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Bifilar Analysis User's Manual—Volume II

Sebastian J. Cassarino
Timoleon Mouzakis

SIKORSKY AIRCRAFT
Stratford, Ct 06602

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Sebastian J. Cassarino and Timoleon Mouzakis

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16. Abstract

This report describes the digital computer program developed to study the vibration response of a coupled rotor/bifilar/airframe coupled system.

The theoretical development of the rotor/airframe system equations of motion is provided in Reference 1 while the fuselage and bifilar absorber equations of motion are discussed in Appendix D.

The modular block approach used in the make-up of this computer program is described in Section 2. Section 3 provides descriptions of the input data needed to run the rotor and bifilar absorber analyses. Sample output formats are presented and discussed in Section 4. The results for four test cases, which use the major logic paths of the computer program, are presented in Section 5. In Section 6, the overall program structure is discussed in detail, including the segmentation procedure (overlay) needed to run the program on the NASA CDC computer system, the routine flow diagrams and a list of the COMMON blocks. Finally, in Section 7, the Fortran subroutines are described in detail.

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SECTION 1

SUMMARY

This report describes the digital computer program developed to study the vibration response of a coupled rotor/bifilar/airframe system.

The theoretical development of the rotor/airframe system equations of motion is provided in Reference 1 while the fuselage and bifilar absorber equations of motion are discussed in Appendix D.

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The work was conducted under NASA contract NAS1-15612, "Bifilar Analysis Study".

SECTION 2

PROGRAM DESCRIPTION

The bifilar absorber analysis was developed to provide the engineer with an analytical tool capable of rapid parametric vibration evaluation of the entire helicopter. A block diagram of the vibration analysis program is shown in Figure 1, where it can be seen that a modular approach has been adopted to form the main program. Modularization is achieved when each component block outputs the mass, damping and stiffness matrices or force vectors with the pertinent degrees-of-freedom (d.o.f.) including the 6 d.o.f. of the attachment point, i.e. 6 d.o.f. at the hub for rotor or bifilar absorber attachment, or 6 d.o.f. in the cabin or cockpit for fixed-system absorber attachment. Where two blocks merge together, the degrees-of-freedom of the attachment point are eliminated and replaced by fuselage modal degrees-of-freedom.

2.1 Analytical Model

Derivation of the coupled analysis is shown in Appendix D.

A description of the analytical model which consists of primarily rotor, fuselage, rotating-system absorbers and fixed-system absorbers is given below.

<u>Rotor</u>

The rotor system is represented by a modal approach which utilizes the equations of motion from Reference 1. The rotor blade degrees-of-freedoms which can be incorporated in the analysis are: up to four blade elastic modes (coupled flatwise/edgewise), up to 2 blade torsional blade modes (first mode represents a rigid blade while the second one is an elastic mode) and rigid blade flapping and lead-lag motions - a total of 8 blade modes which correspond to 24 d.o.f. (each mode has one symmetric and two cyclic components). The rotor/airframe coupling terms are incorporated in the analysis using 5 airframe modes corresponding to uncoupled fuselage longitudinal (x), lateral (y), vertical (z), roll (θ_x) and pitch (θ_y) motions - yaw motion (θ_z) is not included.

The major assumptions made in the development of the rotor system model are listed below:

1. Dynamic and aerodynamic effects assume small perturbations about steady initial values of the system generalized coordinates.

2. Aerodynamic forces are developed using strip theory.

3. Number of rotor blades must be greater than two due to the polar symmetry assumption made in the rotating system generalized coordinate transformations.

4. The following effects are not included in the rotor analysis:

   a. Forward flight aerodynamics
   b. Rotor speed d.o.f.
   c. Variable inflow over the rotor disc
   d. Unsteady aerodynamics
   e. Airframe yaw motion

Fuselage

The fuselage dynamic model is a set of linear modal equations which are provided in Appendix D. The computer program accepts inputs of system modal properties of up to 16 airframe modes.

Rotating-System Absorbers

The bifilar analysis includes linear and non-linear inplane rotorhead absorbers and linear vertical absorbers. The forced response analysis can use up to 5 types of linear absorbers (inplane plus vertical). A maximum of 12 non-linear inplane pendulums can be employed in the time history analysis. Viscous damping of the absorbers is assumed.

Fixed-System Absorbers

Fixed system absorbers are modeled in the analysis as a simple unidirectional spring-mass-damper system. The absorbers attachment point must be at a defined modal vector point. Provisions for up to 5 absorbers are provided for in the analysis.

2.2 Program Execution

The rotor/bifilar coupled program starts by calculating the dynamic and aerodynamic rotor/airframe matrices (assuming that rotor coupling has been requested) and couples them with the bifilar analysis fixed system dynamic matrices. Then, it expands the matrices to include the contributions from the fixed system absorbers and the linear inplane and vertical bifilar pendulums. At this point, a decision is made on the type of solution to be calculated: forced response or time-history. If the forced response solution is requested, then the generalized forces are calculated followed by the evaluation of the forced response solution. If the time-history solution is required, then the program proceeds to calculate the dynamic matrices of the non-linear inplane bifilars, adds them to the matrices from the linear analysis, solves for the acceleration vector and integrates it to obtain the velocity and displacement vectors. The final results are harmonically analyzed (up to 10 harmonics can be obtained) and printed out.

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Multiple cases, in which any number of input variables are changed, can be easily run (a detailed discussion can be found in Section 5 - TEST CASES RESULTS). If the rotor blade characteristics are not changed, then the rotor matrices are calculated only once, stored and used at a later time as needed.

Computer running time is highly dependent on the total system degrees-of-freedom used and the type of solution requested. The time-history solution requires considerably greater computer time than the forced response solution. Typical computer running times for the time history solution range from one to seven minutes while the forced response execution usually requires from a few seconds to one-half a minute (see Section 5.5 for more details).

When a nonsystem error occurs during the execution of a case, the program attempts to continue using the best data available; if the error is too fundamental for the case to be meaningful, then a partial data print-out, followed by an error message, is given before job termination.

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SECTION 3

INPUT DESCRIPTION

3.1 Rotor Aeroelastic Analysis Input

3.1.1 Computer Listing of Input for Rotor Aeroelastic Analysis

Symbol	Location	Input Item	Units
RHO	1	Air mass density	Lb sec**2/ft**4
VS	2	Speed of sound	Ft/sec
TL	3	Tip loss factor	Nd
VIP	4	Rotor axial velocity	Knots
OMEGAI	5	Rotor rotational speed	RPM
RIP	6	Rotor radius	Ft
EIP	7	Blade offset	Ft
BLADES	8	No. of blades - must be greater than 2	Nd
KBETA	9	Blade flapping hinge spring constant	Lb-in/rad
KGAMMA	10	Blade lag hinge spring constant	Lb-in/rad
GAMOI	11	Blade prelag angle - lag positive	Deg
BETOI	12	Blade precone angle - up positive	Deg
THETAO	13	Blade collective pitch angle at 75% radius	Deg
EB	14	Blade Young's modulus	Lb/in**2
YPH2	15	Distance along blade axis from center of rotation to push rod	
PHL	16	Distance from blade elastic axis to push rod - positive toward leading edge	In
ZETGAM	17	Fraction of critical lag damping	Nd
ZETPIT	18	Fraction of critical blade pitch damping - based on rotor speed	Nd
OMGI	19	Reference rotor speed for defining percent critical lag damping in ground resonance studies	RPM
XNEMOD	20	Number of blade bending modes - up to 4	Nd
---	21	Open	
ZETBLD	22-25	Fraction of critical damping of blade bending modes - based on mode frequencies	Nd
---	26-105	Open	
ALPHA1	106	Blade pitch/lag coupling	Nd

Control Switches

DUM1(1)	107	Printout of 30 X 30 rotor dynamic (3) and aerodynamic (2) matrices 0 - no ; 1 - yes
DUM1(2)	108	Printout of KXK (compressed) rotor matrices (dyn. + aero.) 0 - no ; 1 - yes
DUM1(3)	109	Punch out of KXK (compressed) rotor matrices (dyn. + aero.) 0 - no ; 1 - yes
DUM1(4)	110	Use of rotor matrices in bifilar anlaysis 0. do not use 1. use new rotor matrices -1. use previous rotor matrices
ROTEST	111	1. for main rotor 2. for tail rotor
---	112	Open
SYSDEF	113	System definition - ABCDEFGH. A - blade bending B - blade rigid body pitching C - blade rigid body flapping D - blade rigid body lagging E - fixed system modes F - blade elastic pitching G - set to 1 H - set to 1 Element = 0 to include = 1 to exclude
ROTDEF	114	Rotor definition - X. X = 1. - blade hinged flatwise and edgewise = 2. - blade cantilevered flatwise and edgewise = 3. - blade hinged flatwise, cantilevered edgewise = 4. - blade cantilevered flatwise, hinged edgewise = 5. - gimbaled rotor
ARTIC	115	Blade pitch input control - XY. X = 1 - pitch bearing follows blade out-of-plane root slope = 0 - pitch bearing remains in plane of hub or preconed position Y = 1 - pitch bearing follows blade inplane root slope = 0 - pitch bearing remains in vertical plane or prelagged position
---	116-118	Open
PRINT	119	Main printout control - X. X = 3. - A, basic calculations + dyn. and aero. integrals = 4. - B, A + blade frequency input = 5. - C, basic calculations only = 6. - D, B + blade frequency output = 7 or greater - same as for X = 3.

DUM2(1)	120	Propeller moment option in dynamic stiffness matrix 0. include propeller moment (default value) 1. exclude propeller moment
---	121- 124	Open
LAGKII	125	Blade lag damper option 0. include lag damper 1. exclude lag damper
		<u>Lag damper physical characteristics</u>
L1	126	L1, inches
L2	127	L2, inches
L3	128	L3, inches
L4	129	L4, inches
L5	130	L5, inches
L6	131	L6, inches
CLD	132	Damper damping coefficient, lb-sec/in.
KLD	133	Damper stiffness coefficient, lb/in.
ZETELP	134	Blade elastic pitch modal damping, nd
CP	135	Effective radius to cantilevered point in torsion (defaults to value of blade offset in location 7) , in.
---	136- 199	Open
BN	200	Number of elements in blade segment chart - up to 20. These are defined over the length of the blade only - the first segment is at the root of the blade-normally the last segment should be no more than 3 percent radius.
RR	201- 249	Segment lengths, inches

***

The following blade properties are input from the center of rotation to the blade tip at specified radial positions

***

NCHI	250	Number of elements in blade chord chart
CHORD	251- 349	Radius - in : chord - in
NATWI	350	Number of elements in blade aerodynamic twist chart
ATWIST	351- 449	Radius - in : twist - deg (twist down is positive and is zero at 75 percent radius)

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161447

NSTWI	450	Number of elements in blade structural twist chart
STWIST	451-549	Radius - in : twist - deg (twist down is positive. The twist at 75 percent radius is defined by the blade geometry consistent with the definition of aerodynamic twist)
NCGI	550	Number of elements in blade center of gravity chart
CG	551-649	Radius - in : CG - in (CG is positioned relative to and positive ahead of elastic axis)
NACI	650	Number of elements in blade aerodynamic center chart
AC	651-749	Radius - in : AC - in (AC is positioned relative to and positive ahead of elastic axis)
NEAI	750	Number of elements in blade elastic axis chart
EA	751-849	Radius - in : EA - in (EA is positioned relative to and positive aft of semi-chord)
		The following blade properties are defined for a specified radial segment
NGJFI	850	Number of elements in flexbeam torsional stiffness (cross-beam rotor design) chart
RGJF	851-949	GJ - lb-in**2 : segment length - in
NMBI	950	Number of elements in blade weight chart
RMB	951-1049	Weight - lb/in : segment length - in
NIEI	1050	Number of elements in blade edgewise second moment of area chart
RIE	1051-1149	IYY - In**4 : segment length - in
NIFI	1150	Number of elements in blade flatwise second moment of area chart
RIF	1151-1249	IXX - In**4 : segment length - in
NFM	1250	Number of elements in blade flatwise mass moment of inertia chart (about CG)
RFM&FMI	1251-1349	IF - lb-in-sec**2/in : segment length - in
NEM	1350	Number of elements in blade edgewise mass moment of inertia chart (about CG)
REM&EMI	1351-1449	IE - lb-in-sec**2/in : segment length - in
NTM	1450	Number of elements in blade torsional mass moment of inertia chart (about CG)
RTM&TMI	1451-1549	IT - lb-in-sec**2/in : segment length - in
NGJI	1550	Number of elements in blade torsional stiffness - GJ chart
RGJ	1551-1649	GJ - lb-in**2 : segment length - in

XBR Parameters

YA	1650	Distance from center of rotation to outer snubber, in.
YB	1651	Distance from center of rotation to inner snubber, in.
FRR	1652	Distance from center of rotation to flexbeam root, in.
---	1653-1669	Open

Tail Rotor Control System Parameters

PBM	1670	Weight at blade pushrod, lb.
---	1671-1674	Open
C1	1675	Damping associated with weight above, lb-sec/in.
---	1676-1679	Open
PBK	1680	Stiffness associated with weight above, lb/in.
---	1681-1766	Open

Main Rotor Control System Parameters

PBMM	1767	Weight at blade pushrod, lb.
---	1768	Open
C1M	1769	Damping associated with weight above, lb-sec/in.
---	1770-1772	Open
PBKM	1773	Stiffness associated with the weight above, lb/in.
---	1774-1778	Open
RSB	1779	Distance from center of rotation to pushrod connection on swashplate, in.
RS	1780	Distance from center of rotation to servo actuator connections on swashplate, in.
---	1781-1849	Open

Blade Section Aerodynamic Data

DRGDAT	1850-2749	Drag data
LIFDAT	2750-3649	Lift data
PMDAT	3650-4548	Pitching moment data

3.1.2 Description of Input for Rotor Aeroelastic Analysis

Additional information on the input parameters is provided in this section.

Each input quantity is given in the following format:

Location No., Quantity, Units.

Important details and comments.

1. Air Mass Density, lb sec^2/ft^4.

   If set to zero, all aerodynamic calculations are omitted.

2. Speed of Sound, ft/sec.

   Used to compute local blade Mach number for lift, drag, and pitching moment calculations.

3. Tip Loss Factor, Nondimensional.

   Provides lift and pitching moment loss in the blade tip region. Drag is not affected.

   Value should be equal to or greater than one minus the non-dimensional length of the blade tip segment. A value of one constitutes no loss.

4. Rotor Axial Velocity, Knots.

   Represents climb or sideslip velocity for a main or tail rotor respectively. For propellers this is the forward speed of the aircraft. Positive velocities are in the same direction as the thrust.

5. Rotor Rotational Speed, rpm.

   When calculating blade bending frequencies at low rpm's, computer time is greatly increased. It is suggested that rpm not be less than 50 cpm.

6. Rotor Radius, ft.

   Measured from center of rotation.

7. Blade Offset, ft.

   Distance from center of rotation to flap and lag hinge. These hinges are assumed to be coincident.

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8. Number of Blades, Nondimensional.

Any number greater than 2 since the analysis assumes that the rotors have polar symmetry. The analysis will execute for N = 2 but the results are incorrect.

9. Blade Flapping Hinge Spring Constant, lb in/rad.

If the rotor definition, location 114, stipulates a hinged root boundary condition, the flapping spring provides root flapping restraint in the calculation of the blade elastic modes. It is also used in the rigid-body flapping equation.

10. Blade Lag Hinge Spring Constant, lb in/rad.

Same comments as above with "lagging" substituted for "flapping".

11. Blade Prelag Angle, Degrees.

Lag positive.

12. Blade Precone Angle, Degrees.

Up positive.

13. Blade Collective Pitch, Degrees.

Aerodynamic blade angle at 75% radius. Leading edge up positive. If affects blade thrust and blade bending frequencies and mode shapes.

14. Blade Young's Modulus, lb/in^2.

Used in calculation of blade elastic modes and steady elastic deflections. Appears explicitly in steady elastic deflection calculations and therefore must have a value if the blades are flexible.

15. Distance Along Blade Axis From Center of Rotation to Blade Pushrod, in.

Used in calculation of pitch/flap coupling for rigid-body flapping and for pitch/flap and pitch/lag coupling for elastic blades.

16. Distance From Blade Elastic Axis to Pushrod, in.

Used in calculation of pitch/flap coupling for rigid body flapping, pitch/lag coupling for rigid body lagging, and pitch/flap and pitch/lag coupling for elastic blades. Positive toward leading edge.

09949

17. Fraction of Critical Lag Damping, Nondimensional.

Used only in the rigid-body lag equation. Based on uncoupled lag frequency. This value is ignored if the lag damper option is exercised (loc 125 set to 0).

18. Fraction of Critical Blade Pitch Damping, Nondimensional.

Used only in the blade pitch equation. Based on rotor speed. If, for example, at a given rotor speed the blade torsional frequency is 5 per rev and we wish to incorporate 10% critical damping, we would input 0.5.

19. Reference Rotor Speed, RPM.

Holds the rigid-body lag and pitch damping coefficients, C, constant at the value corresponding to that at the chosen reference rotor speed. If actual rotor speed variations are made, the percentage of critical damping will vary. If this is not desired, a zero should be input.

20. Number of Blade Bending Modes, Nondimensional.

Up to four elastic modes can be used. If rigid-body modes are also being used, the program automatically recognizes this and will provide the correct elastic modes. For example, if two bending modes are requested and rigid-body flapping is being used, the program finds the first three blade modes, eliminates that mode which corresponds to rigid-body flapping, and provides the remaining two modes. A zero locks out elastic modes.

22.-25. Fraction of Critical Damping of Blade Bending Modes, Nondimensional.

Used only in the blade bending equations. Based on modal frequencies.

106. Blade Pitch-Lag Coupling, Nondimensional.

Defined as degrees pitch-up per degress lead for rigid-body lagging motion, or degrees pitch-up per degree blade tip inplane lead angle measured at blade root for elastic modes. Lead/pitch-up positive.

97205

107. Printout option for 30 X 30 rotor matrices. The dynamic stiffness, damping and mass matrices and the aerodynamic damping and stiffness matrices are printed out if switch is set to 1.

108. Printout option for compressed rotor matrices. Same as above except that only those degrees-of-freedom requested are printed out if switch is set to 1.

109. Punchout option for compressed rotor matrices. This option was used before rotor was coupled to bifilar anlaysis internally in the program. It is excercised if set to 1.

