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LATERAL DENSITY VARIATIONS IN ELASTIC EARTH MODELS FROM AN EXTENDED MINIMUM ENERGY APPROACH

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Abstract

The gravitational field of the earth shows the existence of lateral density inhomogeneities in the mantle and crust but it does not yield a unique solution for them, that is, there are an infinite number of possible density perturbation fields which will match the observed gravitational field.

Kaula's minimum energy approach has been extended to include the nonhydrostatic gravitational potential energy and the density perturbation field has been obtained to degree and order 8. The depth profiles for the density perturbation show a stratification with density excesses and deficiencies alternating with depth. The results indicate that the addition of the gravitational potential energy in the minimization process does not change significantly the conclusions based on results for the minimum shear strain energy case, concerning the inability of the mantle to withstand the lateral loading elastically.

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1. Introduction

The external gravitational potential of the earth can be expressed in spherical harmonics and its components determined by analysis of satellite data. Such an analysis indicates the presence of lateral variations of density in the earth interior. However, it is not possible to define uniquely what these lateral density variations are by inverting only the gravity data. The question of lateral density variations and the internal stresses they produce has implications with respect to the possible existence of mantle convection within the earth. Kaula (1967), Lambeck (1976), Phillips and Lambeck (1980) among others have provided extensive and detailed discussions of these ideas. A possible approach to the problem consists in using static earth-tide theory in a modified form to include interior density anomalies expressed in spherical harmonics, the parameters which define the anomalies are determined by imposing the condition of minimum strain energy and the solution is constrained to satisfy the external boundary conditions provided by the gravitational potential; the surface topography is also included in the problem as a loading surface density distribution. This is the method followed by Kaula (1963) and Arkani-Hamed (1970, 1972). It provides an estimate of the minimum shear stresses induced by the internal density anomalies and conclusions can be drawn concerning the capability of the mantle to withstand such stresses elastically. The results indicate that the stresses are probably excessive.

Lambeck (1976) has pointed out that the nonhydrostatic gravity field implies the existence of free potential energy within the earth and that there must be a capacity for rearrangements of mass toward distributions of lower energy (assuming no new energy generation and no kinematic constraints). A study by Rubincam (1979) indicates that the maximum nonhydrostatic gravitational potential energy in the mantle and crust is of the same order of magnitude as Kaula's estimate for the minimum shear strain energy.

In the light of the remarks above, it is not without a measure of interest to carry out an investigation similar to Kaula's but with the inclusion of the gravitational potential energy in the minimization process. The object of the study is to determine the distribution of lateral density anomalies and to compare it to the results for the minimum strain energy case to ascertain if any significant changes occur.

2. Differential equations

The equations of motion governing the vibration of an elastic body are given by Love (1944), these equations and the Poisson's equation for the gravitational potential can be expressed in spherical coordinates and with the appropriate set of boundary conditions they will admit an eigenvector solution. The static solution is obtained by setting the frequency equal to zero. Using the transformation of Alterman et. al., (1959) these partial differential equations yield the following set of six ordinary differential equations. The dot denotes derivatives with respect to r.

$$\dot{\vec{y}} = [A]\,\vec{y} + [B]\,\vec{d} \tag{2.1}$$

$$\vec{y} = (y_1, y_2, y_3, y_4, y_5, y_6)^T \tag{2.2}$$

The elements of the matrix [A] are given by Smylie and Mansinha (1971) and Israel et. al., (1973). They are given below for the sake of completeness. The symbols ρ_o, μ and λ denote the unperturbed density and the elastic parameters, g_o stands for the gravitational acceleration and n is the degree of the deformation.

$c = 1/(\lambda + 2\mu)$

$A_{11} = -2\lambda c/r$

$A_{12} = c$

$A_{13} = \lambda n\,(n + 1)\,c/r$

$A_{21} = 4\mu(3\lambda + 2\mu)\,c/r^2 - 4\rho_o g_o/r$

$A_{22} = -4\mu\,c/r$

$A_{23} = -n\,(n + 1)\,[2\mu\,(3\lambda + 2\mu)\,c/r^2 - \rho_o g_o/r]$

$A_{24} = n\,(n + 1)/r$

$A_{26} = -\rho_o$

$A_{31} = -1/r$

$A_{33} = 1/r$

$A_{34} = 1/\mu$

$A_{41} = -2\mu\,(3\lambda + 2\mu)\,c/r^2 + \rho_o g_o/r$

$A_{42} = -\lambda c/r$

$A_{43} = 4n\ (n + 1)\ \mu(\lambda + \mu)\ c/r^2 - 2\mu/r^2$

$A_{44} = -3/r$

$A_{45} = -\rho_o/r$

$A_{51} = 4\pi G\rho_o$

$A_{56} = 1$

$A_{63} = -4\pi G\ n(n + 1)\ \rho_o/r$

$A_{65} = n(n + 1)/r^2$

$A_{66} = -2/r$

The unlisted elements are equal to zero. The y_j are radial functions defined as follows

$$
\begin{aligned}
u_r &= y_1(r)\, S_{nm} \\
u_\theta &= y_3(r)\, (\partial/\partial\theta)\, S_{nm} \\
u_\phi &= y_3(r)\, (1/\sin\theta)\, (\partial/\partial\phi)\, S_{nm}
\end{aligned}
\qquad (2.3)
$$

where u_r, u_θ and u_ϕ denote displacements in the respective coordinate directions and S_{nm} is a surface harmonic of degree n and order m. The stress-strain relations yield:

$$(\delta_{rr}, \delta_{\theta\theta}, \delta_{\phi\phi}, \delta_{r\theta}, \delta_{r\phi}, \delta_{\theta\phi})^T = [R]\ (y_1, y_2, y_3, y_4)^T \qquad (2.4)$$

where the δ's denote stresses and the matrix [R] is given by Arkani-Hamed (1972),

$R_{21} = [2\mu\ c(3\lambda + 2\mu)/r]\ S_{nm}$

$R_{31} = R_{21}$

$R_{12} = S_{nm}$

$R_{22} = \lambda c\ S_{nm}$

$R_{32} = R_{22}$

$R_{23} = (2\mu/r)\ [\partial^2/\partial\theta^2 - n(n + 1)\ \lambda c]\ S_{nm}$

$R_{33} = (-2\mu/r)\ [\partial^2/\partial\theta^2 + 2n(n + 1)\ (\lambda + \mu)\ c]\ S_{nm}$

