ON THE VLBI-SATELLITE LASER RANGING "IRON TRIANGLE" INTERCOMPARISON EXPERIMENT

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ABSTRACT

A very long baseline interferometry (VLBI)-laser intercomparison experiment is planned for the "Iron Triangle" consisting of the stations at Haystack, Owens Valley, and Ft. Davis. The improvements in the variances of the estimable parameters resulting from the addition of the stations at the National Radio Astronomy Observatory (NRAO), Green Bank (Greenbelt, in the laser case), and Goldstone is examined by means of a least squares covariance analysis assuming only random observational errors and disregarding systematic effects. (The latter are clearly aided by additional baseline observations.)

The usefulness of a substitute station at Richmond, Florida, is also examined with the idea of improving the accuracy of polar motion. Although this station is not in operation, it would be possible to use a portable laser and VLBI antenna at this site during the intercomparison experiment.

In the case of VLBI, a covariance analysis is performed on multi-baseline configurations. The parameters examined include baseline related quantities, quasar declinations, quasar right ascension differences, and Earth rotation parameters including polar motion. The variances of these parameters are calculated and compared among the various station configurations.

Laser range observations to LAGEOS are simulated, and the variances of the recovered baselines are examined. For the purpose of establishing a lower bound on the achievable baseline standard deviations independent of dynamical errors and assumptions, the laser observations were also analyzed in the geometric mode.

Although the absolute numbers recovered for the variances are not meaningful in themselves, their relative improvements within each of the two systems are instructive in planning for the intercomparison experiments.

INTRODUCTION

In anticipation of the upcoming VLBI-laser intercomparison experiments scheduled for 1979-1980, simulations were performed to determine the suitability of the proposed station locations. The criterion was a comparison among the possible station configurations of the standard deviations of baseline and Earth rotation parameters estimated from a least squares covariance analysis. Only the relative magnitudes of the standard deviations were addressed in the analysis. Thus, only random errors were assumed and no provision was made for systematic effects.

The "Iron Triangle," consisting of the stations at Westford (Haystack), Massachusetts, Owens Valley, California, and Ft. Davis, Texas, was regarded as the basic structure of the proposed network with options to incorporate either the Goldstone, California, or Green Bank, West Virginia, station or both. In addition, it was decided to include the Richmond, Florida, station in the analysis since it offered more North-South separation and therefore could strengthen the geometry of the network especially in the recovery of Earth rotation parameters. Although the station is not in operation as yet (it is part of the proposed Polar-motion Analysis by Radio Interferometric Surveying (Polaris) triangle), it would be possible to use a portable antenna at the site if its addition was found to be worthwhile. The effect of adding the more precise STALAS laser at Greenbelt was also considered.

The VLBI and laser simulations were done independently. Obviously, no absolute comparison of the numerical results is possible.

MATHEMATICAL MODEL

VLBI

The mathematical model for the time delay "observable" can be written as the inner product of the baseline vector in an Earth-fixed system, and the quasar unit vector rotated from an inertial system into the Earth-fixed frame. The observable, denoted by d, is the product of the time delay and the speed of light, and can be expressed in a simplified manner, suitable for this type of analysis, as follows:

$$d = -[\Delta X_i \; \Delta Y_i \; \Delta Z_i] \begin{bmatrix} 1 & 0 & \xi_j \\ 0 & 1 & -\eta_j \\ -\xi_j & \eta_j & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\delta_k \;\; \cos\alpha_k \\ \cos\delta_k \;\; \sin\alpha_k \\ \sin\delta_k \end{bmatrix}$$

where

$\Delta X_i, \Delta Y_i, \Delta Z_i$ are the coordinate differences of the i^{th} baseline in an Earth-fixed system

α_k, δ_k are the true right ascension and declination of the k^{th} quasar, respectively

θ is the Greenwich Apparent Sidereal Time (GAST)

$$\theta = \theta_o + W_d\,[TAI - (TAI - UT1)]$$
$$= \theta_o + W_d\,[TAI - (TAI - A1) - (A1 - UTC) - (UTC - UT1)_{BIH} + \kappa + \dot{\kappa}t] + \text{Eq. E.}$$

