Atmospheric Turbulence Effects on Aircraft Noise Propagation

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INTRODUCTION

Of all the characteristics of the atmosphere that affect sound propagation, turbulence is the least understood. Attenuation of sound by viscous and heat conduction effects is now well understood and can be calculated (refs. 1, 2, 3). Refraction of sound rays caused by wind and temperature gradients can also be determined if profiles of the mean temperature and wind are known or measured (ref. 4). If the acoustical impedance of the ground surface beneath the atmosphere is known or can be measured, then the sound attenuation caused by the ground can be approximately calculated (ref. 5).

Accounting for atmospheric turbulence effects, however, is much more difficult because there is still uncertainty as to the physical mechanisms by which turbulence affects sound propagation. Several theoretical studies have been based on a physical model which assures that the important aspect of turbulence is the scattering of an initially coherent sound wave by thermal and momentum fluctuations in the atmosphere (refs. 6 through 8). The loss in intensity of the coherent wave manifests itself as an apparent attenuation caused by turbulence.

Other studies of turbulence effects on sound propagation assumed that the important effect is the broadening of a beam of sound caused by phase fluctuations induced by the turbulence (ref. 9). The increase in cross-sectional area of the beam causes a corresponding decrease in sound intensity because the sound-energy flux is spread out over a larger area.

Finally, there have been other studies of sound attenuation due to turbulence which show that for certain conditions, such as for propagation of sound through a highly turbulent jet, actual turbulent absorption might be important (refs. 10 and 11). The absorption comes about through an interaction of a sound wave with a turbulent flow field involving a net loss

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of energy from the sound wave. A process of cascading of energy from large to small turbulent eddies occurs whereby acoustic energy is ultimately dissipated through viscous forces acting on the smallest eddies.

In addition to the various physical mechanisms proposed to explain attenuation caused by turbulence, there have been a variety of mathematical models and simplifying assumptions used in order to carry out an analysis based on a particular assumed physical model. The different simplifying assumptions often lead to different end results (ref. 12).

In an effort to bring some order to the chaotic state of turbulence effects on sound propagation, recourse has been made to experiment. Summaries and critiques of important experimental results can be found in refs. 9 and 13 to 17. In addition, the books by Tatarskii (ref. 13) and Ishimaru (refs. 14 and 13) contain derivations and explanations of principal theoretical methods used for analyzing problems of sound- and electromagnetic-wave propagation through a turbulent atmosphere.

It is unfortunate, as pointed out by Brown and Clifford (ref. 9), that the experimental results have been complicated by extraneous factors, and that crucial parameters needed to validate a particular theoretical model were often not recorded or measured. Nevertheless, the relatively few experiments that have been conducted to study atmospheric turbulence effects have been useful in providing some indication about the relative merits of the competing analyses that have been conducted so far.

It is clear that what is needed before further progress can be made in advancing knowledge of turbulence effects on sound propagation is a new experimental program designed specifically to test the premises that underlie the more promising physical and mathematical models. The objective of the study reported here is to design an experimental program — based on a specific physical and mathematical model — which will provide all of the necessary information needed to validate (or discredit) the model. The experiment must be practical and use state-of-the-art, obtainable equipment. The purpose of the experiment would be to obtain information ultimately

useful for determining turbulence effects on aircraft flyover noise and thereby to supplement other more-fundamental research efforts.

In the remainder of this report, we describe the specific analytical model chosen to represent the effects of turbulence on the attenuation of sound (the model of Brown and Clifford, ref. 9), point out and describe the important parameters needed to employ the model and show what effect variations in the parameters would be expected to have on attenuation of noise (especially from aircraft). We then describe one analytical model for predicting the statistical variance of the sound pressure fluctuations caused by atmopsheric turbulence. And finally, we describe a practical experimental program which would provide the necessary information to validate the analytical models.

ATTENUATION OF SOUND BY TURBULENCE

Choice of Model

To design an experiment capable of assessing the importance of atmospheric turbulence on sound propagation and providing better methods to predict turbulence attenuation, a working model of the physical effects of turbulence on sound propagation must be selected and developed, as required, to make the model applicable to the problem of aircraft flyover noise.

The basic model chosen to predict the effect of atmospheric turbulence on attenuation is that developed by Brown and Clifford. In their paper (ref. 9), Brown and Clifford point out that the simple scattering theories such as those of DeLoach (ref. 15), Lighthill (ref. 19), and Blokhintzev (ref. 20) are not correct in that they invoke a single scatter model that does not conserve energy. The single-scatter models assume that the scattered acoustic energy is lost; they do not take into account the significant amount of energy scattered toward the observer (or microphone) by off-axis turbulence.

The theory proposed by Brown and Clifford, on the other hand, is based on previously developed theories of forward propagation of optical waves

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through turbulence, and appears to rest on a somewhat more solid theoretical foundation. The application to acoustics of results derived on the basis of approximations valid for optics is somewhat tenuous. However, Brown and Clifford argue in their paper (ref. 9) that the results they derive should be valid for acoustic wavelengths of up to a few meters, i.e., to relatively low audible-frequency sounds.

Comparisons between calculated results based on their theory and various available experimental results were made by Brown and Clifford and showed good agreement. However, as we show later, Brown and Clifford made some numerical errors in their calculations. The agreement between theory and experiment now appears worse than reported in ref. 9. Furthermore, good agreement between theory and experiment can also be obtained from other models, such as that of DeLoach, if certain empirical parameters are chosen properly.

However, the fundamental problem, mentioned in the introduction still remains — namely, that the experimental reports invariably lack key information on values of important parameters because the experimenters did not, at the time, know what parameters were most important for determining attenuation by turbulence. That is why new experiments based on a physical model are needed.

The Brown and Clifford Model

In this section we shall outline the derivation of the equation obtained by Brown and Clifford for the attenuation of sound by turbulence. In their model, the principal mechanism for the apparent attenuation caused by atmospheric turbulence is the broadening of a beam of sound. Figure 1 illustrates a beam of sound propagating through a non-turbulent and a turbulent atmosphere. For a non-turbulent atmosphere, diffraction will cause the beam to spread from an initial diameter D_0 to a larger diameter D_f after propagating a distance L. The magnitude of D_f depends on D_0, L, and the frequency f of the sound.

(a) Non-turbulent atmosphere

(b) Turbulent atmosphere
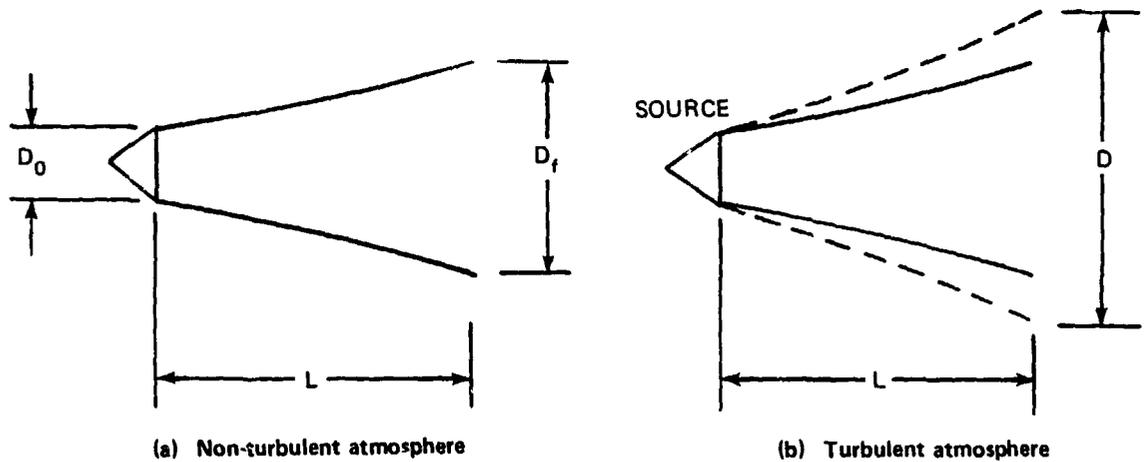
Figure 1.-Geometry of an acoustic beam propagating through a turbulent and a non-turbulent atmosphere.

For a turbulent atmosphere, there is an additional spreading of the beam caused mainly by turbulence-induced phase fluctuations. Thus, after propagating a distance L through a turbulent atmosphere, the diameter of the beam will be equal to D which is larger than D_f. The magnitude of the additional spreading of the beam due to turbulence depends on L, f, and characteristics of the turbulence.

Neglecting atmospheric absorption (which can be accounted for separately), the acoustic power carried by the sound wave is constant through any cross section of the beam. Therefore, the average acoustic intensity over a cross section is inversely proportional to the cross-sectional area and the following relation holds between the intensity I_f in the absence of turbulence and the long-term time-averaged intensity $<I>$ in the presence of turbulence:

$$<I>/I_f = D_f^2/D^2 = D_f^2/(D_f^2 + D_t^2) \tag{1}$$

where $\pi D_t^2/4$ is the additional beam area caused by turbulence-induced beam spreading.

The attenuation A_t, in decibels, caused by turbulence is defined by

$$A_t = 10 \log_{10}(I_f/<I>) = 10 \log_{10}[1 + (D_t^2/D_f^2)] \tag{2}$$

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For the quantity D_t^2, Brown and Clifford use a solution obtained by Yura (ref. 21):

$$D_t^2 = 25 \ k^{2/5} \left[\int_0^L (L - s)^{5/3} \ C_n^2(s) \ ds \right]^{6/5} \tag{3}$$

where $k = 2\pi f/c$ is the wavenumber of the sound, f the frequency of the sound, c the sound speed and s is distance along the sound propagation path from the source to the receiver.

The quantity C_n^2 is the structure function for refractivity fluctuations with units of length to the $-2/3$ power; it is the only parameter in the model that depends on the characteristics of the atmospheric turbulence. It will be defined and discussed in a later section. The integral in eq. (3) represents a sort of weighted average of C_n^2 over the sound path. The integral ranges from the sound source to the receiver, i.e., $s = 0$ is the location of the source and $s = L$ is the location of the receiver.

For the quantity D_f^2, Brown and Clifford use the following expression:

$$D_f^2 = D_0^2 + (16 \ L^2)/(k^2 D_0^2) \tag{4}$$

In the far-field the first term in eq. (4) is negligible compared with the second term, and the equation is an expression of "inverse-square-law" spreading. The factor 16 multiplying the second term is suspect. We believe that it is in error by a factor of 4, i.e., the correct value is 64. In Appendix A we justify the value of 64.

By combining eq. (2), (3), and (4) Brown and Clifford obtained the following equation for attenuation caused by turbulence:

$$A_t = 10 \ \log_{10} \left\{ 1 + 1.56 \ k^{12/5} D_0^2 \left[\int_0^L [(L - s)/L]^{5/3} \ C_n^2 \ (s) \ ds \right]^{6/5} \right\} \tag{5}$$

The factor 1.56 in eq. (5) is the value obtained by Brown and Clifford. If the factor 16 in eq. (4) were changed to 64 then the number 1.56 would be changed to 0.391.

The parameters appearing in the Brown and Clifford model are the structure function for refractivity fluctuations C_n^2, wavenumber k, in. source diameter D_0, and propagation distance L. In this session, we discuss each of the parameters and illustrate what effect varyin each parameter would have on attenuation caused by turbulence.

Structure Parameters.-As mentioned previously, the function C_n^2 is the one variable in the Brown and Clifford model that depends on the characteristics of the turbulence in the atmosphere. It can be related to two measurable parameters which will be defined now. The first of those is the structure parameter for velocity fluctuations C_v^2; the second is the structure parameter for temperature fluctuations C_T^2.

Before the structure parameters can be defined, it is first necessary to define so-called structure functions for turbulent velocities and temperatures. For velocities, the structure function is a tensor defined by the following expression:

$$D_{ik} = \langle (v_{2i} - v_{1i})(v_{2k} - v_{1k}) \rangle \tag{6}$$

where the symbol $\langle \rangle$ denotes a time average. The definition expresses a correlation of velocities, i.e., the relation of velocities at two neighboring points 1 and 2. The velocities at the two points are $\vec{v}_1$ and $\vec{v}_2$. The subscripts i and k denote components of the velocity vectors. For locally isotropic turbulence D_{ik} depends only on the distance r between the two neighboring points. The component D_{rr} is then just the mean-square relative velocity of two neighboring fluid particles along the line joining them. Similarly, the component D_{tt} is the mean-square transverse velocity of one particle relative to another. If the distance r between the two points is large compared with the size of the smallest eddies in the turbulent flow but small compared to the largest eddies (i.e., the so-called "inertial subrange" region) then it can be shown (ref. 22) that $D_{tt} = (4/3) D_{rr}$ and

$$D_{rr} = \text{"constant"} \cdot r^{2/3} = C_v^2 r^{2/3} \tag{7}$$

The structure parameter for velocity fluctuations C_v^2 is thus defined as the "constant" in the "2/3-power law" for the structure function D_{rr}. The quotation marks around the word constant indicate that C_v^2 is actually only a constant over a limited region of the atmosphere and does vary, as shown below, with height above ground level and the condition of the atmosphere.

Similarly, a structure function D_T can be defined as the mean-square relative temperature difference of two neighboring fluid particles:

$$D_T = <(T_2 - T_1)^2> \tag{8}$$

Again, it can be shown that

$$D_T = \text{"constant"} \cdot r^{2/3} = C_T^2 \, r^{2/3} \tag{9}$$

Equation 9 defines C_T^2 which, like C_v^2, is only a "constant" over a limited region of the atmosphere.

The structure parameters can be measured directly by, say, employing two sets of hot-wire anemometers located a distance r apart. However, for practical applications it is often necessary to relate the structure parameters to other quantities such as turbulent energy dissipation, which may be easier to measure, and then deduce numerical values of the structure parameters as a function of height. Practical methods for measuring C_v^2 and C_T^2 are discussed in a later section.

