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EFFECTS OF SUPPLY CONDITIONS ON FILM THICKNESS IN LUBRICATED HERTZIAN CONTACTS

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16. Abstract

This article describes a study to use generalization of the hydrodynamic expression for Hertzian contacts and to give various methods for calculating the thickness of the oil film winder steady-state, isothermal conditions. This is important for engineering applications such as gears and bearings because these new results are closer to real operating conditions.

The theories of lubrication are discussed. The mathematics involved are presented using approximately 30 equations and 13 figures.

The study demonstrated that for lubricated, linear, elliptical or point Hertzian contacts it is possible to calculate the thickness of the oil film at the center of the contact for steady-state isothermal conditions.

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EFFECTS OF SUPPLY CONDITIONS ON FILM THICKNESS IN LUBRICATED HERTZIAN CONTACTS

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1. INTRODUCTION

In a theoretical and experimental study we have shown [1] and [2]:

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- that over a wide range of loads and speeds, a complete oil film can form in a point Hertzian contact defined by a ball and a plane under operating conditions which correspond to those which exist in real mechanisms such as ball bearings;

- that the thickness of the oil film can be calculated from exact or approximate theories which are applicable in the three steady-state, isothermal operating regimes:

  - the iso-viscous hydrodynamic regime in which the contact surfaces are undeformable and the viscosity of the fluid is constant;
  - the piezo-viscous hydrodynamic regime in which the surfaces are still undeformable, but the viscosity varies exponentially with pressure;
  - the elasto-hydrodynamic regime in which the surfaces deform under the high hydrodynamic pressures in the fluid whose viscosity varies with pressure;
- that the agreement between theoretical and experimental results is satisfactory in an operating range which covers the hydrodynamic regime and the beginning of the elasto-hydrodynamic.

We have also seen that for some operating conditions, formation

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**Numbers in margins indicate foreign pagination.

of an oil film in this type of contact becomes very difficult. This difficulty is seen as a sharp drop in the thickness of the oil film, and is characterized by the existence of an air-oil meniscus at the inlet to the contact. Thus, the ordinary boundary conditions on the inlet pressure are no longer applicable because they are directly linked to conditions on the oil supply to the contact.

In this study we propose:

- to use a generalization of the hydrodynamic expression for Hertzian contacts, and to give various methods for calculating the thickness of the oil film under steady-state, isothermal conditions;

- to discuss boundary conditions at the inlet to the contact, to show their effects on film thickness, and also to introduce a calculating method for a variable oil supply;

- in the special case of a sphere-plane contact, to compare the experimental results obtained with various supply conditions, and thus to compare the theoretical and experimental results for this real example;

- to show the importance of supply conditions for engineering applications such as gears and bearings, because these new results are closer to real operating conditions.

2. THEORIES

This study is an extension and generalization of the results reported previously for the special case of a sphere-plane contact [1].

2.1. Model and dimensionless parameters

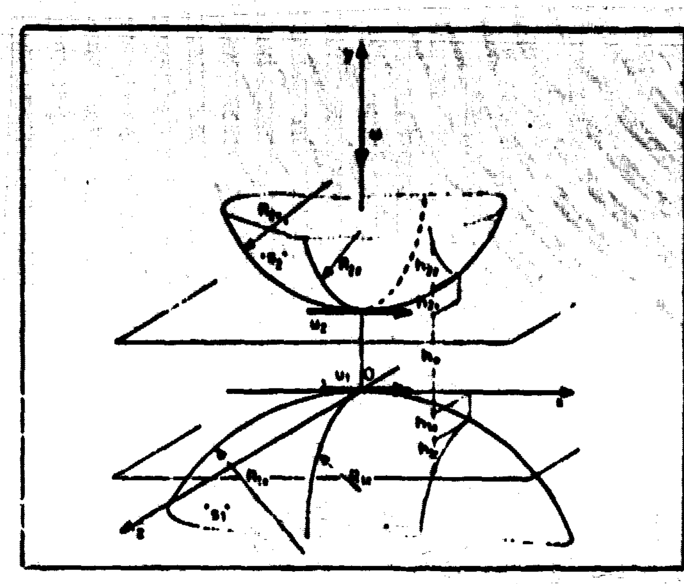

Figure 1. General model of a contact

For the lubricated contact encountered in a bearing or a gear, we can take a simplified model in which the radii of curvature of the surfaces are the principal radii $R_{1x}, R_{1z}, R_{2x}, R_{2z}$, the surface velocities u_1 and u_2 lie along principal direction Ox, and the normal loading or hydrodynamic lift W acts along Oy (Figure 1).

In the neighborhood of the contact, film thickness h(x,y), which is written $h = h_0 + h_{1x} + h_{2x} + h_{1z} + h_{2z}$, becomes:

$$h \simeq h_0 + \frac{x^2}{2}\left(\frac{1}{R_{1x}} + \frac{1}{R_{2x}}\right) + \frac{z^2}{2}\left(\frac{1}{R_{1z}} + \frac{1}{R_{2z}}\right)$$

In this region, the system is geometrically equivalent to a model composed of a solid of revolution whose surface S_2 is generated by the two circles R_x and R_z, and the surface xOz "S1" plane (Figure 2). The film thickness is:

$$h \simeq h_0 + \frac{x^2}{2R_x} + \frac{z^2}{2R_z}$$

where R_x and R_z are the equivalent radii of curvature defined by:

$$R_x = \frac{R_{1x} \cdot R_{2x}}{R_{1x} + R_{2x}} \qquad \text{and} \qquad R_z = \frac{R_{1z} \cdot R_{2z}}{R_{1z} + R_{2z}}$$

This geometric and kinematic model is a good representation for hydrodynamic and elasto-hydrodynamic studies of the operation of a gear or bearing under steady-state or isothermal conditions. We note, however, that it does not take account of rotational motions about the Oy axis which may also occur in these mechanisms.

From dimensional analysis, we shall define the dimensionless parameters, which characterize the steady-state, isothermal operation of a lubricated contact:

$G^* = \alpha E$, the material parameter,

where α is the piezo-viscous coefficient, such that $\mu = \mu_0 \exp(\alpha p)$, and E is the equivalent elastic modulus defined from the Young's moduli and the Poisson's ratios of the two materials:

$$\frac{1}{E} = \frac{1}{2}\left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}\right)$$

$U^* = \frac{\mu_0 (u_1 + u_2)}{E R_1}$ the velocity parameter, /26

which involves the sum of the velocities of the two surfaces S_1 and S_2.

$W^* = \frac{\omega}{E R_1 L}$ the load parameter,

where length L represents the length l of a cylinder or the radius R of a sphere.