110. Coupling switch governing use of rotor matrices in the bifilar analysis.

0. = do not use rotor matrices in the bifilar analysis
1. = calculate and use rotor matrices in the bifilar analysis
-1. = use previously calculated rotor matrices in the bifilar analysis

111. Control Switch for Main or Tail Rotor.

1 = Main Rotor
2 = Tail Rotor

113. System Definition.

Exercises primary control in the program. Overrides any contradictory controls. There is one exception: when location 114 equals 5, i.e., a gimbaled rotor. In this case location 114 has executive control whereby digits 1, 3 and 4 of the system definition are ignored. Up to 8 blade degrees-of-freedom (4 bending modes plus 2 torsional modes plus rigid body flapping and inplane motions) can be used in the rotor analysis.

114. Rotor Definition.

When equal to 1, 3, or 4, motion at the hinges is restrained by the springs in locations 9 and 10 as appropriate.

When equal to 5, gimbaled rotor, the program automatically uses rigid-body flapping, and 4 elastic modes with boundary conditions suitably selected to correctly define the first 5 gimbaled rotor modes. See Reference 1 for a complete explanation of this.

09767

Rotor Definition - X.

X = 1 - Blade Hinged Flatwise & Edgewise
= 2 - Blade Cantilevered Flatwise & Edgewise
= 3 - Blade Hinged Flatwise, Cantilevered Edgewise
= 4 - Blade Cantilevered Flatwise, Hinged Edgewise
= 5 - Gimbaled Rotor

115. Blade Pitch Input Control.

This simply determines whether inboard or outboard blade feathering is being employed. The feathering bearing is always at the root of the blade.

Blade Pitch Input Control - XY.

X = 1 - Pitch Bearing Follows Blade Out of Plane Root Slope
= 0 - Pitch Bearing Remains In Plane of Hub or Preconed Position
Y = 1 - Pitch Bearing Follows Blade Inplane Root Slope
= 0 - Pitch Bearing Remains In Vertical Plane or Prelagged Position

119. Main Printout Control.

Provides a graduated printout capability for debugging, etc.

It is generally good practice to use option 5 for the first case of any run in order to establish that all of the input is correct.

Main Printout Control - X.

X = 3. - A, Basic Calculations + Dynamic & Aerodynamic Integrals
= 4. - B, A + Blade Frequency Input
= 5. - C, Basic Calculations Only
= 6. - D, B + Blade Frequency Output
$\geq$ 7. - A (Same as X = 3.)

120. Propeller Moment Option - X.

The propeller moment contribution to the blade rigid torsion degree-of-freedom can be accounted for or neglected depending on the rotor blade design. Removal of the propeller moment is necessary when the rotor design employs blade counterweight devices. The propeller moment is given by:

$$M_P = \Omega^2 I_P \theta \text{ where}$$

$$I_P = \int_0^1 (I_E - I_F)\, dx$$

The inertia, I_P, is the difference between the blade edgewise and flatwise mass moments of inertia integrated along the blade span. Rotor speed is Ω and blade torsional deflection is given by θ. The moment affects only the dynamic stiffness matrix.

X = 0. - Include Propeller Moment
  = 1. - Do Not Include Propeller Moment

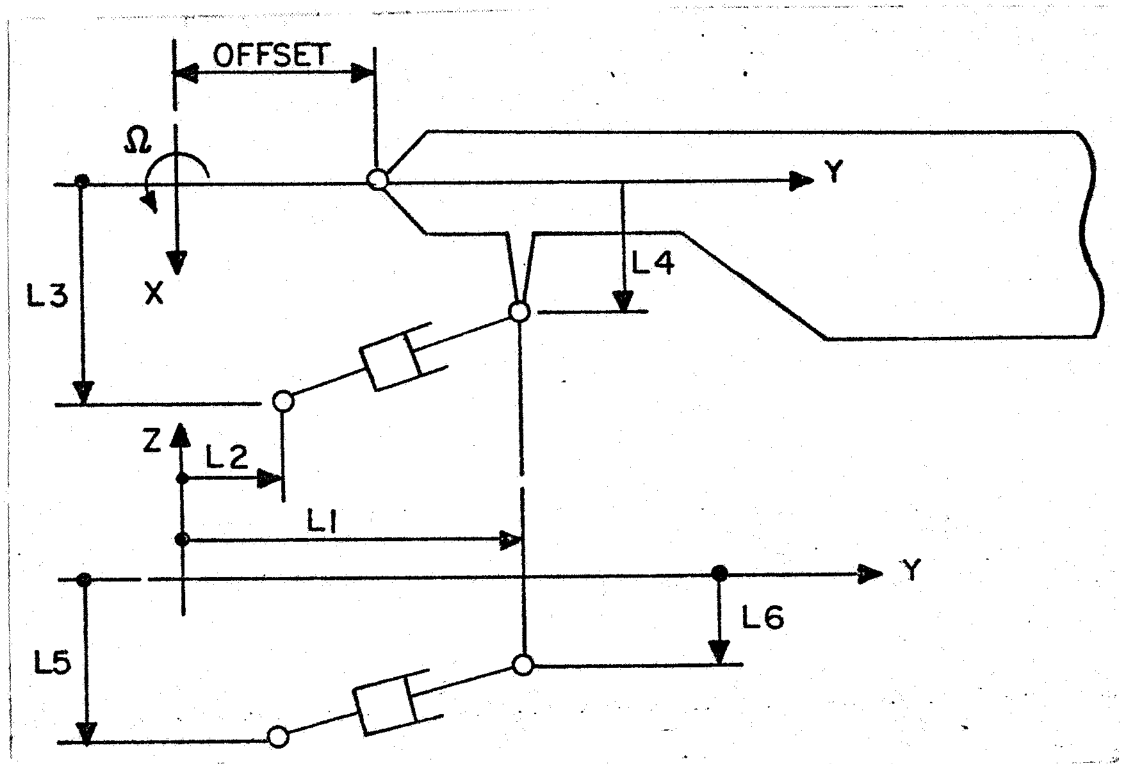
125. Blade Lag Damper Option - X.

X = 0. - Include Lag Damper
  = 1. - Do Not use Lag Damper

This option allows the use of a blade lag damper which includes all blade motions kinematics and damping and stiffness constants.

A typical arrangement is shown in the sketch below.

The positive convention for the lag damper distances is also shown in the sketch.



Blade Lag Damper Schematic

It should be noted that if a lag damper is used, then input to location 17 is ignored.

97232

126.-134. Refer to the lag damper sketch above.

135. Effective radius to cantilevered point in torsion, in.

All elastic deformation of the blade occurs outboard of this point. For example, in a conventional blade this might be the radius to the pitch horn; in a crossbeam design, it may be the radius to the outboard snubber (if the torque tube is assumed rigid) or it may be the radius to the push rod connection for a torsionally flexible torque tube. The user must use his own judgement in deciding where the cantilever point is situated. If the input cantilever point radius is less than the offset, it is set equal to the offset.

200. Number of Elements in Blade Segment Chart.

Up to 20 may be used. The choice of elements is important, particularly in relation to cross-beam type rotors where it is generally necessary to have smaller elements inboard where the spar is highly twisted. Thus, we may have, for example, 10 elements describing the inboard 25% of the blade, and 10 elements describing the remaining 75%.

201.-220. Segment Lengths, in.

These are defined over the length of the blade only. That is, their sum should equal the blade radius minus the offset. The tip segment should generally be of the order of (1-tip loss factor) times the blade radius.

General Comments for Inputs 250 to 849.

All of these quantities are input as values corresponding to a radial position. The first value is always at the center of rotation and the last at the tip. Linear interpolation is used between adjacent values. Each chart can have up to 49 pairs of radius/value coordinates.

250.-349. Blade Chord, in.

350.-449. Blade Aerodynamic Twist, deg.

Defined positive nose down and must be zero at 75% radius. This distribution is used in the calculation of aerodynamic effects.

16 1450

450.-549. Blade Structural Twist, deg.

Defined positive nose down and will have a value at 75% radius consistent with the geometric properties of the blade. This distribution is used in the calculation of the blade elastic modes, steady elastic deflections, and blade center of gravity offset effects.

550.-649. Blade Center of Gravity, in.

Positioned relative to, and defined positive ahead of, elastic axis.

650.-749. Blade Aerodynamic Center, in.

Positioned relative to, and defined positive ahead of, elastic axis.

750.-849. Blade Elastic Axis, In.

Positioned relative to, and defined positive aft of, blade semi-chord (for a normal blade, input = - chord/4).

<u>General Comments for Inputs 850 to 1649</u>.

All of these quantities are input as values over a segment length. If an offset exists, the first pair should be "zero, offset". The total of the segment lengths should equal the rotor radius. The weight and stiffness charts can have up to 49 value/segment length pairs. The mass moment of inertia charts can have up to 24 value/segment length pairs.

850.-949. Flexbeam Torsional Stiffness Distribution.

Stiffness: lb-in^2, Segment Length: in.

If an effective offset is input, stiffness may be set to zero over this distance.

161449

950.-1049. Blade Weight Distribution.

Weight: lb/in, Segment Length: in.

1050.-1149. Blade Flatwise Area Moment Distribution.

Area Mom: in^4, Segment Length: in.

1150.-1249. Blade Edgewise Area Moment Distribution:

Area Mom: in^4, Segment Length: in.

1250.-1299. Blade Edgewise Mass Moment of Inertia Distribution.

Edg. Mass Mom: lb in sec^2/in, Segment Length: in.

1350.-1399. Blade Flatwise Mass Moment of Inertia Distribution.

Flat. Mass Mom: lb in sec^2/in, Segment Length: in.

1450.-1499. Blade Torsional Mass Moment of Inertia Distribution.

Tors. Mass Mom: lb in sec^2/in, Segment Length: in.

1550.-1649. Blade Torsional Stiffness Distribution.

Stiffness: lb-in^2, Segment Length: in.

Zeros may be input up to the cantilever point radius (loc. 135). Do not include flexbeam stiffness in this chart.

1650. Distance From Center of Rotation to Outer Snubber, in.

1651. Distance From Center of Rotation to Inner Snubber, in.

Locations 1650 and 1651 have a value other than zero only when cross-beam type rotors are being treated. Nonzero values instruct the program to apply the collective pitch at the outer snubber location. The spar, or structural, blade angle inboard of the outer snubber location is then made equal to the sum of the collective pitch (which is reduced linearly from the value at the outer snubber to zero at the blade root) and the input structural twist. The aerodynamic blade angle inboard of the outer snubber is made equal to the value at the outer snubber.

The torque tube is assumed to be rigid in calculations of pitch/flap coupling.

1652. Distance From Center of Rotation to Flexbeam Root, in.

This is the radius at which torsional deformations of the flexbeam may be assumed to be zero. This radius is also used in defining the twist distribution for blade bending mode computations.

Tail Rotor Control System Parameters

1670. Weight at Blade Pushrod, lb.

1675. Damping Associated With Weight Above, lb-sec/in.

1680. Series Stiffness of One Arm of Pitch Beam, Pushrod, and Pitch Horn for Pitch Beam Assumed Cantilevered at its Center, lb/in.

Main Rotor Control System Parameters

1767. Weight at Blade Pushrod, lb.

1769. Damping Associated With Weight Above, lb-sec/in.

1773. Stiffness Associated With Pushrod, Swashplate Connection, Pitch Horn, etc..., lb/in.

1779. Distance From Center of Rotation to Pushrod Connection on Swash Plate, in.

1780. Distance From Center of Rotation to Servo Actuator Connection on Swash Plate, in.

Blade Section Aerodynamic Data

Drag, lift, and pitching moment data are each input in sets of CD, CL, or CM pairs corresponding to up to 12 Mach numbers.

Any or all of the data charts can be omitted.

In each chart there are up to 35 pairs of angle-of-attack and CD, CL, or CM data corresponding to each Mach number.

A total of 75 locations are allocated to each Mach number.

Mach numbers and angles-of-attack are input in ascending order.

If charts are employed use no less than 2 Mach numbers and 5 pairs of data per chart.

The input angle-of-attack range should exceed the expected range for the condition being analyzed.

Unsymmetric airfoil data can be used.

Only one set of airfoil data can be used.

Example of input format:

1850	Number of pairs for first Mach number
1851	First Mach number
1852-1924	Angle-of-attack: CD pairs
1925	Number of pairs for second Mach number
1926	Second Mach number
1927-1999	Angle-of-attack: CD pairs
Etc.	

1850.-2749.	Drag Data.
2750.-3649.	Lift Data.
3650.-4549.	Pitching Moment Data.

3.2 Bifilar Analysis Input

3.2.1 Computer Listing of Input for Bifilar Analysis

COMPUTER LISTING OF INPUT TO THE BIFILAR ANALYSIS

SYMBOL	LOCATION	DESCRIPTION	UNITS
NINBF	1	KINDS OF INPLANE BIFILARS =< 5	
NAP	2	NUMBER OF AIRCRAFT STATION POINTS TO EVALUATE RESPONSE =< 4	
WF	3	FORCING FREQUENCY	CYCLES/REV
NIMP	4	DIMENSION OF IMPEDANCE MATRIX (LOC 4 = LOC 9 * 2)	
------	5	OPEN	
XNARM	6	TOTAL NUMBER OF INPLANE BIFILARS =< 10	
OMEGAR	7	ROTOR SPEED	RPM
NFABS	8	NUMBER OF FIXED SYSTEM ABSORBERS =< 5	
NF	9	NUMBER OF FIXED SYSTEM MODES =< 16	
IWRITE	10	=1. GET EXTENSIVE PRINTOUT	
------	11	OPEN	
NVBIF	12	KINDS OF VERTICAL BIFILARS =< 5	
XNARM	13	TOTAL NUMBER OF VERTICAL BIFILARS=< 10	
RHM	14	=1. CALCULATE ROTOR HEAD MOTION ONLY	
ICHECK	15	=1. WRITE M,C,K MATRICES AND IMPEDANCE	
------	16	OPEN	
ROTPNT	17	=0. NO PRINTOUT OF ROTOR MATRICES =1. GET PRINTOUT OF ROTOR MATRICES	
NLBFSW	18	=0. NO NON-LINEAR INPLANE BIFILARS =1. USE NON-LINEAR INPLANE BIFILARS	
------	19	OPEN	
		FIXED SYSTEM ABSORBER DATA	
FABSM	20-29	FIXED SYSTEM ABSORBER MASS ARRAY	BUGS
FABSWN	30-39	FIXED SYSTEM ABSORBER FREQUENCY ARRAY	CPM
FABSD	40-49	FIXED SYSTEM ABSORBER DAMPING ARRAY	
		FIXED SYSTEM MODAL DATA	
XMG	50-69	GENERALIZED MODAL MASS ARRAY	BUGS
XWN	70-89	GENERALIZED MODAL FREQUENCY ARRAY	CPM
XDP	90-109	GENERALIZED MODAL DAMPING ARRAY	
		FORCES AND MOMENTS DATA	
FHC	110-119	COSINE COMPONENT OF HUB FORCES & MOMENTS	LB & IN-LB
FHS	120-129	SINE COMPONENT OF HUB FORCES & MOMENTS	"
FTC	130-139	COSINE COMPONENT OF TAIL ROTOR LOADS	"

```
FTS       140-149  SINE   COMPONENT OF TAIL ROTOR LOADS          "
FFC       150-159  COSINE COMPONENT OF HORIZONTAL TAIL LOADS     "
FFS       160-169  SINE   COMPONENT OF HORIZONTAL TAIL LOADS     "
FEC       170-179  COSINE COMPONENT OF LOADS OF ANOTHER POINT    "
FES       180-189  SINE   COMPONENT OF LOADS OF ANOTHER POINT    "
-----------------------------------------------------------------------
                   INPLANE  BIFILAR  PENDULUMS  DATA
-----------------------------------------------------------------------
BIFM      190-199  INPLANE BIFILAR MASS ARRAY                   BUGS
BARM      200-209  BIFILAR ARM FROM CENTER OF ROTATION          IN
BIFWN     210-219  INPLANE BIFILAR TUNING FREQUENCY ARRAY       CYCLES/REV
BIFDAM    220-229  INPLANE BIFILAR DAMPING ARRAY
-------------------------------------------------------------------
                   VERTICAL  BIFILAR  PENDULUMS  DATA
--------------------------------------------------------------------
BIFM      230-239  VERTICAL BIFILAR MASS ARRAY                  BUGS
BIFARM    240-249  BIFILAR ARM FROM CENTER OF ROTATION          IN
BIFWN     250-259  VERTICAL BIFILAR TUNING FREQUENCY ARRAY      CYCLES/REV
BIFDAM    260-269  VERTICAL BIFILAR DAMPING ARRAY
---------------------------------------------------------------------
------    270-449  OPEN
XPHI      450-549  TRANSFER MATRIX TO MODAL COORDINATES
                   FOR MAIN ROTOR IMPEDANCE (6 X NF)
-----------------------------------------------------------------------
      MODE SHAPES OF 2 AIRCRAFT STATIONS WHERE LOADS ARE APPLIED
-----------------------------------------------------------------------
XPF       550-649  FIRST  AIRCRAFT STATION (6 X NF)
XPE       650-749  SECOND AIRCRAFT STATION (6 X NF)
-----------------------------------------------------------------------
           MODE SHAPES OF FIXED SYSTEM ABSORBERS & TAIL ROTOR HUB
-----------------------------------------------------------------------
XPHABS    750-849  FIXED SYSTEM ABSORBER COUPLING MATRIX (NFABS X NF)
                   (20 SPACES RESERVED FOR EACH ABSORBER)
XPT       850-949  TAIL ROTOR HUB TRANSFER MATRIX
------    950-999  OPEN
-----------------------------------------------------------------------
                MODE SHAPES OF AIRCRAFT STATION POINTS
-----------------------------------------------------------------------
XPHAP   1000-1399  COUPLING MATRIX OF AIRCRAFT STATION POINTS
                             (6 X NF X NAP)
                   1000-1099 FOR FIRST  AIRCRAFT STATION
                   1100-1199 FOR SECOND AIRCRAFT STATION
                   1200-1299 FOR THIRD  AIRCRAFT STATION
                   1300-1399 FOR FOURTH AIRCRAFT STATION
------ 1400-1489   OPEN
-----------------------------------------------------------------------
                      PRINTOUT OPTIONS
-----------------------------------------------------------------------
                   =0. PRINTOUT IS NOT WANTED
                   =1. PRINTOUT IS WANTED
              ************************************
FSPNT    1490      A =     FIXED SYSTEM MATRICES
FSRPNT   1491      B = A + ROTOR MATRICES
FSAPNT   1492      C = B + FIXED SYSTEM ABSORBERS
IBPPNT   1493      D = C + INPLANE BIFILAR PENDULUMS
VBPPNT   1494      E = D + VERTICAL BIFILAR PENDULUMS
IBQPNT   1495      INPLANE BIFILAR  MATRICES ONLY (9 X 9)
VBQPNT   1496      VERTICAL BIFILAR MATRICES ONLY (9 X 9)
IGAMMA   1497      PRINTOUT OF "GAMMAS" (DEGREES-OF-FREEDOM)
ICUSE    1498      =0. DO NOT USE INITIAL CONDITIONS OPTION
```

```
                    =1. USE INITIAL CONDITIONS FROM PREVIOUS CASE
                    (SEE LOCATIONS 1720-1939)
------ 1499         OPEN

***********  END OF INPUT FOR LINEAR BIFILAR ANALYSIS  ****************

********  INPUT FOR NON-LINEAR INPLANE  BIFILARS ANALYSIS  ************

-------------------------------------------------------------------------
           INPLANE BIFILAR PENDULUMS CHARACTERISTICS
-------------------------------------------------------------------------
BM      1500-1519  BIFILAR WEIGHTS                               LBS
DP      1520-1539  BIFILAR DAMPING
WP      1540-1559  BIFILAR NATURAL FREQUENCIES                   CYCLES/REV
BR      1560-1579  ARM DISTANCES FROM CENTER OF ROTATION         IN
PSIR    1580-1599  RELATIVE AZIMUTH LOCATIONS                    DEGREES
-------------------------------------------------------------------------
               HUB FORCES & MOMENTS HARMONICS INPUT
-------------------------------------------------------------------------
               FORMAT IS  " COS(K*PSI) + SIN(K*PSI) "
               WHERE K=1 THROUGH A MAXIMUM VALUE OF 10
     **********************************************************
FXX     1600-1619  LONGITUDINAL HUB FORCE    (X)                 LBS
FYY     1620-1639  LATERAL      HUB FORCE    (Y)                 LBS
FZZ     1640-1659  VERTICAL     HUB FORCE    (Z)                 LBS
XXM     1660-1679  ROLL         HUB MOMENT   (THETAX)            IN-LB
YYM     1680-1699  PITCH        HUB MOMENT   (THETAY)            IN-LB
ZZM     1700-1719  YAW          HUB MOMENT   (THETAZ)            IN-LB
-------------------------------------------------------------------------
GAMMA   1720-1739  INITIAL BIFILAR ANGULAR DISPLACEMENT          RADIANS
GAMMAD  1740-1759  INITIAL BIFILAR ANGULAR VELOCITY              RAD/SEC
PSIP    1760       FORCE RAMP FACTOR( 3. IS RECOMMENDED)
DPSI    1761       DELTA AZIMUTH ANGLE                           DEGREES
FPSI    1762       MAXIMUM AZIMUTH ANGLE                         DEGREES
NBIF    1763       NUMBER OF BIFILARS ( MAX=12)
NHFH    1764       NO. OF INPUT HUB FORCE HARMONICS (MAX=10)
XNH     1765       NO. OF OUTPUT STATE VARIABLES HARMONICS(MAX=10)
PSI     1766       INTEGRATION VARIABLE (NOT AN INPUT ITEM)
NFP     1767       NUMBER OF POINTS ON AIRCRAFT WHERE FORCE
                   IS APPLIED BESIDES ROTOR HEAD (=< 2)
                   (MODE SHAPES ARE INPUT IN  LOC. 550-749)
-------------------------------------------------------------------------
               HUB & STATE VARIABLES INITIAL VALUES
-------------------------------------------------------------------------
HUBD    1768-1773  HUB INITIAL DISPLACEMENTS
HUBV    1774-1779  HUB INITIAL VELOCITIES
STDP    1780-1859  STATE VARIABLES INITIAL DISPLACEMENTS
TVL     1860-1939  STATE VARIABLES INITIAL VELOCITIES
-------------------------------------------------------------------------
    FORCES & MOMENTS HARMONICS INPUT OF 2 ADDITIONAL AIRCRAFT POINTS
-------------------------------------------------------------------------
               FORMAT IS  " COS(K*PSI) + SIN(K*PSI) "
               WHERE K=1 THROUGH A MAXIMUM VALUE OF 10
               (MODE SHAPES ARE INPUT IN LOC. 550-749)
              ***************************************
P1FX    1940-1959  POINT 1  LONGITUDNAL      (X)                 LBS
P1FY    1960-1979     "     LATERAL          (Y)                 LBS
P1FZ    1980-1999     "     VERTICAL         (Z)                 LBS
P1MX    2000-2019     "     ROLL             (THETAX)            IN-LB
P1MY    2020-2039     "     PITCH            (THETAY)            IN-LB
```

```
P1MZ    2040-2059     "      YAW               (THETAZ)          IN-LB
P2FX    2060-2079  POINT 2   LONGITUDINAL      (X)               LBS
P2FY    2080-2099     "      LATERAL           (Y)               LBS
P2FZ    2100-2119     "      VERTICAL          (Z)               LBS
P2MX    2120-2139     "      ROLL              (THETAX)          IN-LB
P2MY    2140-2159     "      PITCH             (THETAY)          IN-LB
P2MZ    2160-2179     "      YAW               (THETAZ)          IN-LB
----    2180-2199  OPEN
------------------------------------------------------------------------
             SUCCESSIVE CASES CONTROL SWITCH
------------------------------------------------------------------------
CODE    2200       =1. COMPLETES LAST CASE
                   =0. GOES BACK TO ROTOR ANALYSIS FOR NEXT CASE
------------------------------------------------------------------------

****************  END OF INPUT LOCATIONS  *****************************

------------------------------------------------------------------------
```

