AUTONOMOUS SATELLITE ORBIT DETERMINATION DURING THE DEVELOPMENT PHASES OF THE GLOBAL POSITIONING SYSTEM*

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ABSTRACT

An onboard navigation system was developed to aid the design and evaluation of algorithms used in autonomous satellite navigation with Global Positioning System (GPS) data. The performance of the algorithms designed for a GPS Receiver/Processor Assembly (R/PA) intended for Landsat-D was investigated during the development phases of the GPS (four to six satellites in the constellation). This evaluation emphasized the effects on the orbit determination accuracy of the expected user clock errors, GPS satellite visibility, force model approximations, and state and covariance propagation approximations. Results are presented giving the sensitivity of orbit determination accuracy to these constraints.

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INTRODUCTION

The Navstar Global Positioning System (GPS) is a Department of Defense program that will provide navigation information to properly equipped users. A constellation of up to 24 satellites in 12-hour orbits will broadcast coded signals from which the user's position can be determined. The application of GPS to onboard satellite navigation has been previously discussed (References 1, 2). As part of the evaluation of the feasibility of autonomous satellite orbit determination using GPS, an experimental GPS Receiver/Processor Assembly (R/PA) will be placed on Landsat-D, and the resultant orbital solution will be compared to that obtained using more conventional ground-based techniques. This experiment will be conducted during the early phases of GPS, during which there will be four to six GPS satellites available.

The R/PA design proposed for spacecraft applications (Reference 3) consists of a dual-channel receiver and a Digital Equipment Corporation (DEC) LSI-11 processor. The R/PA measures pseudorange and delta pseudorange observations from the GPS signals, estimates the corresponding observations using the GPS navigation message, and uses the observation residuals in a UDU^T formulation of the extended Kalman filter (EKF) to determine the user spacecraft's position, velocity, clock bias and bias rate, and satellite drag coefficient.

Simulation studies are in progress to determine the accuracy attainable with this use of the GPS data, to identify and evaluate the primary sources of error, and to examine the algorithms in the proposed R/PA. As an aid to such studies, an onboard navigation package simulator (ONPAC) was developed on a DEC PDP-11/70 computer, which has computational accuracy similar to that of the LSI-11. The simulator is designed for both premission planning and real-time analysis as well as evaluation of the GPS receiver algorithms.

The simulator is being used in this study to determine the factors affecting the optimum performance of the onboard processor that are mission independent.

Some of these results are presented here. The topics studied include data editing, residual smoothing, fading of the filter memory, clock modeling, state process noise covariance modeling, and GPS selection.

As an aid to the use and evaluation of GPS pseudorange and delta pseudorange observations, the capabilities to simulate and use these observation types were built into the Research and Development Goddard Trajectory Determination System (R&D GTDS). GPS observation can be simulated with R&D GTDS for both ONPAC and R&D GTDS use.

An overview of the steps involved in simulation and use of GPS data are summarized in Figure 1. Both truth model information and simulated data are passed to the ONPAC program. There, the orbit estimation is done, and the estimated trajectory is compared to the truth model.

DATA SIMULATION

The force model used in generating the true user ephemeris can be selected from the options available to the R&D GTDS EPHEM program (Reference 4). These include geopotential harmonic coefficients (up to 21-by-21), drag, solar radiation pressure, and perturbations from the Sun, the Moon, and the other planets.

Data simulation options for parameters affecting the data accuracy are listed in Figure 2. It should be noted that GPS satellites can be scheduled for specific subsets of the total simulation time span. If a GPS satellite is scheduled for observations only during periods when it is not visible, it is "scheduled out" of the data set. The default inclination will also be a modifiable option in the future.

The information passed to ONPAC is summarized in Figure 3. Because all the computations in ONPAC are done in the Earth-centered Earth-fixed (ECEF) coordinate system, this information is in ECEF coordinates.

