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INFLUENCE OF MAGNETIC PRESSURE ON STELLAR STRUCTURE:

A MECHANISM FOR SOLAR VARIABILITY

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Abstract .

We propose a physical mechanism that couples the sun's dynamo magnetic field to its gravitational potential energy. The mechanism involves the isotropic field pressure resulting in a lifting force on the convective envelope, thereby raising its potential energy. Decay of the field due to solar activity allows the envelope to subside and releases this energy, which can augment the otherwise steady solar luminosity. Equations are developed and applied to the sun for several field configurations. Our "best estimate" model suggests that uniform luminosity variations as large as 0.02% for half a sunspot cycle may occur. Brief temporal variations or the rotation of spatial structures could allow larger excursions in the energy released.

Introduction

Recent studies of terrestial climate variations by <u>Mitchell et al.</u> (1979) and <u>Dicke</u> (1978, 1979) suggest a link with the state of the sun's activity cycle (see <u>White</u> (1979) for reviews of the subject). Such a link would result if the solar dynamo could modify the total energy output of the sun and this has prompted theoretical investigations into possible effects of the dynamo upon the solar luminosity. <u>Thomas</u> (1979) has examined possible luminosity changes associated with magnetic bouyancy in sunspots; <u>Spiegel and Weiss</u> (1980) have considered the effect of magnetic fields on the temperature structure of the convection zone; and <u>Schatten and Sofia</u> (1980) have suggested that magnetic fields may alter the depth of the convection zone. These investigations were concerned with the effects of magnetic fields on energy transport. In this paper, we consider the direct effect of the isotropic pressure exerted by the dynamo field on the gravitational potential energy of the solar convection zone.

The standard theory of the solar dynamo involves a buried toroidal magnetic field which alternately is wound up by differential rotation and develops into solar activity. This magnetic field is assumed to exert an isotropic pressure which can lift the convection zone and increase its gravitational energy. This potential energy increase will occur during the field amplification phase of the solar cycle. When the field breaks up into activity (solar flares, sunspots, etc.), the convection zone subsides and releases the stored potential energy. This energy will be shown to be much greater than the energy released by solar activity. When added to the otherwise steady solar luminosity, the result is a solar-cycle modulation of the total energy output of the sun. In the following section we develop a simple model for this process. The

estimated amplitude of a uniform luminosity modulation is ∼0.02%,

i.e., about 10^3 times greater than that associated with surface activity.

Potential Energy Changes Due to the Dynamo Field

We are interested in obtaining the uplift and energy change which results from the dynamo field. To do this, we first need to estimate the strength of the toroidal field at the base of the convection zone. This can be done by combining observations of the surface field with physical limits on dynamically stable field configurations.

The observations of Howard (1977) and Sheeley (1976) provide the polar magnetic flux to be $\Phi_p = 1.2 \times 10^{22}$ Mx, with an upper limit of 2×10^{22} Mx. Figure 1 indicates the manner in which the field threads to the base of the convection zone. The toroidal field subtends a latitude range Θ in each hemisphere and the field lines are inclined at an angle α with respect to a latitude circle. We identify Θ with the range in which sunspots occur, while α can be inferred from the tilt of sunspot pairs. We use $\Theta \sim 37^O = 0.65$ radian and $\alpha \sim 6^O = 0.1$ radian (Allen, 1963). This gives

$$\Phi \approx 2\Phi_t = 2\Phi_p \ \Theta \ \cot \alpha / 2\pi \simeq 2.3 \times 10^{22} \text{ Mx}, \tag{1}$$

where Φ is the total magnetic flux through both hemispheres and Φ_t the toroidal flux in each hemisphere.

The flux condition governing the field through the element of area shown in figure 1 is

$$B \ \Delta r \ R_c \ \Theta = \Phi_t, \tag{2}$$

where Δr is the radial thickness of the field layer and R_c the radius at the base of the convection zone.

We can obtain limits on the toroidal field by considering some simple stability conditions. The equation for radial force balance is

$$\frac{d}{dr}(P + \frac{B^2}{8\pi}) = \frac{-GM_i}{R_c^2} \ \rho, \tag{3}$$

where P is the gas pressure, M_i the mass internal to the convection zone, and ρ the gas density. A Rayleigh-Taylor interchange instability can

develop if a positive radial gradient in the gas density occurs   Similarly, a positive gradient of temperature would prevent the outward flow of thermal energy through the field layer and lead to a thermal instability. If both the density and temperature gradients must be less than or equal to zero, the same condition applies to the gas pressure gradient. Thus, equation (3) limits the field gradient to

$$-\frac{1}{8\pi}\frac{dB^2}{dr} = \frac{GM_i}{R^2_c}\rho + \frac{dP}{dr} \le \frac{GM_i}{R^2_c}\rho. \tag{4}$$

Assuming the field outside our toroidal volume is small, we replace $|dB^2/dr| \approx \Delta B^2/\Delta r$ by $B^2/\Delta r$ and find

$$B^2/\Delta r \le 8\pi GM_i\rho/R_c^2 \tag{5}$$

Using the flux condition (2), we get

$$B \le \left(\frac{8\pi GM_i\ \rho\Phi_t}{R^3_c\ \Theta}\right)^{1/3} \text{and}\ \Delta r \ge \left(\frac{\Phi_t^2}{8\pi GM_i\rho\Theta^2}\right)^{1/3} \tag{6}$$

For the conditions at the bottom of the convection zone in the solar model of <u>Endal and Sofia</u> (1980), B is limited to 2×10^5 G and Δr must be at least 13 km.

