CONSIDERATIONS IN THE PLACEMENT OF PHASE CALIBRATOR TONES*

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ABSTRACT

In the use of tones to calibrate unwanted instrumental phases for very long baseline interferometry experiments, certain problems exist which are related to the placement of these phase calibrator tones, for example:

1. A bias exists in an analytically generated stopping function used during correlation if its frequency satisfies the following condition; $f = f_s m/n$, where f_s = sampling frequency, m = any integer, n = any odd integer.

2. Due to the quantized representation of sine waves in the stopping function, odd harmonics of the fundamental frequencies are generated. Several mechanisms are available through which these harmonics can cause errors in the residual phase extracted from the recorded tones.

3. When multiple tones are injected into a pass band, intermodulation products can occur.

The magnitude of these various effects will be discussed along with strategies designed to avoid them.

*This paper represents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract No. NAS 7-100, sponsored by the National Aeronautics and Space Administration.

The very long baseline interferometry (VLBI) development group, at the Jet Propulsion Laboratory (JPL), has been investigating the phase calibrator technique for measuring instrumental delays added to radio data at the receiving station. In the JPL procedure, two or more tones are injected into each channel of incoming data. Multiple tones are required for the adequate phase calibration of spacecraft signals, which are received in different localized regions of the passband from day to day. When recorded with quasar signals, multiple tones provide redundancy, as well as a continuous monitor of the passband phase versus frequency response. The availability of multiple tones simplifies the system of data taking and analysis by allowing the bandwidth synthesis process to be done on a single scan basis, without the need for extensive measurements to be taken at the receiving stations. This is felt to be an important benefit of multiple tone phase calibration for achieving accurate results with an operational program of VLBI experiments.

In our experience with phase calibration, we have noticed some systematic sources of phase errors, some of which were not previously understood among the radio-astronomy community, which are the subject of this talk.

Shown in figure 1 are hypothetical spectra of tones in the recorded data and in the stopping function. The residual tone phase is extracted in the correlation process after it is beat to 0 frequency with an analytically generated stopping function. Systematic errors stem from the coincidence of extraneous tones in the data with tones in the stopping function. These accidental coincidences are of two types. For the first type, the interfering tone in the data occurs at or near the frequency of the phase calibrator tone; while, for the second type, the interfering data tone is stopped by one of the harmonics in the stopping function. I will briefly describe the origin of the extra tones in the data and stopping function, with emphasis on the troublesome case of amplitude occurring at 0 frequency in the spectra of both the recorded data and the stopping function. In particular, I will be describing the JPL/CIT 4 megabits per second systems used for recording and correlating data, but the descriptions can be easily generalized to other systems.

The stopping function is generated in three steps as is shown in figure 2. In the first step, an analytical model is produced with the nominal frequency of the phase calibration tone and a known initial phase. However, before this model is compared with the data it is quantized to three levels and sampled at 4 megabits per second. As a result of the quantization, all odd harmonics of the nominal tone frequency are present, and the sampling causes these harmonics to be aliased into the 2 MHz passband. The tone stopping functions planned for the JPL/CIT Block I/II correlators have 255 levels, which will practically eliminate any harmonics. The design of these correlators will be discussed by Dr. Rogstad in Thursday afternoon's session.

The recorded data also has extra tones resulting from various sources. The 5 kHz modulation used in the cable compensator causes small side bands at ±5 kHz to the injected tones. There is generally a bit-stream bias present; that is, an excess of either 1 bit or 0 bits in the recorded data. For example, this can be caused by dc level offsets in the sampling electronics. Some power feeds into the data from tones in the opposite sideband. When multiple tones are used, intermodulation products can become important. These intermodulation products are formed when the tones in the data are converted to a two-level bit-stream. (See figure 3.)