Once C_T^2 and C_v^2 have been determined as a function of height, C_n^2 can be found from the following equation* (ref. 13):

$$C_n^2 = C_T^2/4T^2 + C_v^2/c^2 \tag{10}$$

where T is the ambient temperature and c is the ambient speed of sound.

_____

*A more exact equation for C_n^2 is the following:

$$C_n^2 = C_T^2/4T^2 + C_v^2/c^2 + 2(0.307)C_{eT}/4pT + (0.307)^2 C_e^2/4p^2$$

where C_{eT} is a parameter for cross correlation of humidity and temperature fluctuations, p is the ambient pressure, and C_e^2 is a structure parameter for vapor pressure fluctuations. The expanded equation was derived by Wesley (ref. 23) and is also discussed in ref. 17. The additional parameters C_{eT} and C_e^2 have not been thoroughly investigated but their effects may be important for sound propagation in humid atmospheres.

Careful measurements of the structure parameters C_v^2 and C_T^2 were made during two extensive experimental programs conducted to measure the characteristics of turbulence in the atmospheric boundary layer (refs. 24 and 25). A typical set of profiles of potential temperature, wind speed, and wind direction, taken from ref. 24 is shown in fig. 2. The thickness of the convective atmospheric boundary layer Z_i is defined by the height of the lowest inversion base. The dashed portion of the curves was obtained from rawinsonde measurements which tend to be less precise than the balloon-borne measurements used to obtain the lower portion of the curves.

By properly nondimensionalizing the structure constants, Kaimal *et al.* in ref. 24 showed that data from all of the convective-atmosphere runs can be collapsed to single curves over a large range of heights. The nondimensionalized data for the structure constants C_v^2 and C_T^2 from ref. 24 are shown in fig. 3. The scaling velocity w_* and temperature T_* are defined by the following equations:

$$w_* = [Q_0 \ Z_i \ (g/T)]^{1/3} \tag{11a}$$

$$T_* = Q_0/w_* \tag{11b}$$

where Q_0 is the surface kinematic heat flux defined by $Q_0 = <w'T'>$ at a height $Z = 4$ m; w' is the fluctuating component of wind in the vertical direction; T' is the fluctuating temperature; and g is the acceleration of gravity. (Note: we have used somewhat different notation from that of ref. 24).

The dashed lines in fig. 3 are predictions for the free-convection portion of the atmospheric boundary layer and are based on the data reported in ref. 25. For heights greater than 0.1 Z_i the following approximate relation holds

$$C_v^2 \ Z_i^{2/3}/w_*^2 = \text{constant} \tag{12}$$

The constant in eq. (12) is between about 1.0 and 1.5.
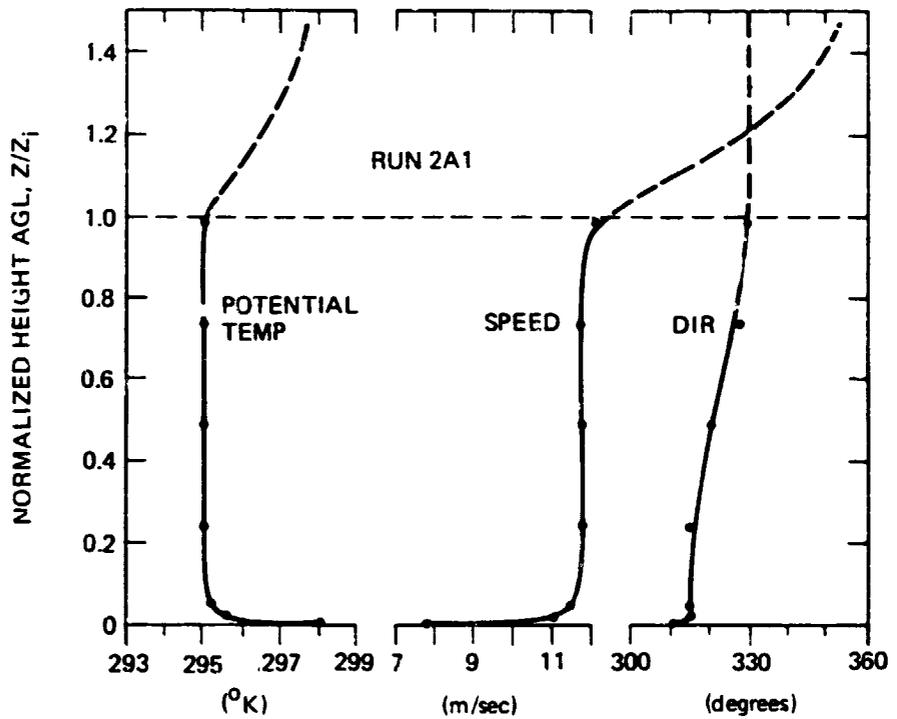
Figure 2.-Profiles of potential temperature, wind speed, and wind direction, for Run 2A1. (From Kaimal et al, ref. 24. Z_i = thickness of convective atmospheric boundary layer.
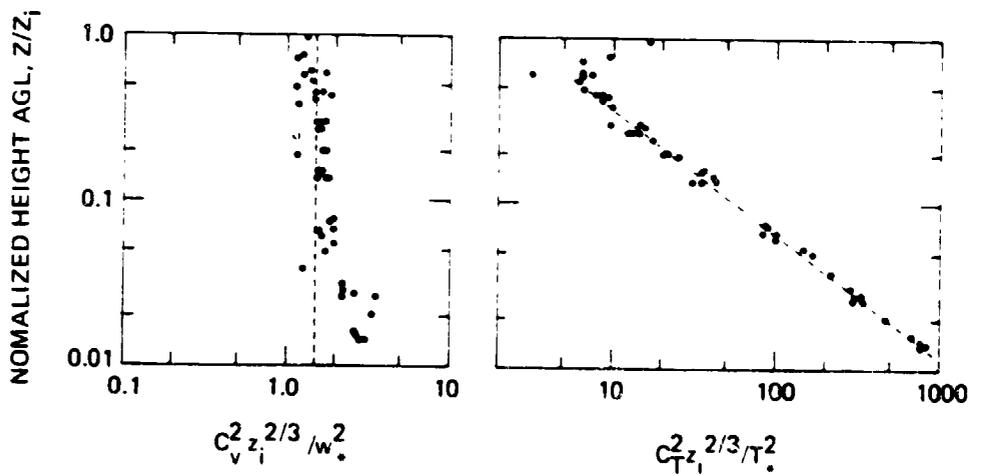

Figure 3.-Vertical profiles of structure functions. (From Kaimal et al, ref. 24).

For the range of heights $0.01 Z_i \leq Z \leq 0.1 Z_i$, Kaimal *et al.* (ref. 24) suggest the following approximation for the normalized C_v^2:

$$C_v^2 \ Z_i^{2/3}/w_*^2 = 1.3 + 0.1 \ (Z/Z_i)^{-2/3} \tag{13}$$

For the normalized C_T^2 they suggest the following approximation which is indicated by the dashed line in fig. 3:

$$C_T^2 \ Z_i^{2/3}/T_*^2 = 2.67 \ (Z/Z_i)^{-4/3} \tag{14}$$

Figure 3 shows that the data for the normalized C_T^2 have little scatter and that eq. (14) holds for heights up to about $Z = 0.7 \ Z_i$.

For the typical run (Run 2A1) for which the wind and potential temperature profiles are shown in fig. 2, the following numerical values for the boundary layer parameters w_*, T_*, and Z_i are given in ref. 24: $w_* = 2.00$ m/s, $T_* = 0.098$ K, and $Z_i = 1250$ m. Substituting those values into eqs. (13) and (14) gives the following approximate expressions for C_v^2 and C_T^2 for a typical midday unstable boundary layer:

$$C_v^2 = 0.045 + 0.40 \ Z^{-2/3}, \ m^{4/3}/s^2 \tag{15}$$

$$C_T^2 = 3.0 \ Z^{-4/3}, \ K^2/m^{2/3} \tag{16}$$

where Z is in meters.

For illustrative purposes it is useful to obtain a typical variation of C_n^2 with height. To do so, we shall assume an atmosphere with a constant temperature of $T = 283$ K. Then substituting eqs. (15) and (16) into eq. (10) we obtain the following expression for a typical variation of C_n^2 with height Z:

$$C_n^2 = 4.0 \times 10^{-7} + 3.5 \times 10^{-6} \ Z^{-2/3} + 9.4 \times 10^{-6} \ Z^{-4/3}, \ m^{-2/3} \tag{17}$$

The values of the coefficients in eq. (17) have been rounded to two significant figures.

Brown and Clifford (ref. 9) also use the data of ref. 23 to obtain a typical variation of C_n^2 with height. Their equations for C_v^2 and C_T^2 are the same as eqs. (15) and (16) but the values of the coefficients are slightly different. If we use their values and a temperature of 283 K, we obtain the following equations for the structure parameters:

$$C_v^2/c^2 = 3.5 \times 10^{-7} + 2.9 \times 10^{-6} \, Z^{-2/3}, \; m^{-2/3} \tag{18a}$$

$$C_T^2/4T^2 = 9.1 \times 10^{-6} \, Z^{-4/3}, \; m^{-2/3} \tag{18b}$$

and the following for the structure function C_n^2

$$C_n^2 = 3.5 \times 10^{-7} + 2.9 \times 10^{-6} \, Z^{-2/3} + 9.1 \times 10^{-6} \, Z^{-4/3}, \; m^{-2/3} \tag{19}$$

In order to be consistent with Brown and Clifford we shall use eq. (19) instead of eq. (17) for sample calculations later. The variation of C_n^2 with Z, according to eq. (19), is shown in fig. 4. Since the curve is just to be used for sample calculations, the range of Z has been extended downwards below the height for which eq. (19) is known to be a good approximation to the variation of C_n^2 with height above ground level.
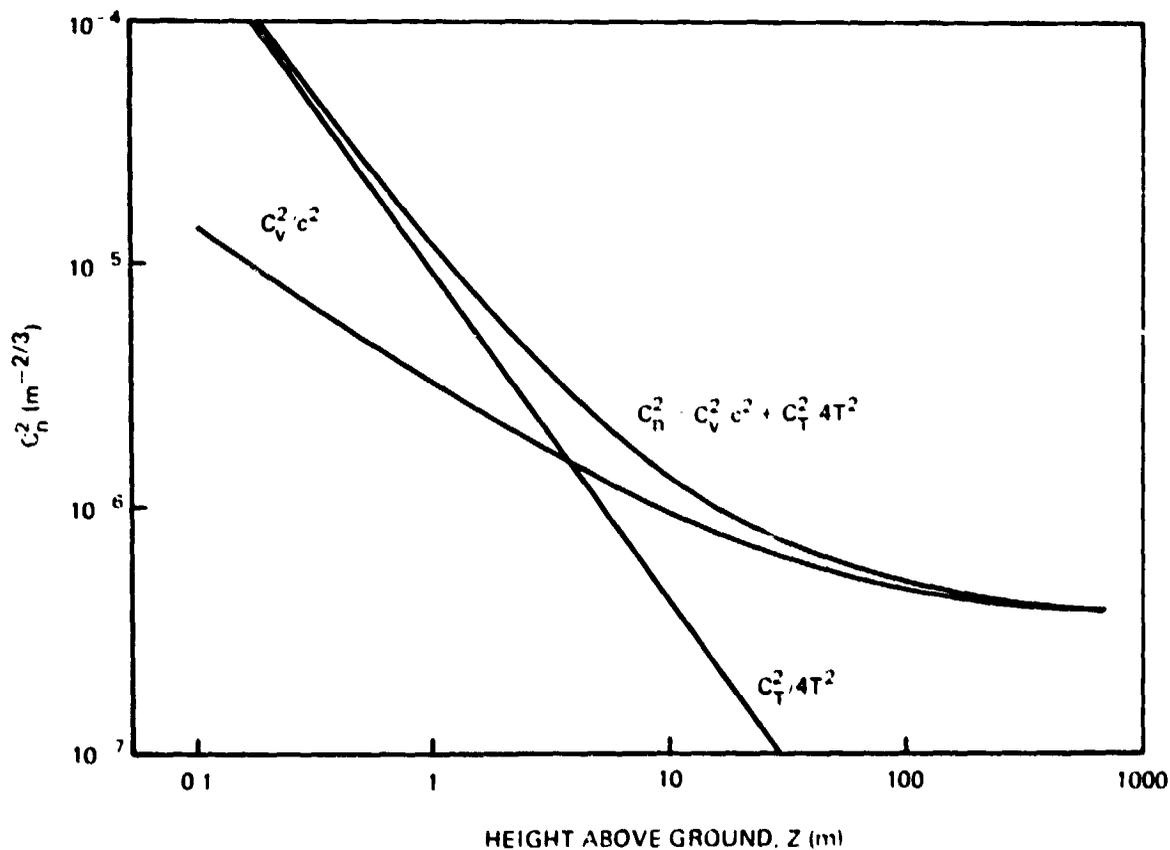

Figure 4. Typical variation of structure parameters with height above ground. Calculated according to eqs. 18 and 19. T = 283K.

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Curves of C_v^2/c^2 and $C_T^2/4T^2$ are also shown in fig. 4. The two curves cross at $Z = 3.75$ m. For heights greater than that $C_T/4T^2$ rapidly becomes much less than C_v^2/c^2. Near the ground, fluctuations in temperature (C_T^2) dominate the value of C_n^2. At heights above about 10 m, fluctuations in wind velocity (C_v^2) control the value of C_n^2.

Parameters k, D_0, and L.-The wave number k and the sound propagation distance L appearing in eq. (5) are well defined and can be determined in a straightforward way. The remaining parameter D_0 — the initial beam diameter — is not well defined except for an experimental situation that closely approximates the beam model on which the analysis of Brown and Clifford is based. Thus, if an experiment were conducted using a tower-mounted-loudspeaker sound source, D_0 would be set equal to the diameter equivalent to the area at the exit plane of the source.

For such an experiment, the frequency of the loudspeaker-generated sound would be controlled through the electrical signal driving the loudspeaker. Thus, the wavenumber k would be independent of diameter D_0.