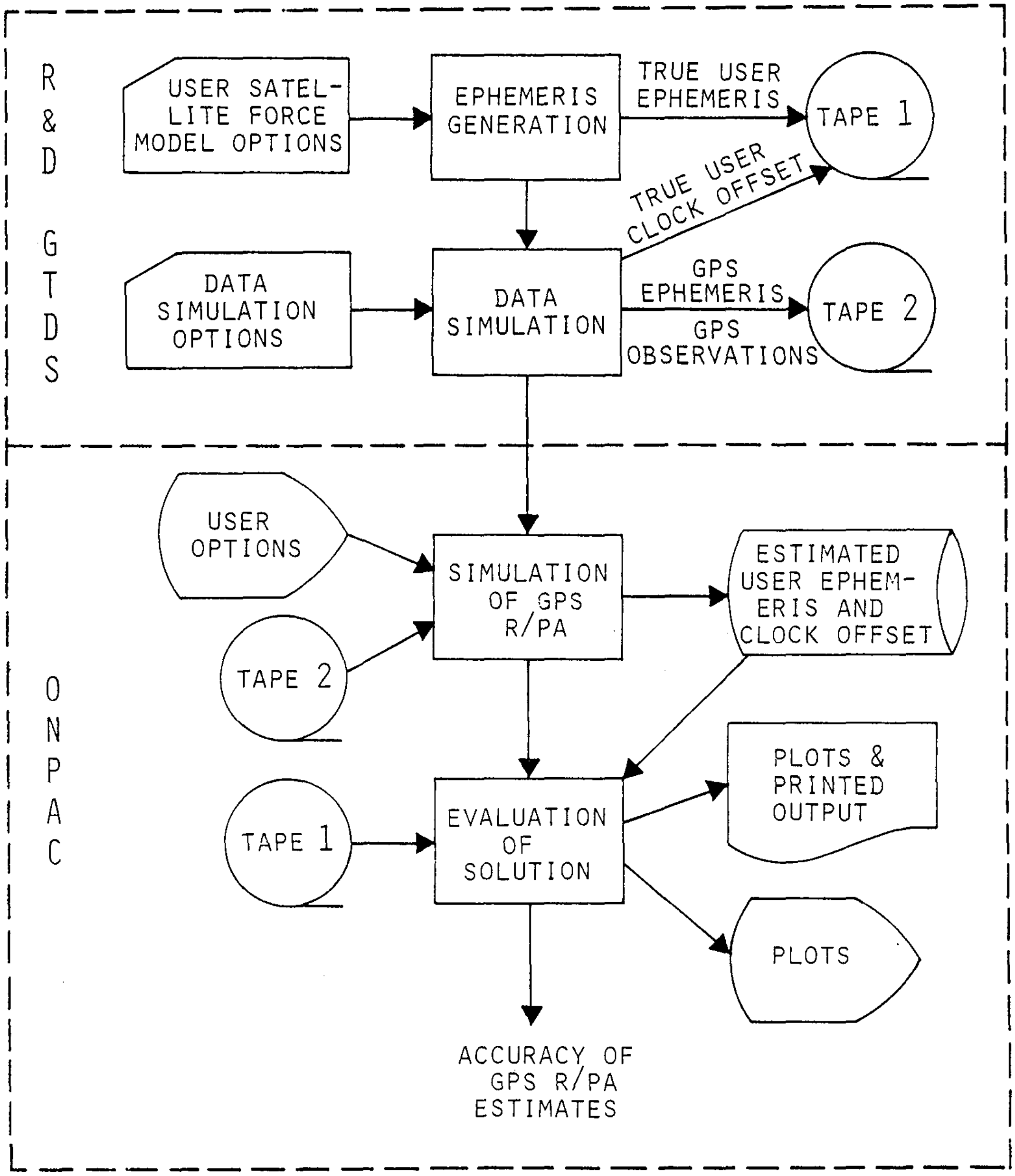

Figure 1. Overview of Analysis Approach

- USER CLOCK ERROR MODEL OPTIONS:
  - No error
  - Quadratic
  - Random walk
- OBSERVATION MEASUREMENT ERRORS
- GPS CLOCK ERROR MODEL OPTIONS:
  - No error
  - Constant bias, uncorrelated
  - Constant bias, correlated (i.e., all GPS clocks have the same error)
- GPS CONFIGURATION OPTIONS:
  - 1 to 24 GPSs (Defaults: 6 in phase I, 12 in phase II, 24 in phase III)
  - 3 orbit planes
  - All orbits circular; inclination = 63 degrees; 12-hour periods
- GPS DATA SPACING OPTIONS:
  - Δt_1 (from Ω to $\Delta\Omega$)
  - Δt_2 (from GPS n to GPS $n+1$)
  - Δt_3 (from last GPS in constellation to GPS #1)
- GPS SELECTION:
  - All observable
  - Geometric dilution of precision (GDOP)
  - Each GPS may be scheduled for subset(s) of the total simulation time span
- GPS EPHEMERIS ERROR OPTIONS:
  - None
  - Random constants for radial and cross-track (H,C); linearly increasing along-track (L) to a randomly selected maximum:
    - -- Uncorrelated
    - -- Orbit-wise correlated
    - -- Totally correlated
  - Sinusoidal:
    - -- Input is H,C,L amplitudes, period (P), and along-track rate ($\dot{L}$)
    - -- Different phase offset for each GPS, computation based on number of GPSs in the configuration

Figure 2. R&D GTDS GPS Data Simulation Options

FOR EACH OBSERVATION:

t_{OBS}	= TIME OF OBSERVATION (INCLUDING USER CLOCK OFFSET)
$T(t_k)$	= USER CLOCK OFFSET = $t_{OBS} - t_k$
$T^D(t_k)$	= USER CLOCK DRIFT AT t_k
ρ_{OBS}	= "OBSERVED" OBSERVATION (PSEUDORANGE)
ρ	= TRUE OBSERVATION (PSEUDORANGE)
$\Delta\rho_{OBS}$	= "OBSERVED" OBSERVATION (DELTA PSEUDORANGE)
$\Delta\rho$	= TRUE OBSERVATION (DELTA PSEUDORANGE)
$\underline{S}_{GPS}$, $\dot{\underline{S}}_{GPS}$	= GPS POSITION AND VELOCITY VECTORS IN ECEF COORDINATES, INCLUDING THE EFFECT OF GPS EPHEMERIS ERRORS
$\underline{r}$, $\underline{v}$	= TRUE USER POSITION AND VELOCITY VECTORS IN ECEF COORDINATES

