APPLICATION OF OPTIMUM SMOOTHING FOR IMAGE DISTORTION CORRECTION

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ABSTRACT

Optimum linear smoothing is utilized to estimate certain distortions in Landsat-D images. Measurements that are processed by the smoother consist of designated control point locations within the images. Image distortions that are estimated by the smoother are those induced by Landsat-D satellite navigation errors and slowly-varying attitude and sensor alignment uncertainties. Preliminary results indicate that optimum smoothing produces substantially more accurate distortion estimates than optimum filtering and that optimum smoothing may reduce the number of control points needed to yield a desired image correction accuracy.

INTRODUCTION

Landsat-D is the next of a series of satellites designed to transmit imagery data to the ground to support earth resources management. The primary payload of Landsat D spacecraft is a thematic mapper (TM) and the secondary payload is a multispectral scanner. The mission objective is to produce high quality images of the earth surface for use in agriculture monitoring. The TM has seven spectral bands and 30 meter resolution. It scans the earth 185 km perpendicular to the spacecraft ground track at 7.4 hz rate; spacecraft motion provides the along-track scan. Digitized image data, along with spacecraft attitude measurements, are telemetered real time to the NASA/Goddard ground station, where the data is processed to produce high precision images: ±5.5 meter (1σ) registration error and ±9.1 meter (1σ) total geometric error.

The raw image data contains distortions due to navigation error, attitude measurement error, and TM misalignment relation to the attitude reference axes. In order to remove these distortions from the image data and thereby achieve the precision images that are required, a Recursive Distortion Estimator (RDE) is designed to estimate the distortions. The measurements used by the RDE are based on locations of control points in the distorted image data, together with their known locations on the ground. The image of each control point is projected onto the ground. Distortion in the image causes the projected position of the control point to differ from its known true position. This difference in position is used by the RDE to estimate the distortion in the image data.

Reference 1 suggests a Kalman filter RDE. This document evaluates an optimum smoother RDE and compares its performance with that of a Kalman filter RDE.

SYSTEM DEFINITION

The system state variables X_i, i = 1 through 6, are defined as follows:

$$\left.\begin{matrix} X_1 \\ X_2 \\ X_3 \end{matrix}\right\} \text{Along-track, cross-track and vertical components of navigated position error}$$

$$\left.\begin{matrix} X_4 \\ X_5 \\ X_6 \end{matrix}\right\} \text{Roll, Pitch, and yaw attitude measurement error plus instrument misalignment}$$

$$\left.\begin{array}{l} x_7 \\ x_8 \\ x_9 \end{array}\right\} \text{Along-track, cross-track and vertical components of navigated velocity error}$$

$$\left.\begin{array}{l} x_{10} \\ x_{11} \\ x_{12} \end{array}\right\} \text{Roll, pitch, and yaw attitude measurement error drift rate plus instrument misalignment rate}$$

The state differential equations are

$$\dot{x}_i = x_{i+6} \quad \text{for } i = 1 \text{ through } 6 \tag{1}$$

$$\dot{x}_i = \sum_{k=1}^{3} \frac{\partial g_{s_{i-6}}}{\partial x_k} x_k + z_i \quad \text{for } i = 7, 8, 9 \tag{2}$$

$$\dot{x}_i = a_i x_{i-6} + b_i x_i + z_i \quad \text{for } i = 10, 11, 12 \tag{3}$$

where g_{s_j} is the j^{th} component of spherical (Keplerian) mass attraction acceleration for $j = 1, 2, 3$ and z_i is Gaussian uncorrelated white noise for $i = 7$ through 12. The coefficients in Equation 3 are $a_i = 0$ and $b_i = -0.00139\ \text{sec}^{-1}$ for $i = 10, 11, 12$. The standard deviation σ_{Zi} of each component of state noise z_i is: $\sigma_{Zi} = 1.52\times10^{-5}\text{m/sec}^{3/2}$ for $i = 7$ and 8, $\sigma_{Z9} = 2.28\times10^{-5}\text{m/sec}^{3/2}$ and $\sigma_{Zi} = 0.0213\ \mu\text{rad/sec}^{3/2}$ for $i = 10, 11, 12$.

The standard deviations σ_{x_i} of initial uncertainty in each state variable x_i is: $\sigma_{x_1} = 250\text{m}$, $\sigma_{x_2} = 50\text{m}$, $\sigma_{x_3} = 17\text{m}$, $\sigma_{x_i} = 291\ \mu\text{rad}$ for $i = 4, 5, 6$,

$\sigma_{x_7} = 0.05$ m/sec, $\sigma_{x_i} = 0.02$ m/sec for i = 8, 9, and $\sigma_{x_i} = 0.4$ µrad/sec for i = 10, 11, 12.