For other types of sound, such as the jet noise of an aircraft engine, the wavenumber k and the parameter D_0 will not be independent of each other. The noise generation region in the jet extends aft from the tailpipe exit. The high frequency part of the jet-noise spectrum is created by sources close to the tailpipe exit and the low frequency part of the spectrum is caused by sources further downstream. Furthermore, the diameter of the jet increases in the downstream direction. Thus the high-frequency (small wavenumber) part of the spectrum is associated with a smaller characteristic dimension for D_0 than is the low frequency (large wavenumber) part of the spectrum. The coupling of k and D_0 can be put on a more quantitative basis by rewriting the factor $k^{12/5} D_0^2$ in eq. (5) as $(D_0^2 k^2)k^{2/5}$. Then by introducing a Strouhal number $St = fD_0/U = kD_0/2\pi M$, where $M = U/c$ is a characteristic Mach number, the factor $k^{12/5}D_0^2$ can be seen to be equal to $4\pi^2 M^2 St^2 k^{2/5}$. Equation (5) can therefore be written in the following form:*

$$A_t = 10 \log_{10} \left\{ 1 + 61.6\ M^2 St^2\ k^{2/5} \left[\int_0^L [(L - s)/L]^{5/3}\ C_n^2\ (s)\ ds \right]^{6/5} \right\} \quad (20)$$

*Again, it must be pointed out that the factor 61.6 in eq. (20) should be divided by four, i.e., it should be 15.4 if the factor 16 in eq. (4) were changed to 64.

Equation (20) is appropriate for application to the propagation of aircraft flyover noise whereas eq. (5) is applicable to noise produced by a loudspeaker. A comparison of the two equations shows that the frequency dependence of turbulence-induced sound attenuation is stronger for sound generated by a loudspeaker than for sound from an aircraft engine because of the $k^{12/5}$ factor in eq. (5) compared with the $k^{2/5}$ factor in eq. (20).

Furthermore, a comparison of the two equations indicates that the magnitude of the turbulence-induced attenuation is much less for an airplane-engine sound source than for a loudspeaker source. That can be seen by taking the ratio of the second term in the braces in eq. (20) to the second term in the braces in eq. (5). For small values of the second term the above ratio is approximately equal to the ratio of the attenuation for an airplane-engine sound source and a loudspeaker sound source. Calling the ratio R we obtain

$$R = 4\pi^2 M^2 St^2 / D_0^2 \, k^2 . \tag{21}$$

For reasonable values of M, St, and D_0 (say $M = St = 1$ and $D_0 = 1$ m) the ratio R will be much less than unity except for small values of k, i.e., low frequencies.

Calculation of Sound Attenuation Induced by Turbulence
(Brown and Clifford Model)

In this section we show the results of some calculations of turbulence-induced sound attenuation based on the Brown and Clifford model.

For sound generated by a loudspeaker-type sound source, eq. (5) is rewritten in the following form:

$$A_t = 10 \log_{10} \left\{ 1 + C_1 \, k^{12/5} \, D_0^2 \, (L \, \overline{C_n^2})^{6/5} \right\} \tag{22}$$

where $\overline{C_n^2}$ is a weighted average of C_n^2 over the sound path; i.e.

$$\overline{C_n^2} \equiv (1/L) \int_0^L [(L - s)/L]^{5/3} \, C_n^2 \, (s) \, ds . \tag{23}$$

The nondimensional constant C_1 in eq. (22) is equal to 1.56 according to the original Brown and Clifford analysis or is equal to $(1.56/4) = 0.391$ if their expression for the far-field spreading of a beam of sound in a non-turbulent atmosphere were incorrect by a factor of four (see Appendix A).

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For aircraft noise eq. (20) is rewritten in the following form:

$$A_t = 10 \log_{10} \left\{ 1 + C_2 M^2 St^2 k^{2/5} (L\overline{C_n^2})^{6/5} \right\} \tag{24}$$

where C_2 is equal to 61.6 if the far-field beam spreading equation used by Brown and Clifford is correct, or to $(61.6/4) = 15.4$ if that equation is off by a factor of four.

According to eq. (23), the turbulence nearer the source is weighted more heavily than the turbulence nearer the receiver in its effect on $\overline{C_n^2}$. The weighting comes about because of the $[(L - s)/L]^{5/3}$ factor in the integrand since $s = 0$ at the source and $s = L$ at the receiver. Therefore, because C_n^2 generally decreases with height (see fig. 4), $\overline{C_n^2}$ would be smaller for the case of vertical sound propagation from a source located a distance L above a receiver than for a receiver located a distance L above a source.

Figure 5 shows some geometrical relationships and defines notation for two cases of vertical sound propagation. One case is that of a sound source above a receiver, i.e., the case of an elevated source. The other case is that of a receiver above a sound source, i.e., the case of an elevated receiver. The weighted average of C_n^2 is given by the following equations for the two cases:

For an elevated source

$$L\overline{C_n^2} = \int_{Z_0}^{Z_L} [(Z - Z_0)/(Z_L - Z_0)]^{5/3} C_n^2(Z) \, dZ \tag{25}$$

For an elevated receiver

$$L\overline{C_n^2} = \int_{Z_0}^{Z_L} [(Z_L - Z)/(Z_L - Z_0)]^{5/3} C_n^2(Z) \, dZ \tag{26}$$

Figure 6 shows calculated curves of average turbulence-induced attenuation coefficient in dB/m for the two cases of elevated source and elevated receiver. The assumed variation of turbulence structure parameter for refractivity fluctuations C_n^2 used for the calculations is that given by eq. (19) and shown in fig. 4. The air temperature was assumed to be 283 K.

(a) Elevated source.

(b) Elevated receiver.
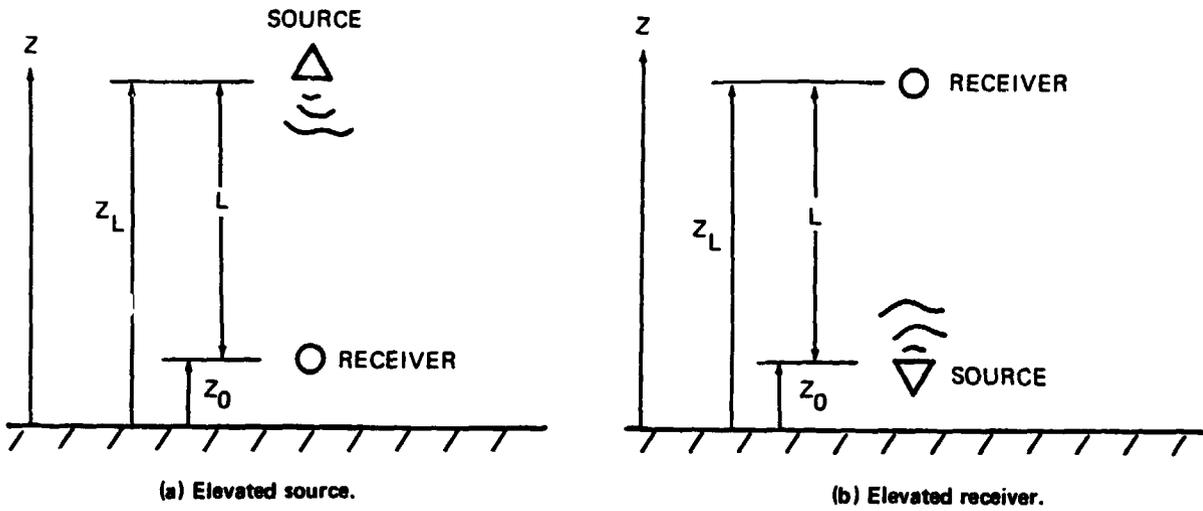
Figure 5.-Vertical sound propagation for a source above a receiver
and for a receiver above a source.

SOURCE OR RECEIVER HEIGHT, Z_0 (m)
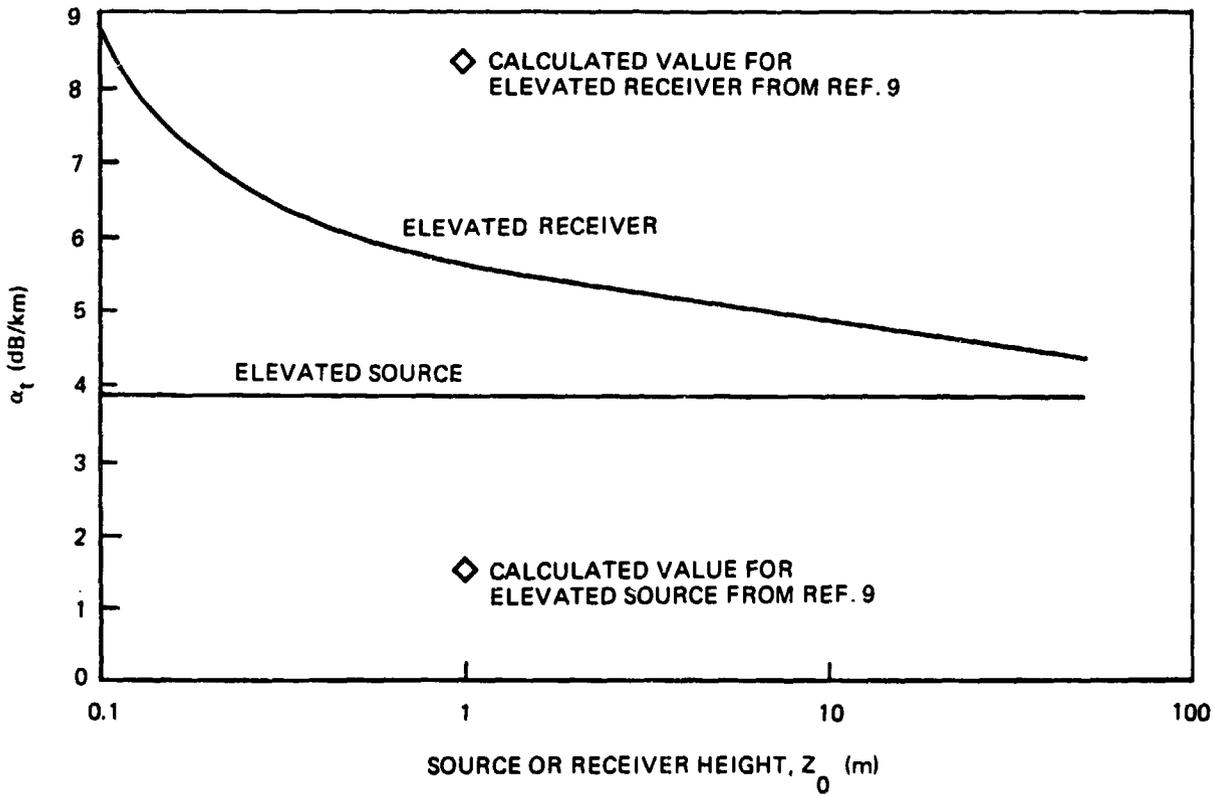
Figure 6.-Average turbulence attenuation coefficient for a vertical path.
$T = 283\,K$, $f = 4000$ Hz, $C_1 = 1.56$, $\alpha_t = (1000/L)\,A_t$ (dB/km).
A_t from eq. (22).

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The variable in fig. 6 is the height above ground Z_0 of the source when the receiver is elevated or of the receiver when the source is elevated. The height Z_L was always kept at 600 m and the path length decreased as Z_0 increased. Equations (22) and (23) with $C_1 = 1.56$ were used for the calculations.

The calculations for fig. 6 were done for a temperature of 283 K, a frequency of 4000 Hz (a wavenumber k of 74.5 m^{-1}), and an initial beam diameter $D_0 = 1$ m. Those values were chosen because they were the same as those picked by Brown and Clifford for some of their examples and they thus provided a check on the calculations. The quantity $\overline{LC_n^2}$ (i.e. the weighted average of the structure parameter C_n^2 over the propagation path multiplied by the path length) was found by numerical integration of eqs. (25) and (26).

The results in fig. 6 indicate that when the source is near the ground plane and the receiver is elevated, the attenuation coefficient decreases by a factor of about 2 as the source is raised from 0.1 to 50 m or out of the region of strong turbulence. However, when the source is elevated at a height of 600 m and the receiver is located near the ground plane, then raising of the receiver does not influence the calculated attenuation.

The difference between the calculated α_t values for an elevated receiver and an elevated source decreased rapidly as the distance above the ground plane was increased, coinciding with the rapid decrease in the C_n^2 function with increasing height in fig. 4. When there was little variation of C_n^2 with height above the ground plane, say for heights $Z > 10$ m, then there was little difference in the effect of turbulence on sound propagation. Note in fig. 6 that the values calculated for α_t at $Z_0 = 1$ m are 3.8 and 5.6 dB/km. Brown and Clifford (ref. 9) calculated corresponding values of 1.5 and 8.0 dB/km for the same case. Their results were found to be incorrect due to errors in their calculations. Thus, the calculated difference in attenuation between the cases of an elevated source and an elevated receiver is much less than stated in ref. 9 even if the value of the coefficient C_1 were equal to 1.56.