GPS SATELLITE IDENTIFICATION

TRUTH MODEL INFORMATION PROVIDED DURING DATA GAPS

Figure 3. Simulated Data Produced for ONPAC From R&D GTDS

ONPAC ESTIMATION

The ONPAC estimation is done with the UDU^T form of the EKF, as described in the mathematical specifications (Reference 5). The estimation process is briefly described in Figure 4. The a priori state and covariance matrix can be either the values from the last observation processed or the input values. The integration of the satellite equations of motion is done with a modified Euler integrator. The state transition matrix is computed with a Taylor series approximation. Studies have demonstrated that these propagation techniques have sufficient accuracy for nearly circular orbits for the filter as long as the propagation stepsize is held small, i.e., less than 10 seconds.

The ONPAC state vector is given in Figure 5. The clock bias and bias rate are estimated in position and velocity coordinates as the clock offset and drift times the speed of light. The user can select all nine members as the solve-for state, drop the drag and estimate only eight parameters, or drop the drag and clock drift and estimate seven parameters.

Parameters that can be varied in the ONPAC program are listed in Table 1. The force model options for the user satellite are the Earth geopotential up to the 5-by-5 harmonics, rotation terms, and drag. The state transition matrix is computed with a Taylor series approximation and has only a two-body geopotential contribution plus rotation and drag terms. The effect of even further limitations to this force model can be studied, as can the effect of the integrator stepsize. A tunable parameter study can be done with variations of the process noise parameters, the fading memory smoothing factor, maximum values for the memory factor and the residual, and the observation measurement noise.

RESULTS

As part of the autonomous orbit determination evaluation, an experimental R/PA will be placed on Landsat-D. The proposed Landsat-D orbit was used in studies of the orbit determination.