We can now determine how far the magnetic field will lift the solar convection zone and raise its potential energy. A magnetic field B, contained in a plasma of volume V and temperature T, exerts the same pressure as a hypothetical mass ΔM given by

$$\Delta M = \frac{B^2 V}{8\pi nkT}, \tag{7}$$

where n is the number of particles per unit mass and, for the toroidal volume element, $V = 8R_c^2\ (\Delta r)\ \Theta$. Since this will be a small perturbation on the global structure, we can picture the magnetic field as displacing this hypothetical mass ΔM at the bottom of the convection zone. This is

equivalent to lifting ~~.he~~ the entire convection zone through a distance

$$h = \frac{\Delta M}{4\pi R^2 c\rho} = \frac{2\Theta}{\pi} \beta^{-1} \Delta r, \qquad (8)$$

where β is the ratio of gas to magnetic pressure. The increase in the gravitational potential energy due to this uplift is

$$\Delta E = gM_{cz} h = \frac{GM_1 M_{cz}}{R_c 2} \left(\frac{2\Theta}{\pi}\right) \beta^{-1} \Delta r, \qquad (9)$$

where M_{cz} is the mass of the convection zone.

With $\Phi_t \simeq 10^{22}$ Mx and B and Δr at the limits (6), ΔE is 1.5×10^{38} ergs; this can provide a 0.02% increase in the sun's luminosity for half a solar cycle. This is the "maximum standard" shown in Table 1. The term maximum reflects that we are using the most favorable values allowed by the limits (6). We are not including possible short temporal variations or spatial structure which could increase the magnitude somewhat.

The energy change ΔE is proportional to the four-thirds power of the polar flux. This is illustrated by the second set of models in Table 1, where we have varied the polar flux to the maximum value (in modern times) given by Howard (1977) and to half the Sheeley (1976) value. A third set of models illustrates the effect of varying the depth of the convection zone. The last set of models is calculated assuming the plasma β is 40 and 4×10^5, rather than relying on the stability condition, which appears to limit β to a value closer to 4×10^3. Although the interchange instability is strong, it is possible that the magnetic field may damp the resulting plasma motions so that the stability criteria may be violated for a short time. Note that, for a given β, Δr is determined by the flux condition (2).

One last calculation will provide an additional check on our model. The energy to inflate the convection zone must first be applied to the magnetic field, as it is the pressure of the field which does the lifting.

The field receives its energy from differential rotation; can the differential rotation supply 1.5×10^{38} erg in 5 years or 2×10^{31} erg s^{-1}? If we use a formula relating the magnetic pressure in a torque law with a differential rotation, we find

$$\text{Power} = \tau \Delta\omega = (B^2/8\pi) \, R_c^{\,2} \, \Delta\omega. \tag{10}$$

In the maximum standard model, this can supply 3.5×10^{31} erg s^{-1}, for $\Delta\omega = 10^{-6}$ s^{-1}, which is more than adequate. Note that most of the 1.5×10^{38} erg released when the convection zone settles must go into luminosity, rather than magnetic activity, as solar activity does not nearly involve this much energy. Further, we have ignored the $\sim 3 \times 10^{37}$ ergs of magnetic energy, $B^2V/8\pi$, which will add approximately 20% to ΔE in the maximum standard case.

Discussion

Although our table 1 provides results for several sets of parameter values, we feel the maximum standard model is close to what would be expected from a large solar cycle. Most of the other models are provided solely to show the sensitivity of luminosity changes to parameter variations. The reasoning is that this standard is based upon an accurate convection zone model, thus the base level of the convection zone is thought to be reliable. Secondly, the model is based upon the Rayleigh-Taylor instability being approached; it is felt that this instability is approached as solar cycles do break up when magnetic buoyancy is reached. It is possible that another instability may limit the growth of the field prior to onset of the Rayleigh-Taylor instability. In this case, the maxim m $B^2/\Delta r$ would be reduced by a factor f, relative to the value given be equation (4). However, the energy release ΔE would only be reduced by $f^{1/3}$ so our computed energy release is rather insensitive to this possibility. Thus, the values cited in the maximum standard are thought be be approached, with the true value falling closer to a larger β (a somewhat smaller energy output).