Figure 7 shows calculated average turbulence-induced-attenuation coefficient for the same conditions used for the calculations displayed in fig. 6. However, the coefficient C_1 in eq. (22) was set equal to 0.391 instead of 1.56. It is seen that the differences between the cases of an elevated source and an elevated receiver are even smaller than those shown in fig. 6 or reported in ref. 9.
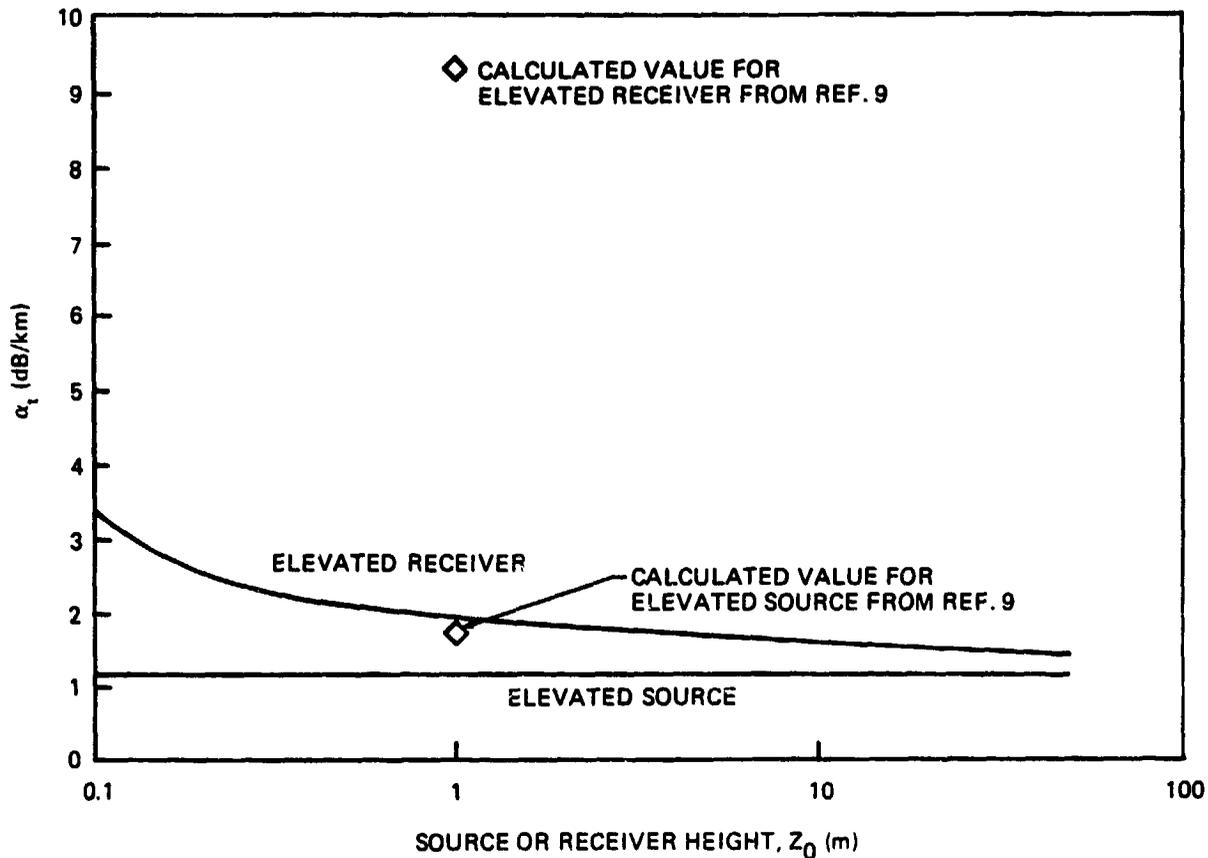
Figure 7.-Average turbulence attenuation coefficient for a vertical path.
$T = 283$ K, $f = 4000$ Hz, $C_1 = 0.391$, $\alpha_t = (1000/L)A_t$ (dB/km) A_t from eq. (22).

Because the structure constants vary with height above the ground, the average turbulence attenuation coefficient for a sound path α_t, in dB/km, varies as the sound source height is varied. Figure 8 illustrates the variation in α_t with source height for a fixed receiver height of 1 m. Separate curves are shown for the two values of the constant C_1. The values of α_t shown in fig. 8 for a source height of 600 m. correspond to those shown in figs. 6 and 7 for an elevated source with the receiver at a height of 1 m.

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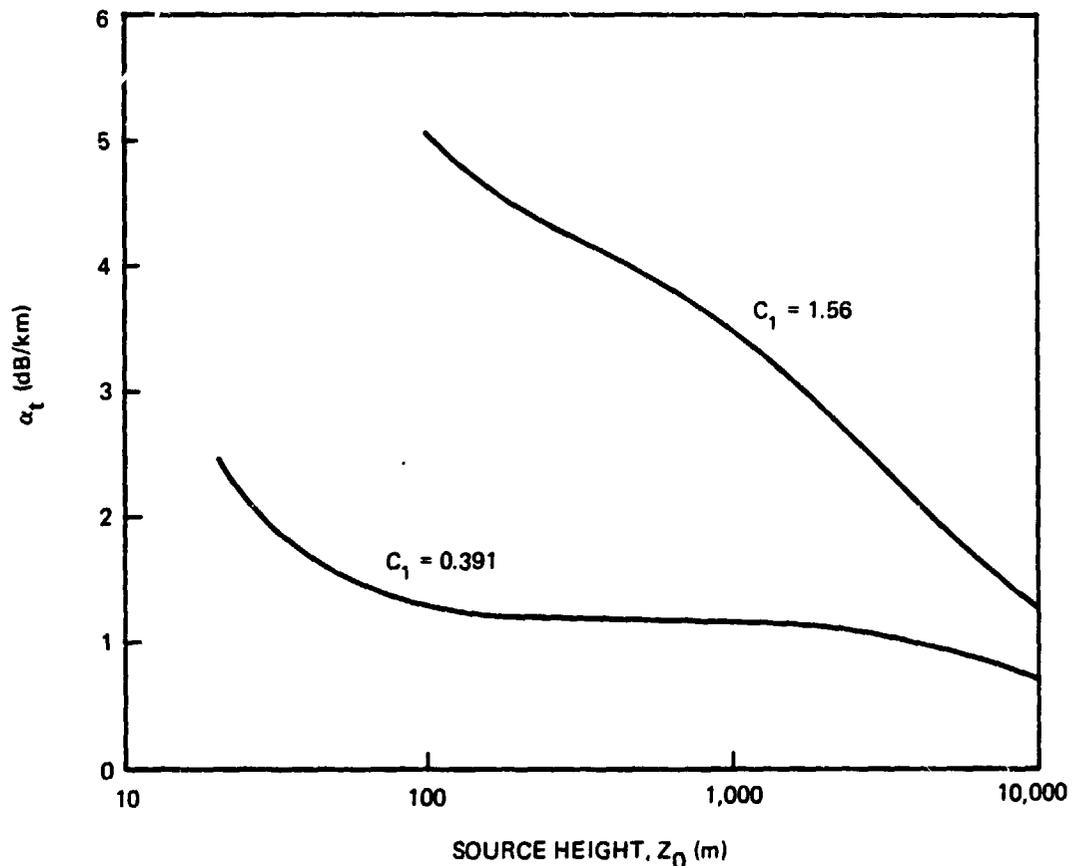
Figure 8.-Average turbulence attenuation coefficient versus source height. Receiver is 1-m above ground level. $\alpha_t = (1000/L)A_t$ (dB/km). T = 283 K, f = 4000 Hz, D_0 = 1 m. A_t from eq. (22).

The variation of α_t with frequency is shown in fig. 9. Separate curves are given for an elevated source and for an elevated receiver and for the two values of the coefficient C_1. Note that α_t is plotted on a logarithmic scale in fig. 9 in contrast to the linear scale in figs. 6 to 8. All four curves have the same general shape. For low frequencies the slope of the curves, on a logarithmic plot, is about 2.3. Thus for low frequencies α_t varies as the 2.3 power of frequency. As the frequency increases the slope of the curves decreases.

Figures 6 to 9 are all for a loudspeaker-type sound source, i.e., a sound source whose effective diameter is independent of frequency. For a jet-propelled airplane, there is a coupling between effective source diameter and frequency, the source size decreasing as frequency increases.
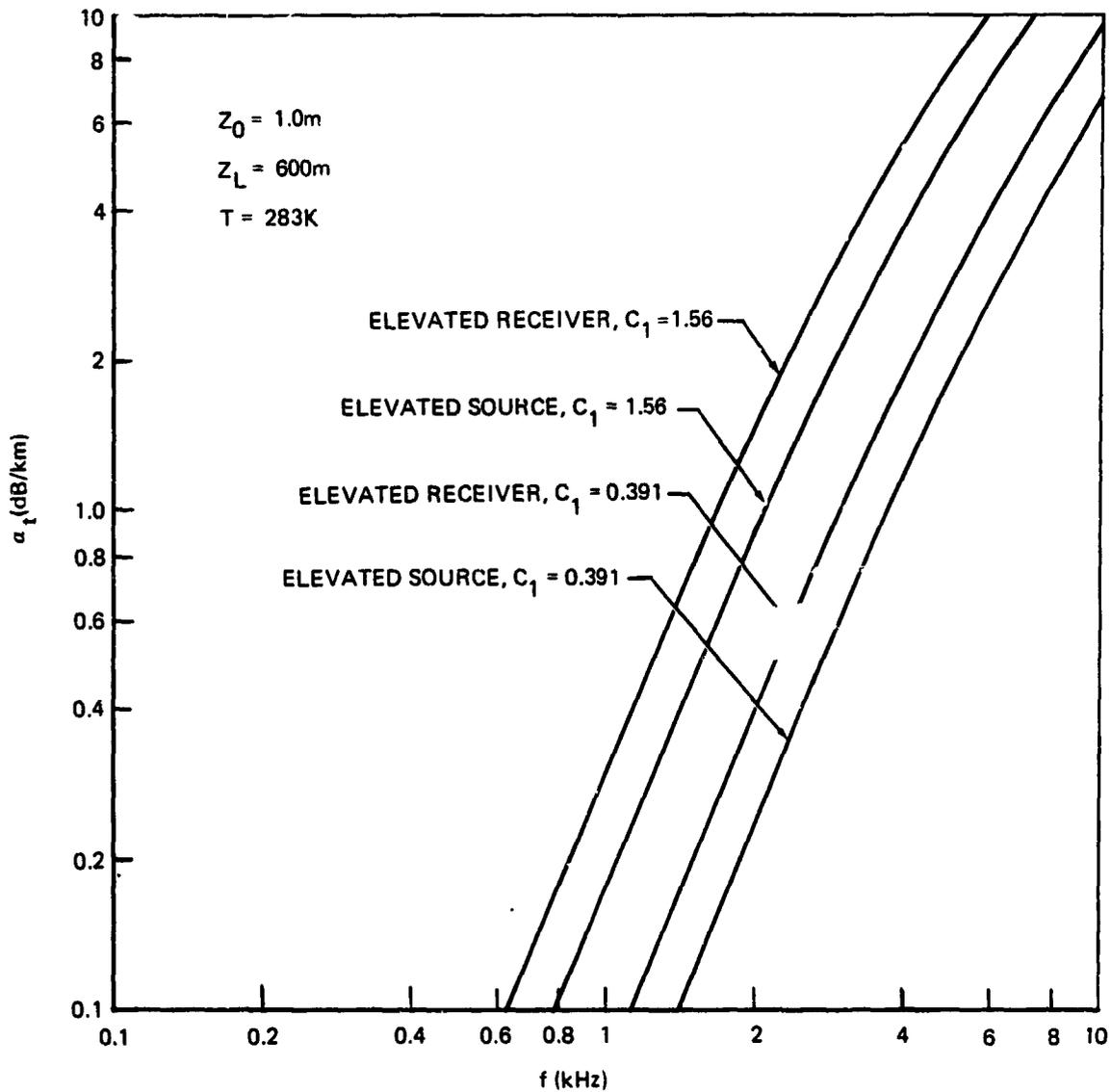
Figure 9.-Variation of turbulence-induced attenuation, α_t, with frequency, f. A_t according to eq. (22).

As described previously, the coupling is through a Strouhal number relationship $St = fD_0/U$. Figure 10 shows the calculated variation of α_t with frequency, according to eq. (24) for an aircraft-type sound source. The characteristic Mach number M and Strouhal number St were both arbitrarily set equal to unity.
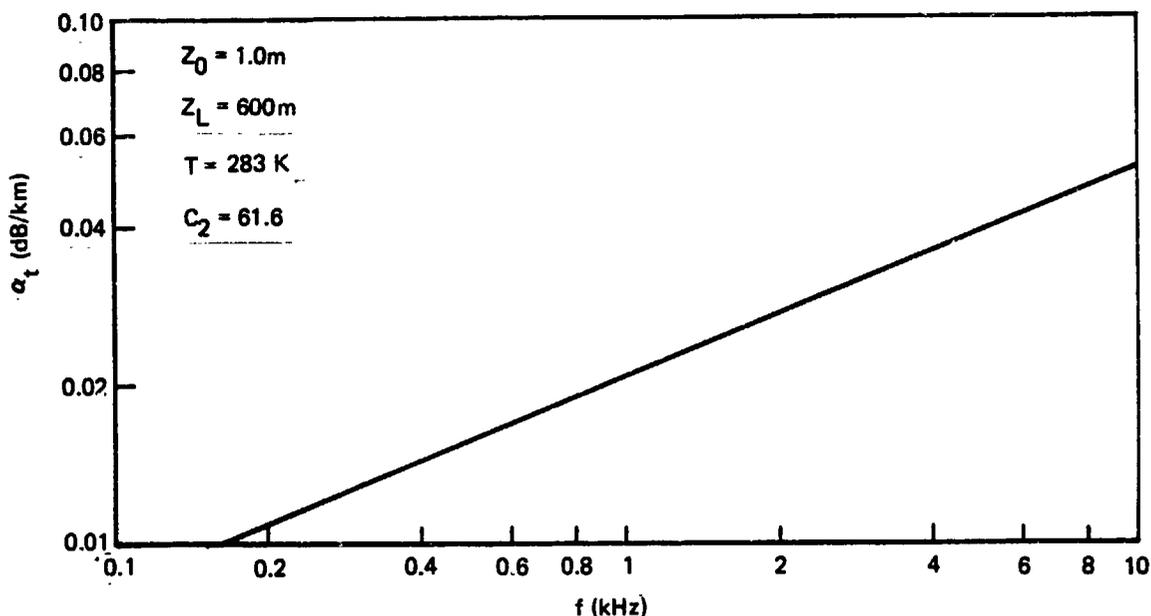
Figure 10.-Variation of turbulence-induced attenuation α_t with frequency, f, for an aircraft-type sound source. M = St = 1.0. α_t = (1000/L)A_t (dB/km.) A_t according to eq. (24).

