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Determination of the Time of Occurrence of Gamma Flares at Different Points in Space

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16. Abstract

This articles considers the problem of determining the time of occurrence of gamma flares at different points in space. This problem arises in the localization of sources of gamma flares by the triangulation method using several spacecraft (SC). A study is made of different methods for determining the time at which the flare reaches the spacecraft and algorithms are given for estimating their accuracy. The results of processing the model information are also given.

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Determination of the Time of Occurrence of Gamma Flares at Different Points in Space

B. L. Noval

1. One of the most interesting results of extra-atmospheric astronomy of recent years is the discovery of powerful flares of space electromagnetic radiation in hard x-ray and gamma-ranges. One very important problem of studying the sources of the flares is the determination of their position in the heavens (localization). The most effective method for solving the problem of localization is to record the time at which the flare arrives or, which is the same thing, the delay in the arrival of the flare at different points in space. This recording is done by sensors which are carried on the spacecraft (SC) at different distances from each other. Thus the determination of the celestial coordinates of the source reduces to statistical processing of recordings of the flare obtained on different SC. /3*

The localization accuracy depends on the accuracy of establishing the position of the SC in space and the accuracy of recording the time of occurrence of the flare by the SC. As follows from [1], for real cases of finding the SC in space the accuracy of determining their coordinates is such that the error in the time of occurrence of the flare can be reduced in every case to the time resolution of the existing sensors. Taking this fact into account, and also the fact that the flare has a finite duration, we must establish what we mean by the flare time of occurrence. Let us /4

* Numbers in margins indicate foreign pagination.

consider the process of recording the flare. Let us assume the sequence $\{n_j^{(i)}\}$ $j = 1, 2, .., M$ represents the reading of the sensor carried on the i-th SC and fixing the calculation (time spectrum) with the resolution Δt . With the very natural and unlimited assumptions regarding the nature of the radiation and the recording, the sequence $\{n_j^{(i)}\}$ may be regarded as a Poisson process with a certain variable, which is unknown previously, with the intensity $\lambda^{(i)}(\cdot)$. If we use identical sensors, then the functions $\lambda^{(i)}(\cdot)$ for different values of i differ only by the displacement of the arguments, and thus we may speak of the function $\lambda(\cdot)$, having in mind all $\lambda^{(i)}(\cdot)$ Thus, the problem of determining the delay for the flare to reach 2 SC (we shall call them the zero one and the first one) is reduced to finding the τ , such that $\lambda^{(1)}(u+\tau) = \lambda^{(0)}(u)$.

If the time spectra $\{n_j^{(1)}\}, \{n_j^{(0)}\}$ coincide, then the problem of identifying them, and consequently determining τ , would be trivial. However, such an agreement does not occur, due to the discreet nature of the recording process and the random nature of the radiation process.

The sensors which are designed to study the flares usually carry special discharge equipment which establishes when the threshold value has been reached in a certain period of time. The signal "flare" is thus given. The arrival time of the flare may be determined in principle, using this equipment. Let us determine the error Δt in determining the delay when using this equipment. We shall confine ourselves to examining only the errors due to the discreet recording of the radiation, i.e., without considering statistical fluctuations. Using a linear approximation in the estimation, we obtain /5

$$\Delta\tau = \frac{\lambda(T_0)\,\delta T}{\lambda(T_0 + \delta T_0)}$$

or

$$\Delta\tau = \frac{(\Lambda T_0 + \lambda_0)\,\delta T}{\Lambda(T_0 + \delta T_0) + \lambda_0}, \qquad \text{(I)}$$

where Λ, λ_0 are the parameters determining the linear approximation of the function $\lambda(\cdot)$; T_0 - value of the argument of the function $\lambda(\cdot)$, corresponding to the beginning of the interval in which the recording reaches the threshold value on the zero SC; δT - difference in the values of the arguments of the function $\lambda(\cdot)$, corresponding to the beginning of the time intervals in which the recording reaches the threshold value on the zero and the first SC; δT - quantity, determined by the logic of the discharge equipment operation.