0996I

3.2.2 Description of Input for Bifilar Analysis

Additional information on the input parameters is provided in this section.

Each input quantity is given in the following format:

Location No., Quantity, Units
Important Details and Comments

Linear Analysis Input (Loc 1-1498)

1. Kinds of Inplane Bifilars, Nondimensional.

Up to 5 kinds of inplane bifilars can be used in the analysis if no vertical bifilars are present. The combined number of kinds of inplane and vertical bifilars cannot be greater than 5. This corresponds to a maximum number of 15 degrees-of-freedom (including collective and 2 cyclic motions). Ex. loc 1=2 if one set of 3P and one set of 5P inplane bifilars are employed.

2. Aircraft Station Points, Nondim.

Due to computer storage requirements, a maximum of 4 points can be used.

3. Forcing Frequency, Cycles/Rev.

Frequency at which the bifilars will respond divided by the rotor speed.

4. Impedance Matrix Dimension, Nondim.

This location is 2 times the number of fixed system modes loaded in location 9.

6. Number of Inplane Bifilars, Nondim.

The actual number of inplane bifilars present in each kind as defined in loc. 1. It should be noted that, if more than one kind of inplane bifilars is used, then the number of bifilars in each kind has to be the same. The maximum number allowed is 10.

7. Rotor Speed, RPM.

Used to non-dimensionalize frequency units.

8. Number of Fixed System Absorbers, Nondim.

The total number of fixed system absorbers is limited to 5.

9. Number of Fixed System Modes, Nondim.

The maximum number of fixed system modes is 16 which allows for 6 rigid body modes (if desired) and 10 flexible modes.

Rigid airframe modes should not be used in the non-linear bifilar analysis.

Program will not execute if number of modes is zero.

10. Extensive Printout Option, Nondim.

Set this location to 1. to obtain additional printout for debugging use.

12. Kinds of Vertical Bifilars, Nondim.

Up to 5 kinds of vertical bifilars can be used if no inplane bifilars are present. Otherwise, the total of the two kinds is 5.

13. Number of Vertical Bifilars, Nondim.

Actual number of vertical bifilars present in each kind as defined in loc. 12. Same number of bifilars is assumed for each kind. Maximum number is 10.

14. Rotor Head Motion Switch, Nondim.

If this control is set to 1. then only the rotor head motion will be calculated.

15. Additional Printout Option, Nondim.

If set to 1., stiffness, damping and mass matrices are printed out for debugging purposes.

17. Rotor Matrices Printout Option, Nondim.

If set to 1., the input matrices to the bifilar anlaysis from the rotor aeroelastic program are printed out. Order is stiffness, damping and mass. The stiffness and damping matrices include aerodynamics if air density is non-zero.

18. Linear/Non-Linear Analysis Option, Nondim.

If set to zero, then linearized inplane bifilar equations of motion are used and the forced response of all components is calculated. Sample runs using 16 and 28 degrees-of-freedom required 3 seconds and 25 seconds respectively.

If set to 1., then the full non-linear inplane bifilar equations of motion are used and a time history solution is calculated. Computer time for the non-linear option is highly dependent on the number of degrees-of-freedom used. A sample run with 17 d.o.f. took 1 minute and 8 seconds requiring 13 rotor revolutions for convergence of the bifilar motions. A sample run with 29 d.o.f. took 7 minutes and 7 seconds of computer time and required 16 rotor revolutions for convergence.

20.-29. Fixed System Absorber Masses, Lb-sec^2/in. or Bugs

Absorber weight in pounds divided by 386.4. Although 10 locations are provided, only 5 can be used (see loc. 8).

30.-39. Fixed System Absorber Frequencies, Cpm

40.-49. Fixed System Absorber Damping, Nondim.

Damping associated with the absorber i.e., .01 corresponding to 1% critical.

50.-69. Fixed System Generalized Masses, Lb-sec^2/in or Bugs

Maximum number of masses is actually 16 (see loc. 9) although 20 computer locations have been allowed.

70.-89. Fixed System Generalized Frequencies, Cpm

90.-109. Fixed System Generalized Damping, Nondim.

Forces and Moments Data - General Comments, Lb & In-lb

110.-189. The Forces and Moments are Defined in the Fixed System.

The order of the forces and moments input data is: x, y, z, θ_x, θ_y, θ_z, which correspond to longitudinal, lateral, vertical, roll, pitch and yaw motions respectively. The coordinate system used is a right-handed (system with the x-axis defined as positive aft and y-axis as positive out of the right wing).

Thus only six locations are needed to define an input force or moment although 10 computer locations have been allocated.

Examples of input of hub forces (loc. 110-129):

a) For a 4P inplane bifilar, to load a pure 5P hub force, then F_X (cosine) = $-F_y$ (sine) (loc. 110 = - loc. 121) and F_y (cosine) = F_X (sine) (loc. 111 = loc. 120).

b) To load a pure 3P hub force, then F_X (cosine) = F_y (sine) (loc. 110 = loc. 121) and F_y (cosine) = $-F_X$ (sine) (loc. 111 = - loc. 120).

Additional forces/moments inputs can be specified for the tail rotor (loc. 130-149), the horizontal tail (150-169) and one other arbitrary point (loc. 170-189).

190.-199. Inplane Bifilar Masses, Lb-sec^2/in. or Bugs

Bifilar weights divided by 386.4 are loaded for each kind of bifilar used (see loc. 1). The limit on the number of bifilar kinds which can be used is 5 although 10 computer locations are available.

200.-209. Inplane Bifilar Arm From Center of Rotation, In.

210.-219. Inplane Bifilar Tuning Frequency, Cycles/Rev

The linear inplane bifilar tuning frequency is defined by

$F^2 = R/r$ where R = bifilar arm (loc. 200-209)
r = bifilar distance from hinge to bifilar center of mass

Example: Given R = 18.22 in, r = 2.02444, then F = 3.0 per rev

220.-229. Inplane Bifilar Damping, Nondim.

230.-239. Vertical Bifilar Masses, Lb-sec^2/in. or Bugs

Same comment as for inplane bifilar (see loc. 12).

240.-249. Vertical Bifilar Arm From Center of Rotation, In.

250.-259. Vertical Bifilar Tuning Frequency, Cycles/Rev

The linear vertical bifilar tuning frequency is defined by
$F^2 = (R+r)/r$ where R = bifilar arm (loc. 240-249)
r = bifilar distance from hinge to bifilar center of mass

Example: Given R = 18.50 in, r = 1.23333 in, then F = 4.0 per rev

260.-269. Vertical Bifilar Damping, Nondim.

450.-549. Transfer Matrix to Modal Coordinates for Main Rotor Impedance, In/in & Rad/in.

The main rotor hub transfer matrix has the dimensions 6 X NF (where NF is the number of fixed system modes defined in loc. 9). Since the maximum value of NF is 16, then 96 computer locations are necessary to define the largest transfer matrix.

The first mode is loaded into locations 450-455, the second mode into locations 456-461, etc. .. until the last or 16th mode into locations 540-545.

For each mode, the order of input is x, y, z, θ_x, θ_y, θ_z. The units of the linear motions are in/in while rotations are in rad/in. The user must be careful to load the mode shapes corresponding to the fixed system modes masses, frequencies and damping values from locations 50-109.

550.-649. Mode Shapes for First Aircraft Station Where Loads Are Applied, In/in. & Rad/in

650.-749. Mode Shapes for Second Aircraft Station Where Loads Are Applied, In/in & Rad/in

Same comments as for the main rotor hub transfer matrix above apply for these inputs.

750.-849. Fixed System Absorber Coupling Matrix, In/in & Rad/in

The fixed absorber coupling matrix has the dimension NFABS X NF (where NFABS is the number of fixed system absorbers from location 8). The modal response in one direction (vertical, lateral or longitudinal) for each fixed system mode (as defined in location 9) is loaded in locations 750-769 for the first absorber, in locations 770-789 for the second, and so on.

850.-949. Tail Rotor Hub Transfer Matrix, In/in & Rad/in

Same comments as above for main rotor hub transfer matrix (loc. 450-549).

46542

1000.-1399. Mode Shapes of Aircraft Station Points, In/in & Rad/in

The response of 4 aircraft stations can be analyzed according to the mode shapes input in locations 1000-1099 for the first station, 1100-1199 for the second, 1200-1299 for the third, and 1300-1399 for the fourth station.

The mode shapes are loaded in the same manner as described above for the main rotor hub (loc. 450-549).

1490.-1494. Printout Options, Nondim.

If any printout option switch is set to 0., then the printout of the corresponding matrices is suppressed. If printout of the matrices is desired, then the proper switch should be loaded as 1.

The build-up of the matrices is as follows:

(loc. 1490)	1.	Fixed system matrices (16 X 16 max)
(loc. 1491)	2.	Add rotor matrices (24 X 24 max → 40 X 40 max total)
(loc. 1492)	3.	Add fixed system absorbers (5 X 16 max → 45 X 45 max. total)
(loc. 1493)	4.	Add linear inplane bifilars
(loc. 1494)	5.	Add vertical bifilars (combined with inplane bifilars, the matrices are 15 X 15 max → 60 X 60 max. total)

Thus, the maximum number of degrees-of-freedom which can be handled at the present time with the linear rotor/bifilar coupled program is 60.

Location 1493 also controls the printout of the "Final Combined Mass Matrix" for the non-linear analysis option.

1495.-1496. Bifilar Pendulums Printout Switches, Nondim.

The individual contributions to the coupled system matrices from the inplane and vertical bifilars can be obtained through the use of the printout options as defined in locations 1495 and 1496 respectively.

1497. The results of the forced response analysis for all the system degrees-of-freedom can be printed out through this switch.

4G541

1498. Initial Conditions Option, Nondim.

This option applies only when the non-linear analysis is requested (loc. 18 = 1.). If set to 1.0, then the initial conditions to be used should be loaded in locations 1720-1939. Input to these locations is printed out at the end of a non-linear analysis run.

Non-Linear Analysis Input (Loc. 1500-2179)

1500.-1519. Inplane Bifilar Pendulum Weights, Lbs.

The weight of each bifilar pendulum in pounds is loaded according to the number of pendulums from loc. 1763. The maximum number of pendulums allowed is 12.

1520.-1539. Inplane Bifilar Pendulum Damping, Nondim.

1540.-1559. Inplane Bifilar Pendulum Frequencies, Cycles/Rev

Same comments as provided for the linear inplane bifilars (loc. 210-219) apply here.

1560.-1579. Inplane Bifilar Pendulum Arms From Center of Rotation, In.

1580.-1599. Relative Azimuth Locations of Inplane Bifilar Pendulums, Deg

Example: If four inplane pendulums are analyzed, then locations 1580-1583 are respectively 0., 90., 180., 270.

1600.-1719. Hub Forces and Moments Input, Lbs and in.-lb

For the non-linear analysis, the fixed system hub forces and moments are input in harmonics format. Up to 10 harmonics can be used. Cosine and sine components of each harmonic are input alternately.

Ex. Loc 1600 - Longitudinal motion - cosine component of first harmonic
Loc 1601 - Longitudinal motion - sine component of first harmonic
Loc 1602 - Longitudinal motion - cosine component of second harmonic
Etc...

Location 1764 is used in conjunction with the input loads.

09789

1720.-1739. Initial Bifilar Angular Displacements, Radians

These locations are used if the initial conditions switch (loc. 1498) is 1. These values are printed out at the end of a converged time history solution for each inplane bifilar.

1740.-1759. Initial Bifilar Angular Velocity, Rad/Sec

Same comments as above for the initial displacements.

1760. Force Ramp Factor, Nondim.

The hub loads are imposed on the rotor/bifilar/fixed system coupled system as a ramp input dependent on the maximum azimuth angle (loc. 1762) for the time history solution and the ramp factor.

Example: If loc. 1762 = 4320 degrees (or 12 revolutions) and loc. 1760 = 3.0, then the hub loads (loc 1600-1719) are applied linearly in the azimuth interval from zero to 4320/3 (which equals 1440 degrees or 4 revolutions).

1761. Azimuthal Increment For Time History Solution, Degrees

A value of 2 degrees is recommended. However, if the time history does not converge, lower values can be tried to eliminate what may possibly be a numerical instability. It should be noted that the computer execution time is directly proportional to the size of this input quantity.

1762. Maximum Azimuth Angle, Degrees

It is recommended that values corresponding to 10 to 20 rotor revolutions (3600 and 7200 degrees respectively) be used in this location. As for loc. 1761 above, the computer execution time is also dependent on this input. If the time history does not meet the convergence criteria, then it will terminate when the integration azimuthal angle equals the input value in loc. 1762.

1763. Number of Inplane Bifilar Pendulums, Nondim.

A maximum number of 12 bifilars can be used.

1764. Number of Input Hub Force Harmonics, Nondim.

A maximum number of 10 harmonics, corresponding to the input loads in loc. 1600-1719, can be used.

1765. Number of Output State Variables Harmonics, Nondim.

This location governs the number of harmonics analyzed and printed out for the following variables:

1. Bifilar pendulum response, degrees
2. Hub response (x, y, z, θ_x, θ_y, θ_z), g's
3. Aircraft stations response (x, y, z), g's

Up to 10 harmonics may be requested.

1767. Number of Aircraft Additional Points Where Loads Are Applied, Nondim.

This number is equal or less than 2. If non-zero, then load appropriate mode shapes in locations 550-749.