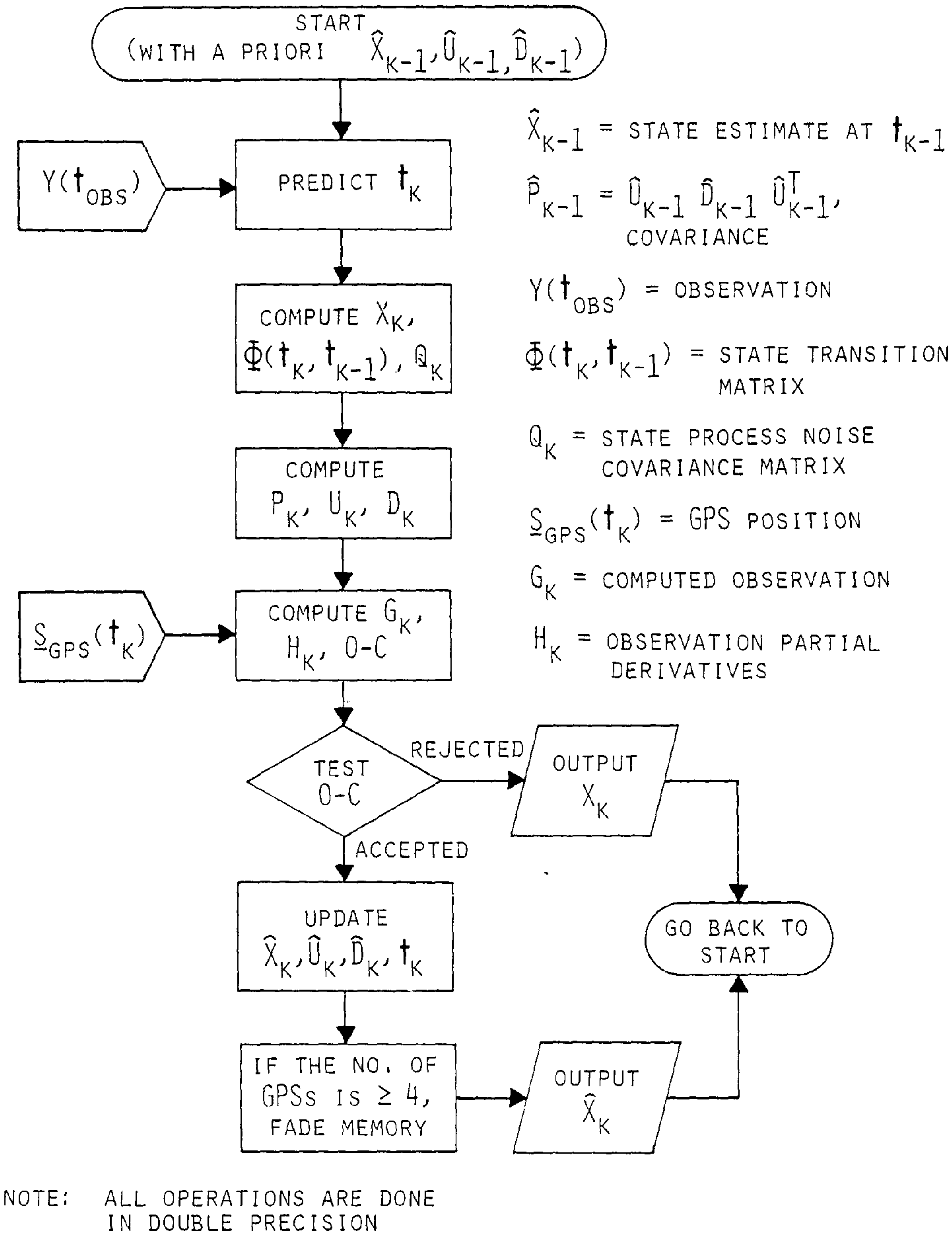

Figure 4. ONPAC Estimation Algorithm

$$\underline{X} = \begin{bmatrix} X \\ Y \\ Z \\ B \\ \dot{X} \\ \dot{Y} \\ \dot{Z} \\ \dot{B} \\ D \end{bmatrix}
\begin{array}{l}
\left.\vphantom{\begin{array}{c}X\\Y\\Z\end{array}}\right\} \text{Cartesian position components in Earth-centered, Earth-fixed (ECEF) coordinates} \\
\text{Clock bias} \\
\left.\vphantom{\begin{array}{c}X\\Y\\Z\end{array}}\right\} \text{Cartesian velocity components in ECEF coordinates} \\
\text{Clock bias rate} \\
\text{Drag coefficient}
\end{array}$$

$$D = \frac{C_D A_{CX}}{2M}$$

WHERE A_{CX} = CROSS-SECTIONAL AREA OF THE SATELLITE

M = MASS OF THE SATELLITE

C_D = CONSTANT COEFFICIENT

Figure 5. ONPAC State Vector

Table 1. User Options in ONPAC

TYPE	OPTION	ADJUSTABLE PARAMETERS
ACCELERATION MODEL		
GEOPOTENTIAL	-	DEGREE & ORDER, 2-BODY TO 5x5
ATMOSPHERIC DRAG	ON/OFF	TIME CONSTANT, τ_d
EKF ALGORITHM		
RESIDUAL TEST FOR ACCEPTANCE	ON/OFF	p_{MAX}
FADING MEMORY	ON/OFF	β, p_2
PROCESS NOISE	ON/OFF	a^2, q_a, q_b, q_d
SOLVE-FOR PARAMETERS	INCLUDE/EXCLUDE DRAG PARAMETER	-
OBSERVATION MEASUREMENT NOISE	-	σ_ρ^2, $\sigma_{\Delta\rho}^2$
STATE TRANSITION MATRIX	-	ORDER OF APPROXIMATION TO Δt^3
INTEGRATOR	-	STEP SIZE
USER CLOCK	-	TIME CONSTANT, τ_f

Initial conditions for Landsat-D and the GPS satellites are given in Figure 6. The GPS satellites constitute the default Phase I configuration. Since the launch of Landsat-D is expected during the early phases of the GPS, efforts were concentrated on orbit determination using the Phase I and subsets of the Phase I configuration.

Sample results are presented with four different sets of simulated data for October 1, 1980, 0 hours to 6 hours Universal Time (UT). The data simulation options used in common for these data sets are given in Figure 7.

The ONPAC options used for four sample cases used with these data sets are given in Table 2. In addition, all runs were done using a 5-by-5 geopotential, drag in the force model, a state transition matrix approximated to Δt^3, and a 3-second stepsize. The level of process noise used was found from tunable parameter studies to give the best results during periods of poor visibility.

Figure 8 shows the root-sum-square (RSS) position error for the baseline case and the Phase I visibility over the 6 hours of the data span. During periods when the fading memory is used and four or more GPS satellites are in view, the RSS position error is less than 10 meters. The curve has a "flat bottomed" appearance found to be characteristic of the cases when fading memory is used.

The studies discussed here have shown that the best results occur when the fading memory is tuned to the periods of good GPS visibility and the process noise covariance to periods of poor GPS visibility. The fading memory multiplies the covariance matrix and inflates the entire matrix, whereas the process noise is additive to certain terms of the covariance. The fading memory swamps the effect of the process noise when they are used together.

Cases 2 and 3, whose RSS errors are shown in Figure 9, are done with data sets B and C, which include GPS clock bias errors. The GPS satellites are selected with the geometric dilution of precision (GDOP) procedure, which, when six or more GPS satellites are in view, picks for observation only those

LANDSAT-D INITIAL CONDITIONS

SEMIMAJOR AXIS	7086.901 KILOMETERS
ECCENTRICITY	0.001
INCLINATION	98.181 DEGREES
LONGITUDE OF ASCENDING NODE	354.878 DEGREES
ARGUMENT OF PERIGEE	180.000 DEGREES
MEAN ANOMALY	0.000 DEGREES
PERIOD	98.956 MINUTES

GPS CONFIGURATION AND INITIAL CONDITIONS

PHASE I CONFIGURATION

INCLINATION	63 DEGREES
ECCENTRICITY	0.0
SATELLITES 1, 2, 3:	
- LONGITUDE OF ASCENDING NODE	120 DEGREES
- MEAN ANOMALIES	100, 140, 180 DEGREES
SATELLITES 4, 5, 6:	
- LONGITUDE OF ASCENDING NODE	240 DEGREES
- MEAN ANOMALIES	60, 100, 140 DEGREES
PERIOD	12 HOURS