Eq. E.	is the equation of the equinoxes
AI	atomic time
TA1	international atomic time
UTC	coordinated universal time
UT1	observed universal time corrected for polar motion
W_d	conversion factor from universal to sidereal time
θ_o	GAST at initial epoch

ξ_j, η_j the components of polar motion that relate the true celestial pole to the terrestrial pole

The estimable parameters whose standard deviations were estimated in these simulations were the following:

A. Quasar parameters

1. Quasar declinations: δ_k
2. Quasar right ascension differences: $\alpha_k - \alpha_o$ where α_o is the adopted right ascension of a low declination quasar, defining the origin of the right ascensions.

B. Earth rotation parameters

1. Polar motion component differences: $\xi_j - \xi_1$, $\eta_j - \eta_1$. The interval of observations (24 hours) was divided into four steps of 6 hours duration. The average values of the first step ξ_1, η_1 are adopted; e.g., they can be taken from other sources (Bureau International de l'Heure (BIH)). The remaining steps were 6-hour averages referred to these initial values. For the purpose of our simulations, $\xi_1 = \eta_1 = 0$.

2. Earth rotation variation parameters: $\dot{\kappa}$ and $\kappa_j - \kappa_1$, where κ and $\dot{\kappa}$ are the first two terms of a polynomial that models the variations in the Earth rotation rate as given by TAI – UT1 in the previous expansion for GAST. The $\dot{\kappa}$ rate parameter was taken as constant over the period of observations. The $\kappa_j - \kappa_1$ earth rotation variation difference parameter was also represented by a step function. The third component of Earth rotation was defined by adopting a value for the first step, κ_1. The remaining steps were 6-hour averages referred to the fixed initial value. The κ_1 value can be obtained from the BIH, but for the purposes of our simulation $\kappa_1 = 0$.

C. Baseline related parameters

1. τ_i, ϵ_i, σ_i. The baseline components ΔX_i, ΔY_i, ΔZ_i are nonestimable quantities being affected by errors in α_o, κ_1, ξ_1, η_1 described above. The differential relationships between these parameters are as follows;

   $d\tau_i = d\Delta X_i + \Delta Y_i\,(d\alpha_o - d\kappa_1) - \Delta Z_i d\xi_1$

   $d\epsilon_i = d\Delta Y_i - \Delta X_i\,(d\alpha_o - d\kappa_1) + \Delta Z_i d\eta_1$

   $d\sigma_i = d\Delta Z_i + \Delta X_i d\xi_1 - \Delta Y_i d\eta_1$

   The subscript i refers to the i^{th} baseline.

2. Baseline distances, ℓ_i. These are estimable parameters.

This completes the list of estimable parameters whose total number is given by:

$$3i + (3(j-1)+1) + (2k-1) = 3i + 3j + 2k - 3$$

i = number of baselines
j = number of steps
k = number of quasars

A least squares covariance analysis was used to estimate the standard deviations of the parameters. These are obtained from the diagonal elements of the inverted normal matrix (the variance-covariance matrix) which is independent of observations. Its values depend on the geometry, the observation schedule, and the anticipated observational noise.

The normal matrix N is derived from the well-known formula

$$N = A^T PA$$

where A represents the partial derivative matrix, A^T its transpose, and P the weight matrix of observables, in this case the time delays. When observing simultaneously from more than two stations, for instance on a triangle, the time delay measurements are correlated and this must be included in the off-diagonal elements of the variance-covariance matrix of the observables, Σ_{L_b}, where

$$P = \sigma_o^2\,\Sigma_{L_b}^{-1}$$

σ_o^2 being the a priori variance of unit weight. On a triangle (and similarly on any closed figure), when observing simultaneously, only two sets of time delays (out of a possible three) should be used. The actual observations are registered on tapes located at the three sites. The time delay is a quasi-observable being derived by cross-correlating the tapes at a later time. Denoting the time delay between stations i, j as τ_{ij}, it follows from the mathematical model that $\tau_{12} + \tau_{23} + \tau_{31} = 0$. Therefore, after choosing any two combinations of time delays, the third combination will be linearly dependent on the other two and thus does not provide new information. Naturally, for any choice of two tapes, the estimated standard deviations of parameters should be identical. However, if the correlations between the time delay "observables" are not included in the Σ_{L_b} matrix, this will not be the case. The weight matrix of observables is diagonal in this case, and the result is three sets of estimated standard deviations instead of one.