1768.-1773. Hub Initial Displacements, In or Rad

At the completion of the time history solution, the hub displacements are printed out to be used for successive cases if desired. The output yields, in order, the longitudinal, lateral, vertical, roll, pitch and yaw motions.

1774-1779. Hub Initial Velocities, In/sec or Rad/sec

Same comments as above for the hub initial displacements.

1780.-1859. State Variables Initial Displacements, In or Rad

The initial displacements of all the degrees-of-freedom (except the non-linear inplane bifilar pendulums) are printed out for use in successive cases. The order of printout is:

1. Fixed system
2. Rotor (if required)
3Fixed system absorbers
4. Linear inplane bifilars
5. Linear vertical bifilars

17310

1860.-1939. State Variables Initial Velocities, In/sec or rad/sec.

Same comments as made above for the initial displacements.

1940.-2179. Aircraft Additional Points Forces and Moments Input, Lbs and in-lb.

Refer to comments on hub forces (locations 1600-1719).

2200. Successive Cases Control Switch, Nondim.

If successive runs are to be made, then location 2200 is set to zero. The program then goes back to the rotor aeroelastic analysis and starts execution of the next case. The last bifilar analysis case must have a 1. in location 2200 for proper termination of the computer run.

The maximum number of degrees-of-freedom which can handled by the linear and non-linear analyses are respectively 60 and 72. A breakdown of the individual component maximum d.o.f. is presented in the chart below.

Component Description	Maximum No. of D.O.F.		Input Location(s)
	Linear	Non-Linear	
1. Linear Bifilars (inplane + vertical)	15	15	1 & 12
2. Fixed System Absorbers	5	5	8
3. Fixed System Modes	16	16	9
4. Rotor Blade	24	24	-
5. Non-linear Inplane Bifilars	-	12	1763
Total →	60	72	

3.3 Input Data Format

All data is input via cards or card-like images with the following (loader) format. Column 2 represents the number of data values on the card, columns 3-6 give the location numbers of the first data values, successive data values are loaded into successive locations, and columns 7-66 contain the data values in fields of 12, the default format being 5E12.4. A minus sign in column 1 indicates the end of a case. Subsequent cases are loaded immediately after this card in the same format. See subroutine LOADIT for a more detailed description of data cards.

19002

SECTION 4

OUTPUT DESCRIPTIONS

4.1 Rotor Aeroelastic Analysis Output

If the rotor coupling option is exercised (location 110 is not zero), then a listing of the input rotor data is first printed out. A sample page of this output can be seen in Figure 2. The input cards are listed out as read by the computer. For successive cases, only the new input items will be printed out to reflect the changes made between runs. This information is provided for all printout options.

The output formats which follow are obtained for all printout options, as provided in location 119. Additional printout of blade bending frequency calculations and dynamic and aerodynamic integrals evaluations can be obtained if the printout switch is not equal to 5. These formats are not shown here for sake of brevity - they are mainly used for debugging purposes.

Input and calculated dynamic and aerodynamic characteristics of the rotor blade and fixed system are presented in Figures 3a through 3h. Additional explanations are provided in Table 1 below. Some differences may appear in the output formats from the input data due to corrections applied by the computer program to eliminate possible inconsistencies in the input control options.

The abbreviations used in the output formats are listed and discussed in the table below. It should be noted that some input quantities have been preset within the computer program since some program capabilities were not needed for the coupling of the rotor and bifilar systems.

TABLE 1. Rotor Analysis Output Description

Output Symbol	Quantity Description	Input Location	Present Value	Figure Number
PHIXPH	Blade Bending Mode Coupling Factors	-	-	3a
PHIZPH	Blade Bending Mode Coupling Factors	-	-	
	..... Lag Damper Quantities			
PHELD	Edgewise Bending Mode at Lag Damper	-	-	
PHEPLD	Edgewise Bending Mode Slope at Lag Damper	-	-	
PHFLD	Flatwise Bending Mode at Lag Damper	-	-	
PHFPLD	Flatwise Bending Mode Slope at Lag Damper	-	-	
QEOLD	Edgewise Steady Deflection at Lag Damper	-	-	

Output Symbol	Quantity Description	Input Location	Present Value	Figure Number
QEOPLD	Edgewise Steady Slope at Lag Damper	-	-	3a
QFOLD	Flatwise Steady Deflection at Lag Damper	-	-	
QFOPLD	Flatwise Steady Slope at Lag Damper	-	-	
PHLD	Torsional Mode Shape at Lag Damper	-	-	
THTLD	Blade Twist at Lag Damper	-	-	
PHOS	Torsional Mode Shape at Outboard Snubber	-	-	
XNAMOD	Number of Fixed System Modes	-	5.	3b
VF	Forward Flight Speed	-	0.	
	..... Control Switches			
ROTEST	Rotor Definition	111	-	
FTEST	Flight Definition	-	1.	
SYSDEF	System Definition	113	-	
ROTDEF	Rotor Definition	114	-	
ARTIC	Blade Pitch Input Control	115	-	
PHASE	Phasing Matrix Printout Control	-	0.	
VECT	Eigenvector Printout Control	-	0.	
TRMASC	Tail Rotor Main Mass Control	-	1.	
SUMASC	Tail Rotor Subsidiary Mass Control	-	111.	
TSERVC	Tail Rotor Servo Control	-	1.	
MRMASC	Main Rotor Mass Control	-	1.	
MSERVC	Main Rotor Servo Control	-	111.	
CIR	Circulatory Unsteady Aerodynamics Control	-	1.	
CIRN	Noncirculatory Unsteady Aerodynamics Control	-	1.	
LAGKII	Lag Damper Control	125	-	
ZETBLD	Fraction of Critical Damping of Blade Bending Modes	22-25	-	
	..... Fixed System Modes			
ZETG	Fraction of Critical Damping of Fixed System Modes	-	0.	
MG	Generalized Mass of Fixed System Modes	-	0.	
OMF	Frequency of Fixed System Modes	-	0.	
PHY	Lateral Fixed System Mode Shape (Second Mode Only Equals 1.0)	-	0. & 1.	
PHX	Longitudinal Fixed System Mode Shape (First Mode Only Equals 1.0)	-	0. & 1.	
PHZ	Vertical Fixed System Mode Shape (Third Mode Only Equals 1.0)	-	0. & 1.	

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Output Symbol	Quantity Description	Input Location	Present Value	Figure Number
PHTY	Pitch Fixed System Mode Shape (Fifth Mode Only Equals 1.0)	-	0. & 1.	3b
PHTX	Roll Fixed System Mode Shape (Fourth Mode Only Equals 1.0)	-	0. & 1.	
R	Radius of the Blade Element Mid-Points From Center of Rotation	201-220	-	3c
AC	Blade Aerodynamic Center	650-749	-	
CG	Blade Center of Gravity	550-649	-	
EA	Blade Elastic Axis	750-849	-	
QEO	Blade Edgewise Steady Deflection		-	
QFO	Blade Flatwise Steady Deflection		-	
QEOP	Derivative of QEO with respect to R		-	
QFOP	Derivative of QFO with respect to R		-	
DT	Elemental Thrust		-	3d
DH	Elemental Drag		-	
DM	Elemental Pitching Moment		-	
$\frac{D}{D(UT)}$	Derivative with Respect to Tangential Velocity		-	
$\frac{D}{D(UP)}$	Derivative with Respect to Vertical Velocity		-	
$\frac{D}{D(\theta T)}$	Derivative with Respect to Angle-of-Attack		-	
CL	Coefficient of Lift	2750-3649	-	3e
CD	Coefficient of Drag	1850-2749	-	
CM	Coefficient of Pitching Moment	3650-4548	-	
$\frac{D}{DA}$	Derivative with Respect to Angle-of-Attack		-	
$\frac{D}{DM}$	Derivative with Respect to Mach Number		-	
θ STRUCTURAL	Structural Twist	450-549	-	
θ AERODYNAMIC	Aerodynamic Twist	350-449	-	
U_P	Hover Inflow Velocity		-	
U_T	Tangential Velocity		-	
U	Total Velocity		-	
Ø	Inflow Angle		-	
ALPHA	Angle-of-Attack		-	
PHE(I)	Edgewise Part of i^{th} Blade Bending Mode	-	-	3f
PHF(I)	Flatwise Part of i^{th} Blade Bending Mode	-	-	
PHEP(I)	Edgewise Slope Part of i^{th} Blade Bending Mode	-	-	
PHFP(I)	Flatwise Slope Part of i^{th} Blade Bending Mode	-	-	

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The rotor blade radial distributions of edgewise, flatwise and torsional mass moments of inertia, mass, and edgewise and flatwise area moments of inertia are shown in Figure 3g. Care must be exercised in the input of these quantities to make sure that the sum of the blade segments equals the blade radius (location 6).

It either rigid blade flapping or inplane motion is used in the rotor analysis, then the rotor blade flapping mass, first and second moments of inertia and the blade lag frequency are printed out as can be seen in Figure 3h. In this figure are also shown calculations of the blade bending mode generalized masses (defined as the blade mass times the sum of the squares of the flatwise and edgewise components) and of other blade parameters.

The total number of degrees-of-freedom used in the rotor analysis is indicated in Figure 3h. The individual degrees-of-freedoms are identified by integers according to the schedule given below.

Number	Degree-of-Freedom	Motion
1-4	Blade Bending Modes (up to 4)	Symmetric
5-6	Blade Torsional Modes (up to 2)	
7	Blade Rigid Body Flapping	
8	Blade Rigid Body Lead-Lag	
9-16	Blade Bending Modes	Cyclic
17-20	Blade Torsional Modes	
21-22	Blade Rigid Body Flapping	
23-24	Blade Rigid Body Lead-Lag	
25-29	Fixed System Modes (Fixed at 5)	-

Thus, for the example in Figure 3h, it is seen that two blade bending modes, rigid body flapping and lead-lag and five fixed system modes are employed for a total of 17 degrees-of-freedom.

The maximum number of rotor/fixed system degrees-of-freedom is 29 (8 blade modes times 3 plus 5 fixed system modes).

A sample output matrix is presented in Figure 4. For this case, location 108 was set to 1. to yield the printout of the compressed (17 X 17) rotor/fixed system matrices. The complete 29 X 29 matrices can be displayed if location 107 is set to 1. The order of the elements in each column follows the schedule shown above. For example, the fourth element in the second row corresponds to the lead-lag symmetric stiffness contribution to the second blade bending mode symmetric equation of motion.

The compressed matrices include rotor aerodynamic contributions. They are coupled directly to the bifilar analysis. If no rotor coupling is desired, then the compressed matrices are stored for future use.

4.2 Bifilar Analysis Output

Typical output formats presenting results from the bifilar analysis are shown in Figures 5 through 8. The output parameters from Figures 5a through 5k are common for both linear and non-linear bifilar analyses. The final linear analysis results are presented in Figure 6 while the non-linear results can be seen in Figures 7 and 8.

Figure 5a shows the number of degrees-of-freedom utilized in a given computer run, the number of aircraft stations where the response is calculated and the printout options requested. For the example shown, it is seen that nine fixed system modes, one fixed system absorber and one kind each of inplane and vertical bifilars are requested. In addition, rotor coupling is employed and the response of four aircraft stations is to be analyzed. All printout switches have been activated (set at 1.0) to show examples of the output generated.

The bifilar analysis starts off with the fixed system degrees-of-freedom. Then, it expands the fixed system stiffness, damping and mass matrices to include in sequence the contributions from the rotor, fixed system absorbers, linear inplane bifilars and finally the linear vertical bifilars. Then, either a forced response or a time-history solution is calculated depending on the input to location 18. The build-up of the degrees-of-freedom is shown in Figures 5b through 5k for the stiffness matrix only. The damping and mass matrices are handled in an identical manner.

The basic fixed system stiffness matrix is presented in Figure 5b. It is seen that for this example the stiffness matrix is a square diagonal matrix of order 9. This printout is governed by location 1490.

The rotor/fixed system stiffness matrix to be coupled with the bifilar analysis is shown in Figure 5c and d and is of order 18. This matrix is basically the same as that from Figure 4 except that now the fixed system degrees-of-freedom appear first and include an extra equation corresponding to the yaw degree-of-freedom (which is not present in the rotor aeroelastic analysis). Consequently, the total number of degrees-of-freedom is 18. This printout option is controlled by location 17.

Every time a new system component is added, the printout shown in Figure 5e is automatically obtained. It shows what the present number of degrees-of-freedom (d.o.f.) is (for this example, 9 fixed system d.o.f.), the number to be added (12 rotor d.o.f.) and the final system d.o.f. (a total of 21 d.o.f.).

The combined fixed system/rotor stiffness matrix is displayed in Figure 5f. Only the first nine equations are shown here for brevity. This output is obtained if location 1491 is set to 1.0.

Next, the fixed system/rotor coupled system is expanded to include the contribution of the fixed system absorber. In this example, only one absorber is used and thus the number of d.o.f. becomes 22, as can be seen in Figure 5g. Location 1492 controls this printout option.

If linear inplane bifilar pendulums are used (see location 1), then the fixed system/bifilar coupled matrices (mass, damping and stiffness) can be obtained for each kind of bifilar (see loc. 1495). The output matrices in all cases are square matrices of order 9. The first 6 d.o.f. correspond to the hub motions (x, y, z, θ_x, θ_y and θ_z respectively) and the next three to the inplane bifilar symmetric and two cyclic modes. In Figure 5h, the coupled stiffness (QQK) matrix is presented.

The system matrices are now expanded to include the contribution of the inplane bifilar pendulums. The stiffness matrix of the fixed system/rotor/fixed system absorber/inplane bifilar coupled system can be seen in Figure 5i. Since only one kind of inplane bifilar is used in this example, the final number of d.o.f. is increased by 3 for a total of 25. This output is generated if location 1493 is set to 1.

If linear vertical bifilar pendulums are employed in the analysis (see loc. 12), the corresponding fixed system/vertical bifilar matrices can be displayed (see loc. 1496) as seen in Figure 5j. The format is the same as that for the inplane bifilars.

Now the vertical bifilar d.o.f. are added to the system. The final stiffness matrix for this example is shown in Figure 5k. The total d.o.f. to be analyzed is 28. This output format is controlled through location 1494.

The results of the linear bifilar analysis are presented in Figure 6. The cosine and sine components of the generalized forces appearing near the top of Figure 6a are obtained by multiplication of the hub forces/moments vectors (see loc 110-119 for the cosine component and loc 120-129 for the sine component) by the transpose of the rotor hub transfer matrix (located in loc. 450-549). The units are lbs and inch-lbs.

The "GAMMAS" printed out in Figure 6a are the generalized coordinates of the rotor/fixed system absorber/bifilar coupled system and are obtained from the forced response solution. First, the cosine component for all d.o.f. is printed out and then the sine component. For this example, the total number of d.o.f. of the rotor, the fixed system absorber and the inplane and vertical bifilars equals 19. Thus, 38 values of "GAMMAS" are printed out; the printout switch is location 1497. The units are in inch for the fixed system absorber and non-dimensional for the rotor and bifilars.

The "GAMMAS" are sorted out according to the system components present and printed out accordingly, as shown in Figure 6a and 6b. The calculated amplitudes are in inch for the fixed system absorber and in degrees for both inplane and vertical bifilar pendulums, all phase angles are shown in degrees. The method used to calculate the bifilar amplitudes is shown below.

OUTPUT FORMAT

	Cosine	Sine	Amplitude	Phase
Symmetric Equation	A_{OC}	A_{OS}	A_N	PHI_N
Cyclic Equation - sine	A_{SC}	A_{SS}	A_{N-1}	PHI_{N-1}
Cyclic Equation - cosine	A_{CC}	A_{CS}	A_{N+1}	PHI_{N+1}

$$A_N = A_{OC}^2 + A_{OS}^2 * 57.30/NB$$

$$A_{N-1} = A_1^2 + A_2^2 * 57.30/NB \text{ where } A_1 = A_{CC} + A_{SS} \ \& \ A_2 = A_{CS} - A_{SC}$$

$$A_{N+1} = A_3^2 + A_4^2 * 57.30/NB \text{ where } A_3 = A_{CC} - A_{SS} \ \& \ A_4 = A_{CS} + A_{SC}$$

and NB = number of bifilar pendulums

The input frequencies are listed out in Figure 6b; the units are in Hz.

The forcing frequency in Hz is obtained by multiplying the forcing frequency in cycles/rev (loc 3) by the rotor speed (loc 7) and dividing by 60.

The conversion factor to g's is obtained by dividing the square of the forcing frequency in rad/sec by the factor 386.40.

The fixed system generalized coordinates are also shown in Figure 6b. The units are inch. They are utilized to calculate the dynamic response of the aircraft stations and of the rotor head which are shown in Figures 6b through 6d. As indicated in the printout, the aircraft and hub response is in g's.

Sample results from the time history solution when non-linear inplane bifilar absorbers are employed (loc 18 is 1.) are printed in Figures 7 and 8. If the total number of degrees-of-freedom exceeds 72, then a message will be printed out to that effect and the non-linear analysis proceeds to the next case to be analyzed.

In Figure 7a, the top line lists out the different components degrees-of-freedom requested in a specific computer run. For the example shown, the total number of d.o.f. is 29 which is obtained as follows:

	System Component	D.O.F.
1.	Fixed system modes (NF)	9
2.	Rotor d.o.f. (KROTOR)	12
3.	Fixed system absorber (NFABS)	1
4.	Kinds of (linear) inplane bifilars (0X3)	0
5.	Kinds of vertical bifilar (1X3)	3
6.	Number of non-linear inplane bifilars	4
		29

The time history solution proceeds at first to collect all the acceleration terms on the left hand side of the equations of motion. The right hand side contains the stiffness and damping terms and the forcing functions. Successive integrations of the accelerations yield the velocity and displacement vectors at the next time increment. Then, the system accelerations are computed again and the procedure continues until a converged time history is obtained or the maximum azimuthal position (specified in location 1762) is reached.

The output formats presented in Figures 7a through 7e are for zero azimuth. The initial right hand side (r.h.s.) terms are displayed in Figure 7a. They are all zero to start since no initial conditions of the state variables displacements and velocities have been located in the input locations 1780 through 1939. This printout appears for azimuth positions up to 3 degrees.

The next printout shown in Figure 7a is that of the left-hand-side mass matrix whose order is the sum of the six hub d.o.f. and the number of non-linear inplane bifilars (loc. 1763) which for this example is four. The bifilar force vector is also listed out in this figure. Small values appear in this vector due to coupling terms between the fixed system and the bifilar pendulums. These outputs are presented for azimuth positions up to 5 degrees.

The rotor head mode shapes appear in Figure 7b. This matrix is an image of the input in locations 450 through 549. It is shwon only for azimuth positions up to 2 degrees.

