MIXING EFFICIENCY IN HETERODYNE SYSTEMS

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ABSTRACT

How the spatial distributions of the signal and local oscillator fields affect the heterodyne signal-to-noise ratio is examined for both a single detector and an array of detectors. For an array, the distribution of gain among the i.f. amplifiers is included. The emphasis is on understanding why the distributions are important and what is the key to maximizing the signal-to-noise ratio in any system. It is shown that for a single detector, the highest signal-to-noise ratio is obtained with an LO distribution the same as that of the signal. For an array, the product of the i.f. gain times the LO field at each detector shall have the same distribution as that of the signal field.

INTRODUCTION

The purpose of this paper is to explain how the spatial distributions of the signal and local oscillator (LO) fields affect the signal-to-noise ratio in heterodyne detection. The detailed derivations of the signal-to-noise formulae are available in References 1 and 2, but to understand the effects here, we only need the dependencies of the signal and noise photocurrents on the signal and LO field strengths over a small detector element. The time-dependent photocurrent is proportional to the power on the detector averaged over times long compared to the optical frequencies, but short compared to the difference frequency:

$$i \propto |E_s + E_\ell|^2 A \qquad (1)$$

where E_s and E_ℓ are the signal and LO field strengths and A is the area of the detector. Eq. (1) can be expanded to

$$i \propto |E_s|^2 A + |E_\ell|^2 A + |E_s||E_\ell| \cos(\omega_{i.f.} t + \phi) A \qquad (2)$$

where $\omega_{i.f.}$ is the angular frequency difference between the signal and LO fields and ϕ is the phase difference between the signal and LO other than that due to the exp($i\omega t$) terms. The second term of Eq. (2) is the LO power and is the largest term; it represents the DC current. The shot noise is proportional to the square root of the number of electrons per second in the current and so is proportional to the square root of this second term:

$$i_n \propto |E_\ell| \sqrt{A} \qquad (3)$$

The third term of Eq. (2) is the intermediate frequency signal current,

$$i_s \propto |E_s||E_\ell|A \quad (4)$$

With these dependencies of the signal and noise currents established, we can examine how the distributions of the signal and LO fields affect the overall signal-to-noise ratio.

THE SIGNAL-TO-NOISE RATIO

Consider the heterodyne detection situation represented by the signal and local oscillator electric field distributions of Fig. 1. As the radius of the detector is increased from 0 to A, the intermediate frequency (i.f.) signal increases. However, as the detector is increased in radius from A to B, the phase reversal of the signal causes a phase reversal in the additional i.f. signal, and the net i.f. signal decreases. Clearly, the detector should not have a radius larger than A, but should the radius even be as large as A? That part of the detector between radius A and, say, radius C collects very little signal, but it collects just as much shot noise from the local oscillator as an equal detector area nearer the center that collects a lot of signal. At some value of C, the additional signal captured by increasing the detector radius from C to A might not be worth the additional noise.

If there is such a point of diminishing returns, can we work around it and usefully capture the signal available between radii C and A? One clue is that we can work around the phase reversal of the signal at A by also reversing the phase of the LO. Then the additional i.f. signal captured between A and B will be in phase with that from 0 to A. This suggests that reducing the strength of the LO field in the C to A region might compensate for the reduction in signal field strength. Reducing the strength of the LO reduces the shot noise, but the LO field strength also multiplies the signal field strength to yield the i.f. current, so reducing it will reduce the signal by the same amount. This is just the physical reason behind the familiar result that as long as the LO is large enough that its shot noise dominates all other noises, changing the strength of the LO does not affect the signal-to-noise ratio. However, changing the strength of the LO can be used in another sense: it can be used as a weighting factor on the information in this poorer signal-to-noise region so that this information is added to that obtained from the better signal-to-noise ratio region, but not counted as heavily.

This can be made quantitative by analyzing the detector network illustrated in Fig. 2 where G_j is the gain, s_j is the signal current, and n_j is the shot noise current of the jth photomixer. Here, the individual photomixers can represent either portions of single detector or separate detectors of an array. If they represent portions of a single detector, the indicated amplifiers correspond to the LO field strength at each portion, for both the i.f. signal current and the shot noise current are proportional to the LO field strength. If the photomixers in Fig. 2 represent individual detectors, the amplifiers represent the product of the LO field strength times the actual amplification. (In either case, the amplifiers can also include a factor for any variation in quantum efficiency.) Note that if the effect of the LO is assigned to the amplifier in the analysis, s_j is not really the signal current, but the signal current divided by the LO field strength, and n_j is the noise current divided by the LO field strength. These may be written as

$$s_j \propto |E_s| A \quad ; \quad n_j \propto \sqrt{A} \tag{5}$$

To have unique signal and LO field strengths, they must be uniform over the photomixer element. That, of course, is no problem for differential elements of a single detector, but it restricts the array analysis to small detectors. In the following, we will assume that either the phase of the LO and/or the phase of the amplifiers are set so that the signal currents add in phase.