Two characteristics of the calculated curve are apparent. First, the variation is very closely approximated by a straight line. The slope of the straight line is 2/5 which implies that α_t varies as the 2/5-power of frequency. The second important characteristic of the plot is that the calculated values of α_t are significantly smaller than those in figs. 6 to 9 for a loudspeaker-type sound source, thus indicating that atmospheric turbulence may have little effect on aircraft noise attenuation. It should be kept in mind, however, that since α_t is approximatley proportional to the product $M^2 St^2$, its value could increase greatly if M or St were increased from their assumed values of unity. Nevertheless, a characteristic value of unity for M and for St is reasonable and even if St were increased to, say, St = 3 the attenuation coefficient α_t would increase by a factor of about ten and would still be rather small.

Calculation of Sound Attenuation Induced by Turbulence
(DeLoach Model)

DeLoach's equation (ref. 15) for turbulence-caused sound attenuation is based on an empirical modification of a single-scatter model. The equation for the attenuation over a sound path (in decibels) is

$$A_t = 1.980 \ k^{1/3} \int_{Z_0}^{Z_L} [(\pi/kL_0) + \sin (\theta_c/2)]^{-5/3} \ \cdot$$

$$[(C_v^2/c^2) + 0.136 \ (C_T^2/T^2)] \ dZ \tag{27}$$

where L_0 is the "outer scale" of the atmospheric turbulence and θ_c is an empirical parameter.

A straightforward derivation of an equation for A_t based on a single-scatter model would yield eq. (27) but with the equivalent angle θ_c equal to zero. For that case A_t would be proportional to the square of the wavenumber. DeLoach's empirical modification leads to the introduction of the new parameter θ_c. If $\sin (\theta_c/2)$ were much greater than π/kL_0 then A_t would be proportional to the one-third power of the wavenumber. Thus, DeLoach's modification allows for a frequency dependence of A_t ranging from a frequency-squared to a frequency to the one-third power. The exact frequency dependence depends on the relative magnitude of the terms π/kL_0 and $\sin (\theta_c/2)$. DeLoach argues that measurements of "excess" attenuation of aircraft flyover noise show a one-third-power-of-the-frequency dependence. Also, he is able to fit experimental results for "excess" attenuation by choosing specific numerical values for the parameters θ_c and L_0.

A deficiency of DeLoach's model is that the quantity θ_c does not seem to be related to any measurable physical quantity (except for A_t itself). Because θ_c cannot be measured, eq. (27) cannot be used to predict the magnitude of A_t even if the structure parameters C_v^2 and C_T^2 have been measured.

In his report, DeLoach gives several examples showing that eq. (27) is consistent with reported measurements of excess attenuation. He defines excess attenuation as the attenuation a sound wave suffers that is in

addition to that caused by "classical plus molecular absorption." DeLoach argues that turbulence scattering is the primary cause of excess attenuation.

Since excess attenuation is defined by DeLoach to be attenuation not accounted for by viscosity and heat conduction effects,* i.e. by atmospheric absorption, it is important in interpreting experimental data to accurately account for the effects of absorption. That fact can be illustrated by looking at some data that DeLoach used as a check of his theory. The data were taken from ref. 26 and represent a composite of measured total attenuation coefficients from several experiments. Temperature and humidity were measured during the experiments but the structure parameters C_v^2 and C_T^2 were not. The outer scale of turbulence L_0 was not measured either.

Figure 11, taken from ref. 15, shows the measured data along with three calculated curves. Two of the curves were calculated by DeLoach, the third was calculated as part of this study. The short-dashed straight lines show the atmospheric absorption calculated by DeLoach according to a method described in ref. 15. The solid-line curves show the sum of the calculated absorption and the attenuation caused by scattering calculated according to eq. (27). The numerical values shown for the structure parameters were assumed values. The parameters θ_c and L_0 were determined by DeLoach by requiring that eq. (27) give a best fit to the data according to the method of least squares.

It can be seen from fig. 11 that the data can be closely matched by a proper choice of θ_c and L_0. On the other hand, the measured attenuation can also be explained on the basis of absorption alone provided that the absorption is calculated according to another method. The long-dashed/ short-dashed curves show absorption calculated according to the method of ref. 3. The procedure of ref. 3 for calculating pure-tone atmospheric absorption coefficients is based on a more up-to-date understanding of the physical phenomena than the procedure used by DeLoach in ref. 15.

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*Viscosity effects include effects associated with both of the two coefficients of viscosity (e.g., see ref. 27) that are in the Navier-Stokes equations. Therefore, both "classical" absorption and molecular relaxation effects are included here as effects of viscosity.

Figure 11.-Variation of attenuation with frequency for flyby data.
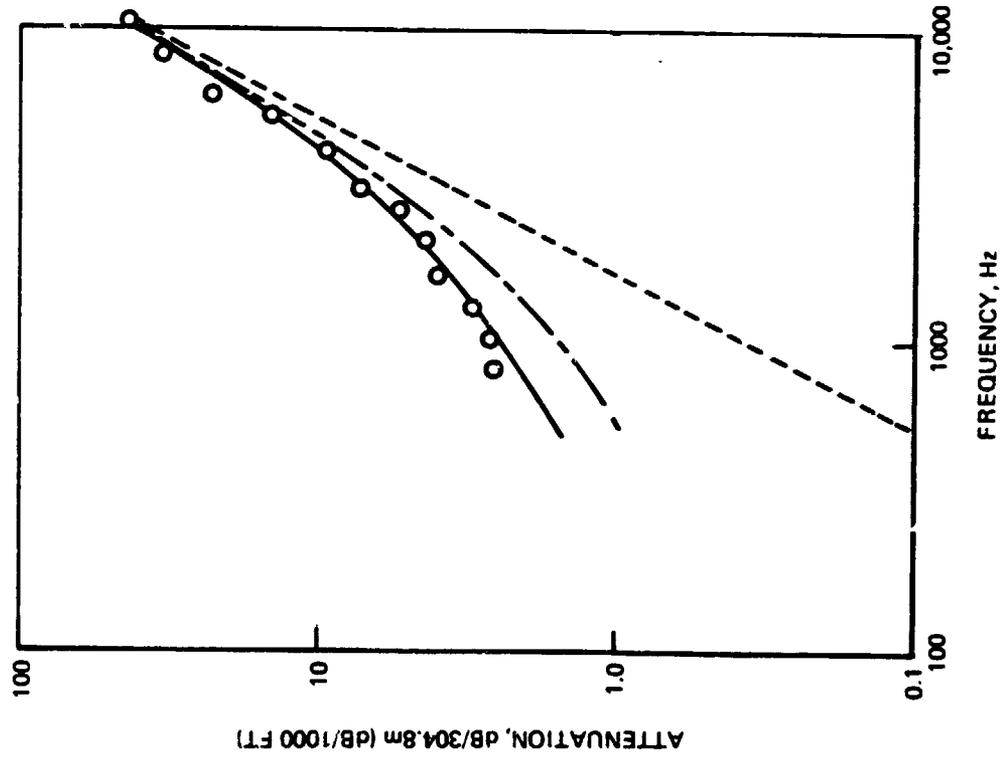
(a) $C_v = 0.34 \text{ m}^{2/3}/\text{sec}$; $C_T = 0.17^\circ \text{ C/m}^{1/3}$; $\theta_c = 1.2^\circ$; $L_0 = 150 \text{ m}$; $T = 21.5^\circ \text{ C}$; and relative humidity = 51.4 percent.
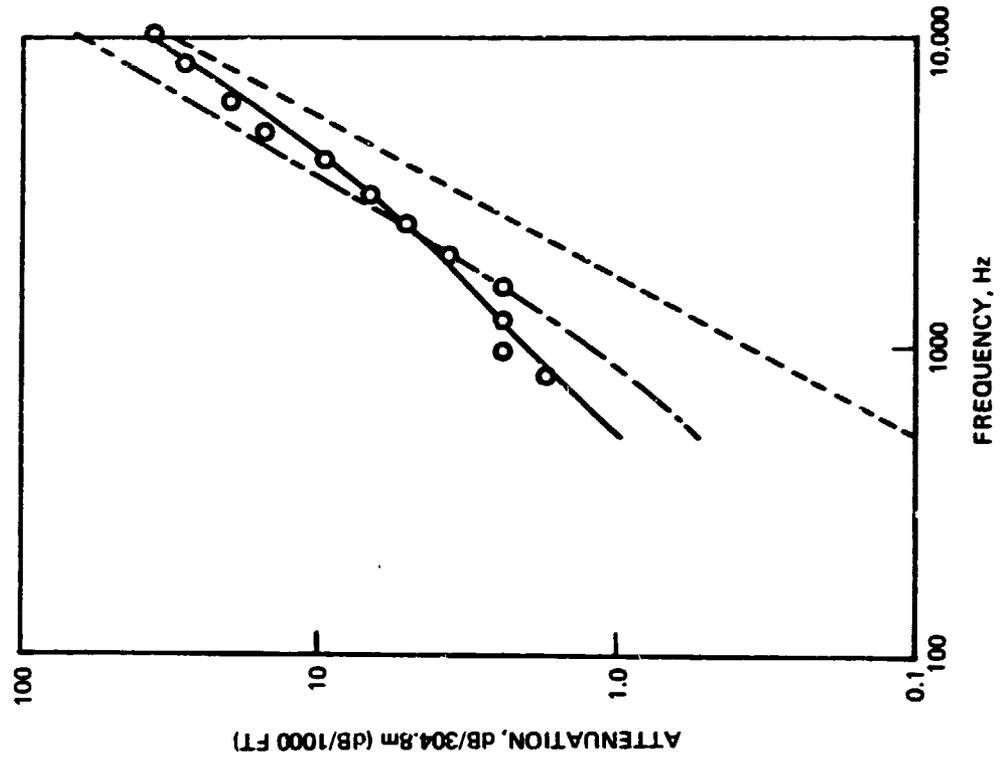
(b) $C_v = 0.15 \text{ m}^{2/3}/\text{sec}$; $C_T = 0.17^\circ \text{ C/m}^{1/3}$; $\theta_c = 0.65^\circ$; $L_0 = 72.7 \text{ m}$; $T = 5.5^\circ \text{ C}$; and relative humidity = 60.8 percent.

Legend:
- O DATA
- – – – ABSORPTION ALONE ACCORDING TO REF. 15
- ——— SUM OF ABSORPTION AND SCATTERING ACCORDING TO REF. 15
- —·—·— ABSORPTION ALONE ACCORDING TO REF. 3

KEY:

O DATA
––– ABSORPTION ALONE ACCORDING TO REF. 15
——— SUM OF ABSORPTION AND SCATTERING ACCORDING TO REF. 15
— – ABSORPTION ALONE ACCORDING TO REF. 3
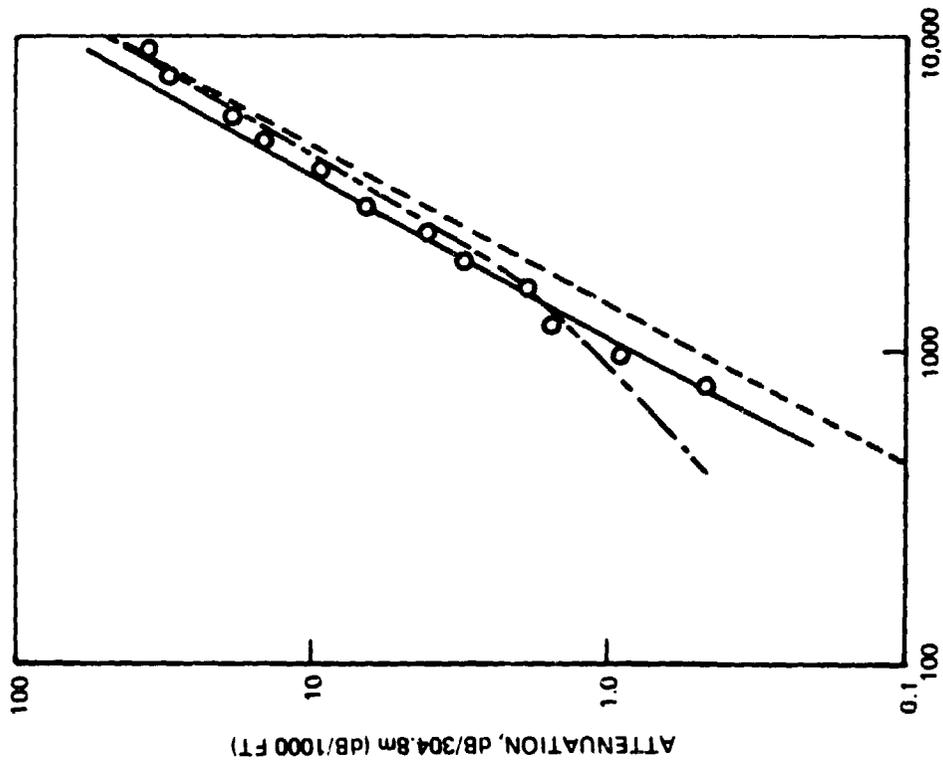
(c) $C_v = 0.15\ m^{2/3}/sec;\ C_T = 0.17^o\ C/m^{1/3};$
     $\theta_c = 0.1^o;\ L_0 = 7.6\ m;\ T = 8.61^o\ C;$
     and relative humidity = 76 percent.
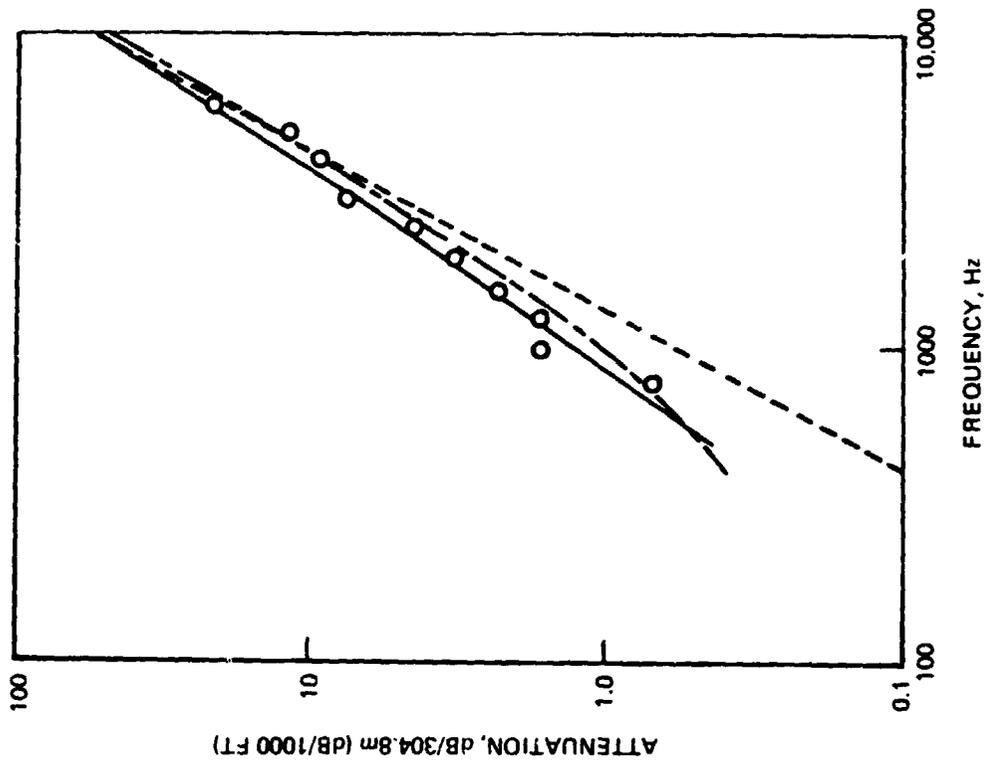
(d) $C_v = 0.15\ m^{2/3}/sec;\ C_T = 0.17^o\ C/m^{1/3};$
     $\theta_c = 0.84^o;\ L_0 = 26.4\ m;\ T = 3.5^o\ C;$
     and relative humidity = 88.3 percent.