$H^* = \frac{h}{R_1}$, the film-thickness parameter

In addition to these parameters involving reference radius R_1 , there is also a shape parameter C^* which is a constant for a given contact, but which can be changed to go from a linear contact of radius R_1 and length l to a point contact of radius $R = R_1 = R_2$ or even to an elliptical contact of radii R_1 and R_2.

2.2. Hydrodynamic problem

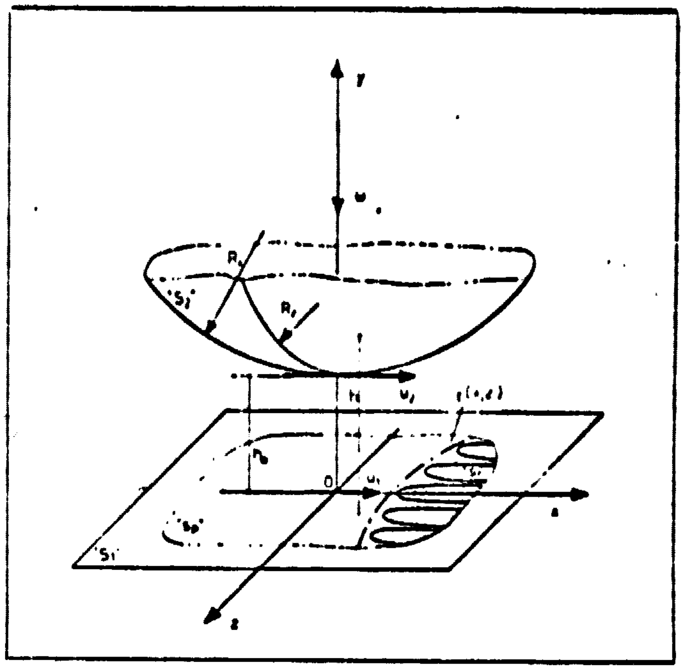

Figure 2. General model of an elliptical contact.

The theory of hydrodynamic lubrication is obtained by integrating the Reynolds equation which represents the differential form of the pressure in thin films. This equation replaces the Navier-Stokes equations and the equation of flow continuity, and for a given thickness of fluid film it gives the pressure distribution if the contour of the pressure domain and the boundary conditions on this contour are known. For laminar flow of an incompressible viscous fluid between the two surfaces S_1 and S_2 under steady-state, isothermal conditions, the Reynolds equation is:

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right) = 6(u_1 + u_2)\frac{\partial h}{\partial x}$$

in which the viscosity μ, the film thickness h, and the pressure p are functions of x and z.

As before, we are thus led to consider an undefined domain s in the xOz plane (which represents surface S_1); this domain includes (Figure 2):

- a pressure region sp within which the pressure satisfies the Reynolds equation. Inside this region the oil film is a homogeneous, continuous medium.

- a rupture region sr in which the pressure is zero (atmospheric pressure or pressure of the saturated vapor), where the film breaks. In this region the divergent configuration of the surfaces and flow continuity provoke rupture of the fluid film and the appearance of threads of air.

- The $\gamma(x,z)$ curve delimits these two regions at the outlet from the contact.

The problem thus reduces to accurately determining the boundary which separates the zones containing a homogeneous, continuous fluid - that is, where high hydrodynamic pressures can exist - from the zones in which the fluid is replaced partly or wholly by another medium, which can be air or vapor.

Two types of schematic supply conditions can be used to define the limits of the sp pressure region at the inlet to the contact:

- the oil supply at the inlet to the contact is superabundant: a change in the supply conditions results in almost no change in the results [of the calculation].

- the oil supply at the inlet to the contact is scanty, but still enough that there is always hydrodynamic operation: a change in the supply conditions thus produces a very large change in the results.

2.3 Results for superabundant supply

Known approximate or exact theories mainly deal with the case of linear, point, or elliptical contacts, and consist of determining the pressure distribution in the contact zone by solving the Reynolds equation. The solution is hydrodynamic when the surfaces defining the contact are indeformable, and elasto-hydrodynamic when the surfaces deform under the high hydrodynamic pressures.

For flow of a viscous film, this pressure distribution is a function of only x and z. The usual boundary conditions on the pressure are:

- along the Ox axis: $p = 0$ for $x = -\infty$.
- along the Oz axis: $p = 0$ for $z = \pm\infty$ for a sphere-plane contact.
  $p = 0$ for $z = \pm l/2$, for a cylinder of finite length l.
  $p =$ constant along Oz for an infinitely long cylinder.
- at the outlet of the film: $p = 0$ on the $\gamma(x, z)$ curve.

The $\gamma(x, z)$ curve can be:

- fixed arbitrarily - this corresponds to the Sommerfeld semi-conditions.

$p = 0$ on the $\gamma(x, z)$ curve, which lies along the Oz axis. The solution obtained this way is approximate. /27

- or determined by the continuity conditions which define the Reynolds conditions:

$p = \frac{dp}{dn} = 0$ on the $\gamma(x, z)$ curve, where n is the normal. The solution obtained this way is exact. We note that this mathematical condition does not explain the formation of thin streaks of air. This aspect of the problem will not be studied here.

At the contact inlet, the amount of oil is still assumed to be in excess of that needed, $p = 0$ for $x = \infty$, which means in practice that the oil thickness at the contact inlet $h_e \sim \infty$.

2.3.1. Hydrodynamic theories

We recall that the film thickness H_0^* at the center of the cylinder-plane contact is given by Martin's solution [3]:

$$H_0^* = 2{,}448 \frac{U^*}{W^*}$$

The thickness H_0^* at the center of a sphere-plane contact is given by Kapitza's approximate analytical solution [4]:

$$H_0^* = 28{,}4 \left(\frac{U^*}{W^*}\right)^2$$

and our exact numerical solution [1] which is a special case of the general problem introduced in Section 2.3:

$$H_0^* = 5{,}66 \left(\frac{U^*}{W^*}\right)^{1{,}77}$$

The thickness H_0^* at the center of a sphere- plane contact is given only by Kapitza's approximate analytical solution:

$$H_0^* = 79 \left(\frac{U^*}{W^*}\right)^2 C^*$$

if $W^* = \frac{\omega}{E R_1^2}$ and if the shape factor C^* is such that:

$$C^* = \left[\left(1 + \frac{2}{3}\frac{R_2}{R_1}\right)^2 \cdot \frac{R_2}{R_1}\right]^{-1}$$

2.3.2. Elasto-hydrodynamic theories

The film thickness at the center of a contact in the elasto-hydrodynamic regime cannot be determined as easily as for the hydrodynamic case. In these theories, the surfaces deform elastically under the high pressures, so that a supplementary equation must be introduced into the calculations to give the elastic displacements of the surfaces. In the approximate solution of the elasto-hydrodynamic problem, the geometric shape of the oil film at the contact is obtained from the displacements resulting from the elliptical Hertz pressure distribution for dry static elastic contacts. As a consequence, the presence of the oil film is assumed to have no effect on the elastic deformation of the contact. The Reynolds equation is integrated to the contact inlet for a piezo-viscous fluid for which $\mu = \mu_0 \exp(\alpha p)$.