The measurements y_1 and y_2 are defined as the along-track and cross-track deviations between the control point image projected onto the ground and true position of the control point. The standard deviation of the noise in each measurement is: $\sigma_{\omega_1} = 3.0$ m and $\sigma_{\omega_2} = 5.0$ m.

In addition to the slowly-varying sensor pointing error (caused by attitude measurement errors and sensor misalignment) that is estimated by the RDE, there is also an uncorrelated (white) pointing error which causes distortion in the image data. The standard deviation of the distortion caused by this random pointing error is 2.55m along-track and 4.73m cross-track.

DESCRIPTION OF SMOOTHING ALGORITHM

The equations for optimum linear smoothing are given in Chapter 6 of Reference 2. The smoothing algorithm utilized for the RDE is called a fixed-interval smoother in Reference 2.

METHOD OF ANALYZING SMOOTHING PERFORMANCE

The RDE performance is evaluated via linear statistical (covariance) analysis. Based on an assumed set of control point locations, the state error covariance matrix is propagated over the smoothing interval by the smoothing equations. The error covariance matrix for along-track and cross-track residual distortions are then computed at each point in the image, based on the state error covariance matrix at that point and the covariances of sensor random pointing errors.

Several cases that were analyzed were repeated assuming that the RDE is a Kalman (optimum) filter. This was done so that Kalman filter performance could be compared with optimum smoothing performance.

SUMMARY OF SMOOTHER PERFORMANCE ANALYSIS RESULTS

The results of this performance analysis show the smoothing algorithm yields substantially more accurate distortion estimation than a Kalman (optimum) filter for the identical case. Furthermore, the smoothing algorithm requires fewer control points to achieve a desired accuracy.

The results also show that the desired distortion compensation accuracy can be achieved with one control point every fourth scene for a series of 40 scenes or by having four control points uniformly distributed over a single scene.

REFERENCES

1. Caron, R. H., Simon, K. W., "Attitude Time-Series Estimator for Rectification of Space-Borne Imagery", Journal of Spacecraft and Rockets (12, 27), January 1975.
2. Meditch, J. S., Stochastic Optimal Linear Estimation and Control, McGraw-Hill Book Co., New York, 1969.

BACKGROUND INFORMATION

- LANDSAT-D SATELLITE TELEMETERS DIGITAL IMAGEY DATA FROM 705 Km ALTITUDE TO NASA/GODDARD GROUND STATION, WHERE IT IS PROCESSED TO PRODUCE PRECISION IMAGES OF THE EARTH SURFACE
- IMAGERY DATA IS PRODUCED BY A THEMATIC MAPER (TM) WHICH SCANS THE SURFACE OF THE EARTH 185 Km AT 7.4 Hz RATE PERPENDICULAR TO THE SATELLITE GROUND TRACK
- THE INSTANTANEOUS FIELD OF VIEW (IFOV) OF THE TM (ONE PICTURE ELEMENT (PIXEL)) IS 30 m x 30 m
- THE RAW IMAGERY DATA CONTAINS SLOWLY-VARYING DISTORTIONS DUE TO NAVIGATION ERROR, ATTITUDE MEASUREMENT ERROR, AND TM MISALIGNMENT, AS WELL AS UNCORRELATED (WHITE) RANDOM POINTING ERRORS
- SLOWLY-VARYING DISTORTIONS ARE ESTIMATED BY THE RECURSIVE DISTORTION ESTIMATOR (RDE) BY COMPARING THE LOCATIONS OF "CONTROL POINTS" IN A SCENE WITH THEIR KNOWN LOCATIONS ON THE GROUND

OBJECTIVES OF RDE

- ESTIMATE AND REMOVE DISTORTIONS FROM IMAGES SO THAT RESIDUAL DISTORTION IS NO GREATER THAN:

  ± 5.5 m (1σ) SCENE-TO-SCENE REGISTRATION ERROR

  ± 9.1 m (1σ) TOTAL GEOMETRIC CORRECTION ERROR

- MINIMIZE THE NUMBER OF GROUND CONTROL POINTS NEEDED TO ACHIEVE ACCURACY REQUIREMENTS

METHOD OF ANALYSIS

- LINEAR STATISTICAL (COVARIANCE) ANALYSIS
  - STATE ERROR COVARIANCE MATRIX PROPAGATED VIA SMOOTHING ALGORITHM
  - ERROR COVARIANCE MATRIX OF RESIDUAL ALONG-TRACK AND CROSS-TRACK DISTORTIONS COMPUTED BASED ON STATE ERROR COVARIANCE MATRIX AND STANDARD DEVIATIONS OF UNCORRELATED POINT ERRORS
- KALMAN (OPTIMUM) FILTER PERFORMANCE EVALUATED AS WELL AS OPTIMUM SMOOTHING PERFORMANCE