We shall use K to designate the integer estimate of the ratio $\frac{\Delta\tau}{\Delta t}$. Using the formula obtained above (I) and the characteristics of the equipment "SNEG-2M" [2], we conclude that K may reach 8. This is an impermissibly large quantity. We therefore turn to the methods based on the more complete use of the measurement information. We should note that K - the integer estimate of the relationship $\frac{\Delta\tau}{\Delta t}$, determined by (I), will be used below.

2. Each of the $n_j^{(i)}$; $i = 0, I$; $j = I, 2, \ldots, M$ is the realization of the random quantity $N_j^{(i)}$, distributed according to the Poisson law with a certain parameter. It is desirable for us to have the sequence of realizations of normal random quantities with unit dispersions. These sequences may be obtained by the following normalization /6

$$x_j'^{(i)} = \frac{n_j^{(i)}}{\sqrt{n_j^{(i)}}} = \sqrt{n_j^{(i)}}$$

This normalization is based on the estimate of $\sqrt{n_j^{(i)}}$ for the mean square deviations $\sqrt{\mu_j^{(i)}}$. For $\mu_j^{(i)}$ we may give the following interval with any given confidence coefficient α

$$n_j^{(i)} - c(\alpha)\sqrt{n_j^{(i)}} \leqslant \mu_j^{(i)} \leqslant n_j^{(i)} + c(\alpha)\sqrt{n_j^{(i)}}$$

whose magnitude depends on the number $C(\alpha)$ [3]. Therefore a study of the difference $\frac{n_j^{(i)}}{\sqrt{\mu_j^{(i)}}} - \frac{n_j^{(i)}}{\sqrt{n_j^{(i)}}}$ may be replaced by the study of

$$d_j^{(i)} = \frac{n_j^{(i)}}{\sqrt{n_j^{(i)}}} - \frac{n_j^{(i)}}{\sqrt{n_j^{(i)} \pm C(\alpha)\sqrt{n_j^{(i)}}}}$$

Using the L'Hopital rule, we obtain

$$\lim_{n_j^{(i)} \to \infty} d_j^{(i)} = \frac{C(\alpha)}{2}$$

This result shows the validity of using $\sqrt{n_j^{(i)}}$, as an estimate for $\sqrt{\mu_j^{(i)}}$. However, to decrease the scatter for finite values of $n_j^{(i)}$ we first smooth the estimate taking 5 points for the smoothing. Thus we obtain the sequence

$$x_j^{(i)} = \frac{n_j^{(i)}}{\sqrt{\hat{n}_j^{(i)}}}$$

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where

$$\hat{n}_j^{(i)} = \frac{1}{9}\left(n_{j-2}^{(i)} + 2n_{j-1}^{(i)} + 3n_j^{(i)} + 2n_{j+1}^{(i)} + n_{j+2}^{(i)}\right)$$

at

$$M-2 \geqslant j \geqslant 3$$

Forming these sequences, we shall look for the minimum

$$z_j = \frac{1}{2}\sum_{s=1}^{m}\left(x_{s+j}^{(i)} - x_{s+K}^{(0)}\right)^2; \quad 0 \leqslant j \leqslant 2K \qquad (2)$$

The minimum of this expression may be written with respect to the index j. The value, which represents the minimum will be designated by j^*

Summation in (2) is performed with respect to the index S from I to m, where m equals $M-K$; K - the integer estimate $\frac{\Delta\tau}{\Delta t}$ determined by the expression (I).