The additional assumption which allows the constant oil-film thickness in the Hertzian region to be calculated was made by Grubin, who assumed that the real pressure in the oil film becomes practically infinite at the inlet to the Hertzian region. In the exact elasto-hydrodynamic solution, deformations of the surfaces are due to hydrodynamic pressures, and an iterative calculation leads to a single hydrodynamic pressure distribution. We note that all solutions obtained from a direct iterative process diverge, and that the exact hydrodynamic solution has been obtained by Dowson and Higginson only for the roller-plane case.

The thickness of the oil film at the center of a sphere-plane contact is given by Grubin's approximate elasto-hydrodynamic solution to be [5]:

$$H_o^* = 1{,}175\ U^{*0{,}727}\ G^{*0{,}727}\ W^{*\,-0{,}091}$$

while Cheng gives [16]:

$$H_o^* = 1{,}131\ U^{*0{,}740}\ G^{*0{,}740}\ W^{*\ 0{,}11}$$

and in the exact elasto-hydrodynamic solution of Dowson and Higginson [7] it is:

$$H_o^* = 0{,}985\ U^{*0{,}7}\ G^{*0{,}6}\ W^{*\,-0{,}13}$$

For the approximate elasto-hydrodynamic solutions obtained by several authors the film thickness at the center of a sphere-plane contact is given uniquely by:

$$H_o^* = k\ U^{*a}\ G^{*a}\ W^{*c}$$

where $k = 3$; $a = 1$ and $c = -0{,}333$ for Cameron and Goher [8]

$k = 0{,}837$; $a = 0{,}740$ and $C = -0{,}074$ for Archard and Cowking [9]

$k = 1{,}022$; $a = 0{,}725$ and $c = -0{,}058$ for Cheng [6]

$k = 1{,}053$; $a = 0{,}714$ and $c = -0{,}048$ for Wedeven et al [10].

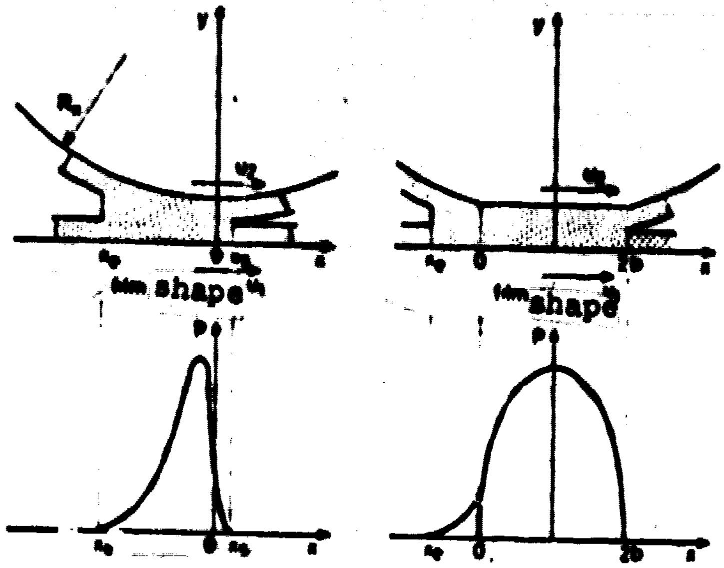

Figure 3 - Models for linear contacts.

All these solutions are very close to one another and are practically equivalent, except for the first one, Cameron and Gohar, which in this simplified form is valid only for high loads.

In the approximate elasto-hydrodynamic solution of Archard and Cowking [9], the film thickness at the center of a sphere-plane contact is given only by:

$$H_0^* = 1{,}222\ U^{*0{,}740} \cdot G^{*0{,}740} \cdot W^{*-0{,}074} \cdot C^{*0{,}740}$$

where C^* is a shape factor such that

$$C^* = \left[1 + \frac{2}{3}\frac{R_y}{R_x}\right]$$

and Cheng's result [6]:

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$$H_0^* = (6N)^{\frac{1}{1-n}} \cdot U^{*\frac{1}{1-n}} \cdot W^{*\frac{n}{1-n}} \cdot C^{*\frac{1}{1-n}}$$

with $C^* = \frac{a}{R_x}\left(\frac{2\pi b}{3R_x}\right)^n$, where N, n and C^* are known constants for the known values of the semi-major axes a and b of the contact ellipse (along Ox and Oy) in Hertz's theory. For the special case where $\frac{b}{a} = [illegible]$ and $\frac{R_z}{R_x} = [illegible]$, Cheng's solution becomes: /28

$$H_0^* = 0.883\, U^{*0.7[illegible]}\, G^{*0.7[illegible]} \left[\frac{3w}{[illegible]\, E}\right]^{-0.[illegible]}$$

The values of the semi-major axes a and b can be determined by Hertz's theory as given by Timoshenko [11] and by calculation making use of the radii of curvature R_x and R_z, the equivalent elastic modulus E, and an auxiliary function which depends on elliptic integrals of the first and second kinds [12].

2.4. Effects of supply conditions on results

The classical theory of lubrication developed above assumes that the abscissa of the inlet represents a boundary at infinity, or that it corresponds to a finite abscissa which gives practically the same result. Solutions consist of decreasing the pressure integration range by decreasing the thickness of the oil film at the inlet to the contact
- that is, by letting x_E, the abscissa of the inlet, tend to 0. The first part deals with the hydrodynamic study of cylinder-plane and sphere-plane contacts.

The oil-film thickness obtained with the classical theory discussed above for superabundant supply is taken as a reference valve (Section 2.3).

The decrease in H_0, the thickness of the oil film at the center of the contact, will be expressed as a function of a single supplementary parameter(given in an appropriate dimensionless form) which characterizes the position of the inlet boundary, and thus the supply conditions. In succession we shall examine the effect of the supply conditions for linear and point Hertzian contacts under hydrodynamic and elasto-hydrodynamic conditions, using the models shown in Figure 3. Study of supply conditions for an elliptical contact has still not been carried out.