The next printout in Figure 7b is that of the "Expanded Bifilar Mass Matrix" whose order is the sum of the number of fixed system modes (loc 9) and the non-linear bifilars (loc 1763). Similarly, the "Expanded Bifilar Force Vector" is printed out in Figure 7c. Both output formats are generated for azimuth angles up to 5 degrees.

The contributions of the remaining system components are now added to the fixed system/non-linear inplane bifilar system. The resulting mass matrix and force vector can be seen in Figures 7d and 7e respectively. The final matrix order is 29, as previously stated in Figure 7a. These printouts are obtained for azimuth angles up to 15 degrees.

The solution vector of the state variables and the non-linear bifilar displacements and velocities at zero azimuth is shown in Figure 7e. It is printed out for azimuth angles up to 30 degrees. This vector is now used to calculate the right-hand-side terms at the next azimuth position (as defined in location 1761) which are displayed at the bottom of Figure 7e.

The time history solution proceeds around the rotor azimuth until either the convergence criterion set on the bifilar motions is satisfied or the maximum azimuth value (loaded in location 1762) is reached. At the end of each rotor revolution, bifilar displacements and hub motions are printed out as exhibited in Figure 7f. For the example shown, 16 rotor revolutions were necessary before the convergence criterion was met, i.e. the angular displacements of the first two bifilars (G1 and G2) for two successive revolutions must be within .002 radian (or .1146 degree).

The rotor/bifilar analysis then proceeds to calculate the harmonic response of the non-linear bifilar pendulums displacements, the hub six degrees-of-freedom (the order is x, y, z, θ_x, θ_y and θ_z) and the aircraft station(s) linear motions (x, y and z), as can be seen in Figures 8a and 8b. The pendulum output is in degrees while the hub and A/C station(s) output is in g's. All phase angles are in degrees. The four rows of output describing the hub and A/C response are respectively the cosine and sine components, the total amplitude and phase angle. For the example shown, the bifilar motions are about 9.8 degrees each; the longitudinal (x) hub response is .13864 g's.

The final output format of the non-linear analysis results consists of the initial values of bifilar, hub and state variables displacements and velocities. These results can be loaded into locations 1720 through 1939 and will be used as initial conditions for the subsequent computer run provided location 1498 is set to 1.0.

Additional information can be printed out if the control switches in locations 10 and 15 are activated. This is only needed if debugging of the bifilar analysis calculations is desired.

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SECTION 5

TEST CASES RESULTS

5.1 Test Cases Description

The rotor/bifilar coupled analysis has been executed for the following four test cases:

Case 1. Includes rotor and uses linear bifilar analysis.
Case 2. Includes rotor and uses non-linear bifilar analysis.
Case 3. Doesn't include rotor and uses linear bifilar analysis.
Case 4. Doesn't include rotor and uses non-linear bifilar analysis.

These cases test the major program logic paths.

For each case, the component degrees-of-freedom utilized are listed in the chart below.

COMPONENT D.O.F.

Test Case	Rotor Blade	Fixed System	Fixed Absorber	Linear Bifilar Inplane	Linear Bifilar Vertical	Non-Linear Bifilars	Total D.O.F.
1	12	9	1	3	3	0	28
2	12	9	1	0	3	4	29
3	0	9	1	3	3	0	16
4	0	9	1	0	3	4	17

The rotor degrees-of-freedom include two blade bending modes and rigid body flapping and lead-lag motion. Only one kind of inplane and vertical linear bifilars is used in the test cases. However, the analysis has been checked out for cases utilizing several kinds of bifilar pendulums. The inplane bifilars are tuned to 4 per rev.

5.2 Job Control Language

The Job Control Language (JCL) needed to execute the coupled program on the IBM 370/169 computer system is presented in Appendix A. This JCL must be modified for use on the NASA CDC computer system. A brief description of the JCL setup is discussed below.

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Card Number	Description
1	Describes job name and characteristics (class, time, etc.).
2	Executes program module E90BCFIN.
3	Locates program module in ET473.SEBBY.LOAD.
4	Reads input data from ET473.BIFILAR.DATA (NASARUN) using Unit 5.
5	Provides paper output using Unit 6.
6	Provides punched cards output using Unit 7.
7	Stores calculated data internally in Unit 8.
8	Stores input data for first case in Unit 11 to be used for successive cases.
9	Ends JCL setup.

5.3 Test Cases Input Data

The input data needed to run the four test cases is listed in Appendix B.

The input format which must be followed to run multiple cases is described in Table 2 below.

TABLE 2. Program Multiple Cases Setup

Data Block	Input Data Description	Case Number
1	Rotor blade data (loc. 1-4549)	1
2	Last rotor blade data card (minus sign in column 1)	↓
3	Title for rotor blade data	↓
4	Title for bifilar data	↓
5	Bifilar data (loc. 1-2199)	↓
6	Last bifilar data card (loc 2200 = 0. - not last case)	↓
7	Rotor blade data	2 (last)
8	Last rotor blade data card (minus sign in column 1)	↓
9	Bifilar data	↓
10	Last bifilar data card (loc 2200 = 1. - last case)	↓

The format above is shown for two cases only for brevity. Data blocks 7 through 10 are repeated for each case.

If the first case does not use rotor data, then the data blocks numbered 1 through 3 above are replaced by a single card as follows:

-1 110 0.

5.4 Test Cases Output

The results of the rotor/bifilar coupled program for the four test cases are presented in Appendix C. Only the important results are shown in the Appendix to minimize the size of this report (the actual run consisted of 165 pages of output with only the most important printout switches being activated).

Some important results from the bifilar analysis are summarized in the Table 3 below.

TABLE 3. Bifilar Analysis Test Cases Results

Test Case Number	Inplane Bifilar Response		Rotor Hub Response					
			Longitudinal (X)		Lateral (Y)		Vertical (Z)	
	Ampl. (deg)	Phase (deg)	Ampl. (g's)	Phase (deg)	Ampl. (g's)	Phase (deg)	Ampl. (g's)	Phase (deg)
1	9.29	-77	.134	113	.094	178	.006	132
2	1. 9.76	96	.139	-84	.100	-20	.008	-53
↓	2. 9.77	-174						
	3. 9.80	-84						
	4. 9.80	6						
3	9.52	-86	.184	113	.081	137	.010	46
4	9.57	88	.193	-79	.086	-56	.010	-134
↓	9.58	177						
	9.63	-93						
	9.62	-2						

For all test cases, the input force is a 4 per rev fixed system force with lateral sine and longitudinal cosine components of 500 pounds. For the non-linear bifilar analysis results, the response of each of the four inplane bifilars is shown in the table above (cases 2 and 4).

5.5 Test Cases Computer Time

The computer total running time for the four test cases was 8 minutes and 43 seconds. The break-down in computer time per case is shown in the following table.

Test Case Number	Number of D.O.F.	Bifilar Analysis	Computer Time	
			Minutes	Seconds
1	28	Linear	0	25
2	29	Non-linear	7	07
3	16	Linear	0	03
4	17	Non-linear	1	08
		Total	8	43

From the table above, a comparison between cases 1 and 2 and between cases 3 and 4 reveals that the time history analysis requires considerable greater computer time for a complete converged solution than the linear analysis. Also, the computer running time increases tremendously as the number of system degrees-of-freedom is increased when comparing cases 1 and 3 and 2 and 4.

SECTION 6

OVERALL PROGRAM STRUCTURE

The rotor/bifilar program is basically made up of two parts: the rotor aeroelastic analysis and the fixed system/fixed absorber/bifilar pendulums analysis. The bifilar anlaysis can be executed with and without coupling with the rotor analysis while the opposite is not possible. In addition, the bifilar portion of the program can use either a forced response solution for linear inplane bifilar pendulums or a time history solution for non-linear inplane bifilars. The main purpose of the rotor aeroelastic analysis is to provide the rotor blade stiffness, damping and mass matrices for coupling with the bifilar analysis.

6.1 Segmentation Structure

Due to the large size of the coupled program, it was necessary to implement a segmentation structure to permit operation within a 64K (decimal) for CDC computer use. A basic breakdown of the 10 control segments needed is shown in Table 4 below and in the schematic of Figure 9.

TABLE 4. Segmentation Structure Description

Segment Number	Leading Fortran Routine	Number of Segment Routines	Segment Description
1	SHAKIT	4	Controls overall program logic and specifically the rotor analysis program flow.
2	PRELIM	34	Reads rotor input, initializes data and performs many basic rotor calculations.
3	DYNMAT	12	Calculates rotor dynamic matrices.
4	AERMAT	17	Calculates rotor aerodynamic matrices.
5	EIGER	2	Combines dynamic and aerodynamic rotor matrices, compresses and links them to the bifilar analysis.
6	MAINSV	3	Controls bifilar analysis program flow.
7	SYSCTL	6	Calculates contributions from fixed system modes, fixed absorber, inplane and vertical linear bifilar and couples rotor matrices.
8	HUBIMP	1	Computes rotor hub impedance and transfer matrices.
9	CMPUTE	4	Controls generalized forces calculations and solves for the forced response for the linear bifilar analysis option.

10	NLBIF	9	Performs time-history solution for the non-linear bifilar analysis option.
		Total = 92	

A total of 92 Fortran routines have been developed for this program: 69 of them comprise the rotor analysis portion while 23 make up the bifilar analysis portion.

From the schematic presented in Figure 9, it seen that the rotor aeroelastic analysis is performed in segments 1 through 5 while the bifilar analysis is handled by segments 6 through 10. There is no lateral transfer of data between any two segments; data can only be transferred in a vertical sense to segment 1 for the rotor portion and to segments 1 and 6 for the bifilar portion.

Table 5 below lists in alphabetical order the Fortran sub-routines and the common blocks needed for each segment.

TABLE 5. Segmentation Structure Routines and COMMON Blocks

Segment Number	Fortran Subroutine Name			COMMON Block Name	
1	SHAKIT			DYNINP	LAGDAM
	INTEG			EOF6	NIMIC
	LOADIT			INDAT	PRNTSW
	QTFG			INEIG	TMDS
				INEIGN	
2	PRELIM	MATEO	PINT	CONT	
	BLIN4	MATF	POUT	DAT	
	ELI	MATR	PRODM	EMATI	
	ELO	MIND	PROUT	EMATO	
	EXTEND	MISC	REMOVE	FMAT	
	E159X	MODES	ROOTX	FREQ	
	FILL	MSHAPE	SECAER	PHTNO	
	FOLL	ORTHOG	SIMLIN	PMAT	
	FREQUN	OVUN	SKIPLN	PRAM1	
	FULL	PFMULT	SORTAB	TORFIN	
	GMPRDD	PICK	STDEFL	WORKA	
	MATEI				
3	DYNMAT	DMATEX	DYNINT	DYNOUT	
	BLELPD	DMDMAT	DYNIN1	NAMIC	
	DISCON	DMMMAT	DYNIN2		
	DISINT	DMSMAT	DYNLST		
4	AERMAT	AERIN5	AEROII	AERO1	
	AERINT	AERIN6	AMATEX	AERO2	
	AERIN1	AERIN7	AMDMAT	AERO3	
	AERIN2	AERIN8	AMSMAT	AERO4	
	AERIN3	AERLST	BLELPA	AERO5	
	AERIN4	AEROI		CDCAER	

Segment Number	Fortran Subroutine Name		COMMON Block Name	
5	EIGER			
↓	COMPRSS			
6	MAINSV		NDOF	TOTMAT
	INCOND		NLDAT1	XFRDAT
↓	INPUTV		PRSWTH	
7	SYSCTL			
	ADDOFR			
	FIXABS			
	FIXSYS			
	LINBIF			
↓	LVBIF			
8	HUBIMP			
9	CMPUTE			
	FORCER			
	GENFOR			
↓	OUTPUT			
10	NLBIF	HARMON	HARM	
	BIFEXP	INTEQ	NLDAT2	
	BIFILR	OUT		
	COMBIN	RHS		
↓	CONVER			

All the Fortran sub-routines listed above except four are computer independent. The computer dependent routines are: SHAKIT (segment 1), LOADIT (segment 1), PRELIM (segment 2) and MAINSV (segment 6). This is due to CDC computer requirements for identification of the main routine, file read error and end of file transfers and word size definition for alpha-numeric read statements. The program coding allows the programmer to convert easily to the IBM 370/168 computer system by commenting out the appropriate block(s) of statements.

6.2 Flow Diagrams

Computer logic flow diagrams are presented in Figures 10 through 16 for the 10 segments. A summary of the flow charts is provided in the table below.

Figure Number	Flow Chart Description	Segment Number(s)
10	Main Program Flow Chart	1 & 5
11	"PRELIM" Flow Chart	2
12	"MODES" Flow Chart	2
13	"DYNMAT" Flow chart	3
14	"AERMAT" Flow Chart	4
15	Bifilar Analysis Flow Chart	6 & 8
16a	"SYSCTL" Flow Chart	7
16b	"CMPUTE" Flow Chart	9
16c	"NLBIF" Flow Chart	10

95185

It should be noted that "IMSL" routines are needed for the calculations performed in "HUBIMP" and "NLBIF". Table 6 below lists the 10 "IMSL" routines used in the bifilar analysis portion of the coupled program:

TABLE 6. IMSL Routines

No.	IMSL Routine Name
1	LEQT2F
2	LINV2F
3	LUDATF
4	LUELMF
5	LUREFF
6	UERTST
7	UGETIO
8	VXADD
9	VXMUL
10	VXSTO

These routines must be supplied by the government for successfull operation of the rotor/bifilar coupled program.

6.3 "COMMON" Blocks

The "COMMON" blocks used in the rotor/bifilar analysis are presented in Figure 17 as they appear in each Fortran sub-routine. Both routines and "COMMON" blocks are listed alphabetically for easy reference.

SECTION 7

SUBROUTINE DESCRIPTIONS

The Fortran subroutines needed to execute the rotor/bifilar coupled program are described in detail in this section. For each routine, the following information is provided, where applicable:

1. Name
2. Purpose
3. Method
4. Usage
5. Subroutines Called
6. Error Returns
7. Restrictions

In addition, an alphabetical listing of the routines and the corresponding page numbers are shown in Table 7 below for easy reference.

TABLE 7. Program Subroutines Listing

No.	Name	Page	No.	Name	Page	No.	Name	Page	No.	Name	Page
1	ADDOFR		24	CMPUTE		47	FORCER		70	NLBIF	131
2	AERINT		25	COMBIN		48	FREQUN		71	ORTHOG	132
3	AERIN1		26	CONVER		49	FULL		72	OUT	133
4	AERIN2		27	DISCON		50	GENFOR		73	OUTPUT	134
5	AERIN3		28	DISINT		51	GMPRDD		74	OVUN	135
6	AERIN4		29	DMATEX		52	HARMON		75	PFMULT	136
7	AERIN5		30	DMDMAT		53	HUBIMP		76	PICK	138
8	AERIN6		31	DMMMAT		54	INCOND		77	PINT	139
9	AERIN7		32	DMSMAT		55	INPUTV		78	POUT	140
10	AERIN8		33	DYNINT		56	INTEG		79	PRELIM	141
11	AERLST		34	DYNIN1		57	INTEQ		80	PRODM	144
12	AERMAT		35	DYNIN2		58	LINBIF		81	PROUT	145
13	AEROI		36	DYNLST		59	LOADIT		82	QTFG	146
14	AEROII		37	DYNMAT		60	LVBIF		83	REMOVE	147
15	AMATEX		38	EIGER		61	MAINSV		84	RHS	148
16	AMDMAT		39	ELI		62	MATEI		85	ROOTX	149
17	AMSMAT		40	ELO		63	MATEO		86	SECAER	150
18	BIFEXP		41	EXTEND		64	MATF		87	SHAKIT	151
19	BIFILR		42	E159X		65	MATR		88	SIMLIN	152
20	BLELPA		43	FILL		66	MIND		89	SKIPLIN	153
21	BLELPD		44	FIXABS		67	MISC		90	SORTAB	154
22	BLIN4		45	FIXSYS		68	MODES		91	STDEFL	155
23	CMPRSS		46	FOLL		69	MSHAPE		92	SYSCTL	157

NAME: ADDOFR

PURPOSE: To add degrees-of-freedom to the mass, damping and stiffness system matrices in the bifilar analysis.

METHOD: The new matrices to be added of order NL X NL are split up in four parts:

1. An upper diagonal matrix of order (NL-NA)X(NL-NA)
2. A lower diagonal matrix of order NA X NA
3. An upper off diagonal matrix of order (NL-NA) X NA
4. A lower off diagonal matrix of order NA X (NL-NA)

The lower diagonal matrices are added to the original matrices of order NP X NP, which now become of order (NP + NA) X (NP + NA). The upper off diagonal matrix is pre-multiplied by the fixed system transfer matrix XPH (locations 450-549) while the lower off diagonal matrix is post-multiplied by XPH. The upper diagonal matrix is pre-multiplied and post-multiplied by XPH.

USAGE: CALL ADDOFR (NL, NA, NP)

NL = Order of matrices to be added

NA = Number of degrees-of-freedom to be added

NP = Present order of matrices (after ADDOFR, order of matrices is NP + NA)

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: None

49022

NAME:	AERINT
PURPOSE:	To set initial values of all blade aerodynamic integrals to zero and to control the calculation of the aerodynamic integrals needed to construct the aerodynamic damping and stiffness matrices.
METHOD:	Aerodynamic integrals independent of the number of bending modes are calculated by calls of AERIN1 and AERIN7 for lift, AERIN3 and AERIN8 for drag, and AERIN4 for pitching moment. Integrals involving bending mode dependent functions are calculated in AERIN2, AERIN5 and AERIN6 for lift, drag and pitching moment respectively.
USAGE:	CALL AERINT
SUBROUTINES CALLED:	AERIN1, AERIN2, AERIN3, AERIN4, AERIN5, AERIN6, AERIN7, AERIN8
ERROR RETURNS:	None
RESTRICTIONS:	Due to computer storage restrictions, the aerodynamic integrals had to be calculated in 8 separate routines, i.e. AERIN1 through AERIN8.

NAME: AERIN1

PURPOSE: To calculate the aerodynamic integrals which contain thrust derivatives and are independent of the number of blade bending modes.

METHOD: These integrals are formed in AEROI from the product of 5 radial functions, and are referred to as un-subscripted ATK and AT1K integrals.

Since all the integrals involve thrust derivatives, the upper limit of integration is the blade radius multiplied by the tip loss factor.