Using our simplified model and assuming that all time delays are "observed" with equal noise, it is easy to construct the Σ_{L_b} matrix. In triangle 1-2-3, the time delays can be written as

$$\begin{aligned} \tau_{12} &= t_2 - t_1 \\ \tau_{23} &= t_3 - t_2 \\ \tau_{31} &= t_1 - t_3 \end{aligned}$$

the differences in the arrival times of a given segment of a wavefront at the two antennas. An error propagation is done using the first two time delays from above to determine Σ_{L_b}.

The complete variance-covariance matrix Σ_{L_b} is composed of 2 x 2 full blocks along the main diagonal, and zeros elsewhere, for each set of observations at a particular epoch, in the following manner:

$$\Sigma_{L_b} = \begin{bmatrix} 2 & -1 & & & & & & \\ -1 & 2 & & & & & & O \\ & & 2 & -1 & & & & \\ & & -1 & 2 & & & & \\ & & & & \ddots & & & \\ O & & & & & & 2 & -1 \\ & & & & & & -1 & 2 \end{bmatrix}$$

This matrix is inverted and scaled according to the assumed observational noise to arrive at the appropriate P matrix.

In the "real world," the Σ_{L_b} matrix will be more difficult to derive, but our tests show that unless the true correlations are known, the results may be very misleading.

LASER

In this simple simulation study, the only quantities which were allowed to adjust were the station positions $\bar{U}$, the initial satellite state-vector $[\bar{X}_o \vdots \dot{\bar{X}}_o]^T$, and the coordinates of the pole ξ, η. The brief presentation of the mathematical model that follows is given in an Earth-fixed coordinate system neglecting the effects of nutation and precession. In compact form, the error equation is as follows:

$$(\rho_o - \rho_c)_i = \sum_j \frac{\partial \rho_{c_i}}{\partial P_j} dP_j - v_i$$

where ρ_{o_i} denotes the i^{th} observed range, ρ_{c_i} the corresponding prediction based on an approximate set of parameters P_j. Based on the above general equation and the aforementioned set of parameters, the linearized observation equation is the following:

$$(\rho_o - \rho_c)_i =$$

$$= \frac{1}{\rho_i}\left\{ -(\bar{X} - \bar{U})^T d\bar{U} + (\bar{X} - \bar{U})^T [R_3(\theta) \vdots\ O] \left[\begin{array}{c|c} \dfrac{\partial \bar{X}}{\partial \bar{X}_o} & \dfrac{\partial \bar{X}}{\partial \dot{\bar{X}}_o} \\ \hline \dfrac{\partial \dot{\bar{X}}}{\partial \bar{X}_o} & \dfrac{\partial \dot{\bar{X}}}{\partial \dot{\bar{X}}_o} \end{array}\right] \left[\begin{array}{c|c} R_3^T(\theta_o) & O \\ \hline \dot{R}_3^T(\theta_o) & R_3^T(\theta_o) \end{array}\right] \left[\begin{array}{c} d\bar{X}_o \\ \hline d\dot{\bar{X}}_o \end{array}\right] \right.$$

$$\left. + [X_1 U_3 \quad -X_3 U_1 \vdots\ X_3 U_2 \quad -X_2 U_3] \left[\begin{array}{c} d\xi \\ \hline d\eta \end{array}\right] \right\} - v_i$$

In the above, the angle θ denotes the GAST at the epoch of observation and θ_o refers to the initial epoch; v_i denotes the random error in the observation. The results presented here are based on a fixed polar motion model; ξ and η were not considered as parameters.