$R_{63} = (2\mu/r\ \sin\theta)\ [\partial^2/\partial\theta\ \partial\phi - \cot\theta\ \partial/\partial\phi]\ S_{nm}$

$$R_{44} = (\partial/\partial\theta)\, S_{nm}$$

$$R_{54} = (1/\sin\theta)\,(\partial/\partial\phi)\, S_{nm}$$

The unlisted elements are equal to zero. Also

$$\begin{aligned} \psi &= y_5(r)\, S_{nm} \\ y_6(r) &= \dot{y}_5 - 4\pi G \rho_o y_1 \end{aligned} \qquad (2.5)$$

where ψ denotes the perturbation in the gravitational potential and G is the gravitational constant.

The density perturbations arise from two different causes, the perturbation caused by the deformation due to the loading and a perturbation due to unspecified effects such as thermal and chemical processes. That is,

$$\rho = \rho_o(r) + \rho_d(r, \theta, \phi) + \rho_A(r, \theta, \phi) \qquad (2.6)$$

where $\rho_o(r)$ is a nominal radial distribution (from seismic data), ρ_d is the density perturbation due to deformation and ρ_A is the added density due to unspecified processes. It is assumed that ρ_A can be expressed in spherical harmonics,

$$\rho_A = \sum_n \sum_m P_{nm}\, [\rho^{(r)}_{Anm1} \cos m\phi + \rho^{(r)}_{Anm2} \sin m\phi] \qquad (2.7)$$

where P_{nm} is an associated Legendre polynomial. Furthermore it is assumed that the coefficients ρ_{Anmi} are given as polynomials in the normalized radius,

$$\rho_{Anmi} = \sum_{j=1}^{6} d_{nmij}\, (r/R)^{j-1} \quad i = 1,2 \qquad (2.8)$$

where the d_{nmij} are constants to be determined and R denotes the outer radius of the earth.

With these assumptions the forcing term in equation (2.1) is specified, the elements of the matrix [B] are given by

$$\begin{aligned} B_{2k} &= g_o (r/R)^{k-1} \\ B_{6k} &= -4\pi G (r/R)^{k-1} \end{aligned} \quad k = 1 \text{ to } 6 \qquad (2.9)$$

$$B_{\ell k} = 0 \quad \ell = 1, 3, 4, 5;\ k = 1 \text{ to } 6$$

The vector $\vec{d}$ includes the constants to be determined

$$\vec{d} = (d_{nmi1}, d_{nmi2}, d_{nmi3}, d_{nmi4}, d_{nmi5}, d_{nmi6})^T \tag{2.10}$$

Equation (2.1) is applicable to the earth mantle and crust, the equations for the liquid core are obtained by assuming hydrostatic equilibrium conditions, they are given by

$$\dot{y}_5 = 4\pi G\rho_0 y_5/g_0 + y_6$$

$$\dot{y}_6 = [n(n+1)/r^2 - 16\pi G\rho_0/g_0 r]\, y_5 - (4\pi G\rho_0/g_0 + 2/r)\, y_6$$

and

$$\begin{aligned} y_1 &= y_5/g_0 \\ y_2 &= 0 \\ y_3 &= (4y_5 + ry_6)/n(n+1)\, g_0 \\ y_4 &= 0 \end{aligned} \tag{2.11}$$

The boundary conditions appropriate for the solution of the equations are specified by the spherical harmonic expansions defining the surface topography and the gravitational potential. Let the surface topography be given by

$$T = \sum_{n=0}^{\infty} \sum_{m=0}^{n} P_{nm} (T_{nm1} \cos m\phi + T_{nm2} \sin m\phi) \tag{2.12}$$

and the gravitational potential by

$$V = \sum_{n=0}^{\infty} \sum_{m=0}^{n} P_{nm} (V_{nm1} \cos m\phi + V_{nm2} \sin m\phi) \tag{2.13}$$

The surface boundary conditions are then

$$\begin{aligned} y_2(R) &= -g_0 (T_{nmi}) \\ y_4(R) &= 0 \\ y_5(R) &= V_{nmi} \\ y_6(R) + [(n+1)/R]\, y_5(R) &= 4\pi G T_{nmi} \end{aligned} \tag{2.14}$$

The initial conditions for the solution of equations (2.11) were approximated by an analytical solution of the surface load problem for a model of the earth having a homogeneous mantle and liquid core, Zharkov (1967). Values of y_5 and y_6 were obtained at $r = R/10$. Equations (2.11) were then integrated numerically up to the core-mantle boundary using the earth model M_3 due to Landisman et. al., (1965) as given by Israel et. al., (1973). This procedure was followed for every degree and order. The boundary conditions adopted at the core-mantle boundary are those developed by Israel et. al., (1973) and Crossley and Gubbins (1975), that is, continuity of the variables y_4 and y_5 only.

Let the solution of equation (2.1) be given by

$$\vec{y} = [M]\vec{c} + [N]\vec{d} \tag{2.15}$$

where $[M]$ and $[N]$ are matrices and $\vec{c}$ is a boundary conditions vector. It follows that

$$\begin{aligned} [\dot{M}] &= [A]\,[M], \quad [M(r_o, r_o)] = [I] \\ [\dot{N}] &= [A]\,[N] + [B], \quad [N(r_o, r_o)] = [0] \end{aligned} \tag{2.16}$$

where $[I]$ is the identity matrix. At some point r,

$$\vec{y}(r) = [M(r, r_o)]\vec{c} + [N(r, r_o)]\vec{d} \tag{2.17}$$

The set of ordinary differential equations has been cast into a transition matrix form to facilitate the application of a least-squares minimization procedure.