USAGE: CALL AERIN1

SUBROUTINES CALLED: AEROII, AEROI

ERROR RETURNS: None

RESTRICTIONS: None

NAME:	AERIN2
PURPOSE:	To calculate the aerodynamic integrals which contain bending mode dependent functions and thrust derivatives.
METHOD:	These integrals are formed in AEROI from the product of 5 radial functions. They are referred to as AT1I and AT2I for doubly subscripted integrals and ATJ, AT1J, AT2J, AT3J for singly subscripted integrals. Since all the integrals involve thrust derivatives, the upper limit of integration is the blade radius multiplied by the tip loss factor.
USAGE:	CALL AERIN2
SUBROUTINES CALLED:	AEROII, AEROI
ERROR RETURNS:	None
RESTRICTIONS:	None

49019

NAME: AERIN3

PURPOSE: To calculate the aerodynamic integrals which contain drag derivatives and are independent of the number of blade bending modes.

METHOD: These integrals are formed in AEROI from the product of 5 radial functions, and are referred to as un-subscripted ADK and AD1K integrals.

Since all the integrals involve drag derivatives, they are computed over the whole blade.

USAGE: CALL AERIN3

SUBROUTINES CALLED: AEROII, AEROI

ERROR RETURNS: None

RESTRICTIONS: None

NAME: AERIN4

PURPOSE: To calculate the aerodynamic integrals which contain pitching moment derivatives and are independent of the number of blade bending modes.

METHOD: These integrals are formed in AEROI from the product of 5 radial functions, and are referred to as unsubscripted AM1K, AM2K, AM3K integrals.

Since all the integrals involve pitching moment derivatives, the upper limit of integration is the blade radius multiplied by the tip loss factor.

USAGE: CALL AERIN4

SUBROUTINES CALLED: AEROII, AEROI

ERROR RETURNS: None

RESTRICTIONS: None

49017

NAME:	AERIN5
PURPOSE:	To calculate the aerodynamic integrals which contain bending mode dependent functions and drag derivatives.
METHOD:	These integrals are formed in AEROI from the product of 5 radial functions. They are referred to as AD1I and AD2I for doubly subscripted integrals and ADJ, AD1J, AD2J and AD3J for singly subscripted integrals. Since all the integrals involve drag derivatives, they are computed over the whole blade.
USAGE:	CALL AERIN5
SUBROUTINES CALLED:	AEROII, AEROI
ERROR RETURNS:	None
RESTRICTIONS:	None

49016

NAME: AERIN6

PURPOSE: To calculate the aerodynamic integrals which contain one bending mode dependent functions and pitching moment derivatives.

METHOD: These integrals are formed in AEROI from the product of 5 radial functions, and are referred to as singly subscripted AM1J and AM2J integrals. Since all the integrals involve pitching moment derivatives, the upper limit of integration is the blade radius multiplied by the tip loss factor.

USAGE: CALL AERIN6

SUBROUTINES CALLED: AEROII, AEROI

ERROR RETURNS: None

RESTRICTIONS: None

49015

NAME: AERIN7

PURPOSE: To calculate the aerodynamic integrals which contain thrust derivatives and are independent of the number of blade bending modes.

METHOD: These integrals are formed in AEROI from the product of 5 radial functions, and are referred to as unsubscripted AT2K and AT3K integrals.

Since all the integrals involve thrust derivatives, the upper limit of integration is the blade radius multiplied by the tip loss factor.

USAGE: CALL AERIN7

SUBROUTINES CALLED: AEROII, AEROI

ERROR RETURNS: None

RESTRICTIONS: None

NAME: AERIN8

PURPOSE: To calculate the aerodynamic integrals which contain drag derivatives and are independent of the number of bending modes.

METHOD: These integrals are formed in AEROI from the product of 5 radial functions, and are referred to as unsubscripted AD2K and AD3K integrals. Since all the integrals involve drag derivatives, they are calculated over the whole blade.

USAGE: CALL AERIN8

SUBROUTINES CALLED: AEROII, AEROI

ERROR RETURNS: None

RESTRICTIONS: None

4 9013

NAME:	AERLST
PURPOSE:	To print the aerodynamic integrals.
METHOD:	If the print option is set at 3 or 4, then the integrals are printed. Otherwise control is returned to AERMAT with no integrals printed. Only those integrals calculated are printed.
USAGE:	CALL AERLST
SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	None

49012

NAME:	AERMAT
PURPOSE:	To control the calculation of the aerodynamic damping and stiffness matrices.
METHOD:	The aerodynamic integrals needed to calculate the matrices are evaluated in AERINT, stored in COMMON blocks AERO1 through AERO5, and printed out by AERLST. AERMAT then makes calls to AMDMAT to calculate the damping matrix, and to AMSMAT to calculate the stiffness matrix. Then, it extends the matrices to include blade torsional bending mode terms in AMATEX and calls BLELPA to calculate elastic torsional contributions.
USAGE:	CALL AERMAT
SUBROUTINES CALLED:	AERINT, AERLST, AMDMAT, AMSMAT, AMATEX, BLELPA
ERROR RETURNS:	None
RESTRICTIONS:	None

49010

NAME: AEROI

PURPOSE: To calculate the integral of the function y = F1(R)*F2(R)*F3(R)*F4(R)*F5(R) in the interval (r1, r2).

METHOD: The 5 radial functions are multiplied together at each radial station specified. It should be noted that the second argument in the AEROI calling sequence is designated complex and corresponds to an aerodynamic derivative function. The integral is evaluated in two parts, real and imaginary, by calls to INTEG. If there are no unsteady aerodynamic corrections, then the imaginary part of the integral is set to zero.

USAGE: CALL AEROI (F1, F2, F3, F4, F5, ANS)

F1, F3, F4, F5 = Real functions defined at the radii in R from AEROII. One or more may be equal to 1.0 at every R.

F2 = Complex function defined at the radii in R. If no unsteady aerodynamic correction has been applied, the imaginary part will be zero.

ANS = The complex integral of the product of F1 through F5. If no unsteady aerodynamic corrections have been applied, then the imaginary part will be zero.

SUBROUTINES CALLED: INTEG

ERROR RETURNS: None

RESTRICTIONS: None

49011

NAME: AEROII

PURPOSE: To set up calculations of the integral of the function y = a(R)*b(R)*c(R)*d(R)*e(R) over the interval (r1,r2).

METHOD: This routine sets the radial values where the functions are defined, the limits of integration, the number of subdivisions to be used in the trapezoidal rule integration method and the switches for unsteady aerodynamics.

Subsequent calls to AEROI supply the 5 functions whose product is to be integrated.

USAGE: CALL AEROII (R, N, RL, RU, K, CIR, CIRN)

R = Array of radii at which the functions are defined.

N = The number of points in R and in each function array.

RL = The lower limit of integration.

RU = The upper limit of integration.

K = The number of subdivisions to be used in performing the integration by the trapezoidal rule.

CIR = 1 No circulatory unsteady aerodynamics used.

CIRN = 1 No noncirculatory unsteady aerodynamics used.

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS:

1. N must be equal or less than 25.
2. CIR and CIRN have been set to 1 for no unsteady aerodynamics.

91783

NAME: AMATEX

PURPOSE: To expand the aerodynamic matrices to include the blade elastic torsional mode.

METHOD: The original aerodynamic damping (AMD) and stiffness (AMS) matrices are increased by 3 rows and columns and redefined as AMDN and AMSN to accomodate one collective and two cyclic modes associated with the blade elastic torsional mode.

USAGE: CALL AMATEX

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: The matrices are limited to 30 x 30.

49009

NAME:	AMDMAT
PURPOSE:	To calculate the aerodynamic damping matrix.
METHOD:	The matrix is calculated from the expressions given in Reference 1 using the aerodynamic integrals calculated in AERINT.
USAGE:	CALL AMDMAT
SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	None

49008

NAME: AMSMAT

PURPOSE: To calculate the aerodynamic stiffness matrix.

METHOD: The elements are calculated from the expressions given in Reference 1 using the aerodynamic integrals calculated in AERINT.

USAGE: CALL AMSMT

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: None

NAME:	BIFEXP
PURPOSE:	To transfer the non-linear bifilar mass matrix and the bifilar and rotor hub force vector to fixed system coordinates.
METHOD:	The bifilar mass (square matrix of order 6+NBIF) and the force vector (of order 6+NBIF) are transferred to the fixed system coordinates by proper multiplications with the fixed system mode shapes vector (loc. 450-549). The final expanded mass matrix and force vector have the dimensions of NF + NBIF. The method used is similar to that discussed in routine ADDOFR. The expanded mass and force vector are passed through labelled COMMON/NLDAT2. They are printed out for azimuth positions up to 5 degrees. The fixed system mode shapes vector is printed out for azimuths up to 2 degrees.
USAGE:	CALL BIFEXP (NRHS, NF) NRHS = Total number of non-linear bifilars plus 6 (maximum is 18). NF = Number of fixed system modes (maximum is 16).
SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	Expanded matrix and force vector cannot have dimensions greater than 28.

NAME: BIFILR

PURPOSE: To calculate the non-linear bifilar mass matrix and the bifilar and rotor hub force vector.

METHOD: For the time history solution, the bifilar acceleration terms are left on the left-hand-side of the equations of motion while the r.h.s. contains the bifilar damping and stiffness terms and the input hub forces. The bifilar mass matrix, S (dimensioned 6+NBIF), is calculated and stored in labelled COMMON/INEIGN. Then, the bifilar damping and stiffness terms are used to develop the r.h.s. vector, T (dimensioned 6+NBIF). Next, the rotor hub forces, loaded in locations 1600 through 1719, are evaluated and added to the bifilar contributions in vector T.

The hub forces are added to the system gradually according to the ramp factor (location 1760) and the azimuth position. The vector T is passed through labelled COMMON/NLDAT2.

The order of the degrees-of-freedom in this routine is: 6 fixed system d.o.f. (x, y, z, θ_x, θ_y, θ_z) and non-linear bifilars d.o.f. (NBIF-location 1763). Thus, the maximum number of d.o.f. is 18.

For azimuth positions up to 5 degrees, the mass matrix and the force vector are printed out.

USAGE: CALL BIFILR (NREV)

NREV = Revolution number-location 1762 divided by 360.

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: Number of input harmonics of rotor hub forces is limited to 10.

86029

NAME:	BLELPA
PURPOSE:	To calculate the blade elastic pitch aerodynamic contributions.
METHOD:	The aerodynamic damping and stiffness matrix elements are calculated using the appropriate expressions for blade pitch in Reference 1. All the terms are multiplied by the torsional mode shape, PH(r), which is a function of blade radius, except the blade pitch terms which are multiplied by the square of the mode shape.
USAGE:	CALL BLELPA
SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	None

NAME:	BLELPD
PURPOSE:	To calculate the blade elastic pitch dynamic contributions.
METHOD:	The dynamic mass, damping and stiffness matrix elements are calculated using the equation for the blade pitch in Reference 1. All the terms are multiplied by the torsional mode shape, PH(r), which is a function of blade radius, except the blade pitch terms which are multiplied by the square of the mode shape.
USAGE:	CALL BLELPD
SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	None

44087

NAME: BLIN4

PURPOSE: To provide a bivariant table lookup with linear interpolation for the airfoil data.

METHOD: The Mach number entries are searched to find K such that

$$M_{K-1} < M < M_K$$

The angles of attack in the M_{K-1} and M_K tables are searched to find I and J such that

$$A_{K-1,I-1} < A < A_{K-1,I}$$

and

$$A_{K,J-1} < A < A_{K,J}$$

Then the coefficient at M_{K-1} and A is given by

$$C_{K-1} = (A - A_{K-1,I-1})*(C_{K-1,I} - C_{C-1,I-1})/(A_{K-1,I} - A_{K-1,I-1}) + C_{K-1,I-1}$$

and the coefficient at M_K and A is given by

$$C_K = (A - A_{K,J-1})*(C_{K,J} - C_{K,J-1})/(A_{K,J} - A_{K,J-1}) + C_{K,J-1}$$

Finally, the coefficient at M and A is obtained from

$$C = (M - M_{K-1})*(C_K - C_{K-1})/(M_K - M_{K-1}) + C_{K-1}$$

$$\frac{DC}{DM} = \frac{C_K - C_{K-1}}{M_K - M_{K-1}}$$

USAGE: CALL BLIN4 (T, M, K, X, Y, Z, D, L)

T = A two-dimensional array containing the coefficient, angle of attack pairs. The first point in each column represents the number of pairs, the second point in each column represents the Mach number, and the remaining points in the column are the pairs for that Mach number.

M,K = The dimensions of T
M is the maximum number of Mach numbers in the table.
K is the maximum column length.

X = Angle of attack.

Y = Mach number.

Z = Returned coefficient.

D = Derivative with respect to M.

L = Error switch.
1 No error
2 Mach number not spanned
3 Angle of attack not spanned

SUBROUTINES CALLED: None

ERROR RETURNS: See above

RESTRICTIONS:

1. Number of Mach numbers specified must be equal or greater than 2 and cannot be greater than 12.

2. Number of coefficient/angle of attack pairs must be equal or greater than 5 and cannot be greater than 35.

49005

NAME: CMPRSS

PURPOSE: To compress a square matrix.

METHOD: The returned matrix of dimension L x L is produced by taking the first L rows and columns of the input matrix of dimension N x N.

USAGE: CALL CMPRSS (A, N, B, L)

A = Input matrix.

N = Dimensions of A.

B = Output matrix, which may have the same location as A.

L = Dimensions of B.

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: If L is equal or greater than N, no matrix compression is done.

86392

NAME:	CMPUTE
PURPOSE:	To control calculations of the generalized forces and of the forced response for the linear bifilar analysis.
METHOD:	First, the generalized forces are calculated in GENFOR. Then, the forced response of the linear system of equations is obtained in FORCER. The results of the analysis are printed out in a call to OUTPUT.
USAGE:	CALL CMPUTE
SUBROUTINES CALLED:	GENFOR, FORCER, OUTPUT
ERROR RETURNS:	None
RESTRICTIONS:	None

NAME: COMBIN

PURPOSE: To combine the non-linear bifilar mass matrix and the force vector developed for the non-linear bifilar analysis with the corresponding matrix and vector from the linear bifilar analysis.

METHOD: The mass matrix and force vector calculated in BIFEXP are added to the results of the linear analysis from SYSCTL. The fixed system d.o.f. are affected. The combined mass matrix and force vector include all the degrees-of-freedom (up to the maximum of 72).

If forces are input to one or two additional aircraft stations (as specified in locations 1767 and 1940 through 2179), their corresponding contributions to the final force vector are included by pre-multiplying the input forces by the appropriate aircraft station mode shapes (see locations 550-749).

The final combined mass matrix and force vector are stored in labelled COMMON/NLDAT2.

USAGE: CALL COMBIN (NBML, NRHS, NF, NREV)

NBML = Fixed system modes plus non-linear bifilars (maximum is 28).

NRHS = Total number of d.o.f. not including non-linear bifilars (maximum is 60).

NF = Number of fixed system modes (maximum is 16).

NREV = Revolution number - loc 1762 divided by 360.

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS:

1. Total number of degrees-of-freedom is 72 (60 from linear analysis and 12 for the maximum number of non-linear bifilars).

2. Harmonic force inputs are possible for only 2 aircraft stations.

863380

NAME: CONVER

PURPOSE: To test on the convergence of the time history solution for two successive revolutions.

METHOD: This routine checks the difference in the displacements of the first two non-linear bifilar pendulums obtained for two successive revolutions. If both differences are within .002 radian (corresponding to .1146 degree), then the convergence criterion is satisfied; IER (see below) is set to 1 and the program returns to NLBIF to calculate one more revolution after which the harmonic analysis is performed and printed out. If one of the differences is greater than .002 radian, the time history analysis proceeds to the next revolution.

The revolution number, the two bifilar displacement differences and the rotor hub x, y, θ_Z, $\dot{x}$, $\dot{y}$ motions are all listed out for each revolution.

USAGE: CALL CONVER (I, IER)

I = Rotor revolution number - location 1762 divided by 360.

IER = Convergence criterion - it is met if equal to 1.

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: None

95176

NAME:	DISCON
PURPOSE:	To adjust a blade stepwise function so that it is defined only over the blade.
METHOD:	The input segments are extended or truncated so that the first segment starts at the offset and the final segment finishes at the blade radius. Function values are unaltered but may be discarded if the segment they refer to is completely outside the blade.
USAGE:	CALL DISCON (KSTAR, DELTAR, NSTAR, E, R, RBAR, KBAR, N1BAR)

KSTAR = Array of input stepwise function values.

DELTAR = Array of segment lengths over which KSTAR is defined.

NSTAR = Number of entries in KSTAR and DELTAR.

E = Blade offset.

R = Blade radius.

RBAR = Array of adjusted segment lengths.

KBAR = Array of stepwise function values defined over RBAR.

N1BAR = Number of entries in RBAR and KBAR.

SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	None

48489

NAME: DISINT

PURPOSE: To perform the integration of the product of a step function and a radial function over the blade length.

METHOD: The stepwise function is first adjusted by DISCON so that it is defined over a segment distribution which starts at the offset and finishes at the blade radius. The integration of the radial function is then performed separately over each of these segments and multiplied by the value of the stepwise function for that segment. The final answer is the sum of these results.

i.e. $$\int_E^{R_N} f(r)S(r)dr = S_1 \int_E^{R_1} f(r)dr + S_2 \int_{R_1}^{R_2} f(r)dr + \ldots$$

$$S_k \int_{R_{K-1}}^{R_k} f(r)dr + \ldots . S_N \int_{R_{N-1}}^{R_N} f(r)dr$$

where

S_K = the value of the stepwise function over the K^{th} segment.

$f(r)$ = the radial function

USAGE: CALL DISINT (COMP, R, N, FPPP, RPPP, NM2, E, CR, SUM)

COMP = Array of radial function values.

R = Array of radii at which COMP is defined

N = Number of points in COMP and R.

FPPP = Array of stepwise function values.

RPPP = Array of segment lengths over which FPPP is defined.

48488

USAGE:	NM2 =	Number of entries in FPPP and RPPP.
	E =	Blade offset.
	CR =	Blade radius.
	SUM =	Integral of the product of the two functions.

SUBROUTINES CALLED: DISCON, INTEG

ERROR RETURNS: None

RESTRICTIONS: None

48490

NAME: DMATEX

PURPOSE: To expand the dynamic matrices to include the blade elastic torsional mode.

METHOD: The original dynamic mass (DMM), damping (DMD) and stiffness (DMS) matrices are increased by 3 rows and columns and redefined as DMMN, DMDN and DMSN respectively to accomodate one collective and two cyclic modes associated with the blade elastic torsional mode.

USAGE: CALL DMATEX

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: The matrices are limited to 30 x 30.