Let us examine the accuracy of identifying the spectra, i.e. finding τ, done in accordance with (I). It is clear that when there are no random fluctuations the accuracy would be determined by the quantity Δt. Actually when finding τ we replace the solution of the continuous problem having the form

$$\int_{t_1}^{t_2}\left(f_2(t+\tau)-f_1(t)\right)^2 dt \to \min_{\tau}$$

by the solution of the discreet problem

$$\sum_{s=1}^{m}\left(f_2(s\Delta t+j\Delta t)-f_1(s\Delta t)\right)^2 \to \min_{j}$$

for functions $f_1(\cdot)$, $f_2(\cdot)$ such that there is a τ for which $f_2(t+\tau)=f_1(t)$. Under the assumption that Δt is small with respect to the time for a large change in $f_1(\cdot)$ and $f_2(\cdot)$, the error of this replacement is Δt. /8

Below we shall estimate the accuracy of identification taking into account the fluctuations. Since there is no important a priori information regarding the function $\lambda(\cdot)$ the estimates will be a posteriori.

3. The first method of accuracy estimation. In accordance with the given normalization, we will calculate each of the $x_j^{(i)}$; $i=0,1$, by the realization of a normally distributed random quantity $\chi_j^{(i)}$ with unit dispersion. Therefore, z_j is the realization of the random quantity Σ_j, having a non-central χ - square distribution with m degrees of freedom and a certain non-central parameter ν_j.

Using the normal approximation for the non-central χ- square distribution, we formulate the confidence interval for ν_j. In view of the approximation $\frac{Z_j-(m+\nu_j)}{\sqrt{2(m+2\nu_j)}}$ is a normal quantity with parameters of zero and one. If we use ζ_α to designate the quantile of order $\frac{2-\alpha}{2}$ of the normal distribution (i.e. $P\{\xi \leqslant \zeta_\alpha\}=\frac{2-\alpha}{2}$ for the normal quantity ξ with parameters of zero and one), then $P\left\{-\zeta_\alpha \leqslant \frac{Z_j-(m+\nu_j)}{\sqrt{2(m+2\nu_j)}} \leqslant \zeta_\alpha\right\} = 1-\alpha$.

Here $1-\alpha$ is the confidence coefficient.

It is apparent that

$$P\left\{-\zeta_\alpha \leqslant \frac{Z_j-(m+\nu_j)}{\sqrt{2(m+2\nu_j)}} \leqslant \zeta_\alpha\right\} = P\left\{0 \leqslant \frac{(Z_j-(m+\nu_j))^2}{2m+4\nu_j} \leqslant \zeta_\alpha^2\right\}.$$

Thus we obtain the relatioship for the confidence values of the parameter ν_j:

$$\nu_j^2 - 2\nu_j(z_j - m + 2\zeta_\alpha^2) + z_j^2 + m^2 + 2z_j m - 2\zeta_\alpha^2 m \leqslant 0$$

and the expression for the upper $\varkappa_j^+$ and lower $\varkappa_j^-$ /9 confidence boundaries.

$$\varkappa_j^{+,-} = z_j - m + 2\zeta_\alpha^2 \pm 2\zeta_\alpha\sqrt{\zeta_\alpha^2 + z_j - \frac{m}{2}} \qquad (3)$$

At first glance it would appear that the expression under the root sign in (3) may be negative and consequently real confidence boundaries may not exist.

We shall show that by considering the probable meaning of the quantities included in this expression and making certain natural assumptions, we can avoid this situation.

Let us consider the solution of the problem

$$\nu_j + m - C(\alpha)\sqrt{2(m+2\nu_j)} \to \min_{\nu_j} \qquad (4)$$

under the condition $\nu_j \geq 0$

The quantity $\nu_j + m - C(\alpha)\sqrt{2(m+2\nu_j)}$ determines, with the selection of the corresponding $C(\alpha)$, the minimum possible value of the random quantity having the non-central distribution χ - square with the non-central parameter ν_j. The solution of problem (4) is zero at $2C(\alpha) < m^3$. Usually in applications of the theory there are not more than 4 probabilities $C(\alpha)$ and therefore the solution is zero for any reasonable number of measurements.

Thus, the minimum value of the random quantity having a non-central χ - square distribution with m degrees of freedom for an unknown ν_j, is determined by the expression $m -$ and $- C(\alpha)\sqrt{2m}$.

It is apparent that if

$$m - C(\alpha)\sqrt{2m} \geq \frac{m}{2} - \zeta_\alpha^2 \qquad (5)$$

then the expression under the root sign in (4) is not negative.