- SUMMARY AND CONCLUSIONS
  - OPTIMUM SMOOTHING BY THE RDE PRODUCES SUBSTANTIALLY MORE ACCURATE DISTORTION ESTIMATION THAN OPTIMUM (KALMAN) FILTERING AND REQUIRES FEWER CONTROL POINTS TO ACHIEVE A DESIRED ACCURACY
  - ONE CONTROL POINT EVERY FOUR SCENES YIELDS ONLY MODEST DEGRADATION IN ACCURACY RELATIVE TO HAVING ONE CONTROL POINT EVERY SCENE
  - DESIRED DISTORTION CORRECTION ACCURACY CAN BE ACHIEVED IN A SINGLE SCENE BY HAVING FOUR CONTROL POINTS UNIFORMLY DISTRIBUTED OVER THE SCENE

SYSTEM DEFINITION

- STATE VECTOR DEFINITION:

$$\left.\begin{array}{l} X_1 \\ X_2 \\ X_3 \end{array}\right\} \text{ALONG-TRACK, CROSS-TRACK AND VERTICAL COMPONENTS OF NAVIGATED POSITION}$$

$$\left.\begin{array}{l} X_4 \\ X_5 \\ X_6 \end{array}\right\} \text{ROLL, PITCH, AND YAW ATTITUDE MEASUREMENT ERROR PLUS INSTRUMENT MISALIGNMENT}$$

$$\left.\begin{array}{l} X_7 \\ X_8 \\ X_9 \end{array}\right\} \text{ALONG-TRACK, CROSS-TRACK AND VERTICAL COMPONENTS OF NAVIGATED VELOCITY ERROR}$$

$$\left.\begin{array}{l} X_{10} \\ X_{11} \\ X_{12} \end{array}\right\} \text{ROLL, PITCH, AND YAW ATTITUDE MEASUREMENT ERROR DRIFT RATE PLUS INSTRUMENT MISALIGNMENT RATE}$$

SYSTEM DEFINITION (Continued)

- STATE DIFFERENTIAL EQUATIONS:

$$\dot{x}_i = x_{i+6} \quad \text{for } i = 1 \text{ through } 6$$

$$\dot{x}_i = \sum_{k=1}^{3} \frac{\partial g_{s_{i-6}}}{\partial x_k} x_k + z_i \quad \text{for } i = 7, 8, 9$$

$$\dot{x}_i = a_i x_{i-6} + b_i x_i + z_i \quad \text{for } i = 10, 11, 12$$

WHERE

a_i = 0, b_i = -0.00139 sec^{-1}

σ_{z_i} = $1.52\text{x}10^{-5}$ $\text{m/sec}^{3/2}$ for i = 7, 8

σ_{z_9} = $2.28\text{x}10^{-5}$ $\text{m/sec}^{3/2}$

σ_{z_i} = 0.0213 $\mu\text{rad/sec}^{3/2}$ for i = 10, 11, 12

SYSTEM DEFINITION (Continued)

- INITIAL STATE UNCERTAINTIES:

$\alpha_{X_1} = 250$ m

$\sigma_{X_2} = 50$ m

$\sigma_{X_3} = 17$ m

$\sigma_{X_i} = 291$ μrad for $i = 4, 5, 6$

$\sigma_{X_7} = 0.05$ m/sec

$\sigma_{X_i} = 0.02$ m/sec for $i = 8, 9$

$\sigma_{X_i} = 0.4$ μrad/sec for $i = 10, 11, 12$

SYSTEM DEFINITION (CONCLUDED)

- MEASUREMENT NOISE (FOR REGISTRATION):

σ_{ω_1} = 3.0 m (ALONG-TRACK)

σ_{ω_2} = 5.0 m (CROSS-TRACK)

- UNCORRELATED RANDOM POINTING ERRORS:

σ_{R1} = 2.55 m (ALONG-TRACK)

σ_{R2} = 4.73 m (CROSS-TRACK)

COMPARISON OF KALMAN FILTERING WITH OPTIMUM SMOOTHING

- REFERENCE CASE REFLECTS TEMPORAL REGISTRATION ACCURACY WITH THE ERROR MODELS DISCUSSED EARLIER AND ASSUMES ONE CONTROL POINT PER SCENE FOR TEN SCENES
- KALMAN FILTERING, AS WELL AS OPTIMUM SMOOTHING, IS EVALUATED FOR THE REFERENCE CASE
- THE STANDARD DEVIATION (IN METERS) OF RESIDUAL DISTORTION AT THE TIMES WHEN THEY ARE MINIMUM ARE SUMMARIZED AS FOLLOWS:

	KALMAN FILTERING			**OPTIMUM SMOOTHING**		
	RDE STATE ESTIMATION ERROR (1σ)	UN-CORRELATED POINTING ERROR (1σ)	TOTAL RESIDUAL DISTORTION (1σ)	RDE STATE ESTIMATION ERROR (1σ)	UN-CORRELATED POINTING ERROR (1σ)	TOTAL RESIDUAL DISTORTION (1σ)
ALONG TRACK	1.89	2.55	3.17	1.28	2.55	2.85
CROSS TRACK	2.81	4.73	5.50	1.85	4.73	5.08