The net power signal-to-noise ratio is given by

$$S/N = \frac{(\Sigma G_j s_j)^2}{\Sigma G_j^2 n_j^2} \tag{6}$$

Now the maximum signal-to-noise ratio may be found by adjusting the G_j. Differentiating S/N of Eq. 6 with respect to G_j and setting the derivative to zero yields

$$Gj = \frac{s_j}{n_j^2} \frac{\Sigma G_j^2 n_j^2}{\Sigma G_j s_j} \tag{7}$$

The ratio of the two sums in Eq. 7 is a constant for all the photomixers once the G_j are set, so the optimum distribution of the G_j is given by the distribution of the s_j/n_j^2. If these optimum settings for the G_j are put into the formula for the overall signal-to-noise ratio of Eq. 6, it becomes

$$S/N = \Sigma(s_j^2/n_j^2) = \Sigma(S/N)_j \tag{8}$$

That is, every additional piece of information (signal) increases the total signal-to-noise ratio no matter how poor the S/N of the additional information, if the additional information is weighted according to Eq. 7.

THE OPTIMUM LO FIELD DISTRIBUTION

It was noted above that s_j is proportional to the product of the signal field strength times the area and n_j in this analysis is not really the noise current, but the noise current divided by the LO field strength. n_j is therefore proportional to only the square root of the area, and the optimum weighting, s_j/n_j^2, is proportional to only the signal field strength at each location. For a single detector, the weighting mechanism is simply the LO field strength, so we have the result that the maximum signal-to-noise ratio is obtained by setting the LO field distribution equal to the signal field distribution. For an array of small detectors, the product of the LO field strength at each element multiplied by the gain of that element's amplifier should have the same distribution as the signal field.

THE GENERAL SOLUTION

References 1 and 2 draw these same two conclusions for the optimum use of a single detector and an array by first deriving the signal-to-noise ratio equations for general signal and local oscillator field and amplifier distributions. For a single detector, the signal-to-noise ratio is given by

$$S/N = \frac{\eta P_S}{h\nu B} \frac{|\int_A |U_S||U_L| \exp(i\phi)\ dA|^2}{\int^\infty |U_S|^2 dA \int_A |U_L|^2\ dA} \quad (9)$$

where η is the quantum efficiency, P_S the total signal power available for detection, h Planck's constant, ν the optical frequency, B the i.f. bandwidth, U_S and U_L the complex field distribution functions for the signal and local oscillator (not including the $\exp(i\omega t)$ dependence), ϕ the phase difference between U_S and U_L, $\int_A$ indicates an integration over the area of the detector, and $\int^\infty$ indicates an integral over the whole detector plane to include all available signal power. The coefficient of $\eta P_S/h\nu B$ is called the heterodyne efficiency, γ.

The equation for an array simply replaces $|U_L|$ with $|U_L|U_j$ and ϕ with $\phi-\psi_j$, where U_j and ψ_j are the gain distribution and phase shifts of the i.f. amplifiers. The integrals over A are then over the sensitive area of the array. For both a single detector and an array, the maximum heterodyne efficiency obtainable is equal to the fraction of the signal power falling on the sensitive area of the detector.

SPECIAL CASES

Reference 1 calculates the heterodyne efficiency for two special cases with the following results:

Case I Signal and LO are matched Airy functions over a circular detector. The heterodyne efficiency is just equal to the fraction of the signal power falling on the detector:

$$\gamma = 1 - J_0^2(x) - J_1^2(x) \quad (10)$$

where J_0 and J_1 are Bessel functions, $x = \pi r/F\lambda$, where r is the radius of the detector, F the f/number of the collection optics, and λ the wavelength of the light. This heterodyne efficiency is plotted in Fig. 3. It is a monotonically increasing function of the detector size and is equal to 0.84 for a detector the same size as the Airy disk.

Case II Signal is an Airy function, LO is uniform over a circular detector. The heterodyne efficiency is given by

$$\gamma = 4\left[1 - J_0^2(x)/x^2\right] \tag{11}$$

where the symbols are as defined for Case I. This heterodyne efficiency is also plotted in Fig. 3. Its peak is 0.72 at a radius of 72% of the radius of the Airy disk. If the detector is increased in size to match the Airy disk, the efficiency drops to 0.54.