Assume the existence of a function H which can be written as

$$H = \begin{Bmatrix} \vec{c} \\ \vec{d} \end{Bmatrix}^T [L] \begin{Bmatrix} \vec{c} \\ \vec{d} \end{Bmatrix} \tag{2.18}$$

where $[L]$ is a matrix of the appropriate dimensions. The function H is to be minimized subject to the following equations of constraint,

$$[E]\begin{Bmatrix} \vec{c} \\ \vec{d} \end{Bmatrix} = \vec{k} \tag{2.19}$$

where $[E]$ is a matrix and $\vec{k}$ is a vector of fixed boundary conditions. It can be shown (Arley and Buch, 1950) that the solution

$$\begin{Bmatrix} \hat{c} \\ \hat{d} \end{Bmatrix} = [L]^{-T} [E] \left[[E] [L]^{-T} [E]^T \right]^{-1} \hat{k} \tag{2.20}$$

minimizes H in a least-squares sense.

3. The elastic and gravitational potential energies

The expression for the total shear strain energy of the mantle is given by Arkani-Hamed (1970),

$$S_{nmi} = 4\pi \int_{R_c}^{R} r^2 \mu \vec{y}^T [P] \vec{y} \tag{3.1}$$

where R_c denotes the core radius and the matrix [P] has the following elements

$P_{11} = 2(3\lambda + 2\mu)^2 c^2/3r^2$ $\quad$ $P_{21} = P_{12}$

$P_{12} = -2(3\lambda + 2\mu) c^2/3r$ $\quad$ $P_{31} = P_{13}$

$P_{13} = -n(n+1) P_{11}/2$ $\quad$ $P_{32} = P_{23}$

$P_{22} = 2c^2/3$

$P_{23} = -n(n+1) P_{12}/2$

$P_{33} = 2n^2(n+1)^2 (3\lambda^2 + 4\mu^2 + 6\mu\lambda) c^2/r^2 - n(n+1)/r^2$

$P_{44} = n(n+1)/2\mu^2$

The unlisted elements are equal to zero.

Making use of equation (2.15) it is possible to rewrite equation (3.1) as follows,

$$S_{nmi} \;\; 4\pi \begin{Bmatrix} \vec{c} \\ \vec{d} \end{Bmatrix}^T \int_{R_c}^{R} r^2 \mu \; \big[[M],[N]\big]^T [P] \; \big[[M],[N]\big] \; dr \begin{Bmatrix} \vec{c} \\ \vec{d} \end{Bmatrix} \tag{3.2}$$

The expression for the gravitational potential energy of the mantle is given by Rubincam (1979),

$$U_{nmi} = [-16\pi^2 G/(2n+1)] \int_{R_c}^{R} \rho(r)\, r^{-n+1} \int_{R_C}^{r} \chi^{n+2} \rho(\chi)\, d\chi\, dr \tag{3.3}$$

where $\rho(r)$ denotes the radial component of the density perturbation which has two components: the one due to deformation (ρ_D) and the one due to unspecified causes (ρ_A). Making use of equation (2.8),

$$U^A_{nmi} = [-16\pi^2 G/(2n+1)]\, \vec{d}^T \int_{R_c}^{R} [r^4 \vec{f}(r) \vec{g}^T(r) - r^{-n+1} R_c^{\,n+3} \vec{f}(r) \vec{g}^T(R_c)]\, dr \cdot \vec{d} \tag{3.4}$$

where

$$\vec{g}^T(r) = (g_1, g_2, g_3, g_4, g_5, g_6)$$

$$g_k = (r/R)^{k-1}/(k + n + 2) \quad k = 1 \text{ to } 6$$

$$\vec{f}(r) = (f_1, f_2, f_3, f_4, f_5, f_6)^T$$

$$f_k = (r/R)^{k-1} \quad k = 1 \text{ to } 6$$

The radial component of the density perturbation field due to deformation is given by

$$\rho_d = \rho_0 \Delta_r - (\partial\rho_o/\partial r)\, y_1 \tag{3.5}$$

where Δ_r denotes the radial component of dilatation,

$$\Delta_r = \dot{y}_1 + (2/r)\, y_1 - [n(n + 1)/r]\, y_3 \tag{3.6}$$

Inserting equation (3.6) into equation (3.5) and making use of the differential equation for $\dot{y}_1$ yields

$$\rho_d = \vec{q}^T \vec{y} \tag{3.7}$$

where

$$\vec{q}^T = (q_1, q_2, q_3, 0, 0, 0)$$

$$q_1 = -\rho_o [A_{11} + (2/r) + (1/\rho_o)\, \partial\rho_o/\partial r]$$

$$q_2 = -\rho_o A_{12}$$

$$q_3 = -\rho_o [A_{13} - n(n + 1)/r]$$

Making use of equations (3.7) and (2.15), equation (3.3) yields

$$U^d_{nmi} = [-16\pi^2 G/(2n + 1)] \begin{Bmatrix} \vec{c} \\ \vec{d} \end{Bmatrix}^T \int_{R_c}^{R} r^{-n+1} \left[[M], [N]\right]^T \vec{q} \int_{R_c}^{r} x^{n+2} \vec{q}^T \left[[M], [N]\right] dx\, dr \cdot \begin{Bmatrix} \vec{c} \\ \vec{d} \end{Bmatrix} \tag{3.8}$$

4. Numerical results and conclusions

Equations (2.16) were solved numerically in order to evaluate the expressions for the shear strain and gravitational potential energies given by equations (3.2), (3.4) and (3.8), then equation (2.20) was used to obtain the best estimate for the vectors $\vec{c}$ and $\vec{d}$ where the function H was representative of three separate cases:

i) minimization of the shear strain energy, $H = S_{nmi}$.

ii) minimization of the gravitational potential energy, $H = U^A_{nmi} + U^d_{nmi}$.

iii) minimization of the sum of the two, $H = S_{nmi} + U^A_{nmi} + U^d_{nmi}$.

The nominal earth model used is the M_3 model of Landisman et. al., (1965) as given by Israel et. al., (1973), the density and elastic parameters are given in table form and a cubic spline interpolation was used to obtain their values as functions of the radius.

The spherical harmonic expansion used for the topography is the equivalent rock set given by Balmino et. al., (1973). The expansion used for the gravitational potential is the Goddard Earth Model 6 by Lerch et. al., (1974) with the hydrostatic components of the second and fourth degree zonals subtracted out (Munk and MacDonald, 1960).