COMPARISON OF KALMAN FILTERING WITH OPTIMUM SMOOTHING (Continued)

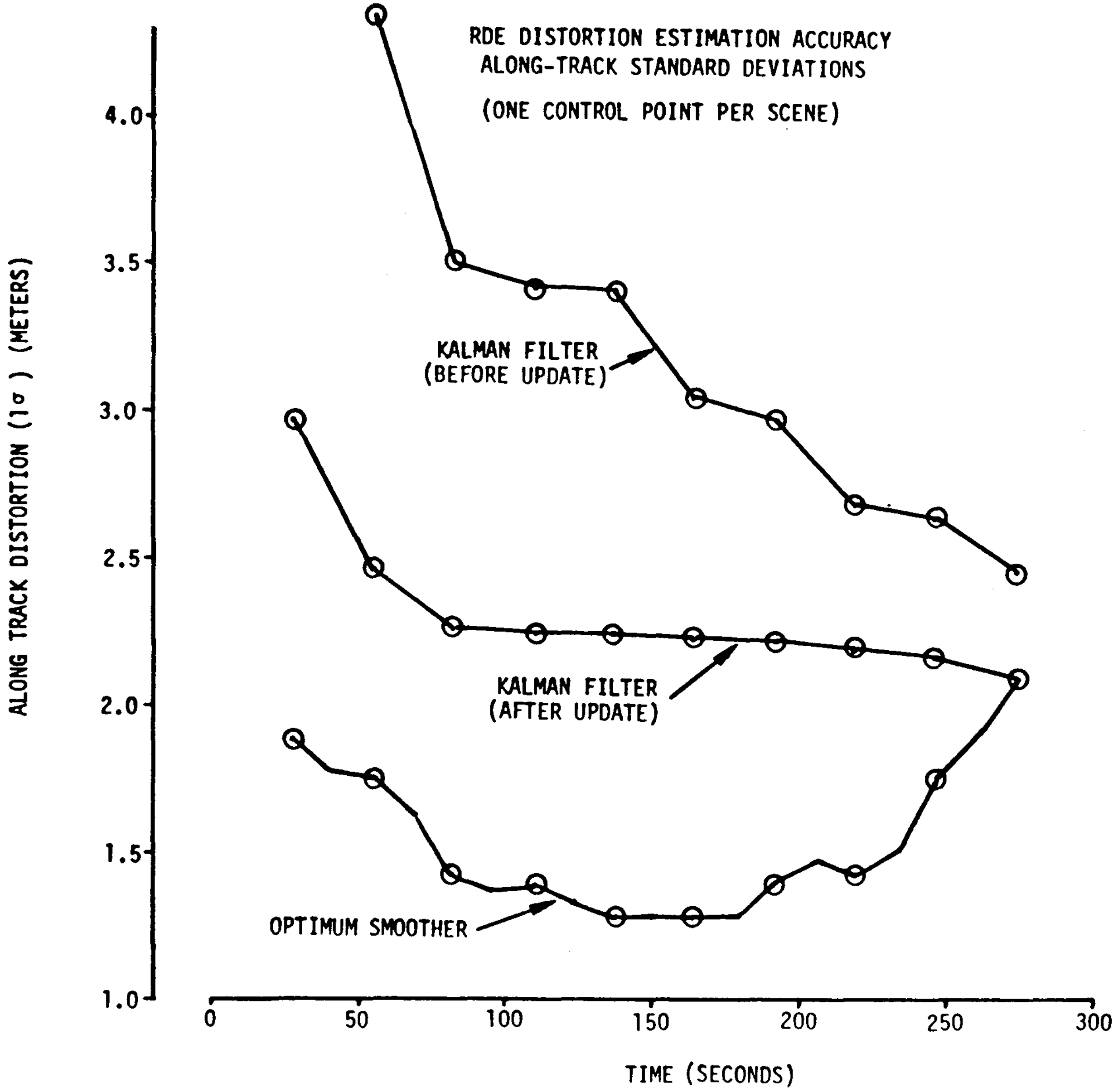
- RESULTS PRESENTED SO FAR INDICATE ONLY MODEST IMPROVEMENT BY SMOOTHING RATHER THAN FILTERING. THIS IS BECAUSE THE STANDARD DEVIATIONS OF RDE ERRORS WERE TAKEN AT THE TIMES WHEN THEY ARE MINIMUM

- THE FIGURES BELOW SHOW DRAMATIC IMPROVEMENT IN RDE ACCURACY WHEN OPTIMUM SMOOTHING IS USED RATHER THAN KALMAN (OPTIMUM) FILTERING

- THESE PLOTS SHOW THAT FEWER CONTROL POINTS ARE NEEDED TO ACHIEVE THE REQUIRED ACCURACY IF THE RDE IS A SMOOTHER RATHER THAN A FILTER