Figure 11.–Continued.

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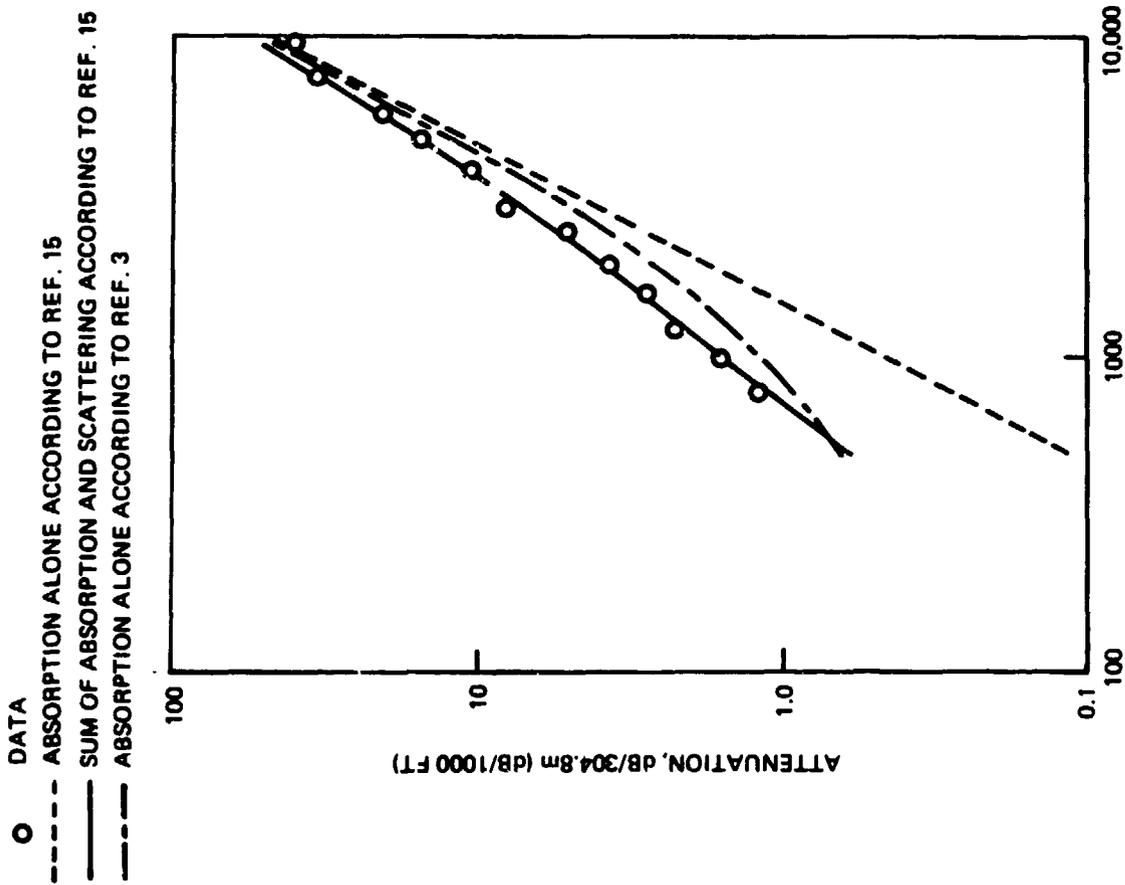
(c) $C_v = 0.15 \text{ m}^{2/3}/\text{sec}$; $C_T = 0.17^\circ \text{ C/m}^{1/3}$;
$\theta_c = 0.67^\circ$; $L_0 = 37.7 \text{ m}$; $T = 12.1^\circ \text{ C}$;
and relative humidity = 68.3 percent;

Figure 11.-Concluded.

TEMPORAL FLUCTUATIONS CAUSED BY TURBULENCE

Turbulence in the atmosphere has two effects on sound measured in the far field of a noise source. The apparent reduction of the mean intensity of the sound has been discussed in the previous section. In this section we discuss the effect of atmospheric turbulence on the variation of sound intensity with time during a measurement.

Large fluctuations in the amplitude of the sound pressure level have been known from common experience and from outdoor sound propagation experiments for some time, though the cause has been a matter of speculation.

In ref. 28, Knudsen describes the results of an experiment conducted on the campus of UCLA in 1934. A transmitter and a receiver were set up about 100 ft apart. The experiment was conducted early on a Sunday morning, before sunrise, with no perceptible wind. With a 4000-Hz tone as a signal, "the level at the receiver fluctuated violently over a range of more than 10 dB with short periods (0.1 s or less) and long periods (several seconds) all jumbled together." In an experiment conducted on a Mojave dry lake in the summer of 1940, the fluctuations were found to increase with sound frequency and distance between transmitter and receiver. The major cause of the fluctuations was found to be inhomogeneities in the temperature and velocity of the atmosphere.

Rudnick (ref. 29), following up the observations of Knudsen and Delsasso, investigated the propagation of sound waves in an anechoic chamber wherein the air temperature was constant at 20°C. He introduced temperature inhomogeneities by heating a 5-m-long resistance wire stretched across the room. He found that the fluctuations in received sound pressure level varied with frequency and angle of incidence of the sound wave onto the sheet of hot air rising from the heated wire, the largest fluctuations occurring near grazing incidence and at higher frequencies. A characteristic of the fluctuations was the presence of short period fluctuations — of the order of seconds and fractions of seconds — superimposed on variations with periods of the order of minutes. For a given angle of incidence, the fluctuations at different frequencies could be correlated statistically by

the ratio of the root-mean-square deviation of the sound pressure about
a given amplitude to the square of the frequency of the sound.

As part of a Symposium on Aircraft Noise in November 1952, Ingard
(ref. 30) presented a review of meteorological effects on sound propaga-
tion including a discussion of the effects of turbulence. Velocity
fluctuations associated with the gustiness of the wind were shown to
produce fluctuations that increased with gustiness, frequency of the sound,
and distance between transmitter and receiver. The fluctuations ranged
from 10 to 20 dB at 2000 Hz and from 5 to 25 dB at 4000 Hz for wind speeds
from 6 to 11 m/s. The source of the fluctuations was thought to be
atmospheric turbulence.

Wiener and Keast (ref. 31) reported the results of an extensive study
of the effects of atmospheric conditions on sound propagation outdoors
from loudspeaker sound sources. At night when there was a strong positive
temperature gradient (or inversion) because of radiation cooling of the
ground, the air near the ground was stable and turbulent wind fluctuations
were limited in amplitude and of low frequency. Under these conditions
the fluctuations in the sound pressure level at the microphones were
small. On sunny, windy days, the air temperature profile showed a large
negative gradient (lapse), the turbulence was strong, and the air near the
ground was unstable. Large fluctuations in the received sound pressure
levels were noted. The fluctuations contained appreciable high-frequency
components.

Defining peak-to-peak fluctuations as the difference between the
maximum and the minimum sound pressure levels in a 30-s observation period
and plotting the results as a function of distance from the sound source
for various octave band center frequencies gave a measure of the effect
of atmospheric turbulence on the fluctuations of the received sound pressure
level. For typical unstable daytime conditions and upwind propagation,
the peak-to-peak fluctuation at 200 ft varied from 12 dB for the 425-Hz
band to 25 dB for the 3400-Hz band; at 3200 and 4500 ft the fluctuations
ranged from 5 to 7 dB. For typical stable nighttime conditions and propa-
gation downwind, there was less variation with distance and the fluctuations

ranged from 3 to 5 dB for the 425-Hz octave band to 5 to 13 dB for the 3400-Hz octave band, with the larger values occurring at the greatest distances. The periods of the fluctuations were generally short for either atmospheric condition.

In a study of the propagation of sound over ocean waters in fog, Wiener (ref. 32) found little effect of atmospheric turbulence on sound propagation. This result was attributed to the very stable condition of the atmospheric boundary layer over water.

Wiener, Malme, and Gogos (ref. 33) describe the results of experiments on the propagation of sound in city streets. They found that the temporal fluctuations in the received sound pressure level were less than noted in open country but were still significant. At night when the atmosphere was reasonably stable, the maximum peak-to-peak fluctuation was about 5 dB and increased with signal frequency and distance from the sound source. Typical day-time fluctuations were about 8 to 10 dB. The period of the largest fluctuations was on the order of a few seconds.

All of the experiments on sound fluctuations that we have just summarized suffer from the same basic defect that existed for the experiments that were conducted to measure attenuation caused by turbulence. That is, because of a lack of a physical model, important parameters that influence the experimental results were not measured.

We shall now give some results, based on weak-fluctuation theory which should provide guidance for experiments to determine atmospheric turbulence effects on sound fluctuations from either a static sound source or an airplane during a flyover noise test.

A statistical measure of the fluctuations in intensity is the variance of the intensity, i.e., the mean-square of the fluctuations in the intensity of a sound wave relative to the mean intensity.

For weak fluctuations, a simple solution exists for the variance of the logarithm of the intensity (e.g., see refs. 14 and 17)

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$$\sigma^2_{\ln I} = <(\ln I - <\ln I>)^2> = 4\sigma^2_\chi$$
$$= 4 \ (0.5628) \ k^{7/6} \ L^{11/6} \ [\overline{C_n^2}]_\chi \tag{28}$$

where σ^2_χ is the "log amplitude variance" and $[\overline{C_n^2}]_\chi$ is different for a plane wave than for a spherical wave.

For a plane wave:

$$[\overline{C_n^2}]_\chi = (1/L) \int_0^L (1 - s/L)^{5/6} \ C_n^2(s) ds \tag{29}$$

For a spherical wave:

$$[\overline{C_n^2}]_\chi = (1/L) \int_0^L (s/L)^{5/6} (1 - s/L)^{5/6} \ C_n^2(s) ds \tag{30}$$

For $\sigma^2_\chi <<1$ it can be shown (ref. 4) that

$$\sigma^2_I = [<I^2> - <I>^2]/<I>^2 = \exp \sigma^2_{\ln I} - 1 \tag{31}$$

The above equations were originally derived on the basis of approximations valid for optical wavelengths. The same is true for the equations used as the basis of the "excess attenuation" model of Brown and Clifford. However, as mentioned previously, Brown and Clifford have shown that their results are also valid for wavelengths appropriate to sound propagation at audible frequencies. It is likely that the equations for the variance of intensity are also applicable to acoustic wave propagation problems.

Inspection of eqs. (28-31) shows that the same parameters appear as in the Brown and Clifford equations for turbulence-induced sound attenuation. Those parameters are the wavenumber k, the propagation path length L, and the structure parameter for refractivity fluctuations C_n^2. On the basis of the discussion in the previous section it would be anticipated that the variance of the intensity would therefore be largest when C_n^2 is largest, i.e., near the surface of the ground. This result agrees with observations from field measurements of a variety of noise sources ranging from aircraft to motor vehicles and railroad trains.

EXPERIMENTAL VERIFICATION PROGRAM

There are several reasons why reports on previous experiments lacked information on key parameters: First, some experiments were carried out without having a theoretical model available when the tests were planned.

Consequently, it was not clear as to what parameters were important to measure. Second, many experiments which were later used to study atmospheric turbulence effects were conducted primarily for some other purpose. Therefore, atmospheric turbulence characteristics were not measured. That was usually the case for flyover noise measurements, for example. And finally, at the time that some of the experiments were conducted, there were no readily available, practical procedures for measuring the necessary turbulence parameters.

In this section of the report we describe two general experiments designed to validate (or discredit) the Brown and Clifford model. The experiments would also be useful for testing other models of sound propagation through a turbulent atmosphere since other models, such as DeLoach's, contain most of the same parameters that are in the Brown and Clifford model.

The two general types of experiments are: (1) tower experiments and (2) aircraft flyover noise experiments. For a tower experiment, a loud-speaker is mounted on top of a tall tower. Meteorological data are obtained from instruments mounted to the tower at various heights. Acoustical data are obtained from microphones on the ground and attached to the guy wires that stabilize the tower. Such a tower experiment is described in ref. 34. The tower used was a 150-m tower near Haswell, Colorado. Although part of the experimental program involved a study of sound pressure fluctuations caused by turbulence, structure parameters were not measured.

