INCORPORATION OF STAR MEASUREMENTS FOR THE DETERMINATION OF ORBIT AND ATTITUDE PARAMETERS OF A GEOSYNCHRONOUS SATELLITE (AN ITERATIVE APPLICATION OF LINEAR REGRESSION)*

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ABSTRACT

Currently on NOAA/NESS's VIRGS system at the World Weather Building star images are being ingested on a daily basis. The image coordinates of the star locations are measured and stored. Subsequently, the information is used to determine the attitude, the misalignment angles between the spin axis and the principal axis of the satellite and the precession rate and direction. This is done for both the 'East' and 'West' operational geosynchronous satellites. This orientation information is then combined with image measurements of earth-based landmarks to determine the orbit of each satellite. The method for determining the orbit is simple. For each landmark measurement one determines a nominal position vector for the satellite by extending a ray from the landmark's position towards the satellite and intersecting the ray with a sphere with center coinciding with the earth's center and with radius equal to the nominal height for a geosynchronous satellite. The apparent motion of the satellite around the earth's center is then approximated with a Keplerian model. In turn the variations of the satellite's height, as a function of time found by using this model, are used to redetermine the successive satellite positions by again using the earth-based landmark measurements and intersecting rays from these landmarks with the newly determined spheres. This process is performed iteratively until convergence is achieved. Only three iterations are required.

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I. Introduction

When the first geosynchronous spin stabilized satellites with spin scan cameras were launched, it was hoped that the image of the earth in these satellite generated images would remain stationary so that the dynamics of the world's weather systems could be observed with respect to an earth's reference frame. This hope was not realized.

Eventually (1970-1971) at SSEC of the University of Wisconsin, software packages were developed by Mr. Dennis Phillips and Mr. Eric Smith which generated a satellite attitude from earth based landmark measurements and satellite orbit parameters and which enabled one to transform earth coordinates to image coordinates and vice versa.

Next, Mr. John T. Young, also at SSEC, skillfully adjusted orbit parameters made available from either NASA or NOAA to align pictures with high precision on a regular basis. However, since this approach requires a highly skilled operator and is time consuming, this approach has essentially never been transferred to other installations.

Consequently, when NOAA/NESS convened with SSEC about the transfer of SSEC's navigational capabilities to NOAA/NESS's operations, it was resolved that a proposal of Mr. Dennis Phillips to develop automatic methods to extract attitude parameters and orbit parameters from earth based landmark measurements and earth edge measurements would be founded. As a result, two software packages, COMORB (compute orbit) and UPGORB (upgrade orbit) were developed at SSEC and transferred (June 1978) along with the VIRGS computer system to NOAA/NESS's World Weather Building. In September, 1978 Dr. Dennis Phillips demonstrated the alignment capability of this system and the software started to be used regularly in the operations around May, 1979.

However, in August, 1978, Dr. Ken Chan, Mr. Ron Gird and Mr. Ben Remondi, demonstrated that star images could be detected and measured in the image frame. It was recognized that star measurements would enable a very precise determination of the satellite's attitude and the misalignment between the satellite's spin axis and the satellite's principal axis. Dr. Dennis Phillips of Scientific Programming and Applied Mathematics, Inc. has subsequently modified the SYSNAV software package to accept these star measurements for attitude determination and changed the UPGORB software package to use these attitude and misalignment parameters to generate a Keplerian set of orbit parameters which predictively aligns satellite images 24 hours in the future. This software will be used in the operations very shortly.

II. Attitude and Misalignment Parameter Determination

Each star measurement (we index the star measurements with the variable i) determines a unit vector (x_i, y_i, z_i) which points parallel to the direction from the satellite to the star. In addition, by using the line number of the position of the star in the image frame, we can determine approximately the angle ϕ_i between satellite's spin axis vector (u, v, w) and the unit vector. We have then, that

$$ux_i + vy_i + wz_i - \cos\phi_i = e_i \text{ for } i=1,\ldots,n$$

where n is the number of star measurements and e_i is the error incurred at each i^{th} measurement.

The mathematical problem is to minimize

$$S = \sum_{i=1}^{n} (ux_i + vy_i + wz_i - \cos\phi_i)^2$$

subject to the constraint $u^2 + v^2 + w^2 = 1$. We do this by iteration and take advantage of the fact that we know that w is always close to the value -1.

We set $(u_0, v_0, w_0) = (0, 0, -1)$

and iterate

(u_n, v_n) = solution of setting

$$\frac{\partial S(u, v, w_{n-1})}{\partial u} = 0$$

$$\frac{\partial S(u, v, w_{n-1})}{\partial v} = 0$$

and normalize by setting

$$w_n = -\sqrt{(1.0-u_n^2 - v_n^2)}$$

until $(u_n-u_{n-1})^2 + (v_n-v_{n-1})^2 \leq 1.0\text{ E-12}.$

Convergence is achieved in 2 or 3 iterations.

To realistically model the problem, we have to introduce the possibility of a pitch misalignment angle. Hence, we consider the problem of minimizing

$$S = \sum_{i=1}^{n} (ux_i + vy_i + wz_i - \cos(\phi_i+\phi))^2$$

again subject to the constraint $u^2+v^2+w^2 = 1$. Instead, we consider the equivalent problem

$$S = \sum_{i=1}^{n} (ux_i+vy_i+wz_i+ a \cos\phi_i + b\sin\phi_i)^2$$

subject to the constraints $u^2+v^2+w^2 = 1$

and $a^2+b^2 = 1$.