SIMULATION PROCEDURE

VLBI

The selection of a quasar observation schedule was guided by two considerations:

- that a quasar be observable simultaneously (maximum zenith distance of 80°) from all stations at a chosen epoch of observation.
- that the final quasar schedule, over the 24-hour period of the simulations, be evenly distributed in right ascensions and declinations in order to achieve a strong geometry and to provide good recovery for low and high quasar declination-dependent parameters. Since the geometry of the experiments shifts by about only 4 minutes every 24 hours, it was decided that a day of observations would adequately encompass the entire geometry of the problem.

In order to simulate real observing conditions, it was decided to observe a quasar every 10 minutes simultaneously from all stations involved in a particular experiment. Although a typical time delay "observation" requires 3 to 5 minutes, a longer period was taken in order to allow time for antenna slewing and switching of tapes. The observational noise was assumed to be 0.1 nanosecond (3 cm).

LASER

For the laser experiments, the observables were simulated ranges to the satellite LAGEOS with a nominal noise level of 10 cm. The observational period was 7 days during which 22 passes were co-observed by all stations. The simulations were performed in two modes: in the dynamic mode involving a short arc solution, and in the geometric mode run for the purpose of recovering standard deviations independent of orbital effects and assumptions. It is recognized that the inherent restriction of having simultaneous observations from at least four distant stations makes this mode impractical. The laser results presented are based on the directly estimable baseline distances as obtained from the short-arc solutions.

RESULTS AND CONCLUSIONS

The various experiments were compared on the basis of the estimated standard deviations of baseline (VLBI and laser) and Earth rotation parameters (VLBI only). However, since the baseline components depend to a certain extent on the coordinate system definition as described earlier, the directly estimable baseline lengths were used as a basis for comparison. As of this time, we have not completed our simulations estimating the recovery of pole coordinates from laser observations.

VLBI

The VLBI results using the assumed correlations are summarized in table 1. Experiment 1 using the tapes from the Iron Triangle configuration is the basis of comparison. In experiments 2-4, the effects of the addition of either Green Bank, Goldstone, or Richmond are examined. In experiment 5, the effect of adding both Green Bank and Goldstone to the basic configuration is listed. The columns headed by IMPRV give the improvement in the estimated standard deviations of an experiment relative to experiment 1.

Experiments 2 and 3 show improvements on the order of 5 to 10 percent over experiment 1. However, the addition of the Richmond station results in improvements of about 25 to 35 percent. Especially apparent is the improvement in the $\Delta\eta$ parameter, because of the significant North-South separation of the Westford and Richmond stations. The closeness of the Green Bank and Goldstone stations to the Iron Triangle configuration implies that the effect of these stations is the same as would be expected from increasing the number of observations in the Iron Triangle itself. Obviously, there is a limit to the number of observations over a given interval of time that can be made especially for the earth rotation parameters which are time dependent. In any case, these stations do not substantially improve the recovery of the parameters of interest. In fact, as seen in experiment 5, the addition of both Goldstone and Green Bank has less effect on the results than the addition of Richmond alone in experiment 4. Thus, it can be concluded that only Richmond would significantly improve the basic configuration, as expected. Naturally, another station providing a similarly favorable geometry such as in Alaska could provide similarly improved results.

LASER

Table 2 summarizes the laser experiments and gives the improvement of average relative precisions of the three baselines of the Iron Triangle (L1). In each column (L2 – L6) the results of adding stations to the Iron Triangle are listed. The numbers in parentheses are the total number of ranges