Numerous other large towers exist and have been used extensively to study the atmospheric boundary layer. In fact, in his review article on the atmospheric boundary layer, Panofsky (ref. 35) calls the lower part of the boundary layer the *tower layer* because of the numerous tower observations that have been conducted to study that region of the atmosphere.

An example of a tall tower that would be suitable for a study of atmospheric turbulence effects on sound propagation is the 300-m instrumented research tower near Boulder, Colorado. The tower is operated by the Wave Propagation Laboratory of the National Oceanographic and Atmospheric Administration (NOAA).

Aircraft flyover noise experiments can be conducted with an aircraft climbing, descending, or flying level. However, level-flight flyovers are easiest to interpret. The aircraft engine (or engines) itself is the primary sound source. Microphones are located near the ground beneath the flight path of the airplane. For a study of atmospheric turbulence effects the easiest data to analyze and interpret would be those data obtained from microphones directly beneath the flightpath and taken at times when the airplane is nearly overhead. For those conditions refraction by steady wind or temperature gradients and ground attenuation effects are minimized.

Meteorological data for aircraft flyover noise tests are ordinarily obtained from instruments carried aboard a light airplane. The meteorological or "met" airplane generally obtains data during a spiraling descent from a height above the greatest height of the test airplane to a height of 5 to 10 m above the ground. Meteorological data from the met airplane should be supplemented by data from a ground-based weather station located in the vicinity of the microphones.

An experiment to verify a model for turbulence-induced sound attenuation should measure the important parameters that enter into the model. For the Brown and Clifford model they are the wavenumber k of the sound, the sound-source diameter D_0, the propagation distance L, the structure parameters C_T^2 and C_v^2, and the temperature T. The sound speed c can be deduced from the temperature; and the wavenumber spectrum can be deduced from the frequency spectrum and speed of sound.

The basic procedure for experimental verification of the model would be to measure the sound pressure level spectrum at microphones located at fixed locations for a range of conditions of atmospheric turbulence and sound propagation path lengths. The range of atmospheric conditions would mean a range of the structure parameters C_T^2 and C_v^2 as well as a range of air temperature and humidity. Corresponding to the different atmospheric conditions would be different measured and predicted attenuations caused by atmospheric turbulence.

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It is also important to measure other atmospheric properties in addition to those that influence attenuation caused by turbulence. That is required because it is unlikely that, from test to test, only the turbulence characteristics of the atmosphere would change. Other quantities that determine the amount of atmospheric absorption of sound will also vary. Since absorption is undoubtedly the principal sound attenuation mechanism (in addition to geometric spreading, of course) it is important to accurately adjust the data to be able to distinguish between the attenuation caused by absorption and that caused by turbulence effects. The primary atmospheric characteristics that determine the amount of absorption are temperature and humidity. Pressure has a relatively small effect. Therefore, it is necessary that accurate measurements be made of temperature and humidity over the entire lengths of the sound propagation paths. A measurement of barometric pressure should be obtained at the surface station near the microphones.

Since temperature and humidity measurements are routinely made and procedures for making the measurements are well established, we need not delve further into methods for their measurement here. Measurements of the structure parameters C_T^2 and C_v^2, however, are not routinely made for sound-propagation experiments, and since they are important parameters influencing sound attenuation by turbulence we shall now describe methods for their measurement.

Measurement of C_v^2

Although it is possible to measure C_v^2 directly by correlating the signals from two hot-wire anemometers located a fixed distance apart and making use of eq. (7), it is much easier to obtain C_v^2 by an indirect method. The indirect method depends on the following relation between C_v^2 and the dissipation rate of the kinetic energy ε:

$$C_v^2 = 4\alpha_1 \varepsilon^{2/3} \tag{32}$$

A derivation of eq. (32) can be found in ref. 36. The quantity α_1 is a nondimensional universal constant which has been found to be about 0.5 from a variety of independent measurements.

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There are several practical ways to measure ε and thus, from eq. (32), to determine C_v^2. A rather direct measurement of ε has been developed by Wyngaard and Coté (ref. 37). They measure the fluctuating velocity $u(t)$ with a single hot-wire anemometer. Next, $u(t)$ is differentiated to obtain $\partial u/\partial t$. Taylor's frozen turbulence hypothesis is then invoked to obtain $\partial u/\partial x$. Finally, ε is obtained from the relation $\varepsilon = 15\nu <(\partial u/\partial x)^2>$ where ν is the kinematic viscosity of the air (see ref. 22, pp. 123-126).

The "direct" measurement of ε is not the simplest way to determine the dissipation. Its primary use has been, rather, for determining the value of the constant α_1 that appears in eq. (32). A simpler way to determine ε is from measurements of the frequency spectrum of the u-component of atmospheric turbulence. The u-spectrum of turbulence can be determined by measuring the velocity component $u(t)$ with a sonic anemometer and using a fast-Fourier-transform technique to compute the spectrum. In the inertial-subrange region, Kolmogorov's law for the u-spectrum is

$$F_u(k_1) = \alpha_1 \varepsilon^{2/3} k_1^{-5/3} \tag{33}$$

where k_1 represents the wavenumber spectrum of the turbulence and α_1 is the same constant that appears in eq. (32).

Taylor's hypothesis of frozen turbulence can be used to convert eq. (33) from an expression for the u-spectrum in wavenumber space to an expression in frequency space. According to Taylor's hypothesis the following relation holds between wavenumber k_1, frequency f_1, and mean wind speed U_0:

$$k_1 = 2\pi f_1/U_0 \tag{34}$$

If we denote the u-spectrum in frequency space by the symbol $S_u(f_1)$, the following equation holds for the mean square of $u(t)$:

$$\int_0^\infty F_u(k_1)dk_1 = <u^2> = \int_0^\infty S_u(f_1)df_1 \tag{35}$$

Therefore, with $dk_1 = (k_1/f_1)df_1$ from eq. (34) we require that

$$k_1 F_u(k_1) = f_1 S_u(f_1) \tag{36}$$

Substituting eqs. (34) and (36) into eq. (33), the following equation is obtained for the u-spectrum in frequency space:

$$S_u(f_1) = (2\pi/U_0)^{-2/3} \alpha_1 \epsilon^{2/3} f_1^{-5/3} \tag{37}$$

Equation (37) can be solved for ϵ in the following form:

$$\epsilon^{1/3} = (2\pi/U_0)^{1/3} \alpha_1^{-1/2} f_1^{5/6} [S_u(f_1)]^{1/2} \tag{38}$$

Equation (38) is the final expression needed to determine ϵ from a measured spectrum of $S_u(f_1)$ vs f_1. For example, for $f_1 = 5$ Hz there would be a corresponding measured value of $S_u(f_1)$ of $S_u(f_1) = S_u(5)$. Then eq. (38) would yield a value of $\epsilon^{1/3}$ of

$$\epsilon^{1/3} = 2\pi/U_0)^{1/3} \alpha_1^{-1/2} 5^{5/6} [S_u(5)]^{1/2} \tag{39}$$

which can be solved knowing the value of U_0 and α_1.

The methods just described for obtaining ϵ rely on instruments that would be difficult to use in a "met" airplane. Therefore, they might be considered for a tower-type experiment. Fortunately, instruments have been developed, and are commercially available, for measuring ϵ in an airborne system. The system is called the Universal Indicated Turbulence System (UITS) by its manufacturer Meteorological Research, Inc. in Altadena, California.

The design of UITS is based on eq. (37). A pressure sensor senses the fluctuating pressure from a pitot tube on the airplane. The electrical output voltage from the pressure sensor is proportional to ρU^2, where $U = U_0 + u(t)$ with U_0 the mean true airspeed of the airplane and $u(t)$ the fluctuation of the true airspeed about the mean. Therefore, the mean output voltage from the sensor is proportional to ρU_0^2 and the fluctuating output voltage is proportional to $2\rho U_0 u + \rho U^2 \simeq 2\rho U_0 u$ for small values of $u(t)$.

The power spectrum of the fluctuating output voltage is thus proportional to $(\rho U_0)^2$ times the power spectrum of the fluctuating velocity u. If we denote the power spectrum of the sensor output voltage by $G(f_1)$, then

$$G(f_1) \propto (\rho U_0)^2 S_u(f_1) \tag{40}$$

where $S_u(f_1)$ is given by eq. (37).

The electrical power at the output of the UITS system is proportional to a quantity R^2 defined by

$$R^2 = \int_0^\infty G(f_1) B^2(f_1) df_1 \tag{41}$$

where $B^2(f_1)$ is the frequency response of the filter through which the fluctuating voltage from the pressure sensor is passed. The filter is a low-pass filter designed so that $B^2(f_1)$ is approximately zero for frequencies outside the inertial subrange and approximately unity for frequencies within the inertial subrange.

Using eqs. (41), (40), and (37) the following equation is obtained for the root-mean-square value of R

$$R = \sqrt{R^2} = \text{constant } (\rho U_0^2)^{2/3} (\rho\varepsilon/\rho_{ref})^{1/3} \tag{42}$$

where ρ_{ref} is a constant reference density.

From eq. (42) it is seen that the output of the "epsilon-meter" should be proportional to $(\rho U_0^2)^{2/3} (\rho\varepsilon/\rho_{ref})^{1/3}$. However, in the UITS equipment, a special electrical circuit is provided which alters the gain of the measuring system by a factor of $(\rho U_0^2)^{-2/3}$. Thus R is proportional to $(\rho\varepsilon/\rho_{ref})^{1/3}$ independent of the airplane speed. The constant of proportionality must be obtained by calibrating the system prior to the test. Then, knowing R, air temperature, air pressure, and ρ_{ref}, the value of $\varepsilon^{1/3}$ can be determined and hence C_v^2 using eq. (32) with $\alpha_1 = 0.5$. The ratio C_v^2/c^2 can then be found for the values of c calculated from the temperature. A more complete description of the UITS design can be found in ref. 38.

Measurements of ε have been made, using UITS, for some flyover noise tests. But little use has been made of the data for atmospheric propagation studies. For example, UITS equipment was used for the DC-9 Refan flight demonstration program (ref. 39), but only a small sample of the measured ε data was reported, and no attempt was made to determine values of the structure parameters from the measured dissipation data. The full set of recorded data represents a valuable store of useful information for a study of the effects of atmospheric turbulence on sound propagation since both meteorological data — including profiles of ε — and sound pressure level data were obtained during the flyover noise tests.

Another flight test that was made, using UITS, is described in ref. 40. The purpose of the test was to evaluate the UITS system by comparing values of ε from the UITS system against values of ε measured from a different type of instrument that was mounted on a tower. The tests were accomplished by flying the airplane carrying the UITS equipment past the tower. The results were that the values of ε, determined by the two instruments, agreed with each other within $\pm$ 10 percent.

Measurement of C_T^2

It is possible to determine the structure function C_T^2 in an indirect way, analogous to that used to obtain C_v^2. The method depends on the following equation which is derived, for example, in ref. 36:

$$C_T^2 = 4\beta_1 N \varepsilon^{-1/3} \tag{43}$$

where N is the dissipation rate of $<T'^2/2>$ and T' is the fluctuating temperature. The factor β_1 is a universal constant whose value has been determined to be close to $\beta_1 = 0.8$. Values of N can be determined from measurements of other quantities by making use of the "budget" of $<T'^2/2>$:

$$0 = - <w'T'> (\partial T/\partial Z) - (1/2) (\partial<w'T'>/\partial Z) - N \tag{44}$$

Below about 50 m from the surface, the second term on the right hand side of eq. (44) is negligible and N can be determined within about 10 percent from $<w'T'> (\partial T/\partial Z)$ (ref. 35).

Although it is useful to use such an indirect method to determine C_v^2, it is usually easier to measure C_T^2 directly, at least for a tower-type of experiment. Equipment is available commercially for the direct measurement. One manufacturer of a C_T sensor is Atmospheric Instrumentation Research Company (A.I.R. Co.) of Boulder, Colorado. The A.I.R. C_T sensor utilizes two rapid-response temperature probes separated a fixed distance apart. The structure parameter C_T^2 is determined from the relations given in eqs. (9) and (10). The A.I.R. type of C_T sensor has been used often for measurements from towers and from tethered balloons. It could not be directly used to measure C_T^2 from a "met" airplane. However, since C_T^2 rapidly becomes small compared to C_v^2 with increasing height above the ground

(as shown in fig. 4), it would ordinarily not be necessary to measure vertical profiles of C_T^2 for a flyover noise experiment, though it would be desirable to measure C_T^2 at locations near the ground.

CONCLUSIONS AND RECOMMENDATIONS

1. In any experiment to evaluate the effects of atmospheric turbulence on sound propagation it is crucial to recognize and to measure the important parameters that influence the results. This study has shown that the two important parameters, for sound propagation, that characterize atmospheric turbulence are the structure parameters C_v^2 and C_T^2. Those parameters can be measured in regions of the atmosphere where they are important by using practical state-of-the-art equipment and procedures described in this report.

2. It is also important to make measurements of other parameters that are needed to adjust the data for extraneous factors. For a study of the apparent attenuation of sound caused by turbulence it is important to measure profiles of temperature and humidity over the sound propagation path so as to be able to accurately account for the effects of sound absorption by the atmosphere. It is also important to use an accurate method to calculate the sound absorption. The method described in ref. 3 is recommended.

3. The model for apparent attenuation of sound by turbulence developed in this study predicts that the magnitude of the attenuation will be less from a flyover noise experiment than for a tower experiment utilizing a stationary sound source. The analysis also predicts a much less rapid increase in attenuation with frequency for a flyover noise test than for a tower test. Both effects are caused by the fact that for a sound source such as the jet of an aircraft engine there is a coupling between the wavenumber of the sound and the effective source diameter. In contrast, for a loudspeaker type of sound source the wavenumber and source diameter can be varied independently of each other within limits.