Figure 6. Initial Conditions

LANDSAT-D FORCE MODEL:

- 8x8 GEOPOTENTIAL
- LUNI-SOLAR PERTURBATIONS
- DRAG
- SOLAR RADIATION PRESSURE

QUADRATIC USER CLOCK ERROR:

- $T_1 = 3.3360 \times 10^{-5}$ SECONDS
- $T_2 = 3.475 \times 10^{-10}$ SECONDS/SECOND
- $T_3 = 5.0 \times 10^{-16}$ SECONDS/SECOND^2

OBSERVATION SPACING:

- $\Delta t_1 = 0.6$ SECOND
- $\Delta t_2 = 6$ SECONDS
- $\Delta t_3 = 6$ SECONDS

OBSERVATION STANDARD DEVIATIONS:

- $\sigma_\rho = 2.0$ METERS
- $\sigma_{\Delta\rho} = 1.7$ CENTIMETERS

OPTIONS VARIED:

- GPS EPHEMERIS ERROR MODEL
- GPS CLOCK BIAS
- GPS CONFIGURATION AND SELECTION

Figure 7. Data Simulation Options Used for Data Sets A, B, C, and D From R&D GTDS

Table 2. ONPAC and Data Simulation Options Used in Sample Cases

CASE	FADING MEMORY PARAMETERS	σ_ρ	DATA SET	GPS EPHEMERIS ERROR MODEL	GPS CLOCK BIAS	SELECTION
BASELINE CASE						
TEST CASE 1 (3507)	$\beta = 0.2$ $p_2 = 2.0$	8.1 M	A	RANDOMLY SELECTED $(\sigma_H, \sigma_C, \sigma_L)$ = (5M, 5M, 10M)	NONE	PHASE I, ALL
FADING MEMORY/GPS CLOCK BIAS						
TEST CASE 2 (4010)	$\beta = 0.2$ $p_2 = 1.05$	7.5 M	B	SINUSOIDAL $(H,C,L,\dot{L})$ = (5M,5M, 10M,0.05M/SEC) P = 24 HOURS	CORRELATED T_G = 3 NS	PHASE I, GDOP
TEST CASE 3 (4011)	NOT USED	7.5 M	C	SINUSOIDAL $(H,C,L.\dot{L})$ = (5M,5M, 10M,0.05M/SEC) P = 12 HOURS	UNCORRELATED σ_{T_G} = 3 NS	PHASE I, GDOP
EFFECT OF FOUR IN CONSTELLATION						
TEST CASE 4 (8102)	$\beta = 0.2$ $p_2 = 2.0$	8.1 M	D	SAME AS C ABOVE	NONE	# 2,3,5,6 IN PHASE I