It follows from (5) that /10

$$\frac{m}{2} - C(\alpha)\sqrt{2m} \geq -\zeta_\alpha^2 \qquad (6)$$

If

$$\frac{m}{2} - C(\alpha)\sqrt{2m} > 0 \qquad (7)$$

then the inequality is satisfied for all ζ_α everywhere.

The inequality (7) leads to the condition

$$m > 8c^2(\alpha)$$

and, again considering the probable meaning of $c(\alpha)$, we obtain

$$m > 128. \tag{8}$$

If it is not necessary to satisfy the condition (7), then, considering that the solution of the problem

$$\frac{m}{2} - c(\alpha)\sqrt{2m} \rightarrow \min_{m},$$

equals the minus $c(\alpha)$, we reach the condition

$$c(\alpha) < \zeta_\alpha \tag{9}$$

Thus, if $m > 128$ for an arbitrary ζ_α or if $c(\alpha) < \zeta_\alpha$ at $m > 2$, then there are real confidence boundaries $\varkappa_j^+, \varkappa_j^-$. We shall assume that any of these unlimited conditions are satisfied.

Using the confidence intervals formulated, we obtain an estimate of the accuracy for determining τ. For this purpose we use the system of intervals $T_j = \{\tau : (j-1)\Delta t \leqslant \tau \leqslant (j+1)\Delta t\}$ and examine $T^{(\alpha)} = \bigcup_s T_s$, where those T_s, are included for which the following condition is satisfied

$$\varkappa_s^- \leqslant \varkappa_{j^*}^+ \tag{10}$$

The estimate of $\Delta\tau$ will be determined by the condition that τ belongs to the set $T^{(\alpha)}$.

4. The second method of estimating the accuracy. To improve the estimate of accuracy of identification, it is necessary to consider the stochastic dependence of the quantities Z_j for different values of j, for which it is necessary to know the concurrent distribution of the quantities.

For this purpose we obtain the expression for the characteristic function of quadratic form $Z = \underset{1\times4}{X'}\underset{4\times4}{A}\underset{4\times1}{X}$ where $\underset{4\times1}{X}$ is normal with

$$\left.\begin{aligned} &\mathcal{E}X=\mu, \\ &\mathcal{D}X=E \quad (E \text{ - unit matrix}). \end{aligned}\right\} \tag{11}$$

The density function of the quantity X is determined by the expression $(2\pi)^{-\frac{4}{2}}\exp(-\frac{1}{2}(x-\mu)^T(x-\mu))$. Thus we obtain the following for the characteristic function of the random quantity Z and

$$\varphi(t)=(2\pi)^{-\frac{4}{2}}\int\cdots\int \exp(x\,itA)\exp(-\tfrac{1}{2}(x-\mu)^T(x-\mu))\,dx, \tag{12}$$

where i is imaginary unity.

It is apparent that the integrand equals

$$\exp\left(x^T(itA-\tfrac{E}{2})x + x^T\mu - \tfrac{\mu'\mu}{2}\right)$$

Let us transform it to the form

$$(x+U)^T W(x+U)+v \tag{13}$$

for U , W v we obtain the relationships

$$\begin{aligned} &W=itA-\tfrac{E}{2}, \\ &2WU=\mu, \\ &U^TWU+v=-\tfrac{\mu^T\mu}{2} \end{aligned}$$

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Thus we have

$$\begin{aligned} &U=\tfrac{1}{2}\left(itA-\tfrac{E}{2}\right)^{-1}\mu, \\ &v=-\tfrac{1}{4}\mu^T\left(itA-\tfrac{E}{2}\right)^{-1}+2E)\mu. \end{aligned}$$