2.4.1. Modification of the hydrodynamic theory

For a cylinder-plane contact, the calculations of Wolveridge et al. [13] amount to using the hydrodynamic model sketched in Figure 3a. The results in Figure 4 give the reduction factor:

$$\beta = \frac{h_0}{(h_0)_\infty} = \frac{H_0^*}{(H_0^*)_\infty}$$

in which $(h_0)_\infty$ is the thickness at the center of the oil film, obtained from Martin's solution, and h_0 is the thickness at the center for an inlet abscissa x_e, as a function of the dimensionless inlet parameter:

$$\chi_E = \frac{x_e}{\left|2R(h_0)_\infty\right|^{\frac{1}{2}}}$$

For a sphere-plane contact, we have shown the effect of the integration range when $x_e = z_e$. The results shown in Figure 5 give the reduction factor:

$$\beta = \frac{H_0^*}{(H_0^*)_\infty}$$

as a function of the dimensionless inlet parameter:

$$\chi_E = \frac{x_e}{R(H_e^*)^n_\infty}$$

when $x_e = z_e$ and $n = 0.41$.

2.4.2. Modification of the elasto-hydrodynamic theory

For cylinder-plane contact, the calculations of Wolveridge et.

al. [13] amount to using Grubin's elasto-hydrodynamic model, as shown in Figure 3b. We see that under the assumptions of the calculation only the amount of fluid at the contact inlet is going to be involved, and this leads to the choice of x_e defined in Figure 3b. The results shown in Figure 6 give the reduction factor:

$$\beta = \frac{H_0^*}{(H_0^*)_\infty}$$

in which $(H_0^*)_\infty$ is the thickness at the center of the contact obtained from Grubin's solution as a function of the dimensionless inlet parameter:

$$X_E = \frac{\alpha^{\frac{1}{3}} x_e}{[2R(h_0)_\infty]^{\frac{2}{3}}}$$

For the exact elasto-hydrodynamic solution, Castle and Dowson [14] obtained a curve which coincides with the one in Figure 6. /29

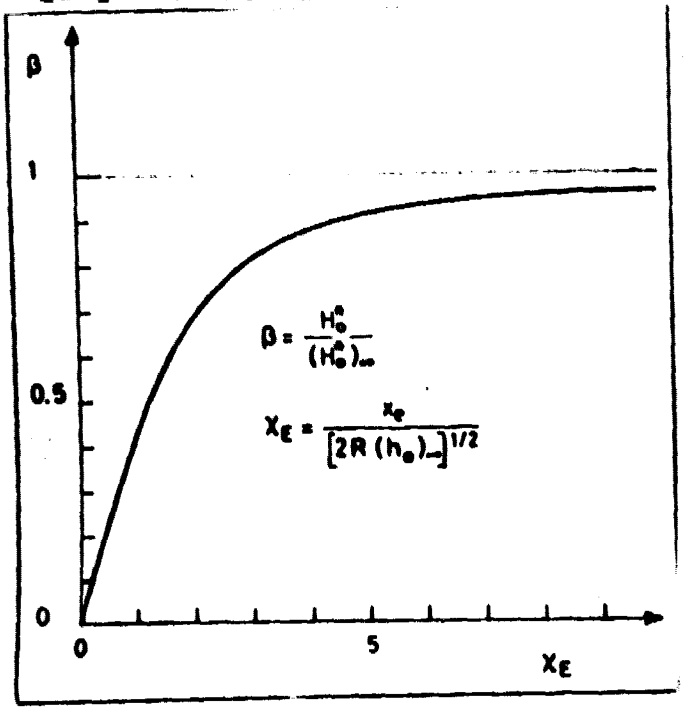

Figure 4: Effect of inlet abscissa on film thickness at contact center for cylinder-plane hydrodynamic contact.

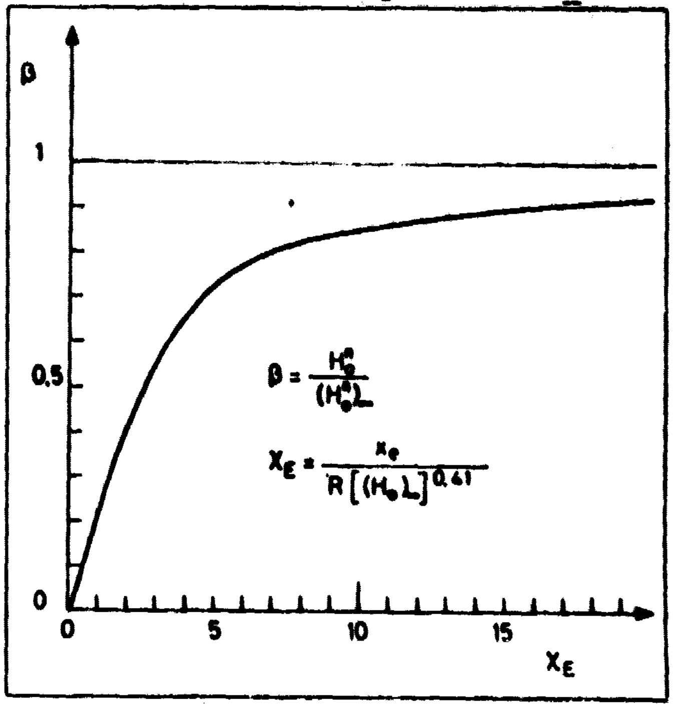

Figure 5: Effect of integration range defined by inlet abscissa x_e on film thickness at contact center for sphere-plane hydrodynamic contact.

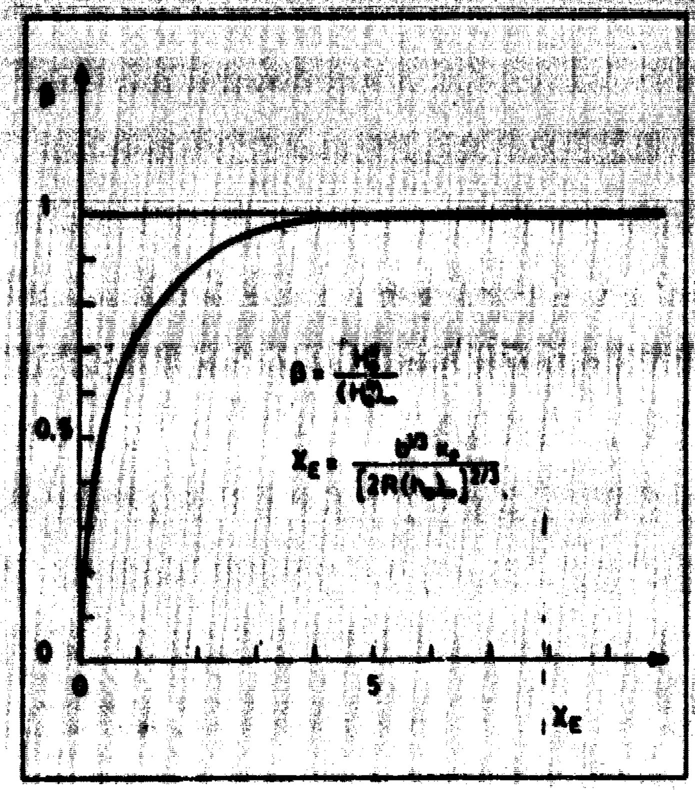

Figure 6: Effect of inlet abscissa on film thickness at contact center for cylinder-plane elasto-hydrodynamic contact.