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4. The same parameters that are important for attenuation of sound by turbulence, namely C_v^2 and C_T^2, are also the important parameters for fluctuations in sound intensity caused by turbulence. Therefore, sound fluctuations can also be studied as part of an experiment designed to study the apparent attenuation of sound caused by turbulence. Because of the transient nature of aircraft flyover noise, it is always desirable to use several microphones and to have several repeat runs in order to obtain an ensemble average which will increase the statistical confidence in the results. For a study of the variance of the fluctuations in the intensity of aircraft noise, ensemble averaging would be a necessity in order to calculate the statistical quantities needed.

5. Aircraft flyover noise data now exist (ref. 39) which include measurements of the important parameters that determine turbulence effects on aircraft sound propagation. The data have not been used to study the turbulence effects. The existing data should be analyzed before further flyover noise tests to study sound propagation are conducted.

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APPENDIX A

THE FAR-FIELD DIAMETER OF A BEAM WAVE

Brown and Clifford (ref. 6) give the following expression for the diameter of a beam of sound propagation in a non-turbulent atmosphere [eq. (4)]

$$D_f^2 = D_0^2 + (16 \ L^2)/(k^2 D_0) \tag{A1}$$

The purpose of this Appendix is to show that the number 16 in the second term is probably too small by a factor of 4; i.e., that the correct number is 64.

First of all, eq. (4) with the number 64 replacing the number 16 agrees with a result derived by Ishimaru (ref. 14) for a beam wave in free-space. He derived an approximate solution for the case of a beam wave having a Gaussian amplitude distribution and a parabolic phase distribution with a radius of curvature R_c, at the source location $x = 0$ [see fig. A1]. Ishimaru's equation for the beam diameter at a distance L from the source is a function of a complex parameter α and is

$$D^2 = D_0^2 \ [(1 - \alpha_i L)^2 + (\alpha_r L)^2] \tag{A2}$$

where $\alpha = \alpha_r + i\alpha_i = (4\lambda/\pi D_0^2) + i(1/R_c)$.

Substituting the expression for α into eq. A2 we obtain the following equation:

$$\begin{aligned} D^2 &= D_0^2 \ (1 - L/R_c)^2 + 16\lambda^2 L^2/(\pi^2 D_0^2) \\ &= D_0^2 \ (1 - L/R_c)^2 + (64L^2/k^2 D_0^2) \end{aligned} \tag{A3}$$

For the collimated beam case, where R_c becomes infinite, eq. (A3) agrees with the expression used by Brown and Clifford in eq. (A1) except that Brown and Clifford's factor of 16 is replaced by the factor of 64 in Ishimaru's equation.

As a further check, consider the problem of an acoustic source consisting of a piston oscillating in an infinite plane baffle (fig. A2).
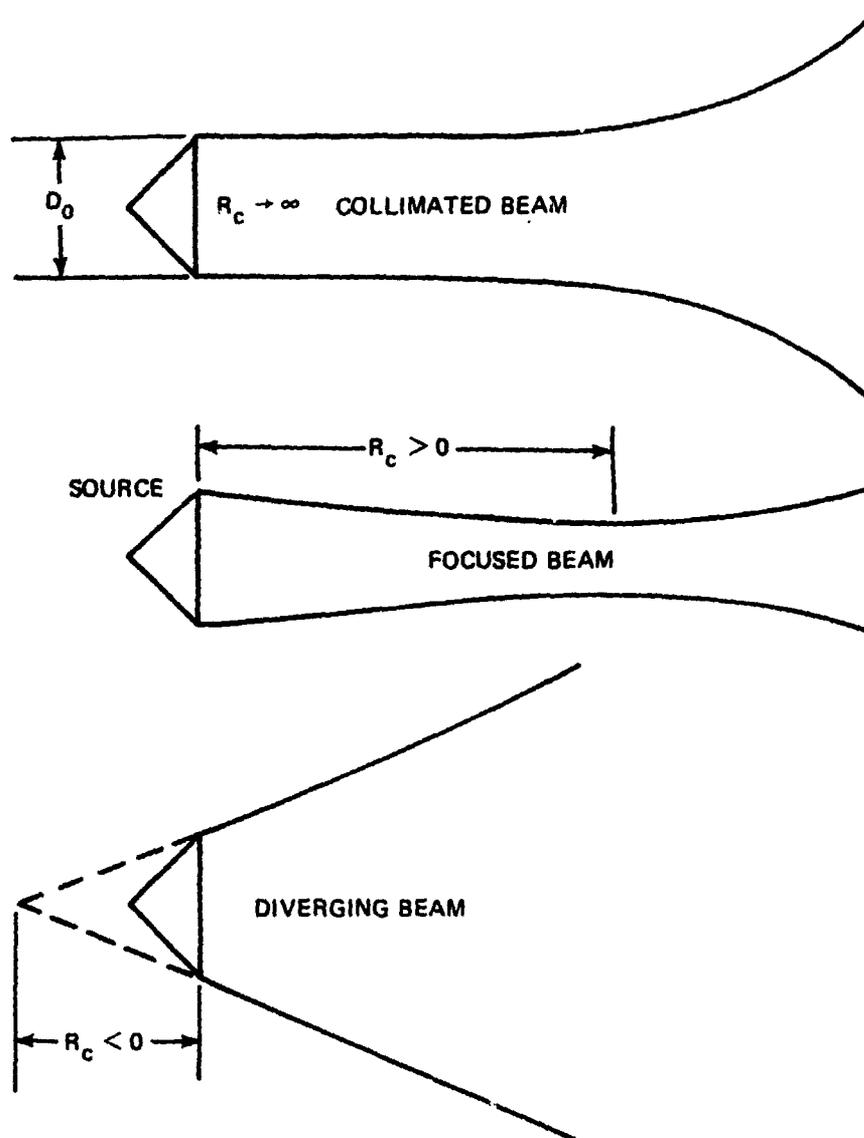
Figure A1.-Geometry of acoustic beams having Gaussian amplitude distributions and parabolic phase distributions with a radius of curvature R_c at the source (after Ishimaru, ref. 14).

That problem has been considered in ref. 41 where the following asymptotic solution for the velocity potential is given:

$$\psi \sim (Va^2/r)e^{ikr-i\omega t} [J_1(ka \sin \theta)/ka \sin \theta] \tag{A4}$$

where the piston is oscillating with a velocity $Ve^{-i\omega t}$, $a = D_0/2$ is the radius of the piston, $\omega = 2\pi f$, and J_1 is a Bessel function of the first kind of order 1. The solution is valid when the distance r is much greater than the radius a.
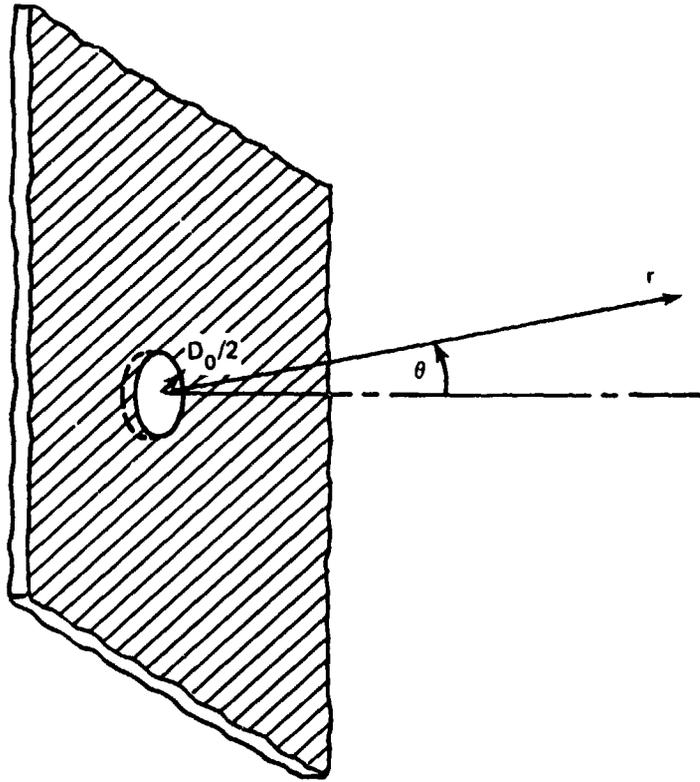
Figure A2.-Piston oscillating in its own plane,
piston radius = $D_0/2$.

From eq. (A4) the acoustic power per unit area can be determined to
be proportional to

$$q(r, \theta) \sim [\omega \rho k V^2 a^4/(2r^2)] \; [J_1(ka \sin \theta)/ka \sin \theta]^2 \qquad (A5)$$

where ρ is the ambient air density.

The beam diameter D is defined by the relation $q(r, \theta_0)/q(r, 0) = 1/e$,
where $D = 2r \sin \theta_0$. From eq. (A5)

$$[J_1(ka \sin \theta_0)/ka \sin \theta_0]^2/(1/4) = 1/e \qquad (A6)$$

Equation A6 is satisfied for $ka \sin \theta_0 = 1.915$. Therefore, the beam
diameter is given by

$$D = 2r \sin \theta_0 = 7.660 \; r/kD_0 \qquad (A7)$$

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For narrow beams, $r \simeq L$. Therefore, from eq. (A7) we obtain the following equation for the square of the beam diameter in the far field

$$D^2 = 58.68 \ L^2/k^2 D_0^2 \qquad\qquad (A8)$$

Comparing eq. (A8) with eq. (A1) for the far field shows that the factor multiplying the second term should probably be closer to 64 than to 16.

APPENDIX B

SYMBOLS

a piston radius $D_0/2$ (fig. A2), m

A_t apparent attenuation caused by turbulence, dB

$B^2(f_1)$ relative frequency response of filter used in UITS system

c speed of sound, ms^{-1}

C_e^2 structure parameter for vapor pressure fluctuations, $N^2m^{-14/3}$

C_{eT} parameter for cross correlation of humidity and temperature fluctuations, $KNm^{-8/3}$

$C_n^2(s)$ structure parameter for refractivity fluctuations, $m^{-2/3}$

$\overline{C_n^2}$ weighted average of C_n^2 over a sound path (eq. 23), $m^{-2/3}$

$[C_n^2]_\chi$ weighted average of C_n^2 over a sound path (eqs. 29 and 30), $m^{-2/3}$

C_T^2 structure parameter for temperature fluctuations, $K^2m^{-2/3}$

C_v^2 structure parameter for velocity fluctuations, $m^{4/3}s^{-2}$

C_1 nondimensional constant (eq. 22)

C_2 nondimensional constant (eq. 24)

D_f beam diameter in absence of turbulence (fig. 1), m

D_{ik} ik-component of velocity structure function tensor, m^2s^{-2}

D_{rr} rr-component of velocity structure function tensor, m^2s^{-2}

D_T structure function for temperature, K^2

D_t increase in beam diameter caused by turbulence, m

D_0 initial beam diameter (fig. 1), m

$F_u(k_1)$ u-spectrum of turbulence in wavenumber space, m^3s^{-2}

f frequency of sound wave, Hz

f_1 frequency of turbulent velocity fluctuations, Hz

g acceleration of gravity, ms^{-2}

I acoustic intensity, Wm^{-2}

$<I>$	long-term term-averaged acoustic intensity, Wm^{-2}
I_f	acoustic intensity in the absence of turbulence, Wm^{-2}
J_1	Bessel function of the first kind of order 1
k	acoustic wave number, m^{-1}
L	sound propagation distance, m
L_0	outer scale of atmospheric turbulence, m
M	characteristic Mach number
N	dissipation rate of $<T'^2/2>$, $K^2 s^{-1}$
p	ambient pressure, Nm^{-2}
Q_0	$<w'T'>$ at $Z = 4m$, surface kinematic heat flux, Kms^{-1}
r	distance between two fluid particles, m
R_c	radius of curvature of acoustic beam (fig. A1), m
s	distance along soundpath from source to receiver, m
St	Strouhal number
$S_u(f_1)$	u-spectrum of turbulence in frequency space, $m^2 s^{-1}$
T	ambient temperature, K
T_1	temperature at "point 1", K
T_2	temperature at "point 2", K
T'	fluctuating temperature, K
T_*	scaling temperature (eq. 11b), K
U	$U_0 + u(t)$, true airspeed, ms^{-1}
$u(t)$	fluctuating component of true airspeed, ms^{-1}
U_0	mean true airspeed, ms^{-1}
V	amplitude of piston velocity, ms^{-1}
v_{1i}, v_{1k}	i'th and k'th velocity component, respectively, at point 1, ms^{-1}
v_{2i}, v_{2k}	i'th and k'th velocity component, respectively, at point 2, ms^{-1}
w'	fluctuating component of wind in vertical direction, ms^{-1}

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w_*	scaling velocity (eq. 11a), ms^{-1}
z	height above ground, m
z_i	thickness of the convective atmospheric boundary layer, m
z_L	height of elevated source or receiver (fig. 5), m
z_0	height of source or receiver (fig. 5), m
α	complex parameter in equation for beam diameter, (eq. A2), m^{-1}
α_i	imaginary part of α, m^{-1}
α_r	real part of α, m^{-1}
α_t	$(1000/L)A_t$, average turbulence attenuation coefficient, dB m^{-1}
α_1	nondimensional constant (eq. 32)
β_1	nondimensional constant (eq. 43)
ε	dissipation rate of kinetic energy, m^2s^{-3}
θ	angular coordinate (fig. A2), rad
θ_c	empirical parameter (eq. 27), rad
θ_0	value of θ at "edge" of acoustic beam
λ	acoustic wavelength, m
ν	kinematic viscosity, m^2s^{-1}
ρ	ambient density, kg m^{-3}
ρ_{ref}	reference density, kg m^{-3}
σ_I^2	normalized variance of acoustic intensity (eq. 31)
σ_{lnI}^2	variance of the logarithm of acoustic intensity (eq. 28)
σ_χ^2	log amplitude variance (eq. 28)
ψ	velocity potential, m^2s^{-1}
ω	$2\pi f$, angular frequency of oscillating piston, rad s^{-1}