After transformation to the form (13) we may readily see that the integrand in (12) represents the product of the distribution density of the normal quantity with the mathematical expectation minus U and the covariational matrix W^{-1} and $\frac{e^{v}}{|W|^{\frac{1}{2}}}$. Thus

$$\varphi(t)=|E-2itA|^{-\frac{1}{2}}e^{-\frac{1}{4}\mu^{T}((itA-\frac{E}{2})^{-1}-E)\mu}$$

or

$$\varphi(t)=|E-2itA|^{-\frac{1}{2}}e^{-\frac{1}{2}((E-2itA)^{-1}-E)\mu} \qquad (14)$$

Using expression (14) we may readily find the first two moments of the random quantity Z

$$\begin{aligned} \mathcal{E}Z &= \frac{1}{i}\varphi'(0)=SpA+\mu^{T}A\mu \quad , \\ \mathcal{E}Z^{2} &= \frac{1}{i^{2}}\varphi''(0)=(\mu^{T}A\mu)+4\mu^{T}A^{2}\mu+ \\ &+2\,Sp\,A\times\mu^{T}A\mu+(SpA)^{2}+\sum_{s}a_{ss}^{2} \quad , \end{aligned} \qquad (15)$$

where a_{ss} are the diagonal elements of the matrix A .

Thus $\mathcal{D}Z=\mathcal{E}Z^{2}-(\mathcal{E}Z)^{2}=4\mu^{T}A^{2}\mu+2\sum_{s}a_{ss}^{2}$

Let us set

$$x=col\,(x^{(0)}_{1+K},x^{(0)}_{2+K},\dots x^{(0)}_{m+K},x^{(1)}_{1},\dots x^{(1)}_{M}) \, ,$$

$$\mu=col(\mathcal{E}x^{(0)}_{1+K},\mathcal{E}x^{(0)}_{2+K},\dots \mathcal{E}x^{(0)}_{m+K},\mathcal{E}x^{(1)}_{1},\dots \mathcal{E}x^{(1)}_{M}) \, ;$$

$$A_{j}=\left(\begin{array}{c|c} E & -B_{j} \\ \hline -B_{j}^{T} & B_{j}^{T}B_{j} \end{array}\right) .$$

/13

where B_{j} . is the dimensionality matrix $m\times M$, such that its element $b_{l,l+j-1}$ equals unity at $l=1,2,\dots,m$, and the remaining elements equal zero.

Then x satisfies the conditions (11) and $z_j = \frac{1}{2} x^T A x$. The difference z_j and z_{j*} is the quadratic form $\frac{1}{2} x^T(A_j - A_{j*})x$. On the other hand, z_j and z_j, in view of the assumptions made have a non-central χ - square distribution. Therefore $\mathcal{E}(z_j - z_{j*})$ equals $2(\nu_j - \nu_{j*})$, and $\mathcal{D}(z_j - z_{j*})$, according to (15), equals

$$\mu^T(A_j - A_{j*})\mu + \frac{1}{2}\sum_{s=1}^{m+n}\left(a_{ss}^{(j)} - a_{ss}^{(j*)}\right)^2.$$

Let us assume

$$\frac{z_j - z_{j*} - 2(\nu_j - \nu_{j*})}{\sqrt{\mu^T(A_j - A_{j*})^2\mu + \frac{1}{2}\sum_{s=1}^{m+n}\left(a_{ss}^{(j)} - a_{ss}^{(j*)}\right)^2}} \qquad (16)$$

We shall assume that a normal approximation is valid for the difference of the quadratic forms included in the denominator. Then expression (16) represents the realization of the normal random quantity with zero mathematical expectation and unit dispersion.

Since μ is not known, we make an estimate for $\mu^T(A_j - A_{j*})^2\mu$. We may readily obtain the following from (15)

$$\mathcal{E}\, x^T(A_j - A_{j*})^2 x - Sp\,(A_j - A_{j*})^2 = \mu^T(A_j - A_{j*})^2\mu$$

And we use $x^T(A_j - A_{j*})^2 - Sp(A_j - A_{j*})^2$ as the estimate for $\mu^T(A_j - A_{j*})\mu$. We shall assume that replacement in (15) does not disturb the normal approximation.