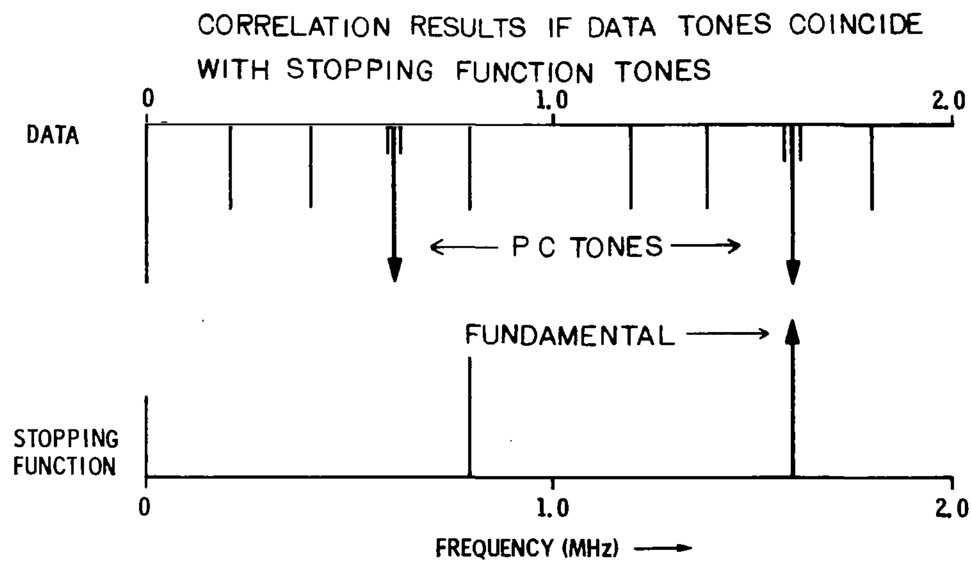
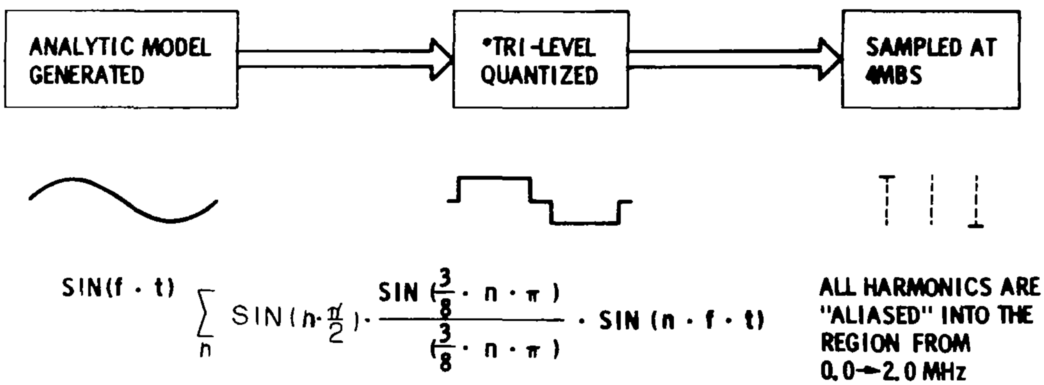


Figure 1. Systematic error sources.



$$\sum_{n} \mathrm{SIN}\left(n \cdot \frac{\pi}{2}\right) \cdot \frac{\mathrm{SIN}\left(\frac{3}{8} \cdot n \cdot \pi\right)}{\left(\frac{3}{8} \cdot n \cdot \pi\right)} \cdot \mathrm{SIN}\,(n \cdot f \cdot t)$$

*A 256 LEVEL QUANTIZED STOPPING FUNCTION FOR TONES IS PLANNED FOR THE JPL/CIT BLOCK II CORRELATOR

Figure 2. Stopping function.