- ALL THE RESULTS THAT FOLLOW ARE BASED ON THE ASSUMPTION THAT OPTIMUM SMOOTHING IS UTILIZED IN THE RDE

COMPARISON OF KALMAN FILTERING WITH OPTIMUM SMOOTHING (Continued)



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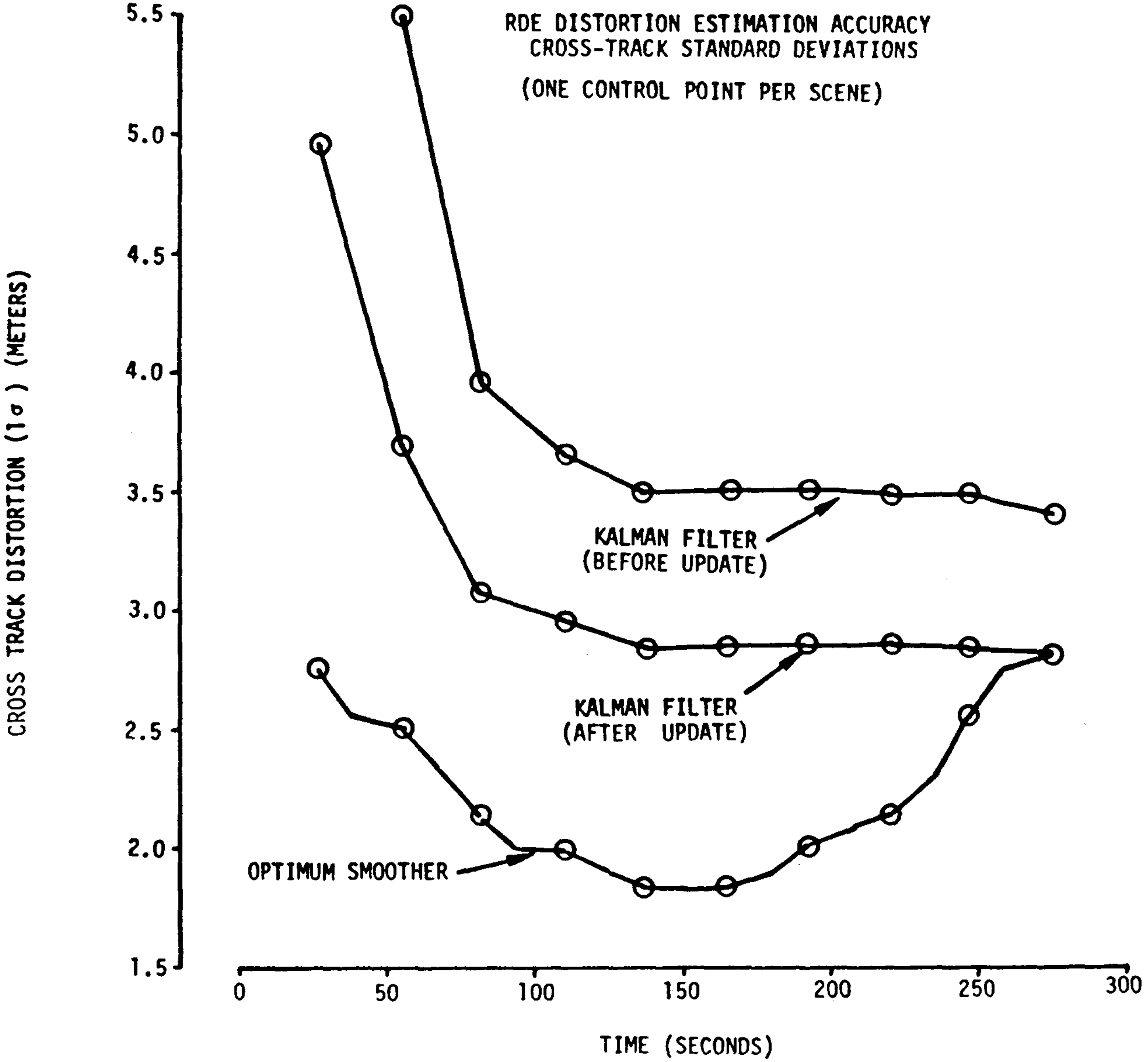
COMPARISON OF KALMAN FILTERING WITH OTPIMUM SMOOTHING (Continued)