For a sphere-plane contact, Wedeven et al [10] have shown the influence of integration range on the results, using Grubin's approximate elasto-hydrodynamic model. These results, shown in Figure 7, give the reduction factor:

$$\beta = \frac{H_o^*}{(H_o^*)_\infty}$$

as a function of the dimensionless inlet parameter $\chi_a \frac{x_e}{x_f}$, normalized to a finite abscissa:

$$x_f \simeq \frac{3{,}52\,[R(h_o)_\infty]^{\frac{2}{3}}}{a^{\frac{1}{3}}}$$

This showed a very small change in film thickness:

$$\beta = \frac{(h_o)_f}{(h_o)_\infty} = 0{,}86.$$

In conclusion, we see that for steady-state isothermal conditions we can use an expression for H_o, the oil-film thickness at the center of the contact, which is of the general form:

$$H_0 = A^{a} . C^{b} . U^{c} . G^{d} . W^{e}$$

where A, C, U, G, and W are parameters describing the supply, shape, speed, material and loading respectively.

3. EXPERIMENTAL RESULTS /30

For cylinder-plane contact, several authors - Crook [15], Lauder [16], and Boness [17] - have shown that for low loads which did not produce a large change in viscosity or induce elastic deformation of the surfaces, and for experiments in which edge effects are negligible, the oil-film thickness measured experimentally was less than the value calculated by Martin's theory; they were thus able to show the effect of an inlet abscissa x_e on the results. For point contacts, Wedeven et al. [10] have shown that under elasto-hydrodynamic conditions and in pure rolling, a change in the amount of oil at the contact inlet produces a decrease in the thickness of the oil film at the center of the contact which follows the curve shown in Figure 7. For sphere-plane contact [18] in both the hydrodynamic and elasto-hydrodynamic regimes, we have also revealed a supply failure for pure sliding using an experimental device which allows application of low and medium loads and simultaneous measurement of load, friction force, speed, and oil temperature at the contact inlet, and determination of the shape of the oil film by an interferometric method [2]. In this device, a glass plane is held fixed, and a 30-mm ball rolls about one of its axes, bathed in the oil which it brings into the contact. The experimental results obtained with this device show that a supply failure, which is seen as a meniscus between the air and the oil, completely changes the thickness and geometric shape of the film, the distribution of hydrodynamic pressures, and the friction force.

3.1. Thickness and shape of the oil film /31

The results shown in Figures 8 and 9 for dimensionless variables show that when there is no supply failure, the film thickness H_0^*, in-

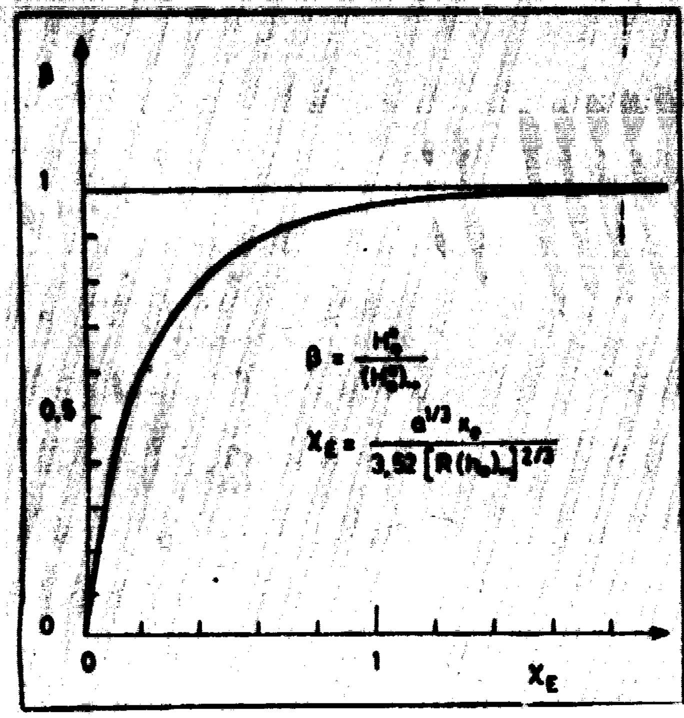

Figure 7: Effect of integration range defined by relative inlet abscissa [illegible] on film thickness at contact center for sphere-plane elasto-hydrodynamic contact.

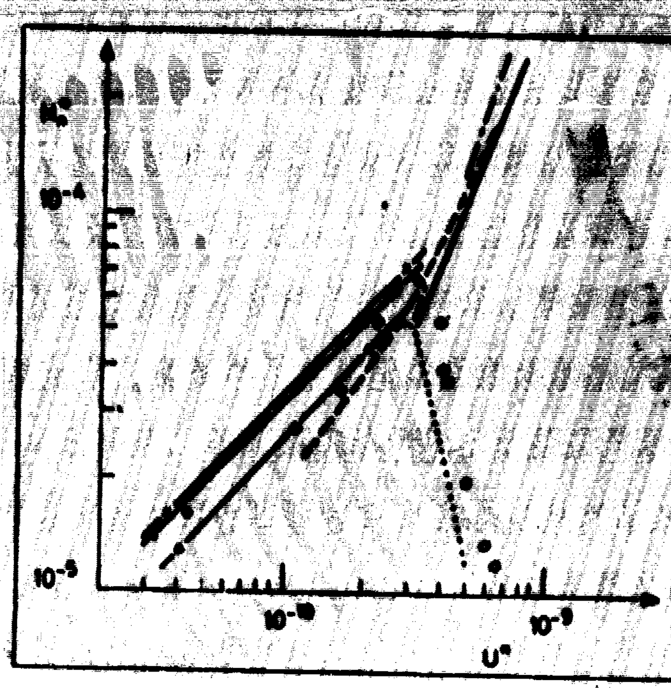

Figure 8: Variation of film thickness propagating into contact at a load [illegible]. Experiments: ✦ elasto-hydrodynamic, o hydrodynamic, ● supply failure.

Theories: hydrodynamic: ___, iso-viscous; _ _ piezoviscous; -.-.- Kapitza. elasto-hydrodynamic: -x- Wedeven et al.; -.- Archard and Cowking; -o- Cheng. modified hydrodynamic: ------.