The density perturbation field due to unspecified causes was obtained for each degree and order by means of equations (2.7) and (2.8), equation (2.7) yields the total sum which in this case was truncated at degree and order 8. The surfaces of equal density perturbation evaluated at the surface of the earth are shown in figures (4.1), (4.2) and (4.3) for each of the three cases mentioned above. Figures (4.4), (4.5) and (4.6) present the surfaces of equal radial displacements at the surface. Figures (4.7), (4.8) and (4.9) show the surfaces of equal density perturbation evaluated on an equatorial cross-section of the earth. The magnitudes of the density perturbations and displacements at the surface is larger by about a factor of two for the case of minimum shear plus gravitational as compared to the minimum shear case. The same result is observed for the density anomalies as a function of depth. The minimum gravitational potential energy case yields much smaller density anomalies but unrealistically large displacements. The depth profiles for the density perturbation show a stratification with density excesses and deficiencies alternating with depth, this internal compensation has been mentioned by Phillips and Lambeck (1980) and others; on the other hand Arkani-Hamed (1970) refers to an oscillation along the radius as a numerical instability which he eliminates by introducing a weighting factor for the amplitudes. The minimum gravitational energy case yields abrupt lateral variations also.

Table (4.1) gives the global energy magnitudes for each case. The minimum total energy is obtained when minimizing only the shear strain contribution.

The results indicate that the addition of the gravitational potential energy in the minimization process does not significantly alter the lateral density perturbation field, as compared to the results obtained for the minimum shear strain energy case; certainly not in a way which would change the existing conclusions concerning the inability of the mantle to withstand the resulting stresses elastically.

Table 4.1
Global energy magnitudes, units of 10^{30} ergs

DEGREE	Min. Shear		Min. Grav.		Min. (Shear + Grav.)	
	Grav.	Shear	Grav.	Shear	Grav.	Shear
N = 2	- 7.912	0.178	-0.086	94.639	-27.599	5.640
N = 3	- 5.966	0.167	-0.120	204.381	-13.683	1.120
N = 4	- 6.020	0.566	-0.237	586.165	-11.624	1.039
N = 5	-13.935	0.225	-0.321	1062.685	- 7.443	0.688
N = 6	- 1.412	0.070	-0.174	719.578	- 1.645	0.062
N = 7	- 0.672	0.043	-0.274	1249.120	- 3.510	2.204
N = 8	- 0.432	0.029	-0.324	1307.191	- 2.623	2.266
$\sum_N$	-36.351	1.281	-1.539	5223.762	-68.130	13.021
Total	-35.070		5222.222		-55.108	

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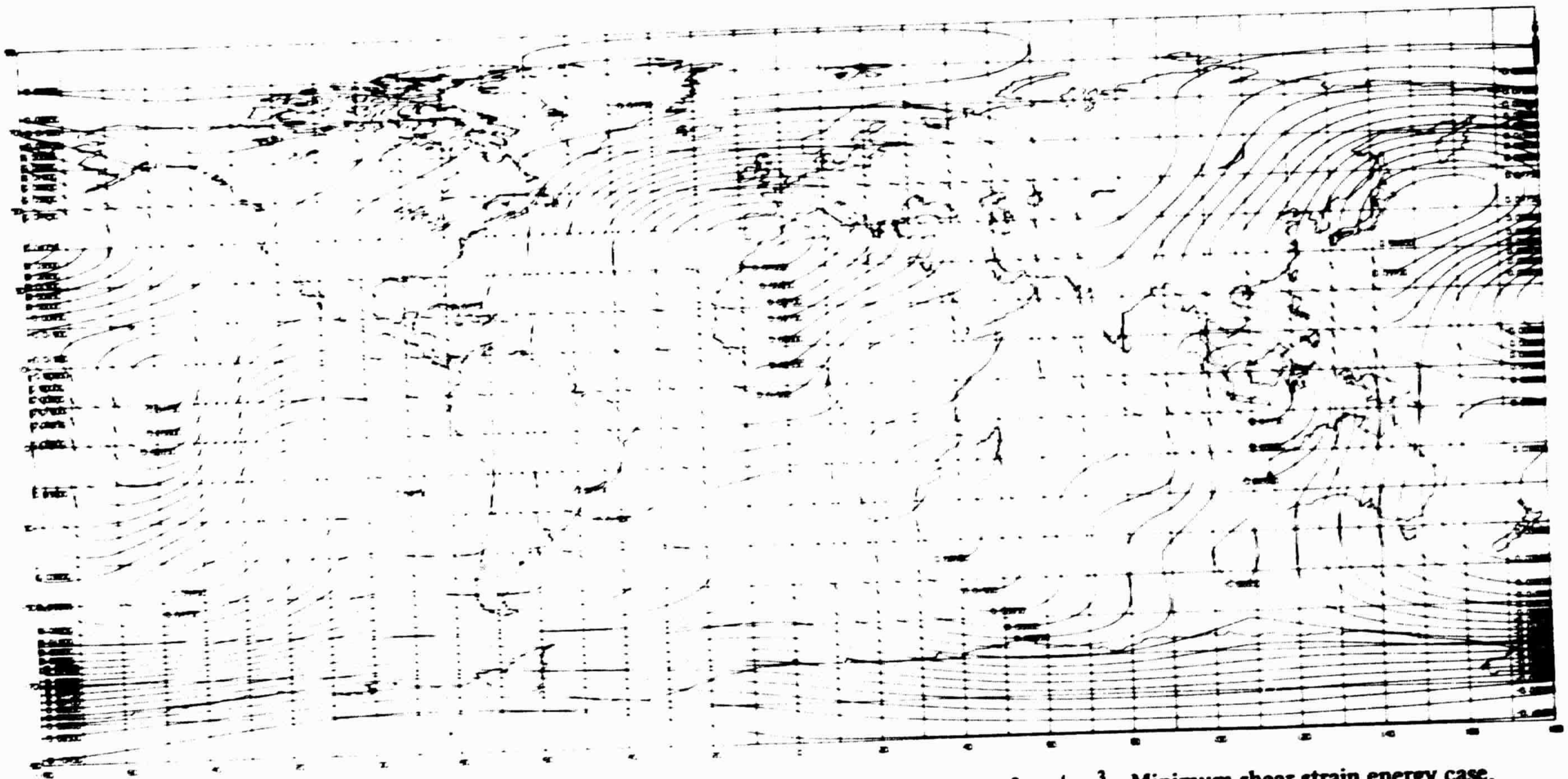

Figure 4.1. Surfaces of equal density perturbation at the surface of the earth, units of gm/cm^3. Minimum shear strain energy case.