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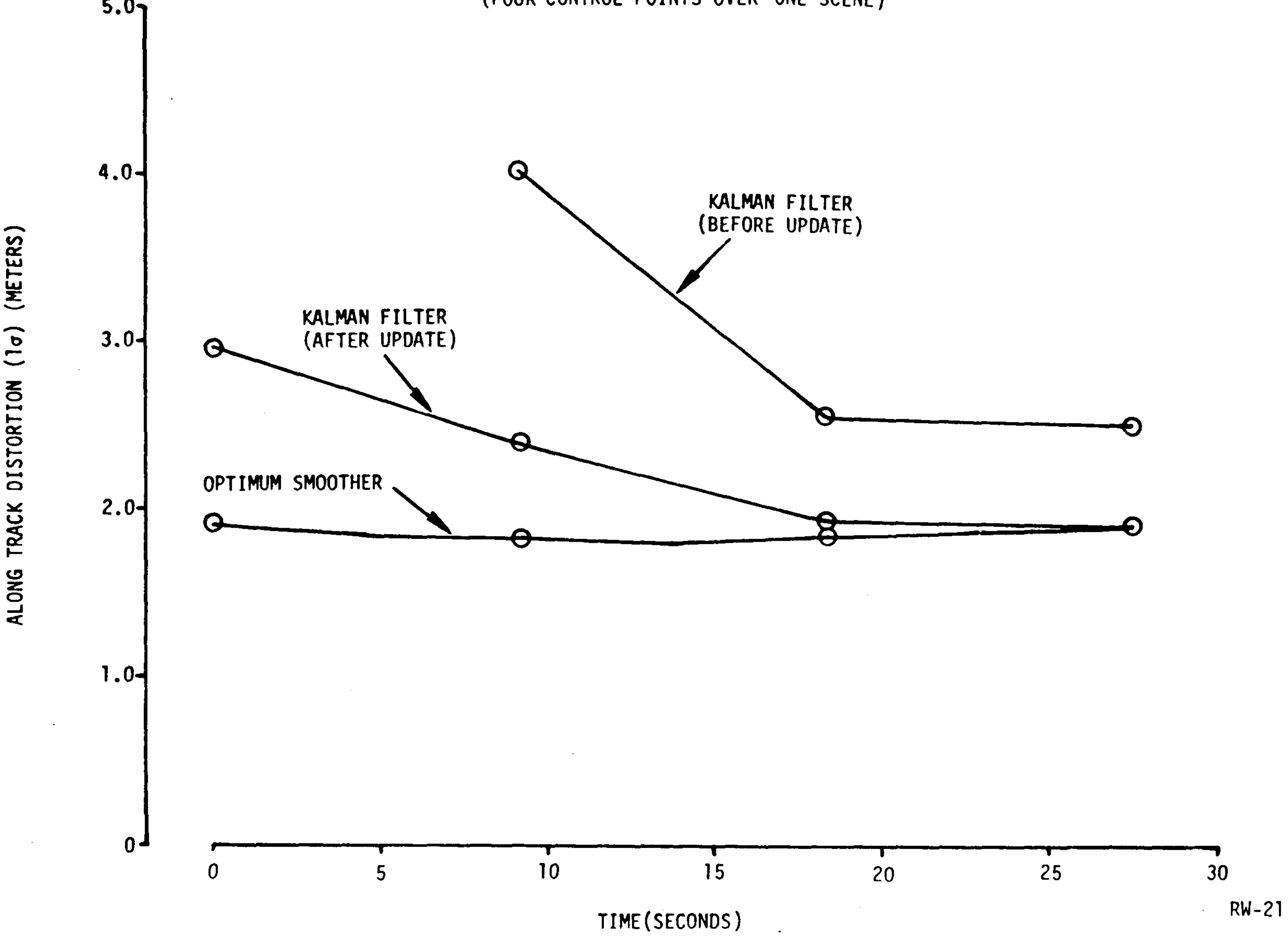
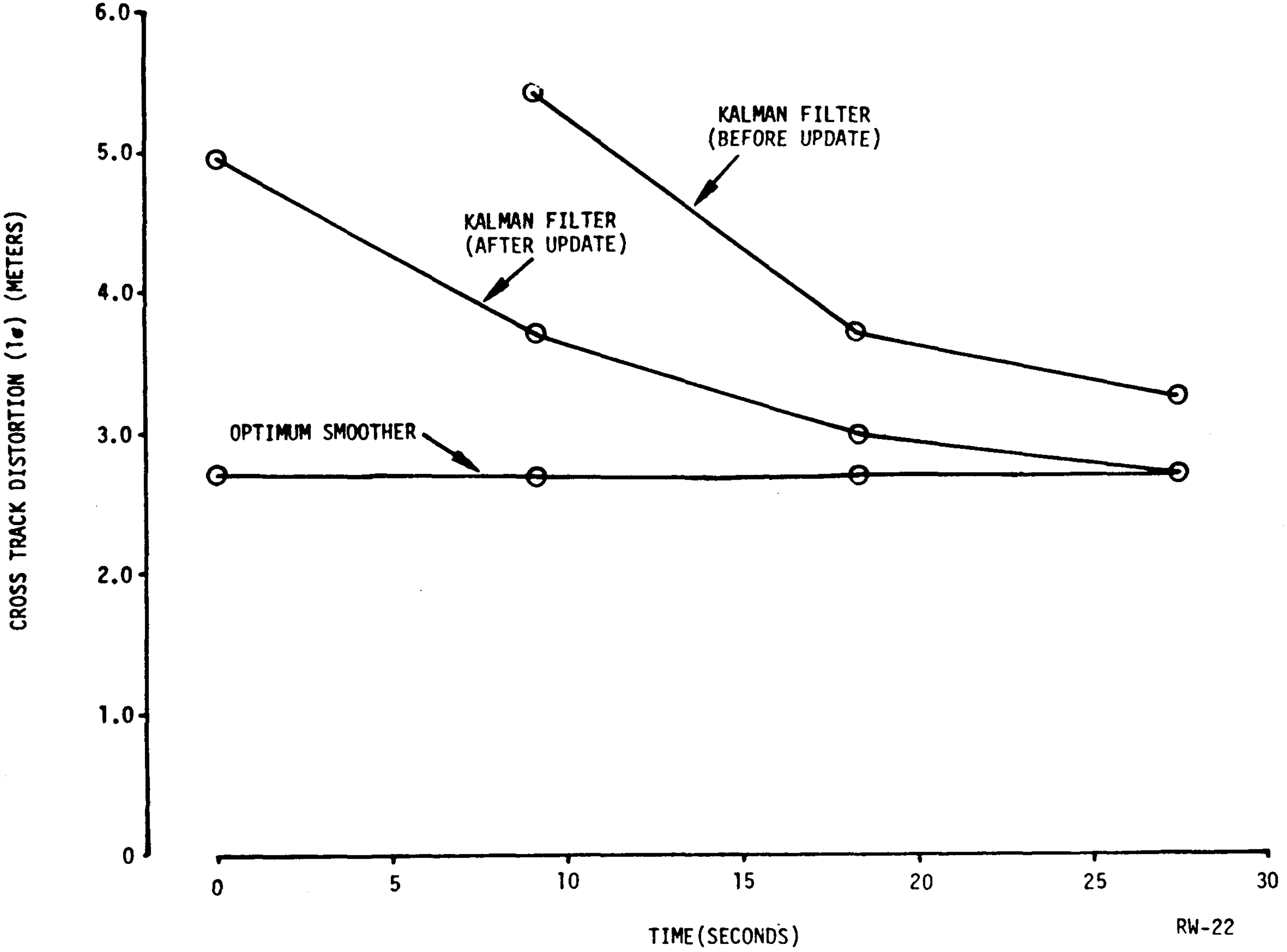
COMPARISON OF KALMAN FILTERING WITH OPTIMUM SMOOTHING (Continued)