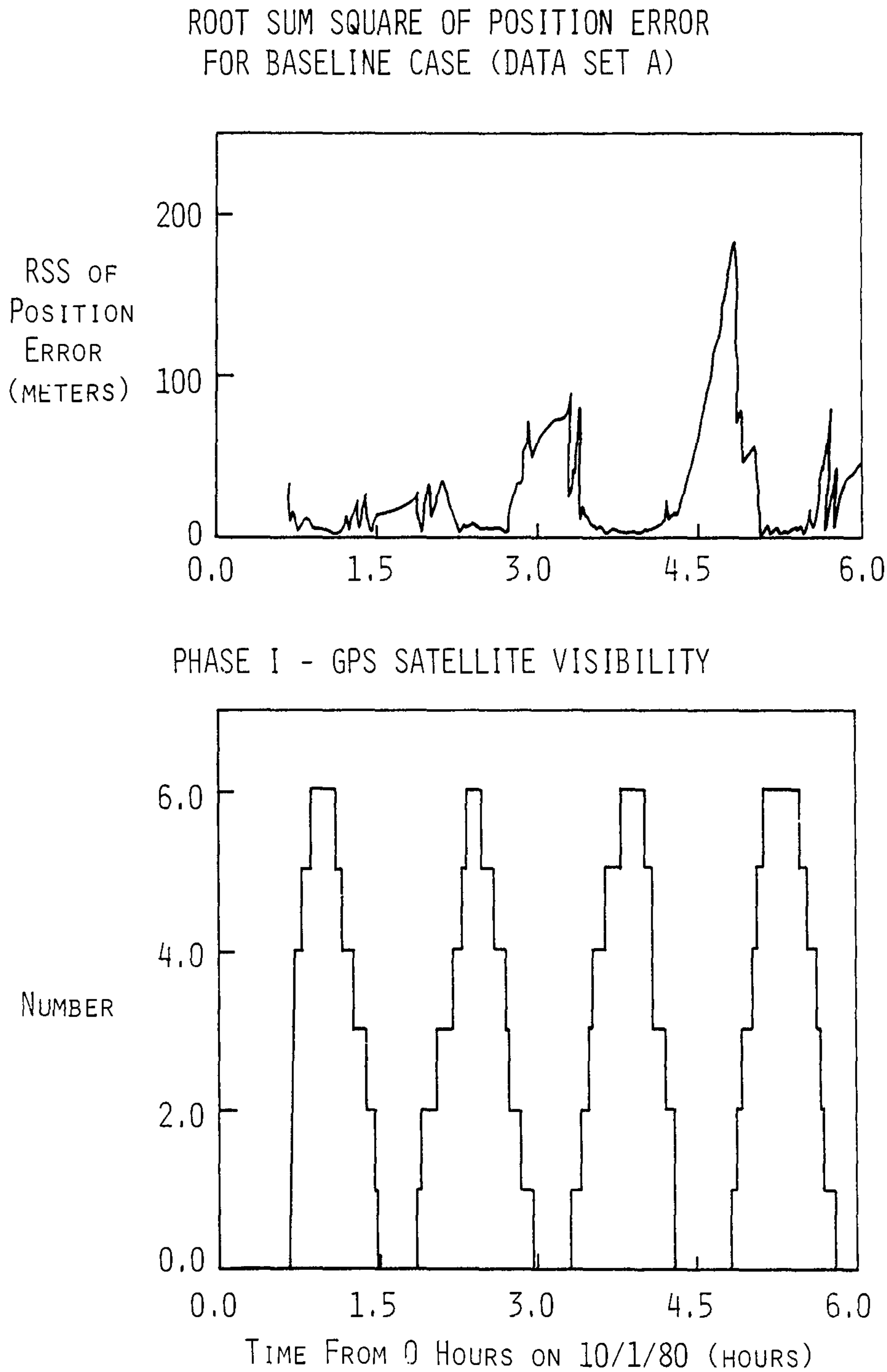

Figure 8. Baseline Case and Visibility

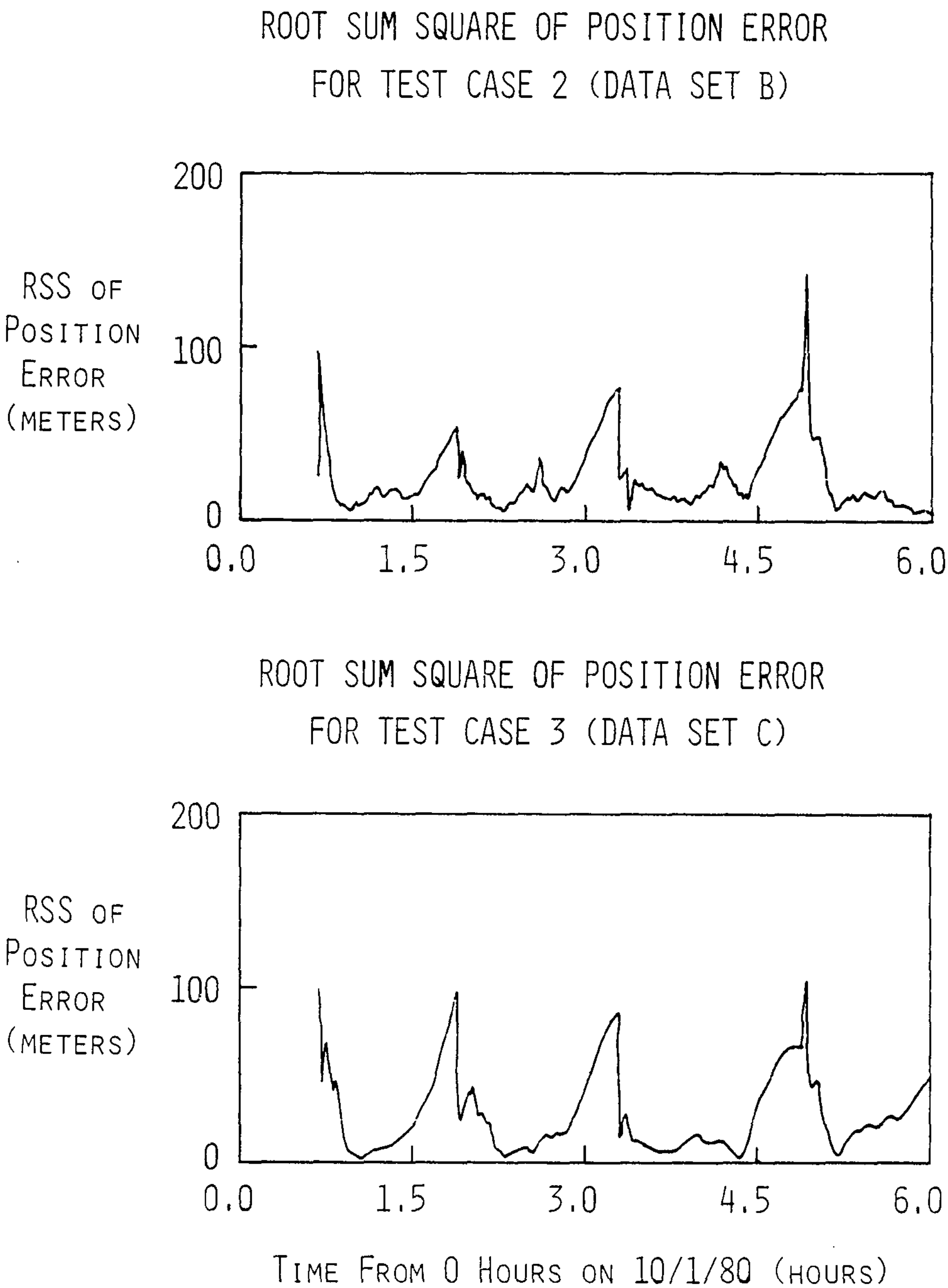

Figure 9. Root-Sum-Square Position Errors of Test Cases 2 and 3

four with the best geometric distribution. If four or fewer are visible, those seen are picked for observation.