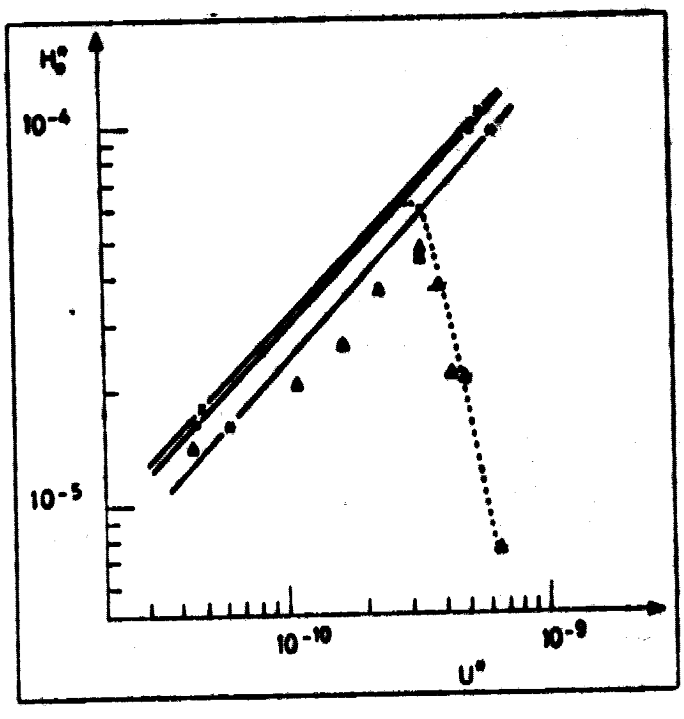

Figure 9. Variation of film thickness at contact center with speed: supply failure propagating into contact at load [illegible]. Experiments: ▲ elasto-hydrodynamic; △ supply failure. Theories: elasto-hydrodynamic: -x- Wedeven et al.; -·- Archard and Cowking; -o- Cheng; --- modified.

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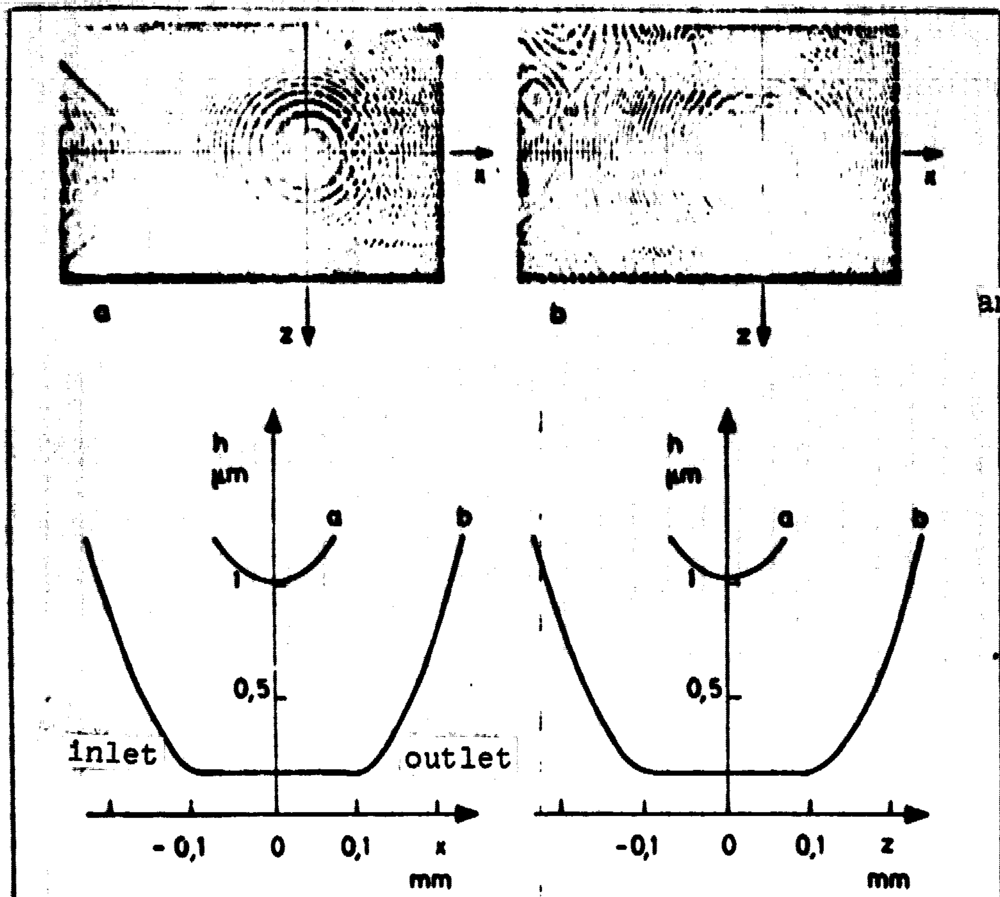

Figure 10: Newton's rings and geometric shape of oil film for O° = 1 74[illegible] ; μ_0 = 1.3 Pl ; R = [illegible] mm ; w = 3.9 N ; u = (a) [illegible] cm/sec and (b) [illegible] cm/sec

