$p_c \propto B_c^{6/7} L^{-1}. \tag{5.6}$$

It is notable that the index 6/7 (0.86) appearing in the scaling relation (5.6) is quite consistent with the value 0.77 ± 0.23 derived from the data of Golub et al. (1980), as noted in equation (1.1), and is in fact a better fit than the value 12/7 which they derived from their theoretical analysis.

It will be interesting to see whether new data, such as may be obtained from the Solar Maximum Mission, will substantiate the above scaling laws.

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REFERENCES

Allen, C.W. 1973, *Astrophysical Quantities* (London: Athlone Press), p. 188.

Athay, R.G., and White, O.R. 1979, *Ap. J. Supp.* 39, 333.

Biermann, L. 1948, *Z. Astrophys.* 25, 161.

Billings, D.E. 1966, *A Guide to the Solar Corona* (New York: Academic Press), p. 9.

Bruner, E.C. 1979, *B.A.A.S.* 11, 697.

Gold, T., and Hoyle, F. 1960, *M.N.R.A.S.* 120, 7.

Golub, L., Maxson, C., Rosner, R., Serio, S., and Vaiana, C.S. 1980, preprint.

Osterbrock, D.E. 1961, *Ap. J.* 134, 347.

Rosner, R., Tucker, W.H., and Vaiana, G.S. 1978, *Ap. J.* 220, 643.

Schwartzshild, M. 1948, *Ap. J.* 107, 1.

Spitzer, L. 1962, *Physics of Fully Ionized Gases* (New York: Interscience), p. 143.

Stein, R.F., and Leibacher, J. 1974, *Ann. Rev. Astron. Astrophys.* 12, 407.

Sturrock, P.A., ed. 1979, *Solar Flares* (Boulder: University of Colorado Press).

Tarbell, T.D., Title, A.M., and Schoolman, S.A. 1979, *Ap. J.* 229, 387.

Uchida, Y., and Kaburaki, O. 1974, *Solar Phys.* 35, 451.

Vesecky, J.F., Antiochos, S., and Underwood, J.H. 1979, *Ap. J.* 223, 987.