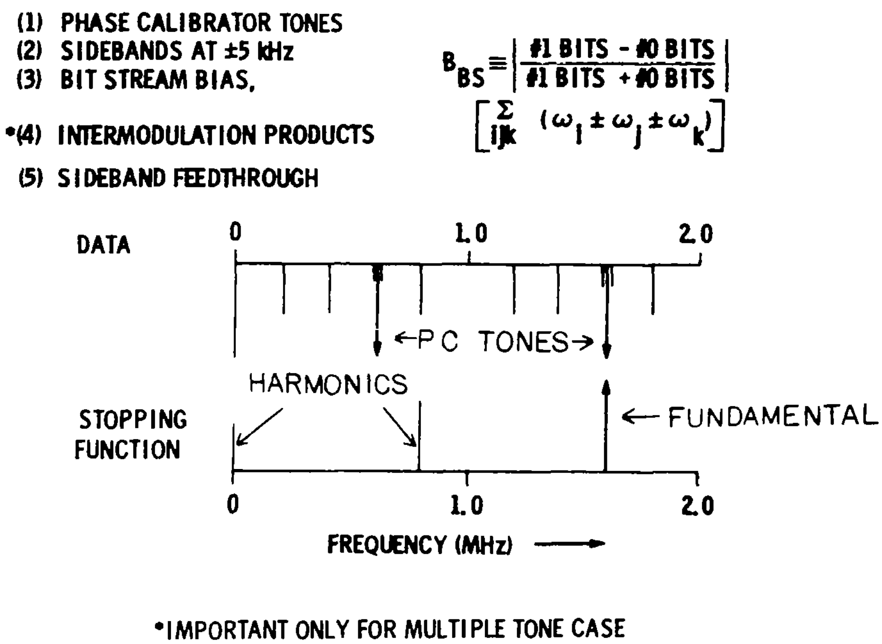


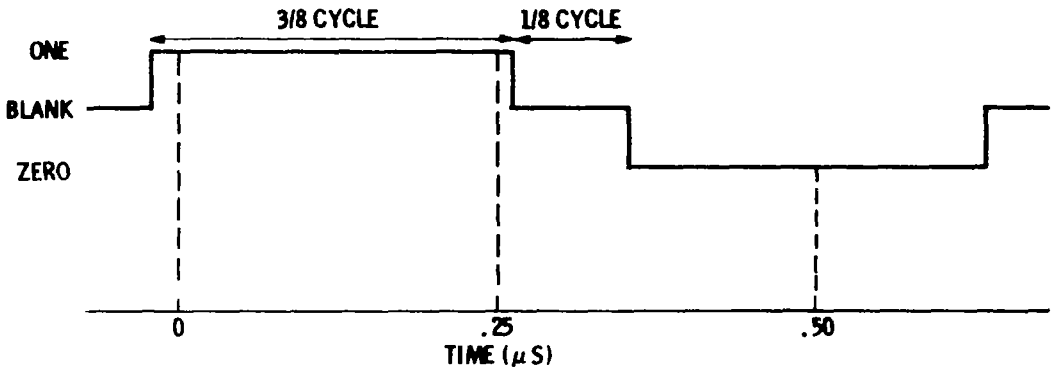
Figure 3. Tones present in recorded data.

Because the bit-stream bias is frequently comparable in amplitude to the phase calibrator tones, let us consider the origin of amplitude in the stopping function at 0 frequency. Figure 4 pictures the tri-level quantized stopping function for a tone at 4/3 MHz. It can be seen that this frequency is commensurate with the sampling rate in such a way that there are exactly 3 bits per cycle, and for the initial phase shown, there are 2 bits at the one level for each bit sampled at the zero level. A stopping function bias (B_{SF}) occurs for some initial phase whenever the stopping function frequency can be expressed as $F = S \cdot m/n$, where S is the sampling rate, m is any integer, and n is any odd integer. This is simply the condition for an odd number of bits per cycle and for an odd harmonic to fall at, or alias to, 0 frequency.

The effect of this bias in the stopping function is that it correlates with the bit-stream bias (B_{BS}), giving an erroneous addition to the amplitude and phase of the stopped phase calibrator tone.

Figure 5 illustrates how an approximate angular error can be attributed to this erroneous amplitude. Here, the correlator output has been drawn as a vector in the complex plane representing the phase and amplitude of a correlated signal. The resultant vector $\vec{A}$ is seen to be displaced in angle from the phase due to the phase calibrator tone by $\Delta\phi$, where, for $\Delta\phi << \pi/2$, $\Delta\phi \leq \mathrm{TAN}^{-1}$ (erroneous correlated amplitude/P.C. tone correlation amplitude).