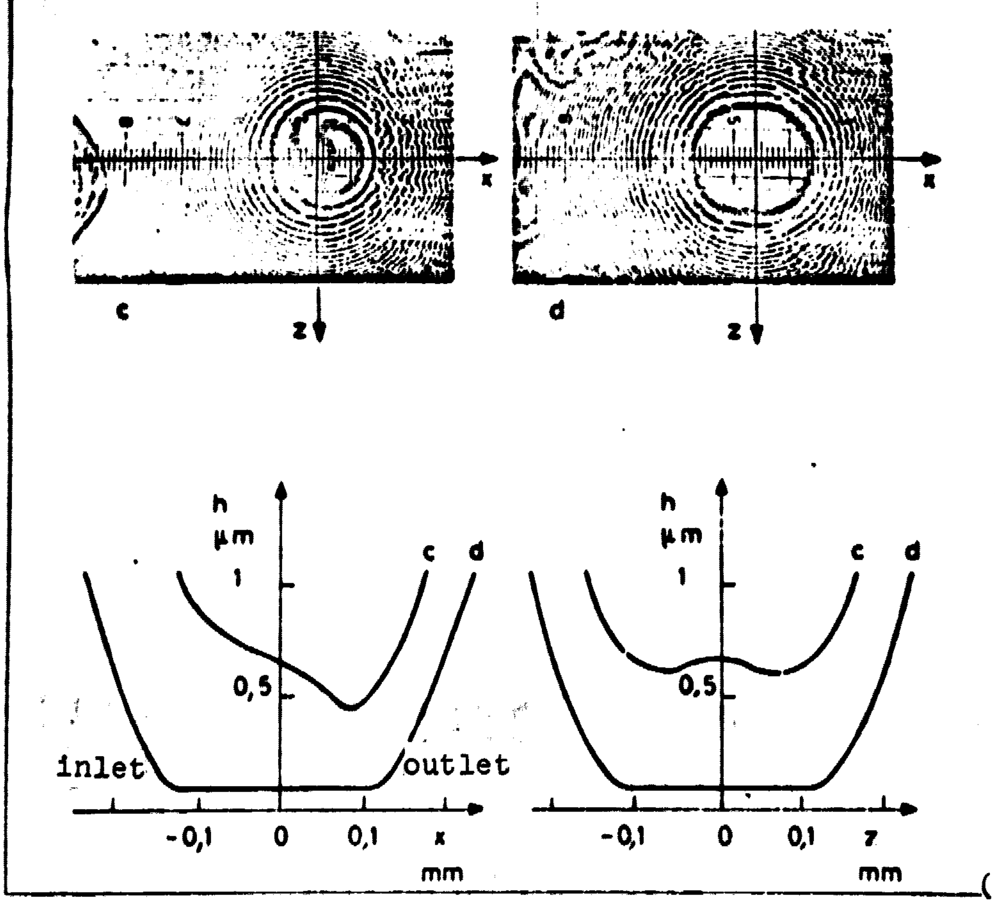

Figure 11: Newton's rings and geometric shape of oil film for O° = 1 740 ; μ_0 = 1.3 Pl ; R = 15 mm ; w = 7.8 N ; u = (c) 23 cm/sec and (d) 42 cm/sec

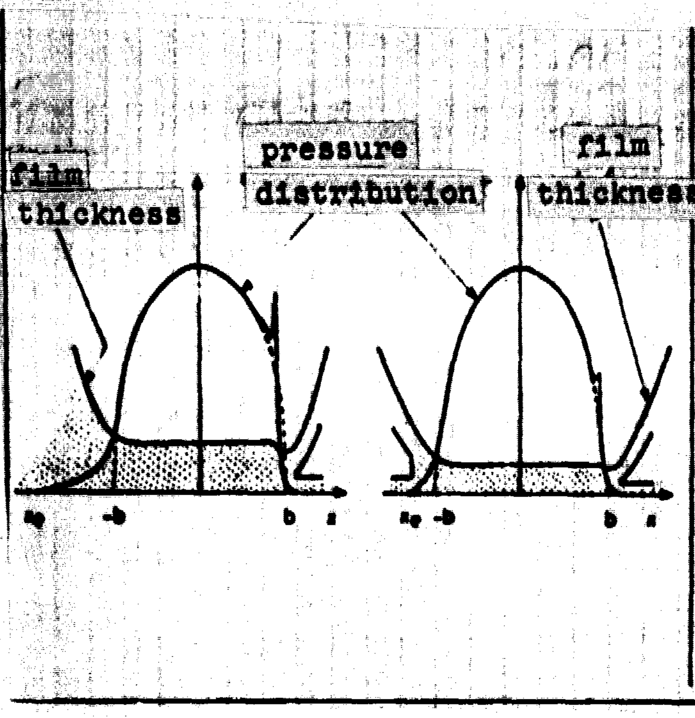

Figure 12: Pressure distributions and geometric shapes of typical films in an elasto-hydrodynamic cylinder-plane contact, as a function of inlet abscissa a_e .

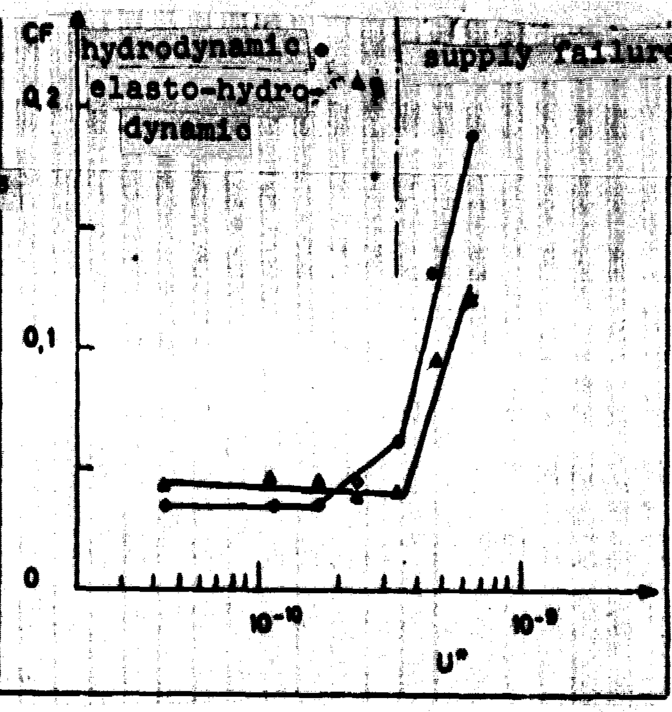

Figure 13: Variation of coefficient of friction as a function of speed for supply failure propagating in a contact at applied loads W' = [illegible] and Δ [illegible]

creases with speed, as the hydrodynamic and elasto-hydrodynamic theories predict. But when the meniscus enters the contact it makes the film thickness drop abruptly. The theoretical curves which include the change in film thickness as a function of inlet abscissa come close to the experimental results, and allow the decrease in oil-film thickness in the contact to be followed. The increase in speed which is favorable in ordinary hydrodynamics becomes very unfavorable when air is entrained in the contact. The film thickness thus shows a maximum. The appearance and progress of the meniscus in the contact depend mainly on the value of the product of speed and viscosity. The geometric shapes of the meniscus and the oil film are visible in the photographs and curves of Figure 10 for hydrodynamic conditions, and in Figure 11 for elasto-hydrodynamic conditions.

3.2. Pressure distribution

As the film thickness decreases, a complete oil film always /32
separates the two surfaces of the contact, but its geometric shape

changes: it flattens out. This flattening can be explained either by assuming non-Newtonian behavior [19] or - closer to reality - by considering the effect of inlet conditions. For linear contacts, Castle and Dowson [14] have shown that when the meniscus penetrates into the contact the height of the elasto-hydrodynamic peak decreases, its position moves toward the outlet of the film, and the hydrodynamic pressure distribution approaches the Hertzian distribution for dry elastic contacts (Figure 12). It is thus reasonable to think by analogy that the pressure distribution for operation with little oil in the contact, Grubin's approximations and assumptions lead to approximate solutions which are found to be very close to reality.