The effect of the GPS clock bias error is to increase the baseline case RSS position error during periods of good visibility. The fading memory option used in case 2 causes the RSS error to drop to the minimum value more quickly than in case 3 at periods of good visibility and makes the curve plotted flatter than that in case 3. Study of the correlated versus the uncorrelated GPS clock errors shows very little difference in their effects.

Data set D was simulated using only four GPS satellites from Phase I: satellites 2 and 3 in one plane and satellites 4 and 5 in another. Figure 10 shows the visibility and GDOP for this data set. Test case 4, whose RSS error is shown in Figure 11, was run using this data set. The RSS position error grows to more than 300 meters during the data gaps, and the user clock is poorly estimated when fewer than four GPS satellites are visible. Given that the error in the data and the GPS ephemeris is approximately 7 meters, the GDOP from Figure 10 would predict an error in the position determination of 35 meters or more when four GPS satellites are visible. The results in test case 4 are in the 25-through-35-meter range at times of good visibility, within the range predicted by the GDOP.

CONCLUSIONS

The conclusions of the studies are given below.

- The algorithms used are sufficient for accurate orbit determination.
- The errors in orbit estimation are less than those predicted from the GDOP.
- Accurate orbit determination is possible with only four GPS satellites in the constellation.
- The orbit determination accuracy is limited by the GPS ephemeris and clock accuracies.