To solve we set

$(u_0, v_0, w_0, a_0, b_0) = (0,0,-1,0,-1)$

and iterate

(u_0,v_0,b_0) = solution of setting

$$\frac{\partial S}{\partial u}(u,v,w_0,a_0,b) = 0$$

$$\frac{\partial S}{\partial v}(u,v,w_0,a_0,b) = 0$$

$$\frac{\partial S}{\partial b}(u,v,w_0,a_0,b) = 0$$

and normalize by setting

$$w_n = \sqrt{1.0-u_n^2-v_n^2}$$

and $a_n = \sqrt{1.0 - b_n^2}$

until $(u_n - u_{n-1})^2 + (v_n - v_{n-1})^2 + (b_n - b_{n-1})^2 \leq 1.0E-12$

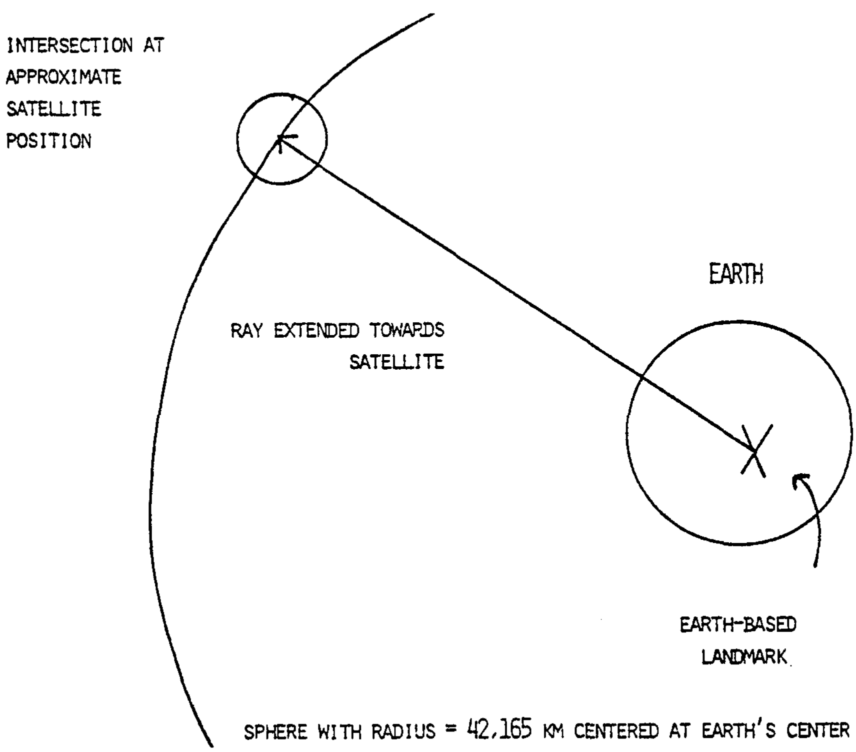
Convergence is still achieved within 2 or 3 iterations.

The roll and yaw misalignment angles are determined by a method which in a mathematical sense is virtually identical to the approach used to find the attitude of the spacecraft.

III. Orbit Determination

Once the attitude and misalignment of the spacecraft are determined, the determination of a set of orbit parameters describing the motion of the spacecraft is relatively straightforward. By using the attitude and misalignment parameters along with the line and element numbers of the measurement of image location of earth-based landmark, one can determine a unit vector in inertial coordinates which is parallel to the vector from the satellite to earth-based landmark.

By extending a ray from the landmark towards the satellite and intersecting that ray with an earth centered sphere whose height approximately equals the height of a geosynchronous satellite, one obtains an approximate satellite position vector $P_i = \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}$ at time t_i and indexed by i.



To determine the orbit plane perpendicular we minimize

$$S = \sum_{i=1}^{n} (ux_i+vy_i+wz_i)^2$$

subject to the constraint $u^2+v^2+w^2 = 1$

where here (u,v,w) is the orbit plane perpendicular and n is the number of approximate satellite positions.

The quantities

$$ux_i+vy_i+wz_i$$

should be close to zero by the definition of a perpendicular. The sum S is minimized by using exactly the same method used to find the spin axis vector.

All that is left to be determined is the motion of the satellite within its orbital plane. We model this motion with equation $t_i = c_1 + c_2\alpha_i + c_3 \sin\alpha_i + c_4 \cos\alpha_i$ where the t_i's are the times the approximate satellite position vectors are determined, the α_i's are the angular positions of the approximate satellite position vectors around center of the earth with respect to some arbitrary reference axis and the C_i's are to be determined. This model is exactly Keplerian within .03 km for eccentricities less than .01.

The C_i's are determined by using linear regression to minimize

$$S = \sum_{i=1}^{n} (C_1+C_2\alpha_i+C_3 \sin\alpha_i+C_4 \cos\alpha_i-t_i)^2.$$

A time span of 18 hours is necessary to determine C_2 and the other C_i's can be determined within a time span of 10 hours.

Estimates of the satellite's orbital variation of height as a function of time are obtained from the C_i's and used to recalculate the satellite approximate position vectors from the earth-based landmark measurements. This is done iteratively until a convergence criteria is satisfied. This requires 5 to 6 iterations. Finally, the orbit plane perpendicular and the constants C_i's are converted to standard Keplerian constants.

IV. Evaluation Criteria for Attitude and Orbit Generating Software

1. The amount of training and background required for each system operator
2. The relative convenience and ease of use of the system
3. The total man and computer resources necessary to operate the system
4. Current operation status
5. Accuracy
6. Time required to recover operational accuracy after maneuvers
7. Future development prospects

V. Future Developments

A. Sun pulse documentation information will be used to detect and measure the effects of nutation and these effects will be removed

B. Attitude precession will be determined automatically

C. The orbit model will be improved to increase accurate propagation periods; eventual goal is to propagate accurately up to 7 to 10 days.