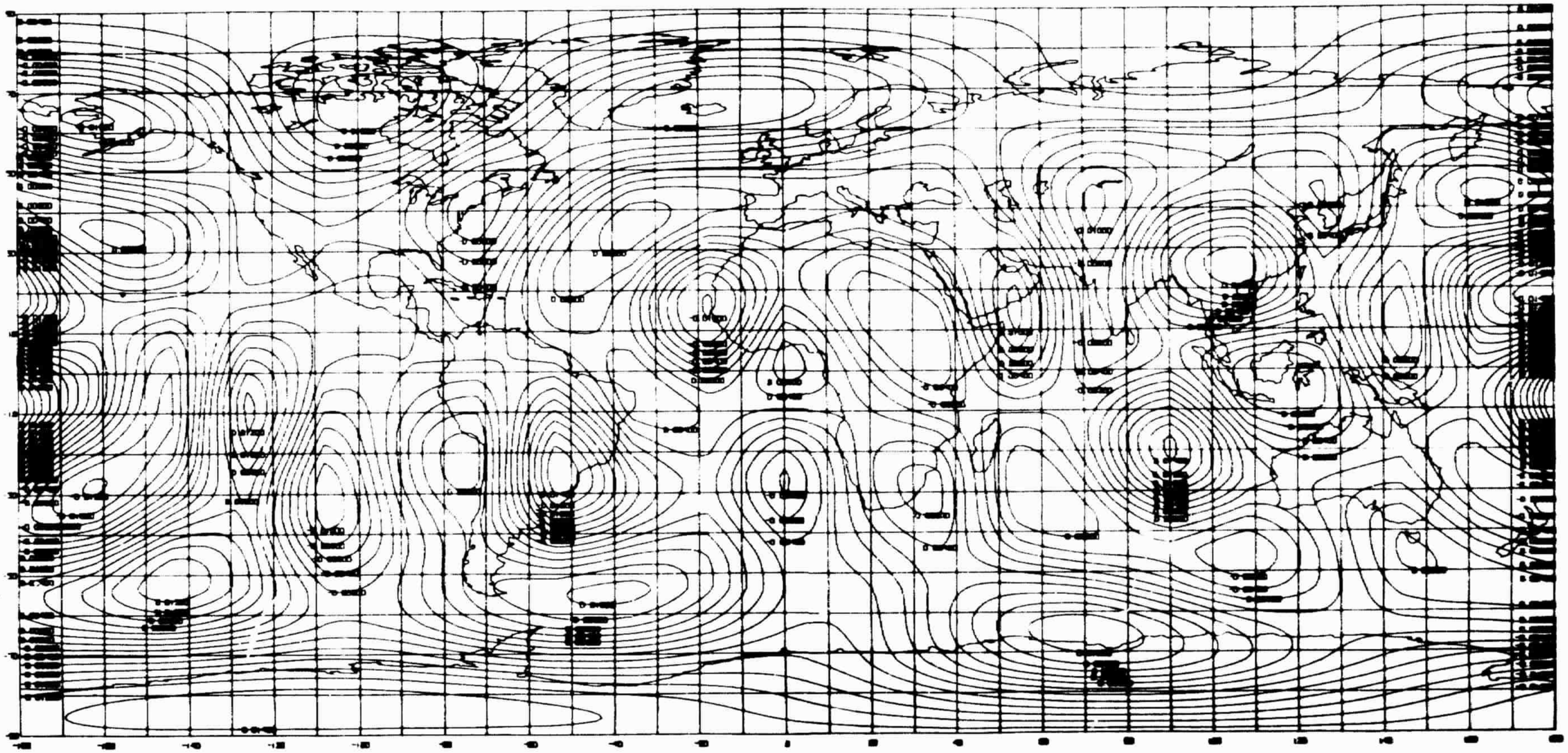

Figure 4.2. Surfaces of equal density perturbation at the surface of the earth, units of gm/cm^3. Minimum gravitational potential energy case.