Let us set $$t_j(x) = \frac{x^T A_j x - x^T A_{j*} x \ldots}{\sqrt{x^T(A_j - A_{j*})^2 x - Sp(A_j - A_{j*})^2 + \frac{1}{2}\sum_{s=1}^{m+n}\left(a_{ss}^{(j)} - a_{ss}^{(j*)}\right)^2}}.$$

Using the statistics $t_j(x)$, we follow the procedure for estimating the accuracy, based on verifying the statistical hypothesis $\gamma_j - \gamma_{j*} > 0$. To solve the problem of verifying this hypothesis, it is sufficient to use the results of the most valid non-biased criteria for unilateral alternatives /14 for the exponential sets [3]. Our problem is a very particular case of this more general problem. Using these results, we obtain the following procedure.

Let us assume z_β is the quantile of order $1-\beta$ of the normal distribution; β - error of the second kind; $\mathcal{T}_j = \{\tau : (j-1)\Delta t \leqslant \tau \leqslant (j+1)\Delta t\}$, as in the previous section. Then, if

$$t_s(x) \leqslant z_\beta , \qquad (17)$$

the corresponding interval $\mathcal{T}_s$ is included in the union $\mathcal{T}^{(2)} = \bigcup \mathcal{T}_s$. It is apparent that only those values of j , for which the $\mathcal{T}_j$ corresponding to them are included in $\mathcal{T}^{(1)}$ must be verified.

The error $\Delta\hat{\tau}$ is determined the same way as was done in the previous section, by the condition of belonging to the set $\mathcal{T}^{(2)}$. We should note that both the set $\mathcal{T}^{(2)}$, and the set $\mathcal{T}^{(1)}$ may be comprised of non-intersecting segments. This case may be interpreted as the existence of several solutions of the problem for determining $\hat{\tau}$.

5. Computational algorithms and computer programs were formulated using the procedures given above.

In formulating the algorithms, we considered the possible singularities having the form of measurement information, particularly the occurrence of incorrect measurements. Measures making it possible to process this information were taken into account in the algorithm.

The programs were used to process the results of Earth-based tests of the "SNEG-2M" equipment. This equipment they used for the Soviet-French experiment on studying the flares /15 of gamma radiation in space [2]. The total number of measurements M by the equipment equalled 1024. This was sufficient for the use of the approximations employed in the procedures mentioned.. The measurement information was obtained by using a flare model.

We shall give the results of processing the measurement information. The minimum value of z_j was obtained when the indexes in the first and zero time spectra coincided. This pointed to the fact that in this case the signal "flare" was processed very accurately.

When estimating the accuracy by the first method, the confidence level was assumed to equal 0.9. This moderate value was selected due to the large rigidity of the solution rule. The use of the procedure given in this method resulted in 9 intervals of $\mathcal{T}_s$, for which the condition (10) was satisfied. The set $\mathcal{T}^{(1)} = \bigcup_s \mathcal{T}_s$ was the interval of the quantity $13\Delta t$, and the possible deviations from the most probable solution were determined by the quantities $\Delta\tau^{+} = 4\Delta t$, and $\Delta\tau^{-} = 9\Delta t$.

This accuracy of determining τ was inadequate and therefore the second method of accuracy estimation was used. Based on the same considerations as in the use of the first method, the error of the second type was chosen to equal 0.1. The application of this procedure resulted in the formation of two mixed intervals $\mathcal{T}_s$ and the set $\mathcal{T}^{(2)} = \bigcup_s \mathcal{T}_s$ - the interval of the quantity $3\Delta t$. The possible deviations from the most probable solution $\Delta\tau^{+} = 2\Delta t$, $\Delta\tau^{-} = \Delta t$.

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References

1. G.A. Mersov. Algorithms for localizing celestial sources of x-ray and gamma-bursts using several spacecraft. Preprint. Moscow, IKI AN SSSR, 1977, Pr-344.

2. M. Niel et al. Astrophys. and Space Sci., 42, 99, 1976.

3. M. Kendall, A. Stewart. Statistical conclusions and relationships. Moscow, Nauka press, 1973.