REFERENCES

1. D. Fink, Coherent Detection Signal-to-Noise, Appl. Opt., 14, 689 (1975).

2. D. Fink and S. N. Vodopia, Coherent Detection SNR of an Array of Detectors, Appl. Opt., 15, 453 (1976).

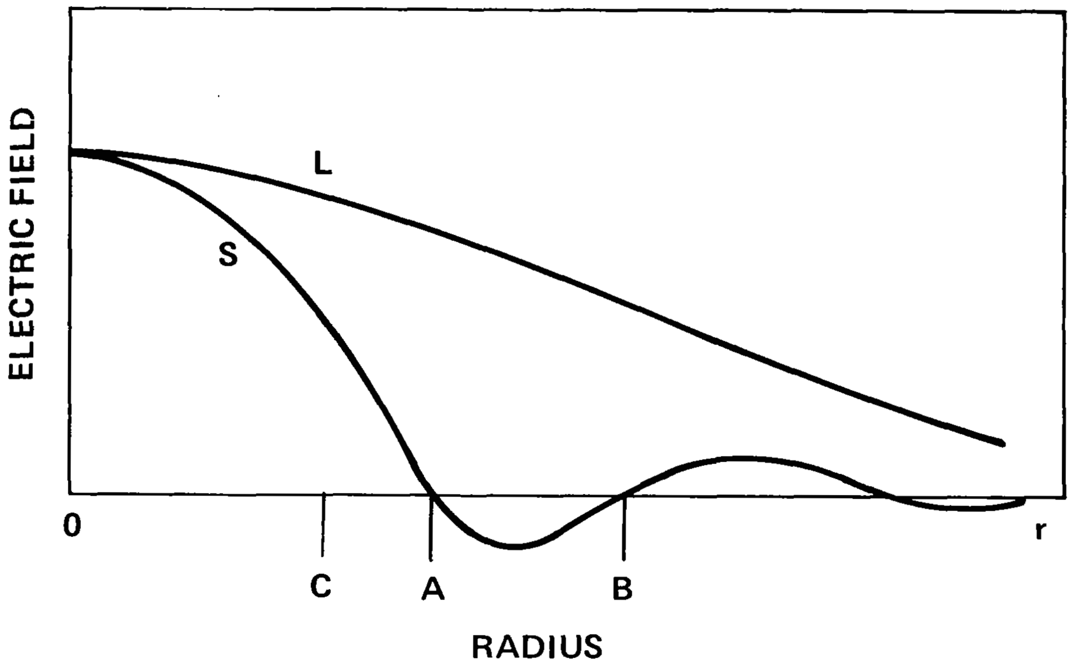

Figure 1.- Representative electric field distributions of signal (S) and local oscillator (L).

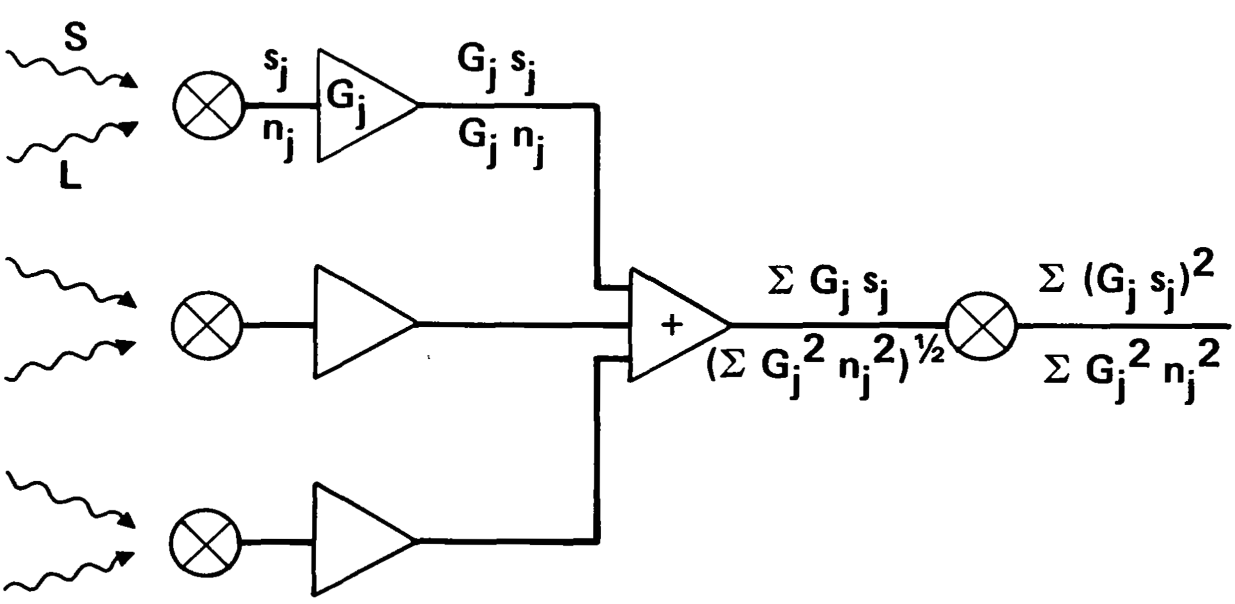

Figure 2.- Photomixing detector network.