RDE DISTORTION ESTIMATION ACCURACY
ALONG-TRACK STANDARD DEVIATIONS
(FOUR CONTROL POINTS OVER ONE SCENE)



COMPARISON OF KALMAN FILTERING WITH OPTIMUM SMOOTHING (Concluded)

RDE DISTORTION ESTIMATION ACCURACY
CROSS-TRACK STANDARD DEVIATIONS
(FOUR CONTROL POINTS OVER ONE SCENE)



REDUCING THE NUMBER OF CONTROL POINTS PER SCENE

- A TM TEMPORAL REGISTRATION CASE REFLECTING ONE CONTROL POINT EVERY FOURTH SCENE WAS ANALYZED

- THE STANDARD DEVIATIONS (IN METERS) OF RESIDUAL DISTORTIONS FOR THIS CASE ARE COMPARED WITH THOSE FROM A CASE WITH ONE CP PER SCENE AS FOLLOWS:

	ONE CP PER SCENE			ONE CP EVERY FOUR SCENES		
	RDE STATE ESTIMATION ERROR (1σ)	UNCORRELATED POINTING ERROR (1σ)	TOTAL RESIDUAL DISTORTION (1σ)	RDE STATE ESTIMATION ERROR (1σ)	UNCORRELATED POINTING ERROR (1σ)	TOTAL RESIDUAL DISTORTION (1σ)
ALONG TRACK	1.28	2.55	2.85	1.77	2.55	3.11
CROSS TRACK	1.85	4.73	5.08	2.41	4.73	5.31