Table 1
VLBI: Improvement of Standard Deviations Relative to Iron Triangle

Experiment No.		1	2		3		4		5		
Configuration		Iron Triangle	+ GB	IMPRV	+ GS	IMPRV	+ RM	IMPRV	+ GB + GS	IMPRV	
Common Baseline Parameters (cm)	τ_1	1.6	1.5	5%	1.5	5%	1.2	27%	1.4	12%	OV - WS
	ϵ_1	1.9	1.7	9%	1.7	9%	1.4	25%	1.5	18%	
	σ_1	2.1	2.0	8%	2.0	7%	1.7	22%	1.8	16%	
	τ_2	1.7	1.6	6%	1.6	5%	1.3	23%	1.5	12%	WS - FD
	ϵ_2	1.8	1.7	7%	1.7	7%	1.2	35%	1.6	14%	
	σ_2	1.7	1.6	6%	1.6	6%	1.4	21%	1.5	14%	
Baseline Distances (cm)	ℓ_1	1.6	1.5	6%	1.5	5%	1.2	27%	1.4	12%	
	ℓ_2	1.6	1.5	4%	1.5	4%	1.1	27%	1.2	23%	
Earth Rotation Rate of Change (μs/hr)	$\dot{\kappa}$	10.1	9.0	11%	9.1	10%	8.1	20%	7.9	22%	
Earth Rotation Variation ($10^2\ \mu$s)	$\Delta\kappa$	1.7	1.6	10%	1.6	9%	1.3	24%	1.4	20%	
Polar Motion Variations (cm)	$\Delta\xi$	5.2	4.8	7%	4.9	7%	4.0	23%	4.4	16%	
	$\Delta\eta$	6.2	5.9	4%	5.9	4%	4.2	33%	5.6	10%	

Simultaneous observations every 10 min. over a 24-hr. period to a total of 8 quasars.
Time delay noise: 0.1 nanoseconds (3 cm).

Table 2
LASER: Improvement of Average Relative Precisions

Standard of Comparison Iron Triangle: L1* (3969)	Experiment					
	L2 GB, GS, RM (7843)	L3 GB, GS (6522)	L4 GB (5337)	L4A GB (3 cm) (5337)	L5 RM (5290)	L6 GS (5154)
Total Improvement % Over L1	35.6	21.6	13.0	44.6	25.9	11.6
Improvement % Due to Increase of Observations	28.9	22.0	13.8	13.8	13.4	12.3
Improvement % Due to Network Extension	6.7	-0.4	-0.8	30.8	12.6	-0.7

* L1 average baseline precision: 1.7×10^{-8}

for each experiment. The first row shows the percent improvement for each solution due to the combined effect of adding more stations and, therefore, at the same time, increasing the number of observations. The second row gives the percent increase due simply to the increase in the number of observations. Note that in the VLBI experiments only the equivalent of the first row was presented since in that case the numbers in the second row would be less meaningful. The difference of the first two rows in the laser table, depicted in the last row, indicates the net improvement due to network geometry only.

As can be seen from experiment L4A, the addition of the high quality laser at Greenbelt provides the most dramatic improvement, about 31 percent, even though Greenbelt is not part of the Iron Triangle. This is due to the improvement of the LAGEOS orbit from the Greenbelt STALAS observations.

The addition of the proposed Richmond station in experiment L5 gives the best improvement, about 13 percent, when all stations have a laser of the same precision such as MOBLAS. In fact, the addition of all three stations to the Iron Triangle in experiment L2 gives poorer results than the addition of Richmond alone, or a similarly located station. This is due to the fact that Green Bank and Goldstone introduce six new unknowns to the adjustment which are not compensated for by improved geometry.

As can be seen by the negative percentages in experiments L3, L4, and L6, the addition of Greenbelt with the MOBLAS precision, or Goldstone, or both provides no improvement. Although not presented here, the results from the geometric mode solutions lead to identical conclusions.

SUMMARY

Examining the results, we suggest that the STALAS laser at Greenbelt definitely be part of the intercomparison experiments as it is already planned, or of any other experiment. The Richmond site or a similarly suitable one should be considered as a useful addition to the Iron Triangle. From the point of view of random errors, the station at Goldstone or the VLBI at Green Bank is not considered particularly useful. At this time, the study on the effect of systematic errors on both VLBI and laser is not complete. In some cases, these systematic errors may be of great importance in deciding on a particular station configuration as opposed to purely geometric considerations. The elimination of systematic errors is likely to be aided by redundant baseline observations. Other factors may also be considered, such as antenna parameters. Finally, we would like to stress the importance of working as much as possible with estimable parameters such as baseline distances and including the VLBI observation correlations between simultaneously "observed" time delays in a multi-baseline configuration. Neglecting these correlations results in a different set of estimated standard deviations from each possible baseline combination. These estimates are generally overly optimistic compared to the unique set obtained using proper correlations.

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