3.3. Friction force

We have shown [18] that under hydrodynamic conditions the coefficient of friction increases when the speed increases, and that it decreases slightly under elasto-hydrodynamic conditions. When the meniscus penetrates into the center of the contact, the shear domain decreases and the friction force also decreases. This is found to be true when air near the contact has almost no effect on the thickness of the oil film. On the other hand, the friction force increases considerably when the meniscus penetrates into the contact (Figure 13). This is due first of all to a large decrease in the thickness of the film, and then to an increase in the region of high shear rates which results from flattening of the contact zone.

4. DISCUSSION OF INLET CONDITIONS

Definition of inlet conditions depends mainly on the physical aspect of the problem, and principally on two of the fundamental assumptions of lubrication:

- homogeneous single-phase fluid,
- laminar flow.

The boundary conditions used at the inlet deal with the pressure and the speed:

Case 1: the pressure is zero at infinity: $p = 0$ for $x = -\infty$
Case 2: the pressure is zero at a finite inlet abscissa:

$p = 0$ for $x = x_0$ with $\frac{\partial p}{\partial x} \neq 0$ at this point.

Case 3: the velocity $\vec{V}(u, v, w)$ of a fluid particle goes to zero somewhere in the flow. For the laminar flow of an incompressible Newtonian fluid under steady-state isothermal conditions, the Navier equations reduce to:

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial v}{\partial p} = 0$$

$$\frac{\partial p}{\partial z} = \mu \frac{\partial^2 w}{\partial y^2}.$$ /33

for a thin film.

Integration of these equations gives the u and w compounds of the velocity. Constants of integration are determined by the boundary conditions, which correspond to assuming zero shear of the fluid on the contact surfaces:

for $y = 0$ $u = u_1$ and $w = w_1 = 0.$
for $y = h$ $u = u_2$ and $w = w_2 = 0.$

Thus, the components of the velocity $\vec{V}$ of a particle are:

$$u = \frac{1}{2\mu} \frac{\partial p}{\partial x} (y^2 - yh) + (u_2 - u_1) \frac{y}{h} + u_1$$

$$v = 0$$

$$w = \frac{1}{2\mu} \frac{\partial p}{\partial z} (y^2 - yh)$$

We can also show from these equations that the velocity $\vec{V}$ of a particle and its derivative $\frac{\partial u}{\partial y}$ go to zero at a point in the flow which does not lie on a wall when:

$$\frac{\partial p}{\partial x} = \frac{2\mu}{h^2}(u_1 + u_2 + 2\sqrt{(u_1 u_2)}$$

$$\frac{\partial^2 p}{\partial x^2} = 0$$

With the velocity conditions of Case 3, there is thus this last condition on the derivatives of the pressure. All these conditions can be, and have been, used for linear contacts, without always corresponding to real physical conditions.

Case 1, which is satisfied under some experimental conditions, is not compatible with the assumptions of viscous thin films, which are applicable only in a very reduced finite region. Practically speaking, this condition, the one most often used, is never encountered in a real mechanism.

Case 2, which is more likely and more realistic, allows the Reynolds equation to be integrated in a region in which there is a reverse flow at the inlet; the thin-film assumptions are valid this time. The reverse flow at the contact inlet allows a high pressure to be established there, and thus considerably increases the pressure at the center of the contact.

Case 3 gives the limit on the flow and the region where the Reynolds equation, limited to the fluid zone, can be applied when there is insufficient oil flow. It defines the point of zero velocity, which in this case is virtually identical to the meniscus separating the air and the oil [16] [20].

Note that this condition is not acceptable when u_1 and u_2 have opposite signs.

The conditions at the physical boundaries are those which appear in the photographs of Figures 10 and 11, for example, when we can no longer consider the problem of lubrication by oil [alone], but come to the two-phase problem generally encountered: air-oil-air. The conditions to be used at the inlet can thus no longer be considered everywhere as an imaginary theoretical boundary, but must be treated locally as a discontinuity, here the surface separating the air and oil.

5. TECHNOLOGICAL VALUE

Knowing the thickness of the oil film lets us explain the operating conditions of a bearing, and is needed to determine its life.

We have seen that under hydrodynamic or elasto-hydrodynamic conditions the thickness of the oil film decreases appreciably only for very low values of the supply parameter [illegible]. In practice, this means that the hydrodynamic lift is the same as long as [illegible], the thickness of the oil film at the contact inlet, is greater than about 100 h_o - that is, of the order of R/100 for a cylinder or a sphere of equivalent radius R. We can thus see that a very small amount of oil is enough to provide hydrodynamic lift comparable to that obtained when the test pieces defining the contact are completely immersed. We have also shown previously [2] that the friction force, which corresponds to the integral of the shear stresses, varies much faster as a function of integration range than does the load. A very small amount of oil at the contact decreases the friction force considerably, and improves the operating conditions of the mechanism. Lubrication in which the meniscus between the air and the oil is quite close to the center of the contact is therefore desirable. The amount of oil at the contact must be enough to prevent contact between asperities in the surfaces, however, because under these conditions hydrodynamic lubrication would become limiting or mixed lubrication. This is why lubrication of high-speed bearings requires only a small amount of oil for optimum operation.

The behavior of the oil film also explains some results related to fatigue and surface damage in the lubrication of bearings. In a recent theoretical and experimental study, Andreason and Lund [21] have shown that the operating life of a bearing is related to a lubrication factor. If the parameter is taken to be the minimum oil-film thickness divided by the average value of the surface roughnesses, the regimes of hydrodynamic, elasto-hydrodynamic, or mixed operation are easily characterized by this parameter. The experimental results have shown that the life of a bearing operating under hydrodynamic or elasto-hydrodynamic conditions is twice that of a bearing operating under mixed conditions. This example thus shows the importance of precisely determining the thickness of the oil film in mechanisms, and especially the effect of supply conditions in defining and predicting these operating conditions.

6. CONCLUSION

For lubricated, linear, elliptical, or point Hertzian contacts it is possible to calculate the thickness of the oil film at the center of the contact for steady-state isothermal conditions. Under hydrodynamic conditions, this calculation requires knowledge of parameters defining the geometric shape, the speeds, the normal applied load, and the viscosity and piezoviscous coefficient of the fluid. Under elasto-hydrodynamic conditions, the elastic properties of the two solids composing the contact surfaces are also involved. /34

This study has shown us that we must introduce an additional parameter to take account of the conditions under which fluid is supplied to the contact. Through a visualization technique, we have looked at a supply failure at the contact inlet, followed its progress in the contact, and shown the abrupt decrease in film thickness which results. This additional parameter has also been introduced in the calculations, and we have thus been able to propose a more general expression for calculating the film thickness in a Hertzian contact by including the supply parameter.

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