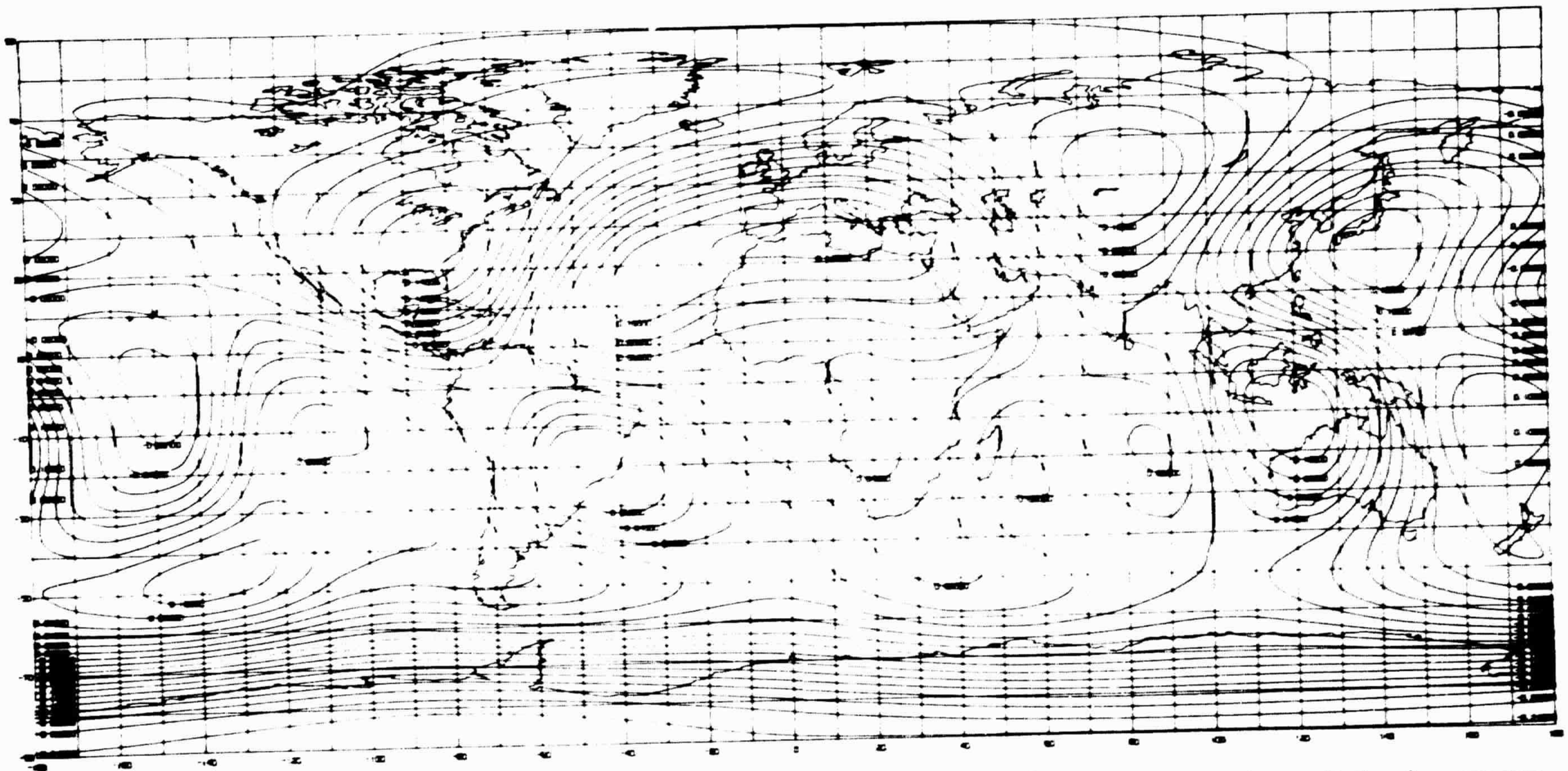

Figure 4.3. Surfaces of equal density perturbation at the surface of the earth, units of gm/cm^3. Minimum shear plus gravitational energy case.

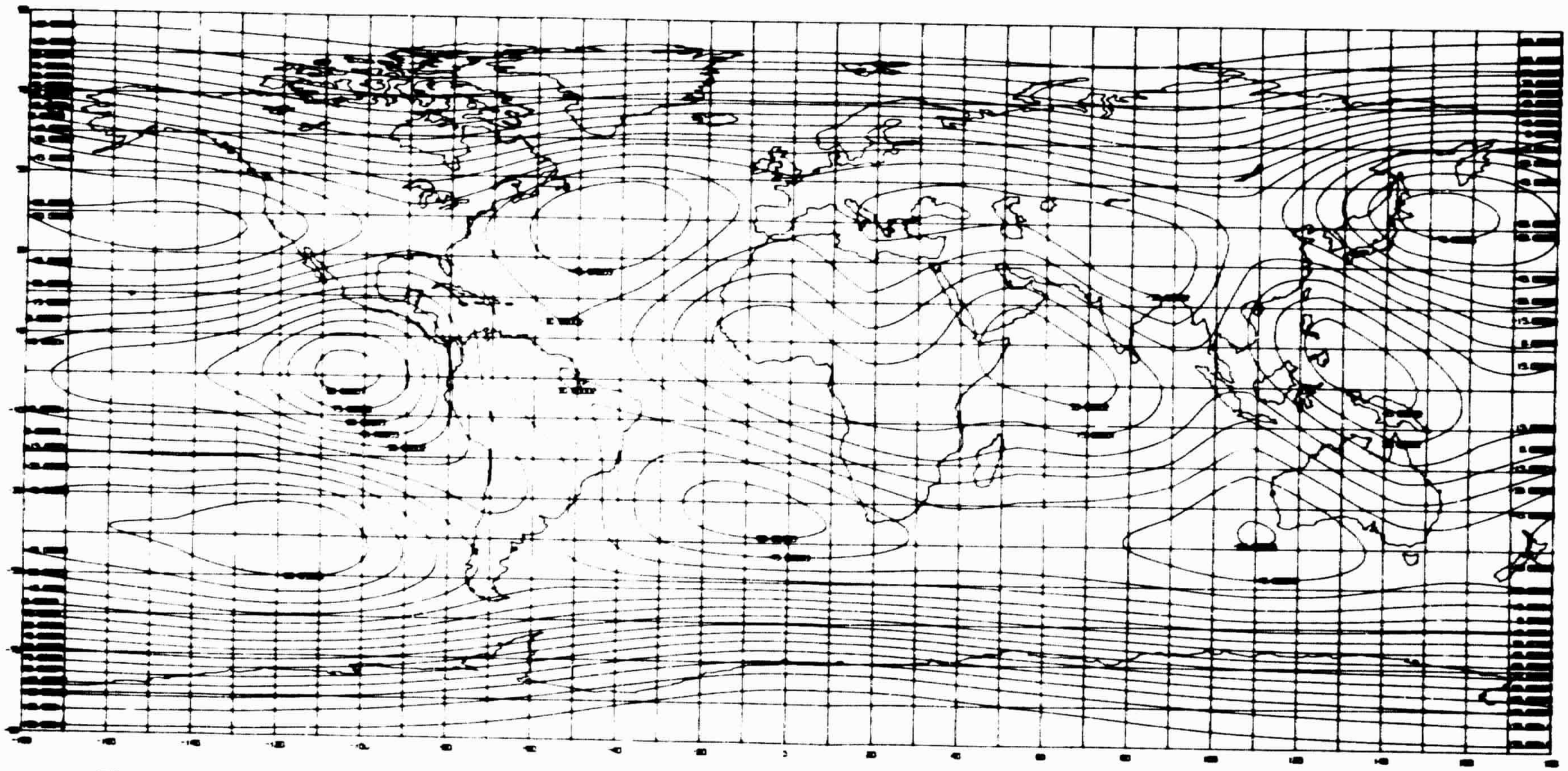

Figure 4.4. Surfaces of equal radial displacement at the surface of the earth, units of meters. Minimum shear strain energy case.