Figure 6 contains formulas to express the limits on systematic phase errors mentioned in this talk. These formulas are valid only if the phase error is much less than $\pi/2$. In short, we have found that some care must be taken to prevent systematic phase errors due to the incorrect placement of phase calibrator tones, but, with the use of the formulas presented in figure 6, as well as calculations of B_{SF}, we have found this to be a straightforward task.



$$B_{SF} \equiv \frac{\#1\ \text{BITS} - \#0\ \text{BITS}}{\#1\ \text{BITS} + \#0\ \text{BITS}} = \frac{1}{3}$$

IN GENERAL, FOR $F = \frac{S \cdot m}{n}$, n ODD, m ANY INTEGER

$$B_{SF} = \frac{1}{n - 2\left[\frac{n+3}{8}\right]} \quad \text{INTEGER PART}$$

CORRELATED D.C. AMPLITUDE, $A^{BIAS} = B_{SF} \cdot B_{BS}$

Figure 4. Stopping function for P.C. tone at F = 4/3 MHz, sampling rate S = 4 MHz.

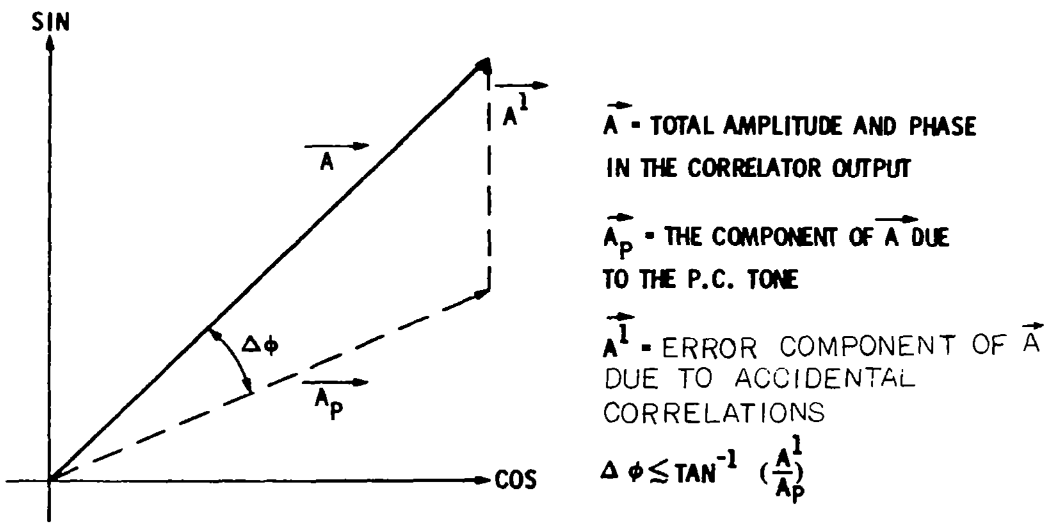


Figure 5. Phase error.

I. D.C. COMPONENTS

$$\Delta\phi^{BIAS} \lesssim \mathrm{TAN}^{-1}\left[\frac{1.6 \cdot B_{BS} \cdot B_{SF}}{\sqrt{P_T}}\right]$$

P_T IS THE FRACTIONAL PASSBAND POWER IN EACH TONE.

II. STOPPING FUNCTION HARMONICS

$$\Delta\phi^{SFH}(f) \lesssim \mathrm{TAN}^{-1}\left[\sum_n \frac{A_{SF}(f_n)}{A_{SF}(f)} \cdot \frac{A_D(f_n + \Delta f)}{\sqrt{2 P_T}} \cdot \frac{\mathrm{SIN}\,(\pi \cdot \Delta f \cdot \tau_{int})}{(\pi \cdot \Delta f \cdot \tau_{int})}\right]$$

$A_{SF}(f)$ IS THE STOPPING FUNCTION AMPLITUDE AT f, $A_D(f)$ IS THE DATA AMPLITUDE AT f, AND τ_{int} IS THE CORRELATION INTERVAL.

III. INTERMODULATION PRODUCTS

$$\Delta\phi^{INTER}(f) \lesssim \mathrm{TAN}^{-1}\left[\frac{P_T \cdot N_{int}}{12}\right]$$

N_{int} IS THE NUMBER OF INTERMODULATION PRODUCTS WHICH FALL AT THE FREQUENCY f.

Figure 6. Phase error (continued).