PHASE DELAY ASTROMETRY AND ITS GEODETIC APPLICATIONS

J. J. Wittels
Massachusetts Institute of Technology

The goal of our experiments was to exploit the accuracy inherent in the fringe phase in the determination of source positions.* That accuracy can be utilized by applying differencing techniques in order to circumvent such systematic effects as clock drift and delay due to the propagation media. The best cancellation of systematic effects could be attained by simultaneous observation of two or more sources with the same interferometer. However, having only one antenna at each end of the baseline, we chose to observe alternately two closely spaced sources, allowing a short cycle time and hence good cancellation of systematic effects by the differencing of the measured fringe phases.

The nonsimultaneous nature of our measurements created two difficulties. First, for the best accuracy, the arbitrary number of phase rotations between sequential observations of the same source had to be removed; or at a minimum, no rotations were to remain between sequential measurements of the phase difference between the two sources. Second, we had to assess the effects of systematic errors from the less than ideal cancellation of nonsimultaneous observations of the two sources.

We conducted four experiments between October 1971 and May 1974, each time, cycling between observations of the quasars 3C 345 and NRAO 512, which are only one-half degree apart. Typically, the integration time on each source was about 100 seconds, the cycle time about 4 or 5 minutes, and the full span of the observations 3 to 5 hours. All four experiments were conducted at 3.8 cm using the Haystack "MA" 37-meter antenna and the Goldstone CA 64-meter NASA antenna; the May 1974 experiment also used the NRAO "WV" 43-meter antenna. All data were recorded and correlated with the Mark I system.

The removal of the $2\pi n$ phase ambiguities – a process termed phase connection – was acheived for each source separately by manipulating a residual fringe phase, φ^r, calculated for each observation of a source from the observed total phase, φ^t, by the formula:

$$\varphi^r = \varphi^t - \varphi^g - \int_{t_o}^{t} \dot{\varphi}^{r\ poly}(t)\,dt$$

where φ^g is the phase from a simple geometric model, and $\dot{\varphi}^{r\ poly}$ is the residual fringe rate calculated from a four or five term polynomial fit to the entire ensemble of residual fringe rates of that source during that experiment. The residual fringe rates are those found by subtracting, for each observation of the source, the fringe rate calculated from the same simple geometric model from the total measured fringe rate.

---

*See Wittels, J. J. (1975), Ph.D. Thesis, Massachusetts Institute of Technology; and Shapiro, I. I., et al. (1979), A. J., 84, 1459.

The ambiguities were then removed by a simple computer algorithm that considered pairs of adjacent observations, and, finally, were checked visually. The definitive test of proper phase connection emerges only after parameterized modeling of the differenced phases (This will be discussed later.). Figure 1 shows schematically the desired results of proper phase connection. Clearly, the noisier the individual measurements of phase, the less reliable the phase connection.

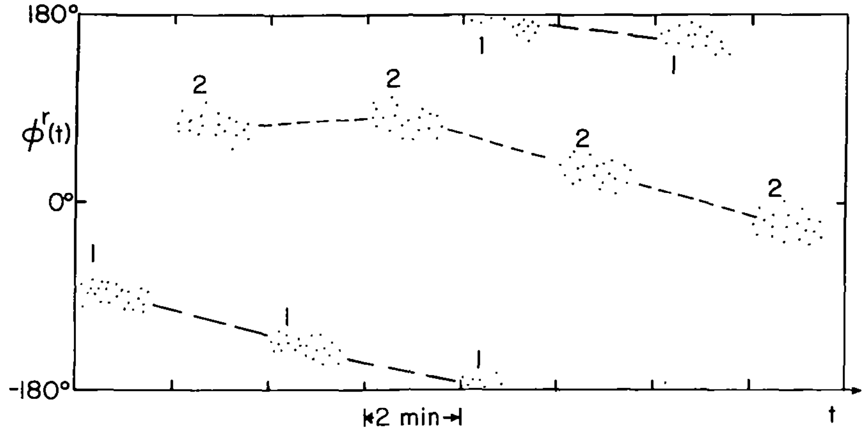

Figure 1.

After removing the ambiguities, the total phase for each observation of each source is reconstructed from the residual phases and the model phases. However, before the phases from the two sources are differenced, one other noncancelling contribution to the total phase should be removed: that due to the structure of the observed source. For noncylindrically symmetric sources, any structure resolvable by the interferometer contributes to the total measured fringe phase. Hence, the best accuracy can only be achieved by observing the source structure and removing its contribution.

For our experiments, NRAO 512 could be considered unresolved. However, 3C 345, as we are now all aware, had extended and time varying structure, which was only inadequately sampled, especially during the first three single-interferometer experiments. Using a simple two-component model for the structure of 3C 345, we corrected the measured phases by removing the structure phase with respect to the model's center of brightness. Hence, the separation determined by subsequent analyses is that between NRAO 512 and the center of brightness of the two components in the model of 3C 345. The source 3C 345 was sufficiently resolved that the structure phase calculated from the model reached a value of about a third of a phase rotation.