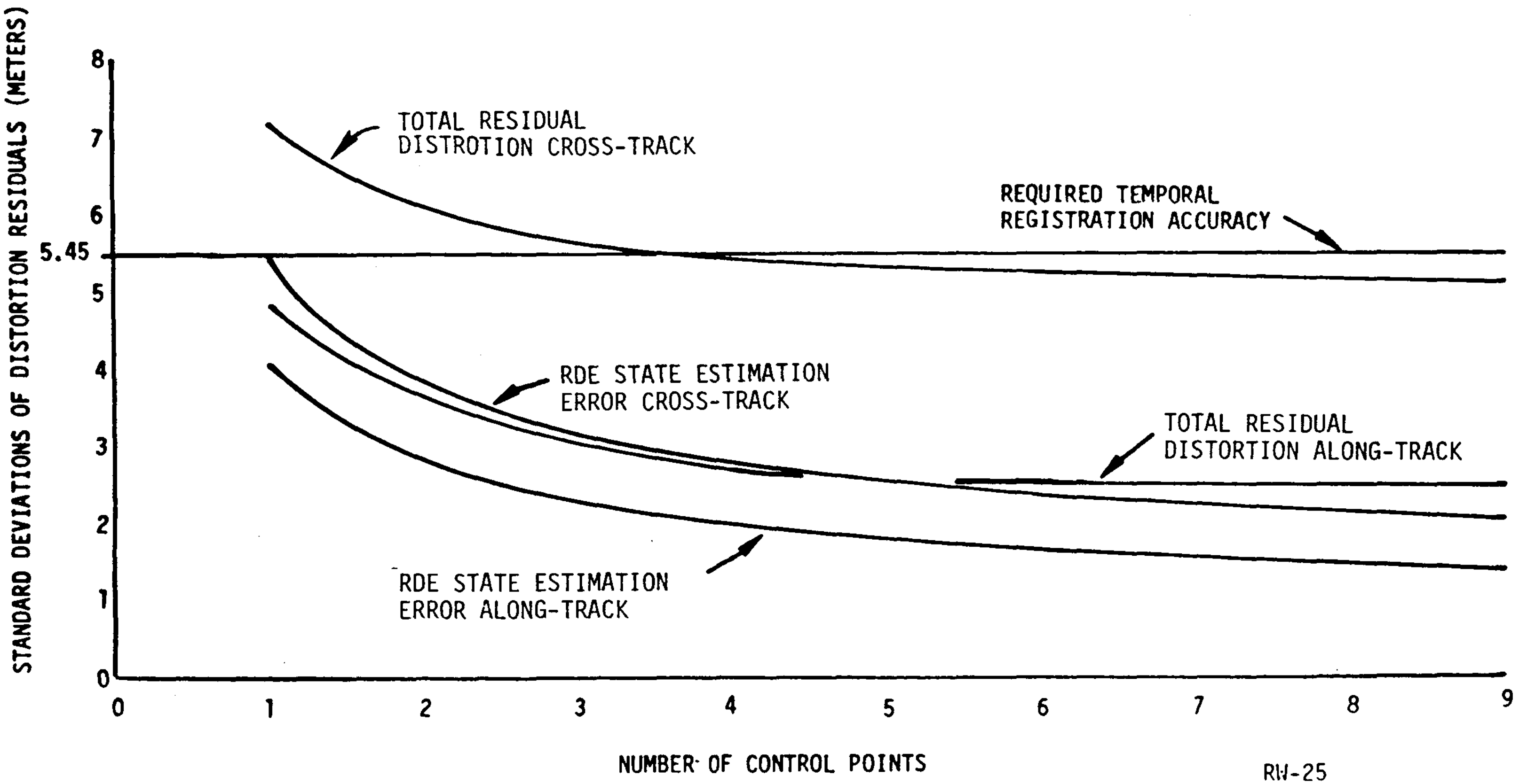
- THESE RESULTS SHOWS THAT REDUCING THE NUMBER OF CP's TO ONE EVERY FOURTH SCENE DEGRADES TOTAL ACCURACY ONLY SLIGHTLY, AND THE TM TEMPORAL REGISTRATION ACCURACY REQUIREMENTS [5.45 M (1σ)] IS STILL SATISFIED

LIMITING THE CONTROL POINT REGION TO ONE SCENE

- SEVERAL TM TEMPORAL REGISTRATION CASES WERE ANALYZED THAT REFLECT UTILIZING VARYING NUMBERS OF CP's UNIFORMLY DISTRIBUTED OVER A SINGLE SCENE IN ORDER TO REMOVE DISTORTIONS FROM THE SCENE
- THE FIGURE BELOW SHOWS HOW THE STANDARD DEVIATIONS OF RESIDUAL DISTORTIONS IN THE SCENE VARY WITH THE TOTAL NUMBER OF CP's UTILIZED TO CORRECT FOR DISTORTIONS
- BASED ON THESE RESULTS, AT LEAST FOUR CP's (DISTRIBUTED OVER THE SCENE) ARE NEEDED TO SATISFY THE TM TEMPORAL REGISTRATION ACCURACY REQUIREMENT
- THESE RESULTS ALSO SHOW THAT FEWER THAN FOUR CP's CAN BE UTILIZED WITH ONLY MODEST DEGRADATION IN REGISTRATION ACCURACY

LIMITING CONTROL POINT REGION TO ONE SCENE (Concluded)

TM TEMPORAL REGISTRATION ACCURACY VS. NUMBER OF CP'S UNIFORMLY DISTRIBUTED OVER ONE SCENE



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