Another consideration is the period of these luminosity oscillations. The references cited in the introduction discuss, for the most part, a 22 year (Hale double cycle) variation in climate, whereas we have discussed the matter in a way which would suggest an 11-year variation. We feel that should a subsurface _directed_ magnetic field be frozen-in to the core material, it could modulate the 11-year solar cycle fields in such a way as to result in an offset of field strengths. This could result in the double cycle (22 year) predominating. This matter deserves further study. Another point to note is that numerous 11-year variations are found! See Herman and Goldberg (1977) for a review of the subject, or Nastrom and Belmont (1980) for recent findings.

Conclusions

We have found that a reasonable solar cycle magnetic field can uplift the convection zone. When the cycle deteriorates during the declining phase, the resulting collapse of material may provide temporal variations up to about 0.02% of the solar luminosity. Larger energy excursions may occur associated with the non-uniform release of energy in the declining phase, however, it is difficult to estimate the size of these irregularities. The value calculated is about 10^3 times larger than the average energy released by conventional forms of solar activity. An 11 year (or possibly a 22 year) variation may result.

Table 1

Solar Convection Zone Uplift Models and

Associated Energy Changes

Model	ϕ_p (Mx)	Rc (km)	B(G)	Δr(km)	ΔE(ergs)	ΔL(L⊙)
1. Maximum Standard	1.2^{22}	5.4^5	2.3^5	13	1.5^{38}	0.02%
2. Flux Variation (max.)	2^{22}	*	2.8^5	18	3.0^{38}	0.05%
(min.)	0.6^{22}	*	1.8^5	8	6.0^{38}	0.01%
3. Varying Field Depth (max.)	*	4.0^5	6^5	6	1.8^{39}	0.29%**
(min.)	*	6.0^5	1.4^5	17	2^{37}	0.01%
4. $\beta^{-1} = 2.5^{-2}$ (max.)	*	*	2.3^6	1	1.5^{39}	0.2%**
$\beta^{-1} = 2.5^{-6}$ (min.)	*	*	2.3^4	130	1.5^{37}	2^{-3}%

(Note: Exponents refer to powers of 10, thus $5^5 \equiv 5 \times 10^5$)
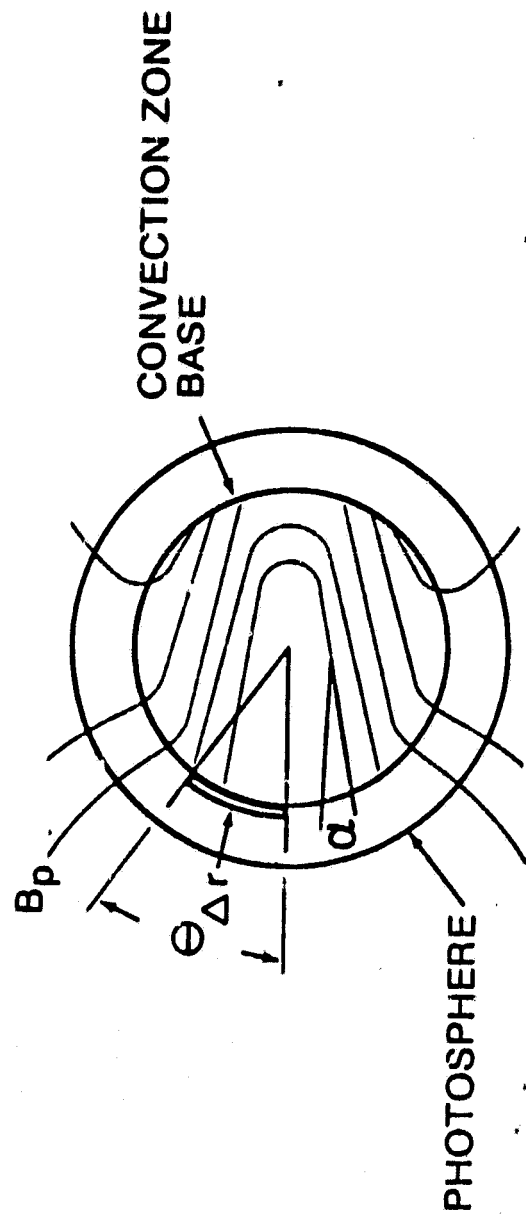
* indicates the maximum standard value is used.

**these values are unrealistic and serve only to show the sensitivity of the model to parameter variations.

Acknowledgement

FIGURE CAPTIONS

Figure 1 Shown is the geometry of the threading of the sun's polar magnetic field to the base of the convection zone, where it is twisted into a toroidal field. The area at the left has a width, $R_c \Theta$, and a thickness Δr, the field lines make an angle, α, with respect to a circle of latitude. The geometry is used to estimate the magnitude of the sun's subsurface field. The field is spread into a toroidal volume, $V = 4\pi R_c^2 \Delta r (2\Theta)$.

CONVECTION ZONE
BASE

B_p

Θ
Δr

α

PHOTOSPHERE

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