91782

NAME:	DMDMAT
PURPOSE:	To calculate the dynamic damping matrix.
METHOD:	The matrix is calculated from the expressions given in Reference 1, using the dynamic integrals calculated in DYNINT.
USAGE:	CALL DMDMAT
SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	None

2011

NAME:	DMMMAT
PURPOSE:	To calculate the dynamic mass matrix.
METHOD:	The matrix is calculated from the expressions given in Reference 1, using the dynamic integrals calculated in DYNINT.
USAGE:	CALL DMMMAT
SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	None

90333

NAME: DMSMAT

PURPOSE: To calculate the dynamic stiffness matrix.

METHOD: The matrix is calculated from the expressions given in Reference 1, using the dynamic integrals calculated in DYNINT.

USAGE: CALL DMSMAT

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: None

98330

NAME:	DYNINT
PURPOSE:	To set the initial values of the dynamic integrals to zero and to control the calculation of the integrals needed to construct the dynamic mass, damping, and stiffness matrices.
METHOD:	The dynamic integrals are obtained from the integration of the product of a radial function and a stepwise function over the blade length. All integrals have the blade mass or the edgewise, flatwise, and torsional mass moments of inertia as their stepwise function. If the radial function is independent of the blade bending modes, then the integrals are calculated in DYNIN1. Integrals with singly and doubly subscripted radial functions are handled in DYNIN2.
USAGE:	CALL DYNINT
SUBROUTINES CALLED:	DYNIN1, DYNIN2
ERROR RETURNS:	None
RESTRICTIONS:	None

10172

NAME:	DYNIN1
PURPOSE:	To calculate the dynamic integrals whose setpwise function is either the blade mass or the edgewise, flatwise, or torsional mass moment of inertia and whose radial functions contain no blade bending mode dependent quantities.
METHOD:	These integrals are formed in DISINT from the product of a radial function and a stepwise function integrated over the blade length. The radial function may itself be a product of radial functions. The integrals which are a function of blade mass are referred to as unsubscripted BMK. The integrals dependent on the blade mass moments of inertia are referred to as unsubscripted BIK.
USAGE:	CALL DYNIN1
SUBROUTINES CALLED:	DISINT
ERROR RETURNS:	None
RESTRICTIONS:	None

1G1439

NAME: DYNIN2

PURPOSE: To calculate the dynamic integrals whose stepwise function is either the blade mass or the edgewise, flatwise, or torsional mass moment of inertia and whose radial function contains one or two blade bending mode dependent quantities.

METHOD: These integrals are formed in DISINT from the product of a radial function and a stepwise function. The integrals which are a function of blade mass are referred to as BMJ and BMI for singly and doubly subscripted values respectively. The integrals dependent on the blade mass moments of inertia are referred to as BIJ and BII for singly and doubly subscripted values respectively.

USAGE: CALL DYNIN2

SUBROUTINES CALLED: DISINT

ERROR RETURNS: None

RESTRICTIONS: None

19888

NAME: DYNLST

PURPOSE: To print out the dynamic integrals.

METHOD: If the print option is set to 3 or 4, the integrals are printed. Otherwise, control is returned to DYNMAT.

Only those integrals calculated are printed.

USAGE: CALL DYNLST

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: None

85777

NAME:	DYNMAT
PURPOSE:	To control the calculation of the dynamic mass, damping and stiffness matrices.
METHOD:	The dynamic integrals needed to calculate the elements of the matrices are evaluated in DYNINT, stored in labelled COMMON/DYNOUT, and printed out by DYNLST. DYNMAT then makes calls to DMMMAT, DMDMAT and DMSMAT to calculate the mass, damping and stiffness matrices respectively. Then, it expands the matrices in DMATEX to include the blade torsional elastic mode terms, which are calculated in BLELPD. In addition, several blade parameters are calculated and printed out according to the degrees-of-freedom being used.
USAGE:	CALL DYNMAT
SUBROUTINES CALLED:	DYNINT, DYNLST, DMMMAT, DMDMAT, DMSMAT, DMATEX, BLELPD
ERROR RETURNS:	None
RESTRICTIONS:	None

95175

NAME:	EIGER
PURPOSE:	To compress and link the rotor matrices to the bifilar analysis.
METHOD:	The degrees-of-freedom not utilized are eliminated from the rotor dynamic and aerodynamic matrices. The dynamic and aerodynamic damping and stiffness matrices are added together. Then, the 30 x 30 matrices are compressed to K X K in CMPRSS. The final compressed matrices (3) are stored in labelled COMMON/INEIG for coupling with the bifilar analysis. The matrices can be printed out and/or punched out in cards if desired.
USAGE:	CALL EIGER
SUBROUTINES CALLED:	CMPRSS
ERROR RETURNS:	None
RESTRICTIONS:	None

10154

NAME:	ELI
PURPOSE:	To calculate the forces and moments needed to create the elastic matrix associated with the inboard half of a segment.
METHOD:	The forces and moments are calculated using the expressions derived in Reference 2.
USAGE:	CALL ELI(I) I = Blade segment number.
SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	None

10170

NAME: ELO

PURPOSE: To calculate the forces and moments needed to create the elastic matrix associated with the outboard half of a segment.

METHOD: The forces and moments are calculated using the expressions derived in Reference 2.

USAGE: CALL ELO(I)

I = Blade segment number.

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: None

NAME: EXTEND

PURPOSE: To find the values of a function at a set of points, given the function values at a different set of points.

METHOD: The value of the function at a new point is obtained by linear interpolation between the two closest old points.

USAGE: CALL EXTEND (F, R, M, RSTAR, N)

F = On input, the old function values. On output, the new function values.

R = The set of points at which F is defined on input.

M = Number of points in F and R.

RSTAR = The set of points at which F is defined on output.

N = The number of points in RSTAR and F on output.

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: N must be less than or equal to 400.

86001

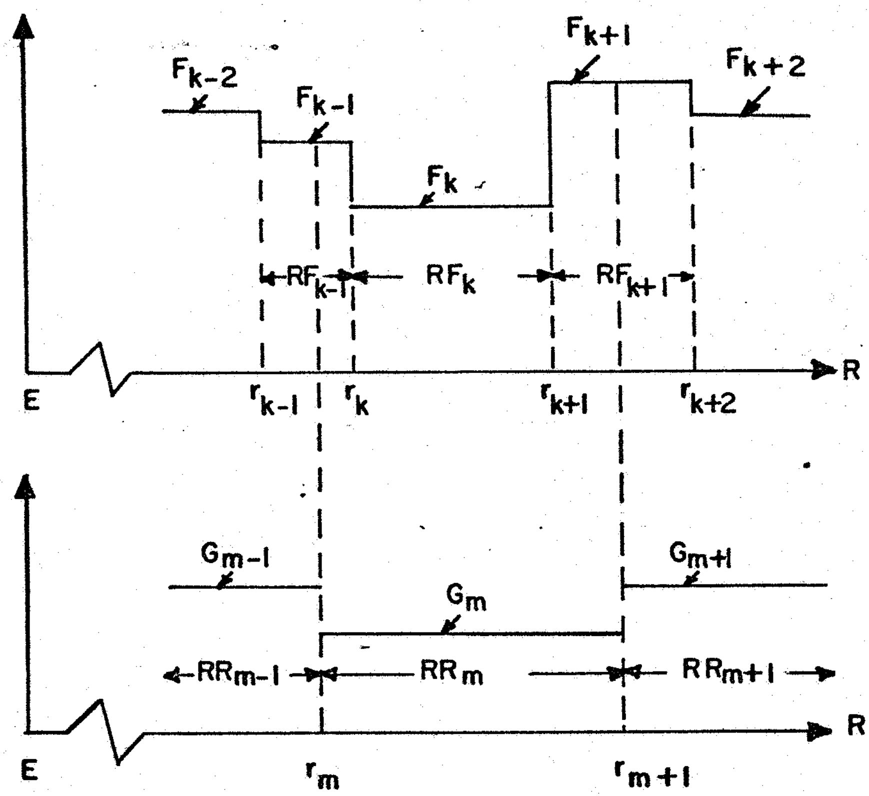
NAME:	E159X
PURPOSE:	To calculate the blade elastic torsional frequency.
METHOD:	The torsional frequency is found from repeated calls to ROOTX which calculates the torsional mode shape. The frequency trials start at zero frequency and proceed in steps of 10 rad/sec up to a maximum of 1990 rad/sec. After each trial, a check is made on the sign of the root mode shape. If a change in sign is found, then the frequency has been found. Three more iterations are performed to zero-in the frequency value (difference in 2 successive values of the root mode shape is within .0001).
USAGE:	CALL E159X
SUBROUTINES CALLED:	ROOTX
ERROR RETURNS:	If after 200 trials no sign change in the root mode shape has been found, then an error message is printed out as follows: 'OUT OF RANGE' followed by the last frequency trial value and the root mode shape value.
RESTRICTIONS:	1. Frequency upper limit is 1990 rad/sec. 2. Only the first elastic torsional frequency is found. 3. Three trials allowed to zero-in the final frequency value.

13588

NAME: FILL

PURPOSE: To find the value/length of a second moment of area function over each of the blade segments, given the function/length values over some other segment distribution.

METHOD:



The redistributed function G_m is defined below for new segment RR_m:

$$\frac{RR_m}{G_m} = \frac{r_k - r_m}{F_{k-1}} + \frac{r_{k+1} - r_k}{F_k} + \frac{r_{m+1} - r_{k+1}}{F_{k+1}}$$

Where:

F_k = Old function values
RF_k = Old blade segments
r_k = Radial positions of function from hinge (E)
G_m = New function values
RR_m = New blade segments

97246

USAGE: CALL FILL (RR, NSEG, F, RF, NF, E)

RR	=	An array containing the blade segment lengths.
NSEG	=	The number of segments in RR.
F	=	On input, the function values over RF. On output, the function values over RR.
RF	=	An array containing the segment lengths over which F is defined on input.
NF	=	The number of segments in RF.
E	=	The offset of the blade from the center of rotation.

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: NSEG must be less than or equal to 25.

86387

NAME: FIXABS

PURPOSE: To calculate the fixed system absorber matrices in the bifilar analysis.

METHOD: This routine initializes the matrices to zero. Then, it defines needed parameters from the input vector, V, and proceeds to evaluate the mass damping and stiffness elements according to the equations from Reference 3.

Printout of final matrices is controlled by location 10.

USAGE: CALL FIXABS

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: Number of fixed system absorber cannot be greater than 5.

161436

NAME: FIXSYS

PURPOSE: To calculate the fixed system modes matrices in the bifilar analysis.

METHOD: The fixed system modal mass, damping and stiffness elements are calculated according to the expressions in Reference 3.

A printout of the matrices can be obtained if location 10 is set to 1.

USAGE: CALL FIXSYS

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: None

97245

NAME: FOLL

PURPOSE: To convert a blade function from value/length to an equivalent radial function.

METHOD: The input function is first converted into a cumulative function with values at the end points of the input segment distribution. The value then assigned to a radial point is derived by finding the cumulative values at the end points of the segment that the radial point represents and subtracting them.

USAGE: CALL FOLL (RAD, F, NF, RB, NB, E, RR)

RAD = Segment lengths of input distribution.

F = Input stepwise function.

NF = Number of elements in RAD and F.

RB = The radial points at which values are required.

NB = The number of elements in RB.

E = The blade offset.

RR = The segment lengths to be associated with the radial points in RB.

SUBROUTINES CALLED: EXTEND

ERROR RETURNS: None

RESTRICTIONS: None

86394

NAME: FORCER

PURPOSE: To solve for the forced response of the linear bifilar analysis.

METHOD: At first, this routine combines the generalized force cosine and sine vectors and prints out the final vector, FRC, of dimension 2*NF. Then, it inverts the hub impedance matrix, T, by a call to LINV2F and forms the fixed system generalized coordinates vector, XQ, of dimension 2*NF. The generalized coordinates vector of all the system degrees-of-freedom, GAMMA, of dimension 90, is calculated from a multiplication of the transfer matrix, TRANSF, and the XQ vector. The final vector GAMMA is used to calculate and print out the forced response amplitudes and phase angles of the fixed system absorbers and bifilar pendulums.

The GAMMA vector is printed out if location 1497 is set to 1.0.

USAGE: CALL FORCER

SUBROUTINES CALLED: LINV2F

(This is an "IMSL" package routine which must be supplied by the Army).

ERROR RETURNS: None

RESTRICTIONS: Refer to subroutine HUBIMP.

90847

NAME: FREQUN

PURPOSE: To calculate the blade natural frequencies.

METHOD: The natural frequencies are the eigenvalues of the matrix formed by applying suitable boundary conditions to the matrix relating forces, etc., at the blade tip to the blade root.

The eigenvalues are found by assuming an initial frequency and then increasing it by a predetermined increment until a change in sign of the determinant occurrs. Newton's method is then used to improve the answer until $|F_{i+1} - F_i| < .001$.

The search for further natural frequencies continues until either the requested number has been found or the upper frequency limit is reached.

The starting values for the frequency trials is $.15\Omega$ where Ω is the rotor speed in rad/sec.

The frequency scan interval is $.10\Omega$.

USAGE: CALL FREQUN

SUBROUTINES CALLED: PRODM, MIND

ERROR RETURNS:

1. 'EXCEEDED UPPER FREQUENCY LIMIT'

   The program will search for the number of frequencies requested up to a frequency of 75 cycles/rev.

   The program continues using the number of natural frequencies that have been found.

2. 'DID NOT CONVERGE AFTER SIGN CHANGE - 100 TRIALS'

   Having located a change in sign of the determinant, the program was unable to improve the eigenvalue sufficiently to satisfy the convergency test in 100 trials. The program continues using the last estimate.

3. 'FAILED TO LOCATE SIGN CHANGE AFTER 500 TRIALS'

An eigenvalue was not detected in the frequency range 0.0 to 75 cycles/rev. The program continues with a frequency of 75 cycles/rev.

4. 'AN EVEN NUMBER OF FREQUENCIES HAVE BEEN MISSED'

Two frequencies differ by less than $.0015\Omega$ - $.00001$ (Ω is the rotor speed). The program treats them as one frequency and continues.

RESTRICTIONS:

1. Frequencies cannot be calculated at zero rotor speed. It is suggested that rotor speed be lowered gradually to about 50 rad/sec. Computer time increases tremendously as rotor speed decreases since the scan interval is set at $.10\Omega$.

2. See notes on "ERROR RETURNS".

NAME:	FULL
PURPOSE:	To find the values of an area function over each of the blade segments, given the function/length values over some other segment distribution.
METHOD:	Same method as discussed for subroutine "FILL".
USAGE:	CALL FULL (RR, NSEG, F, RF, NF, E)
	RR = An array containing the blade segment lengths.
	NSEG = The number of segments in RR.
	F = On input, the function values/length over L. On output, the function values/length over RR.
	RF = An array containing the segment lengths over which F is defined on input.
	NF = The number of segments in RF.
	E = The offset of the blade from the center of rotation.
SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	NSEG must be less than or equal to 25.

80393

NAME: GENFOR

PURPOSE: To calculate the generalized forces for the linear bifilar analysis.

METHOD: The generalized force vectors for the cosine (FQC) and sine component (FQS) are calculated from the force input vectors (locations 110-289) provided for the main and tail rotor hubs and 2 additional aircraft points and their appropriate mode shapes (locations 450-749 and 850-949).

The force vectors of dimension NF (location 9) are passed through labelled COMMON/XFRDAT.

USAGE: CALL GENFOR

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: None

42885

NAME: GMPRDD

PURPOSE: To multiply two general matrices to form a resultant general matrix.

METHOD: The M X L matrix B is premultiplied by the N X M matrix A and the result is stored in the N X L matrix R.

RESULT:

$$\underset{NXL}{R} = \underset{NXM}{A} \times \underset{MXL}{B}$$

USAGE: CALL GMPRDD (A, B, R, N, M, L)

A = First input matrix.

B = Second input matrix.

R = Output matrix.

N = Number of rows in A.

M = Number of columns in A and rows in B.

L = Number of columns in B.

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: None

NAME: HARMON

PURPOSE: To perform harmonic analysis of the time history solution.

METHOD: The solution acceleration vector calculated in INTEQ and passed through labelled COMMON/NLDAT2 is harmonically analyzed once the convergence criterion is met in routine CONVER.

The harmonics are stored in labelled COMMON/HARM.

USAGE: CALL HARMON (IC, N1)

IC = Rotor revolution number - location 1762 divided by 360.

N1 = Total number of degrees-of-freedom (maximum is 72).

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: The number of harmonics output is limited to 10.

86391

NAME:	HUBIMP
PURPOSE:	To develop the transfer and hub impedance matrices for the linear bifilar analysis.

METHOD:

At first, the final mass (XMFC) and stiffness (XKFC) are combined to form matrix A (= XKFC $-W_F^2$*XMFC), where W_F is the forcing frequency. Then, the damping matrix (XCFC) is used to form matrix B (= W_F*XCFC). If no fixed system absorbers and no linear bifilars are present in the system, the routine calculates the hub impedance matrix, E, from matrices A and B and then returns to MAINSV; otherwise, it proceeds to calculate the transfer matrix, TRANSF, after a call to LINV2F, and subsequently the hub impedance matrix E. (Additional information can be obtained in Ref. 3).

The transfer and hub impedance matrices are passed through labelled COMMON/XFRDAT.

Throughout the matrix calculations performed, the resulting matrices can be printed out using the control switches in locations 10 and 15.

USAGE: CALL HUBIMP

SUBROUTINES CALLED: LINV2F

(This is an "IMSL" package routine which must be supplied by the government)

ERROR RETURNS: None

RESTRICTIONS:

1) Number of degrees-of-freedom of the final system matrices (XMFC, XKFC, XCFC) is limited to 60 as follows:

a.	Fixed system modes	16
b.	Fixed system absorbers	5
c.	Linear bifilars (5X3)	15
d.	Rotor Modes	24
		60 = Total

2) The transfer matrix (TRANSF) maximum dimensions are (90X32) where 90 represents 2 times the maximum d.o.f. of the fixed system absorber plus the bifilar and rotor modes and 32 is 2 times the maximum number of fixed system d.o.f.

3) The hub impedance matrix (E) has maximum dimensions of (32X32) obtained from 2 times the maximum fixed system d.o.f.

85870

NAME:	INCOND
PURPOSE:	To initialize quantities to zero in the bifilar analysis.
METHOD:	The system mass, damping and stiffness matrices (XMFC, XCFC, XKFC respectively) of dimensions 60X60 are set to zero. In addition, the matrices (10X75) used to store the harmonic results (CHARM and SHARM) are zeroed out. The input location 1498 controls the initialization of the bifilar pendulums, the rotor hub and the state variables displacements and velocities.
USAGE:	CALL INCOND
SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	None

09940

NAME: INPUTV

PURPOSE: To provide a description of the input locations to the bifilar analysis and to read the input data.