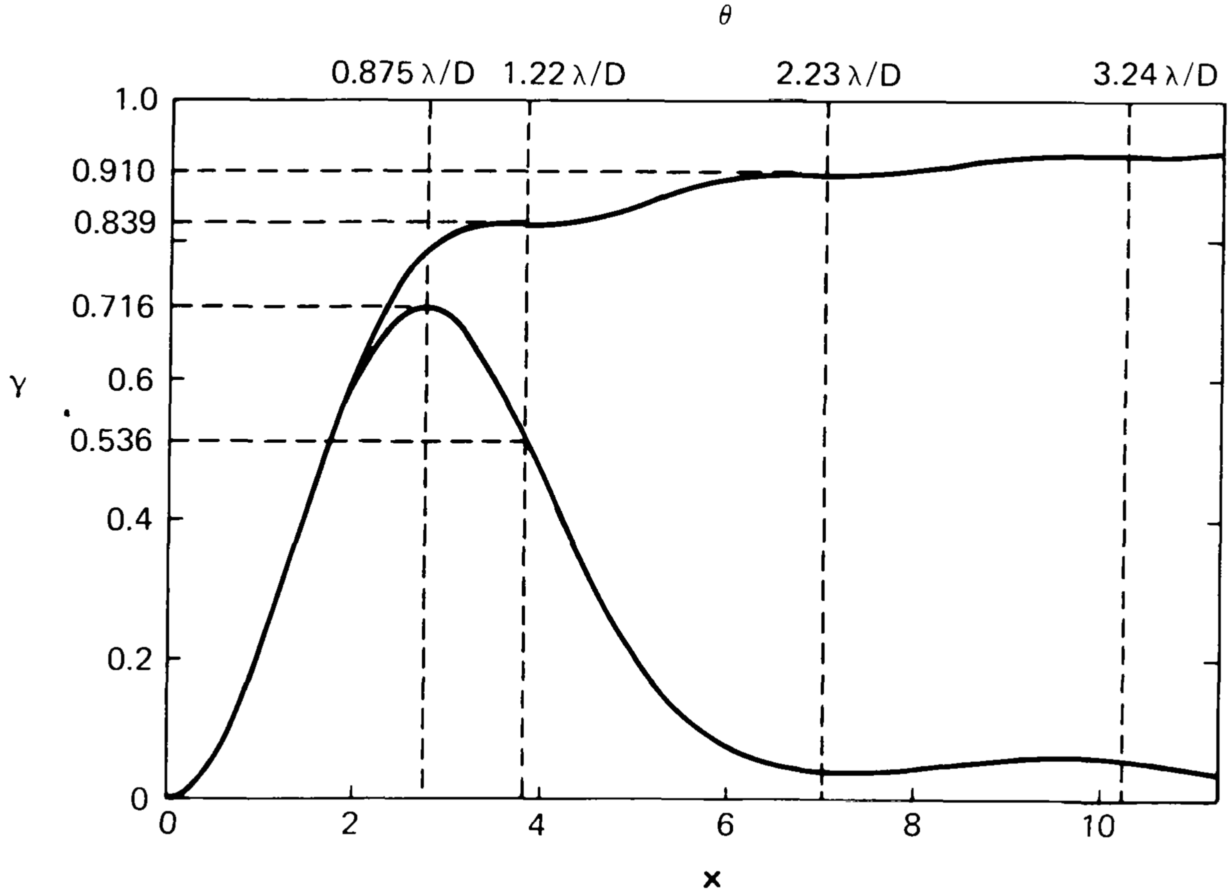

Figure 3.- Heterodyne efficiency for Airy function signal distribution on circular detector. Both curves are for circular detector of radius $r = F\lambda x/\pi$ and Airy function signal field distribution. Upper curve is for matched local oscillator field distribution on detector, which gives maximum possible signal-to-noise ratio. Lower curve is for usual case of uniform local oscillator field distribution on detector. Angular radius detedtor is indicated on upper axis for peak of lower curve (x = 2.75) and first three Airy dark rings (x = 3.83, 7.02, and 10.2).