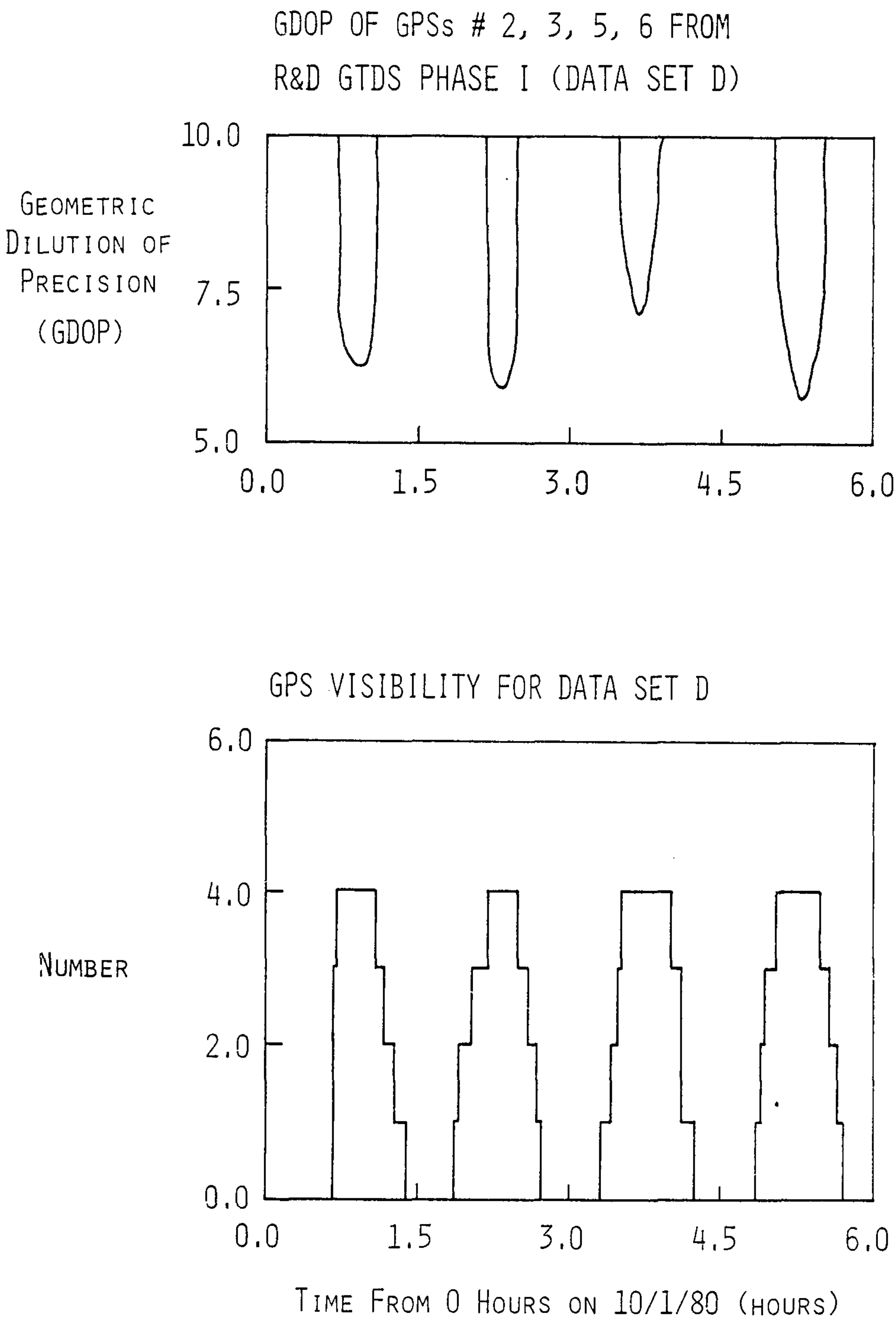

Figure 10. GDOP and Visibility of Data Set D

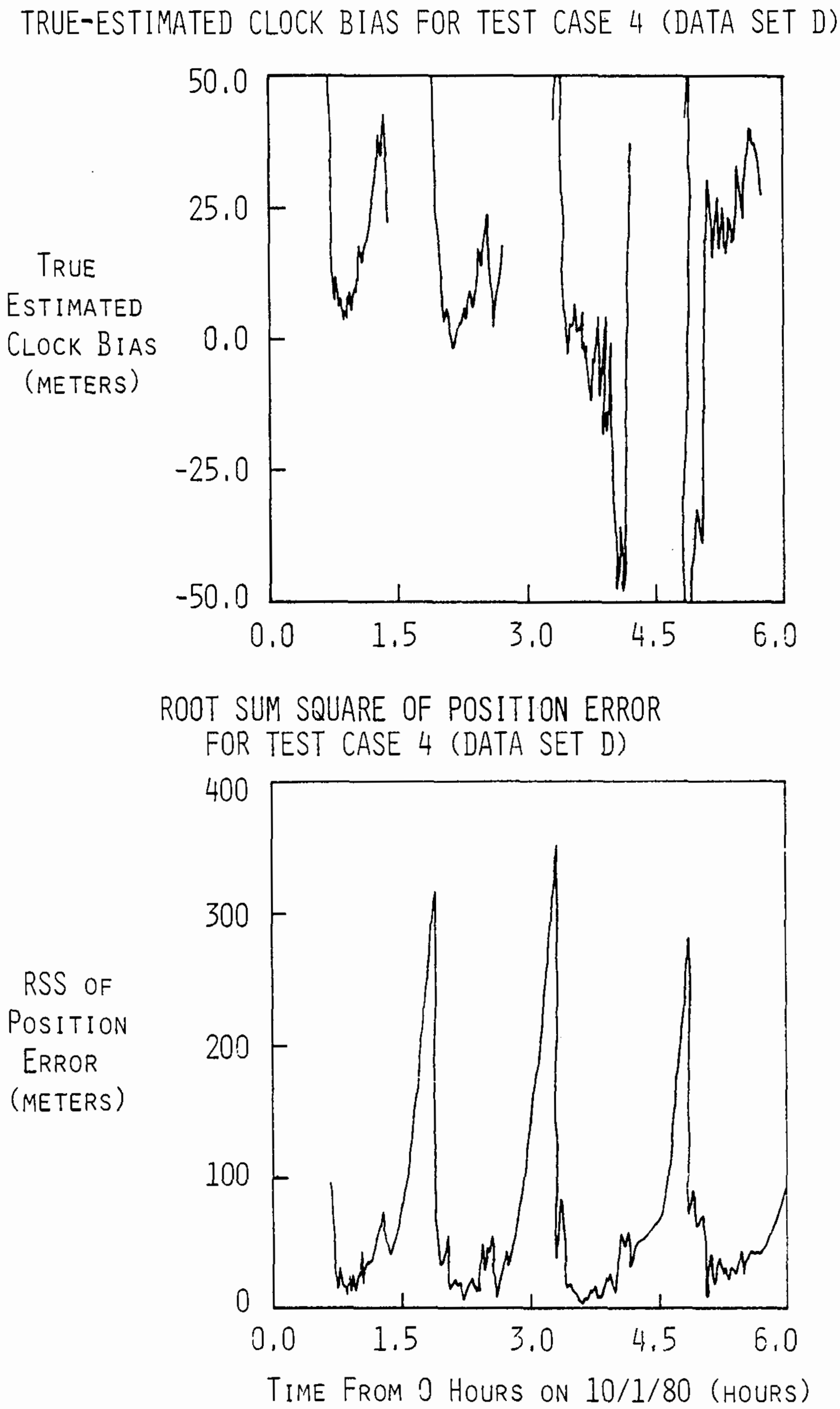

Figure 11. Clock and Position Errors of Test Case 4

- The fading memory enhances the orbit determination accuracy, especially when the a priori knowledge of the user clock offset is poor.

All computations have been done in double precision; the effect of performing some operations in single precision has not yet been investigated.

The studies discussed here have shown that the filter is not overly sensitive to the tunable parameters. The results presented are typical of the results gathered from a range of parameter values.

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