It must be emphasized here that the realization of centimeter accuracy in baseline determinations or of tenth milliarcsecond or better accuracy in relative source positions using fringe phase observations is crucially dependent on knowledge of the structure of the sources. Also, since a very large fraction of observed compact radio sources are time-dependent, the structure must necessarily be observed at the same epoch as the geodetic and astrometric results.

Having removed the ambiguities and source structure, for each experiment, we differenced each measured fringe phase of one source with the following measured fringe phase of the other source. The ensemble of differenced phases for each experiment was then subjected to a least squares analysis using a comprehensive parameterized model.* The only parameters varied were the right ascension and declination separations of the sources and a constant delay offset, referred to as a constant clock; the parameters for the antenna locations, atmospheric models, earth tides, UT1, etc. were kept fixed. Examination of the post-fit residuals was the final determining test of proper phase connection: When all ambiguities had been properly removed, the post-fit residuals would scatter approximately randomly between about ±20 psec, where one full rotation comprised 127 psec. If a phase rotation was inserted between observations, such as in a gap in the October 1971 experiment, the post-fit residuals became highly systematic and spanned most of a full rotation, clearly pointing to an error in phase connection.

In testing the sensitivity of the results from the least squares analyses to changes in the values of the fixed parameters and other effects, we found that the greatest change in the solution values resulted from reversing the order of the two sources in the differencing of the phases. Clearly, this sensitivity resulted from the imperfect cancellation of clock drifts and/or propagation media effects due primarily to nonsimultaneous observations. The problem could be best treated by using both the sum and difference of the phases in the solution. However, due to a computer program limitation, we combined instead the differenced variable with the measured phases from only the stronger source 3C 345. The set of 3C 345 phases alone primarily determined a four or five term polynomial "clock", while the differenced phases determined the right ascension and declination separations of the two sources. The constant clock offset was now fixed at the integral number of phase rotations closest to the value determined for this clock by the earlier solution, in which it had been varied.

The values determined for the right ascension and declination separation, $\Delta\alpha$ and $\Delta\delta$, respectively, from each of the four experiments combine to give a weighted mean of:

$$\Delta\alpha_{\mathrm{M}} = 2^{\mathrm{m}}\ 29\overset{\mathrm{s}}{.}436680 \pm 0\overset{\mathrm{s}}{.}000008$$

$$\Delta\delta_{\mathrm{M}} = 1'\ 40\overset{\prime\prime}{.}72630 \pm 0\overset{\prime\prime}{.}00020$$

for the difference in the position of NRAO 512 from that of 3C 345. The distribution of the four values about this mean is shown in table 1, along with the root-mean-square of the post-fit residuals for both the differenced and the undifferenced observations. Also shown are the comparable values obtained from the group delay data for the two experiments in which wide band synthesis** was used.

---

*See Robertson, D. S. (1975) Ph.D. Thesis, Massachusetts Institute of Technology.

**Whitney, A. R., et al. (1976), Radio Science, 11, 421.

Table 1

Experiment Date	$\Delta\alpha - \Delta\alpha_M$ (x 10^6 sec)		$\Delta\delta - \Delta\delta_M$ (x 10^5 arcsec)		Post-fit RMS	
	Group	Phase	Group	Phase	Difference (psec)	Single (psec)
10/71		5 ± 4		5 ± 11	12	15
05/72	-40 ± 80	4 ± 8	50 ± 40	-7 ± 9	19	36
07/72		28 ± 4		-14 ± 19	13	18
05/74	150 ± 430	-45 ± 5	180 ± 260	12 ± 15	14	18

The values in table 1 show that the distribution in $\Delta\delta$ is consistent with the formal errors for the individual experiments and for the mean. However, the formal errors for $\Delta\alpha$ are too small compared with the actual scatter. The likely explanation for this discrepancy underlines the importance of simultaneous measurements of source structure: 3C 345 is extended primarily in the right ascension direction. The structure of this source is only poorly determined for the epochs of the first three experiments yet structural scale changes during the 2½ years spanned by the experiments are of the same order as the scatter in $\Delta\alpha$. Taking the conservative view of errors, we estimate our accuracy as 0″.0003 in both coordinates. A conservative limit on the relative proper motion of the two sources is 0″.00025 ± 0″.00020/year along a position angle of 220° ± 60°.

In conclusion, the accuracy of the differenced phase technique could be maximized by the following:

1. Full sky coverage for each experiment (11 hours in the case of these two sources with these interferometers) would significantly decouple the determination of $\Delta\alpha$ and $\Delta\delta$.

2. Simultaneous source mapping would keep the measured separations consistently defined. Of course, the reverse application to understanding of the motions within the variable radio sources is also enticing.

3. Good models and values for earth tides, UT1, polar motion, the atmosphere, etc.

4. Simultaneous observations and, or at least, more stability in clocks and receivers, short cycle times, and cooperative propagation media. Fortunately, indications are that we are presently more limited by the equipment than the uncontrollable whims of the propagation media.