Figure 4.5. Surfaces of equal radial displacement at the surface of the earth, units of meters. Minimum gravitational potential energy case.

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Figure 4.6. Surfaces of equal radial displacement at the surface of the earth, units of meters. Minimum shear plus gravitational energy case.

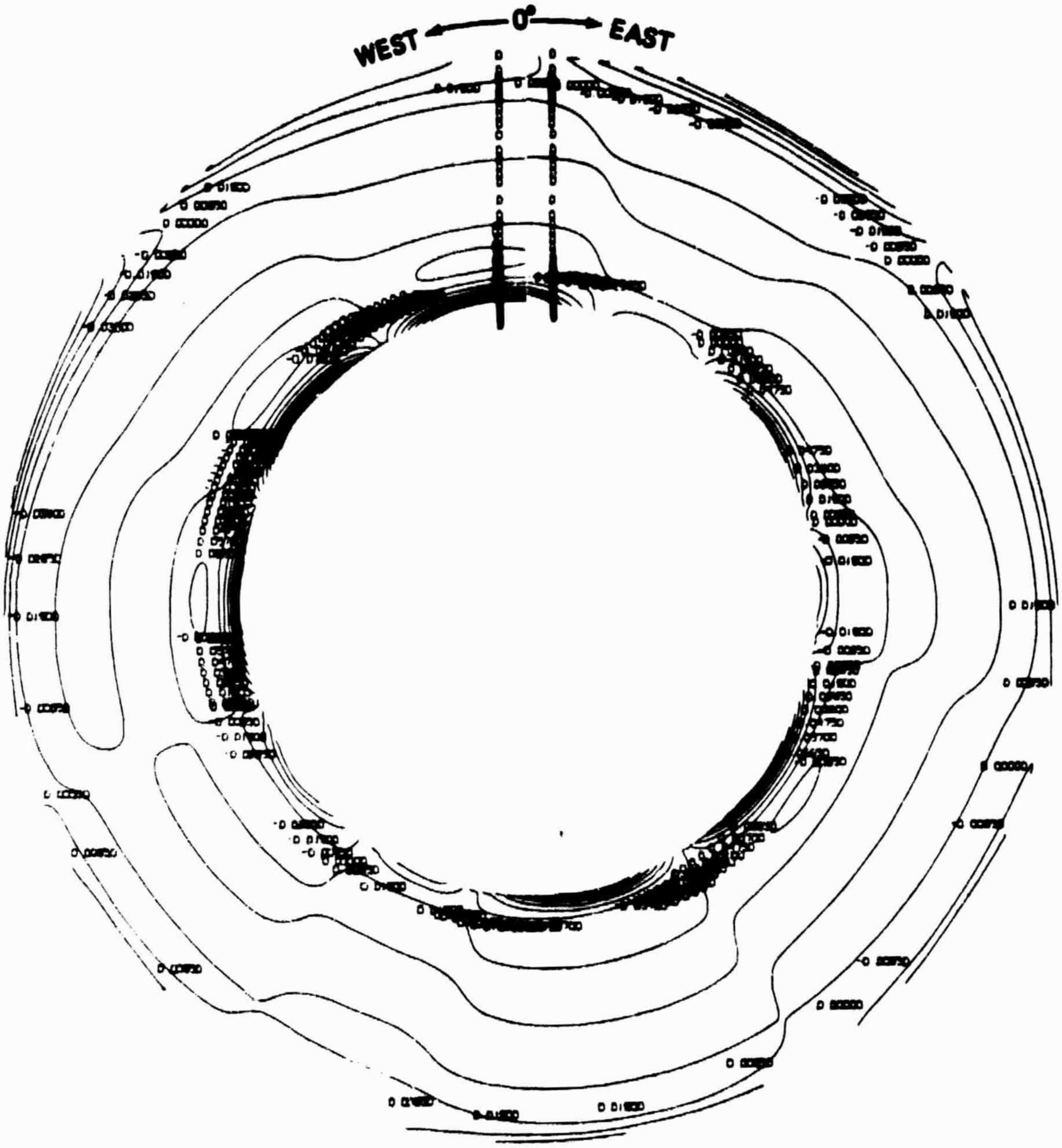

Figure 4.7. Surfaces of equal density perturbation on an equatorial cross-section of the earth, units of gm/cm^3. Minimum shear strain energy case.

