HETERODYNE SIGNAL-TO-NOISE RATIOS IN ACOUSTIC MODE SCATTERING EXPERIMENTS

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SUMMARY

A study has been made of the relation between the SNR obtained in heterodyne detection of radiation scattered from acoustic modes in crystalline solids and the scattered spectral density function. It is shown that in addition to the information provided by the measured frequency shifts and line widths, measurement of the SNR provides a determination of the absolute elasto-optical (Pockel's) constants. Examples are given for cubic crystals, and acceptable SNR values are obtained for scattering from thermally excited phonons at 10.6 microns, with no external perturbation of the sample necessary. The results indicate the special advantages of the method for the study of semiconductors.

ANALYSIS AND DISCUSSION

The ratio of mean square signal current to mean square noise current obtained at the frequency f when laser radiation of frequency ν_o is mixed on the surface of a photoconductor with a spectrum $G(\nu)$ can be expressed (ref. 1) in the form

$$\frac{S}{N} = \eta G(\nu_o \pm f). \tag{1}$$

G is expressed in numbers of photons per second per frequency interval into a coherence area and η is the photoconductor quantum efficiency. If I_1 represents the number of electrons produced per second by light with frequencies in a bandwidth $\Delta\nu$ and solid angular spread Ω falling on area A, then

$$G(\nu) = \frac{I_1 \lambda^2}{A\Omega\Delta\nu} . \tag{2}$$

I_1 is the total energy incident per second, P, divided by the energy per photon, multiplied by the number of electrons produced by one photon, i.e.,

$$I_1 = \frac{P\eta}{h\nu} . \tag{3}$$

If the source of P is the scattering of laser radiation from acoustic waves or phonons propagating through a crystalline sample, then equation (1)

can be written in the convenient form

$$\frac{S}{N} = \eta P_o \frac{P_s/P_o}{A\,\Omega} \frac{\lambda^2}{h\nu} \frac{1}{\Delta\nu} , \tag{4}$$

where P_o represents the total power incident on the sample, P_s the total scattered power, and $\Delta\nu$ the width of the scattered line. The scattering of radiation by propagating thermally excited phonons in crystalline solids has been analyzed by Born and Huang (ref. 2), specialized for birefringent crystals by Gammon and Cummins (ref. 3), and for cubic crystals by Benedek and Fritsch (ref. 4). For small angles the power scattered into solid angle Ω at the point R by the acoustic fluctuation of wavevector $\vec{K}$ is related to the autocorrelation function of the scattered field,

$$P_s(\vec{K},R) = \frac{C}{8\pi} \left\langle \left| E(\vec{K},t) \right|^2 \right\rangle R^2\Omega, \tag{5}$$

where (ref. 4, equation 46)

$$\left\langle \left| E(\vec{K},t) \right|^2 \right\rangle = E_o^2 \left(\frac{\omega_o}{C}\right)^4 \frac{\varepsilon_o^4}{R^4} \frac{V}{4\pi^2} \frac{kT}{\rho} \sum_{\mu=1}^{3} \left|\xi^\mu\right|^2 \frac{K^2}{\omega_\mu^2(K)} \tag{6}$$

Here μ denotes a particular acoustic mode (μ = 1, 2, 3 for cubic crystals), ω_μ the frequency of mode μ, V the scattering volume, ε_o the dielectric constant, and ρ the density of the sample. ξ^μ is in effect a weighting factor which determines the relative intensity of the radiation scattered by mode μ, and is itself determined by the absolute elasto-optical constants. If the illuminated volume of the crystal is written as $V = LA_i$, where A_i is the cross-sectional area of the incident beam and L the scattering length, then for mode μ

$$\frac{P_s/P_o}{\Omega} = \pi^2 \left(\frac{\nu_o}{C}\right)^4 \frac{\varepsilon_o^4 LkT}{\rho} \frac{\left|\xi^\mu\right|^2}{v_\mu^2} , \tag{7}$$

where

$$P_o = \frac{C}{8\pi} E_o^2 A_i . \tag{8}$$

The final expression for the SNR can then be written

$$\frac{S}{N} = \eta P_o \left(\frac{\pi}{C}\right)^2 \nu_o \frac{\varepsilon_o^4 LkT}{hA\rho} \frac{\left|\xi^\mu\right|^2}{v_\mu^2} \frac{1}{\Delta\nu_\mu} . \tag{9}$$

For a given experimental configuration all quantities in this expression are known except $\Delta\nu_\mu$, ξ^μ, and v_μ. $\Delta\nu_\mu$ is directly observable, while v_μ is determined by the measured frequency shift of the scattered radiation for a given scattering angle (ref. 5). Thus a measure of the SNR allows ξ^μ to be found, which in turn leads to values of the absolute elasto-optical constants. For

example, inverting the relation between the elasto-optical constants and the ξ^{μ} given by Benedek and Fritsch (ref. 4), and using numerical values for scattering at 3^{o} from acoustic waves propagating in the [110] plane of potassium chloride (the elastic constants are needed in the calculation), results in the relations for the elasto-optical constants p_{11}, p_{12}, and p_{44}

$$\begin{aligned} p_{11} &= 2\,\xi^{1} - 56.8\,\xi^{2} + 1.72\,\xi^{3} \\ p_{12} &= 1.72\,\xi^{3} - 56.8\,\xi^{2} \\ p_{44} &= 2\,\xi^{1} - 113.6\,\xi^{2} + 1.43\,\xi^{3} \end{aligned} \qquad (10)$$

Potassium chloride is one of the few substances for which the absolute elasto-optical constants are known (ref. 6), and substitution of the known values into equations (9) and (10) result in an expected SNR of 4.6 for scattering of 40 watts of 10.6 micron radiation from the longitudinal ($\mu = 3$) mode. A value of 0.4 has been used for the quantum efficiency for purposes of calculation; in addition, an inverse square frequency dependence for sound wave attenuation has been assumed in order to estimate the expected line width (about 12 kc), such a dependence being characteristic of relaxation phenomena in solids, especially in the megacycle range.

This optimum value of the SNR will be obtained only if the distance z from the scatterer to the detector is related to the detector aperture radius b through the wavevector k according to the expression

$$b \simeq \sqrt{\frac{z}{k}} \quad ; \qquad (11)$$

otherwise the above SNR must be multiplied by an appropriate reduction factor (ref. 7). For a detector of 3 mm diameter the optimum SNR would be obtained for a source detector distance of 1.3 meters, a typical experimental configuration.

Thus a careful determination of the SNR provides a unique method for obtaining the absolute elasto-optical coefficients of crystalline samples, with no external perturbation or acoustic excitation required. The favorable results at 10.6 microns imply the method will be useful for the study of semiconductors having band gaps less than 0.12 eV, which are inaccessible to study by the usual optical methods.

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