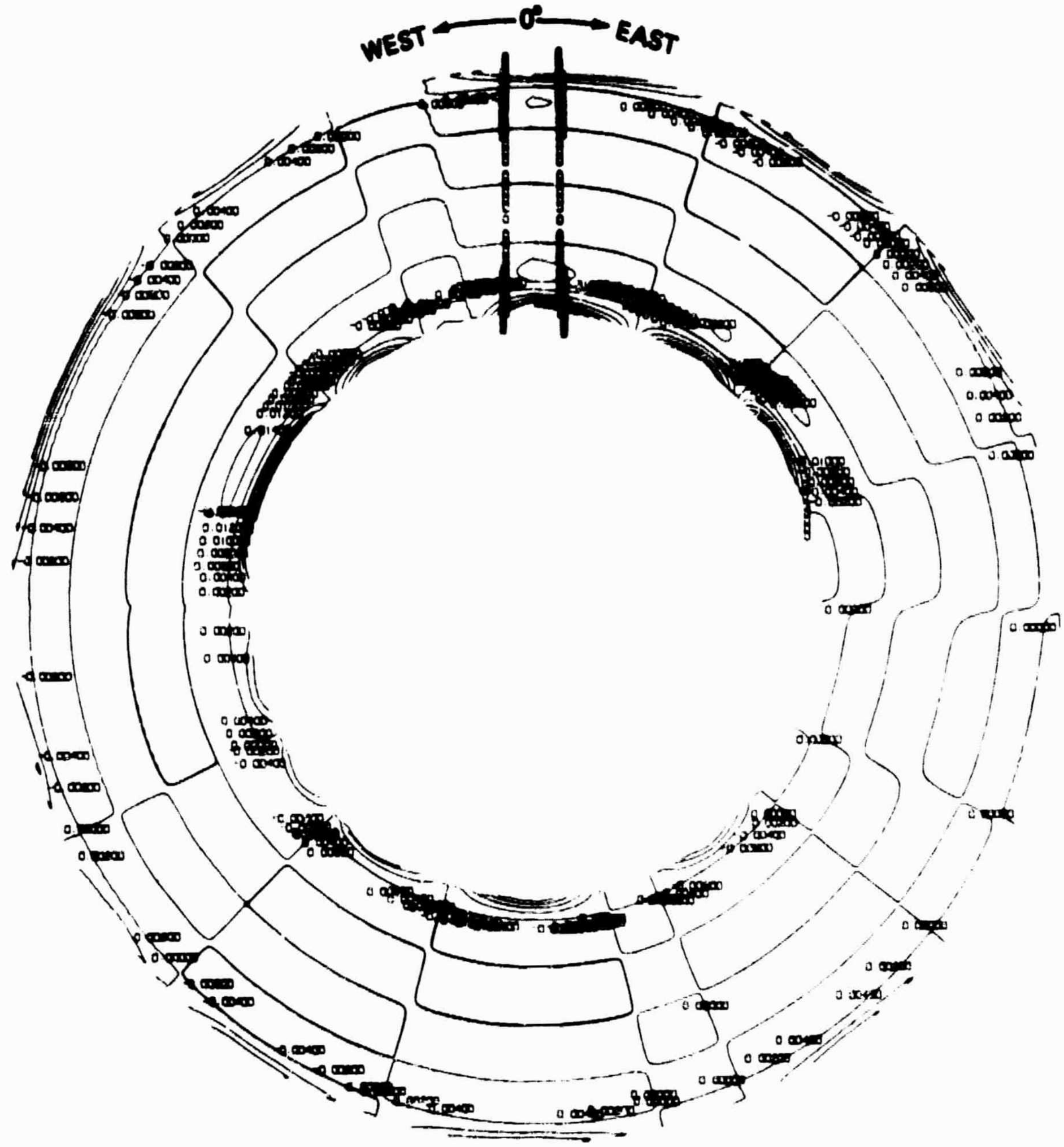

Figure 4.8. Surfaces of equal density perturbation on an equatorial cross-section of the earth, units of gm/cm^3. Minimum gravitational potential energy case.

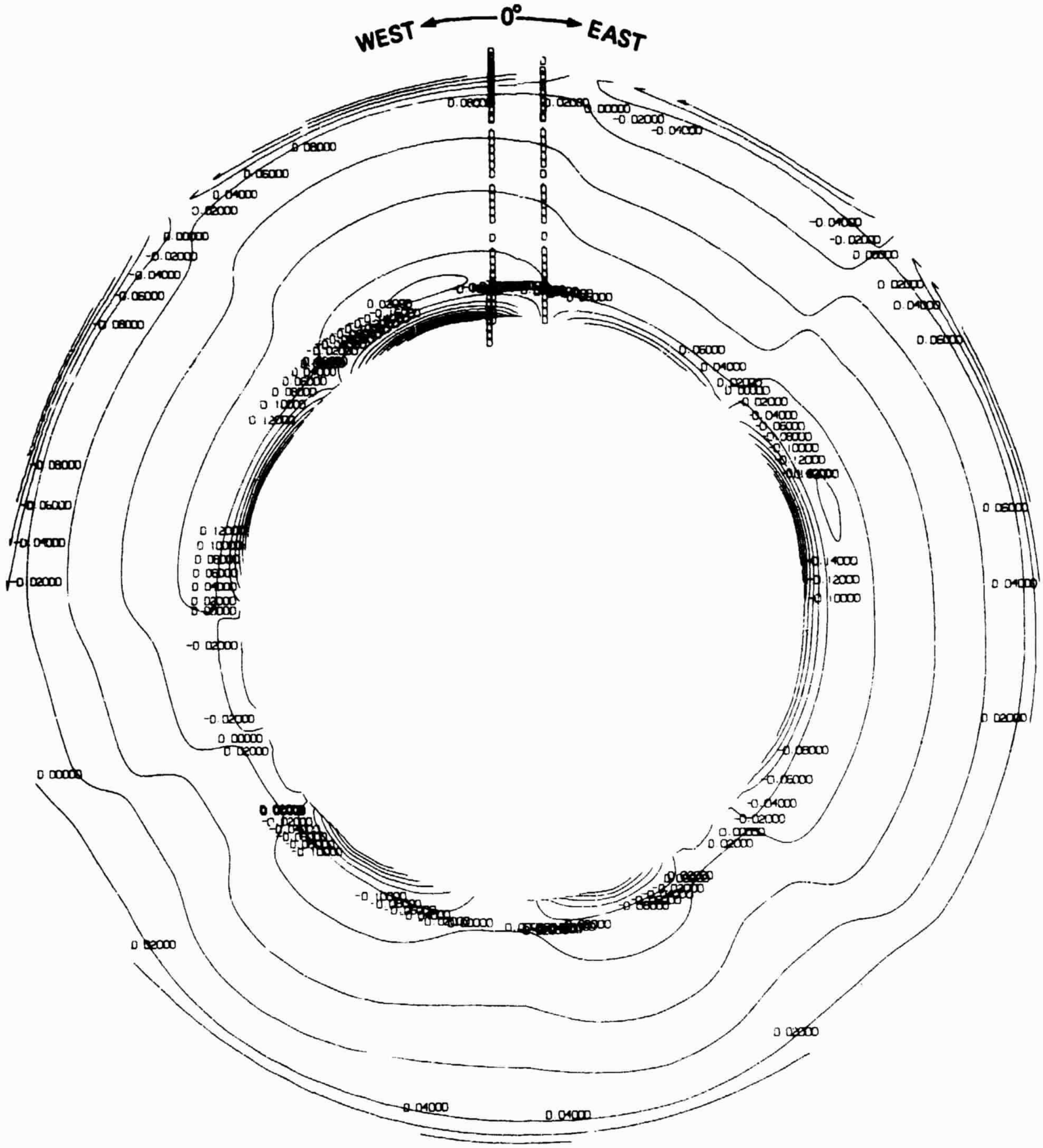

Figure 4.9. Surfaces of equal density perturbation on an equatorial cross–section of the earth, units of gm/cm^3. Minimum shear plus gravitational energy case.