METHOD: Locations 1 through 2200 are listed and discussed - all lines of code are of course commented out. Then, a call is made to LOADIT to read the input data.

USAGE: CALL INPUTV

SUBROUTINES CALLED: LOADIT

ERROR RETURNS: None

RESTRICTIONS: None

858912

NAME: INTEG

PURPOSE: To convert a function defined at unequally spaced argument points, to an equal function defined at a set number of equally spaced points in a given interval and integrate it using the trapezoidal rule.

METHOD: The length of the interval is divided by the number of divisions required to obtain the argument spacing. Function values at these subdivision points are obtained by linearly interpolating between the two nearest existing function values. The redistributed function is then passed to QTFG for integration.

USAGE: CALL INTEG (F, R, NSEG, RBAR1, RBAR2, N, XI)

F = Array of function values.

R = Array of argument values at which F is defined.

NSEG = Number of blade segments

RBAR1 = Lower limit of the interval of integration.

RBAR2 = Upper limit of the interval of integration.

N = Number of subdivisions to be used.

XI = Integral.

SUBROUTINES CALLED: QTFG

ERROR RETURNS: None

RESTRICTIONS: N must be less than or equal to 100.

NAME: INTEQ

PURPOSE: To calculate the time history solution of the non-linear equations of motion.

METHOD: The combined mass matrix (XMT) and force vector (FT) of dimensions NT developed in COMBIN are used to obtain the time history response of the non-linear system. The procedure is discussed below.

Given $\left[XMT\right]\left\{\ddot{q}\right\} = \left\{FT\right\}$,

first, the matrix and vectors are partitioned as follows:

$$\left[\begin{array}{c|c} XMT_A & XMT_B \\ \hline XMT_C & I \end{array}\right]\left[\begin{array}{c} \ddot{q}_1 \\ \hline \ddot{q}_2 \end{array}\right] = \left[\begin{array}{c} FT_1 \\ \hline FT_2 \end{array}\right]$$

where the square matrix, $\left[XMT_A\right]$, the acceleration vector part, $\left\{\ddot{q}_1\right\}$, and the force vector part, $\left\{FT_1\right\}$, have dimensions (NT-ND) and include only the degrees-of-freedom associated with the fixed system and rotor modes, while the unity matrix, $\left[I\right]$, the acceleration vector part, $\left\{\ddot{q}_2\right\}$, and the force vector part, $\left\{FT_2\right\}$, have dimensions ND and include the rest of the d.o.f. of the system.

Next, the accelerations are solved for as shown below:

1) $\left[XMT_A\right]\left\{\ddot{q}_1\right\} + \left[XMT_B\right]\left\{\ddot{q}_2\right\} = \left\{FT_1\right\}$

2) $\left[XMT_C\right]\left\{\ddot{q}_1\right\} + \left[\ I\ \right]\left\{\ddot{q}_2\right\} = \left\{FT_2\right\}$

Solve for $\left\{\ddot{q}_2\right\}$ from equation 2) above.

3) $\left\{\ddot{q}_2\right\} = \left\{FT_2\right\} - \left[XMT_C\right]\left\{\ddot{q}_1\right\}$

Then, the solution vector $\ddot{q}_1$ is given by substituting 3) into 1) above.

4) $\left(\left[XMT_A\right] - \left[XMT_B\right]\left[XMT_C\right]\right)\left\{\ddot{q}_1\right\} =$
$\left\{FT_1\right\} - \left[XMT_B\right]\left\{FT_2\right\}$

which can be written as,

5) $\left[XXT\right]\left\{\ddot{q}_1\right\} = \left\{XT\right\}$

The routine forms the matrix $\left[XXT\right]$ and the vector $\left\{XT\right\}$ (note that the maximum dimensions of (NT-ND) are 40).

The acceleration vector, $\left\{\ddot{q}_1\right\}$, is solved for in the IMSL routine LEQT2F. Subsequently, the acceleration vector, $\left\{\ddot{q}_2\right\}$, is evaluated from expression 3) above.

The analysis proceeds next to integrate the combined acceleration vector, $\left\{\ddot{q}\right\}$, to obtain the velocity and displacement vectors using the expressions below:

6) $\left\{\dot{q}\right\}_{t+\Delta t} = \left\{\dot{q}\right\}_t + \Delta t\left\{\ddot{q}\right\}_t$

7) $\left\{q\right\}_{t+\Delta t} = \left\{q\right\}_t + \Delta t\left\{\dot{q}\right\}_t$

where the time increment, Δt, is defined from

8) $\Delta t = \dfrac{\Delta\psi \text{ (deg-loc 1761)}}{\Omega \text{ (rpm-loc 7) *6}}$, seconds

The resulting velocity vector is loaded into the input vector, V, in locations 1740-1759 for the non-linear bifilars and in locations 1860-1939 for the remaining d.o.f., while the displacement vector is loaded similarly into locations 1720-1739 and locations 1780-1859. The rotor hub velocities and displacements are also calculated by pre-multiplying the resulting vectors by the transfer matrix (input in locations 450-549). The resulting velocity and displacement vectors are printed out at every azimuth position up to 30 degrees. The final step in INTEQ is to increment the azimuthal angle and return to NLBIF.

The final results are passed through labelled COMMON/INDAT for the input vector V and through labelled COMMON/NLDAT2 for the solution acceleration vector $\left\{\ddot{q}\right\}$.

USAGE: CALL INTEG (NT, ND)

NT = Total number of d.o.f. (72 maximum).

86363

ND = NT minus number of d.o.f. of fixed system (16 maximum) and rotor (24 maximum); maximum value of ND is 32.

SUBROUTINES CALLED: LEQT2F

(This is an "IMSL" package routine which must be supplied by the Army).

ERROR RETURNS: None

RESTRICTIONS: The maximum order of the solution vector (as obtained from LEQT2F) is 40.

NAME: LINBIF

PURPOSE: To calculate the linear inplane bifilar matrices

METHOD: This routine defines needed parameters from the input vector, V, to start. Then, it initializes the matrices to zero and proceeds with the calculations of the mass, damping and stiffness elements according to the expressions published in Ref. 3. The matrices are of order 9X9 and consist of the following degrees-of-freedom:

1. Fixed system longitudinal
2. Fixed system lateral
3. Fixed system vertical
4. Fixed system roll
5. Fixed system pitch
6. Fixed system yaw
7. Bifilar symmetric mode
8. Bifilar cyclic (sine) mode
9. Bifilar cyclic (cosine) mode

A printout of the bifilar matrices can be obtained by setting location 1495 to 1.0.

USAGE: CALL LINBIF

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS:

1. Number of bifilars in each kind is limited to 10.

2. Number of different bifilar kinds (inplane and vertical) cannot be greater than 5.

42861

NAME:	LOADIT
PURPOSE:	To read a data card, and then after checking the characters, to store the data in the specified locations.
METHOD:	Each card in the input stream is read and printed out before any interpretation is attempted. Each character on the card is then checked for validity. Column one must be -, +, 0 or blank; column two must be 1 through 5; columns three through six must be +, 0 - 9, or blank; and columns seven through sixty-six can be -, +, 1, 0 - 9, E or blank. A minus in column one indicates end of input data for that case. Column two contains the number of values to be read in. Columns three through six contain the input location at which to start storing the values. If the address is zero or blank, the next location is used. Columns seven through sixty-six contain the values to be stored. A format of 5E12.4 is assumed.
USAGE:	CALL LOADIT(X,NFILE) X = An array into which the data is placed. In the rotor analysis, X = INPUT (which is equivalent to blank COMMON). In the bifilar analysis, X=V (which is equivalent to labelled COMMON/INDAT. NFILE = 5. The read unit file for both rotor and bifilar analyses.
SUBROUTINES CALLED:	None
ERROR RETURNS:	Any error results in the card that caused the error being ignored.

RESTRICTIONS:

*1) LOADIT is one of four routines which are computer dependent. Coding for both IBM and CDC computer systems is retained with appropriate lines commented out.

2) The maximum number of input locations is 8100 for the rotor analysis and 2200 for the bifilar analysis.

NAME: LVBIF

PURPOSE: To calculate the linear vertical bifilar matrices

METHOD: This routine defines needed parameters from the input vector, V, to start. Then, it initializes the matrices to zero and proceeds with the calculations of the mass, damping and stiffness elements according to the expressions published in Ref. 3. The matrices are of order 9X9 and consist of the following degrees-of-freedom:

1. Fixed system longitudinal
2. Fixed system lateral
3. Fixed system vertical
4. Fixed system roll
5. Fixed system pitch
6. Fixed system yaw
7. Bifilar symmetric mode
8. Bifilar cyclic (sine) mode
9. Bifilar cyclic (cosine) mode

A printout of the bifilar matrices can be obtained by setting location 1496 to 1.0.

USAGE: CALL LVBIF

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS:

1. Number of bifilars in each kind is limited to 10.
2. Number of different bifilar kinds (inplane and vertical) cannot be greater than 5.

09941

NAME: MAINSV

PURPOSE: To control the principal logic flow of the bifilar analysis portion of the rotor/bifilar coupled program.

METHOD: MAINSV first calls INPUTV which calls LOADIT to read the input data for the case at hand. Then, the bifilar matrices and other quantities are set to zero in INCOND. The contributions of the fixed system modes, the rotor (if used), the fixed system absorbers, and the linear inplane and vertical bifilar pendulums to the final system mass, damping and stiffness matrices are added in SYSCTL. The program then proceeds to calculate the hub impedance and transfer matrices in HUBIMP, and the generalized forces and the forced response in CMPUTE for the linear analysis case.

If non-linear inplane bifilars are to be analyzed, the program bypasses HUBIMP and CMPUTE and instead it activates NLBIF to calculate the time-history response of the system.

USAGE: CALL MAINSV (I927SW, NCASE)

I927SW	=	$\neq 0$	Include rotor contributions.
		0	Do not include rotor contributions.
NCASE	=	1	Input vector, V(2200), is initialized to zero and the bifilar analysis title (second one) is read in.
		≥ 2	Input vector is not set to zero and the second title card is not read.

SUBROUTINES CALLED: INPUTV, INCOND, SYSCTL, HUBIMP, CMPUTE, NLBIF

ERROR RETURNS: None

RESTRICTIONS: None

NAME:	MATEI
PURPOSE:	To calculate the elastic matrix associated with the inboard half of a rotor blade segment.
METHOD:	The 13 X 13 matrix is calculated using the expressions derived in Reference 2 and the forces and moments calculated in ELI.
USAGE:	CALL MATEI(I) I = Blade segment number.
SUBROUTINES CALLED:	MIND, GMPRDD
ERROR RETURNS:	None
RESTRICTIONS:	None

428718

NAME:	MATEO
PURPOSE:	To calculate the elastic matrix associated with the outboard half of a rotor blade segment.
METHOD:	The 13 X 13 matrix is calculated using the expressions derived in Reference 2 and the forces and moments calculated in ELO.
USAGE:	CALL MATEO(I) I = Blade segment number.
SUBROUTINES CALLED:	MIND, GMPRDD
ERROR RETURNS:	None
RESTRICTIONS:	None

13572

NAME: MATF

PURPOSE: To calculate the mass transfer matrix which relates the blade forces across a concentrated mass.

METHOD: The 13 X 13 matrix is calculated using the expressions derived in Reference 2.

USAGE: CALL MATF(I,W)

I = Blade segment number.

W = Rotational speed.

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: None

13563

NAME:	MATR
PURPOSE:	To calculate the transformation matrix associated with a discontinuity in blade twist.
METHOD:	The 13 X 13 matrix is calculated using the expression derived in Reference 2 and represents the change in twist from segment I to segment I-1.
USAGE:	CALL MATR(I) I = Blade segment number.
SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	None

13562

NAME: MIND

PURPOSE: To invert a matrix and calculate its determinant.

METHOD: The routine is an adaption of the routine MINV from the IBM Scientific Subroutine Package, which uses the standard Gauss-Jordan reduction to obtain the inverse. The value of the determinant is obtained simultaneously as the product of the pivot.

To avoid possible overflow conditions, the value of the determinant is expressed in the form D X 10^I, where D is between 1.0 and 10.0, by repeated calls to OVUN.

USAGE: CALL MIND (A, N, D, L, M, IE)

A = The matrix to be inverted; it is destroyed during the computation.

N = The dimensions of A.

D = Determinant value.

L = Work vector of length N.

M = Work vector of length N.

IE = Power of 10 to be associated with D.

SUBROUTINES CALLED: OVUN

ERROR RETURNS: A determinant of zero indicates a singular matrix.

RESTRICTIONS: None

40018

NAME:	MISC
PURPOSE:	To perform initial calculations on the input data for the blade frequency and mode shape calculations.
METHOD:	The input segment lengths are used to construct an array of radii to the segment centers, and the local twist distribution is used to construct the twist discontinuities.
USAGE:	CALL MISC
SUBROUTINES CALLED:	None
ERROR RETURNS:	The sum of the segment lengths is checked against the blade radius; a difference of more than 10% produces the following warning: 'INCOMPATABILITY BETWEEN RADIUS AND SUM OF SEGMENTS'. The program continues with the input values.

4G532

NAME: MODES

PURPOSE: To control the calculations of the rotor blade fully-coupled frequencies and mode shapes.

METHOD: The blade is assumed to consist of a number of spanwise segments with the inertial loading on each segment concentrated at the center. Each segment is then divided into two parts: one inboard and one outboard of the concentrated mass. The half segments are treated as weightless, with the concentrated mass being located at the junction between them. The elastic properties are assumed to be constant within the inboard and outboard halves of the segment. The built-in blade twist is incorporated in the model by permitting angle changes at the junction between segments.

Relationships between blade forces, moments, and elastic deformations at the tip and at the root of the blade can be obtained by considering the changes in these variables over the blade segments. Elastic matrices are used to give the change across the inboard and outboard sections of the segments. Transformation matrices account for the change due to an abrupt change in twist at the segment junctions, and mass transfer matrices give the change from a position just outboard to just inboard of a concentrated mass at the center of a segment. The product of these matrices for all segments will relate the variables at the tip to those at the root.

By applying suitable boundary conditions and iterating on frequency, the blade natural frequencies and hence the mode shapes can be derived.

USAGE: CALL MODES

SUBROUTINES CALLED: MISC, PINT, FREQUN, MSHAPE, POUT, ORTHOG

ERROR RETURNS: None

RESTRICTIONS: None

90831

NAME:	MSHAPE
PURPOSE:	To calculate the blade mode shapes and their first and second derivatives.
METHOD:	This routine first determines whether the mode shape will be predominantely flatwise, edgewise, or torsional by examining the size of the determinant of the matrix relating the tip properties to the root properties at a frequency just less than the natural frequency. This knowledge is used in setting up a series of simultaneous equations whose solutions yield the required mode shape on substitution into expressions derived in Reference 2.
USAGE:	CALL MSHAPE
SUBROUTINES CALLED:	PRODM, MIND, OVUN, SIMLIN, MATF, ELO, MATEO, MATR, ELI, MATEI, GMPRDD
ERROR RETURNS:	None
RESTRICTIONS:	None

8G346

NAME: NLBIF

PURPOSE: To control the non-linear bifilar analysis calculations.

METHOD: The non-linear bifilar analysis is performed if the control switch in location 18 is set to 1 and the number of non-linear bifilars in location 1763 is greater than zero.

At first, the routine calculates the total number of degrees-of-freedom and proceeds with the calculations if it is equal or less than 72. Then, it calculates the time history response by calling, in sequence, the routines RHS, BIFILR, BIFEXP, COMBIN, INTEQ for each rotor revolution until either the maximum azimuth angle (input location 1762) is reached or the convergence criterion is met, as specified in CONVER. In either case, the analysis proceeds to analyze the harmonic response of the bifilar pendulums, the rotor hub and the aircraft stations in HARMON. The results are printed out in the routine OUT. The initial values of the bifilar, hub and state variables displacements and velocities are then listed to be used as starting values for the next case.

USAGE: CALL NLBIF

SUBROUTINES CALLED: RHS, BIFILR, BIFEXP, COMBIN, INTEQ, HARMON, CONVER, OUT

ERROR RETURNS: If the number of degrees-of-freedom is greater than 72, then the non-linear bifilar analysis is not activated and the following message is printed out:

'TIME HISTORY SOLUTION WAS NOT PERFORMED SINCE TOTAL NUMBER OF D.O.F. =   , I.E. > 72'.

RESTRICTIONS:

1. Number of non-linear inplane bifilars is limited to 12.
2. Total number of d.o.f. is 72 (60 from linear analysis plus 12 for non-linear analysis).

86049

NAME: ORTHOG

PURPOSE: To test the orthogonality of the blade mode shapes.

METHOD: The orthogonality relation is shown below.

$$ORTH(i,j) = \frac{\Sigma_k \, m_k \, \phi_{i,k} \, \phi_{j,k}}{\sqrt{\Sigma_k \, m_k \, (\phi_{i,k}^2 + \phi_{j,k}^2)}}$$

where m_k = Mass of segment k.

$\phi_{i,k}$ = i^{th} mode shape for segment k.

$\phi_{j,k}$ = j^{th} mode shape for segment k.

USAGE: CALL ORTHOG

SUBROUTINES CALLED: None

ERROR RETURN: None

RESTRICTIONS: None

86381

NAME: OUT

PURPOSE: To print out the harmonic results for the non-linear bifilar analysis.

METHOD: At first, this routine calculates some parameters from the input vector V. Then, it uses the harmonic analysis results from HARMON to calculate the amplitudes and phase angles of the harmonic response of the non-linear bifilar pendulums, the rotor head and the aircraft stations. Appropriate multiplications with the mode shapes input for the fixed system and the aircraft stations are performed.

USAGE: CALL OUT (NTOT, N1)

NTOT = Total number of system d.o.f. excluding the number of non-linear bifilars.

N1 = Total number of system d.o.f.

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS:

1. Maximum number of output harmonics is 10.
2. Maximum number of aircraft stations whose response is harmonically analyzed is 4.

NAME:	OUTPUT
PURPOSE:	To calculate and print the aircraft and rotor hub forced response.
METHOD:	This routine uses the results from FORCER (specifically vector XQ) to calculate the forced response at specific aircraft stations (up to 4) where mode shapes are loaded in locations 1000 through 1399 and at the rotor head using the mode shape in locations 450 through 549.
USAGE:	CALL OUTPUT
SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	The response of the aircraft can be evaluated at a maximum of 4 stations.

86048

NAME: OVUN

PURPOSE: To express A in the form B x 10^{I}, where $1.0 \le B \le 10.0$.

METHOD: If A is greater than 10.0, it is continually divided by 10.0 until it is less than 10.0. If A is less than 1.0, it is continually multiplied by 10.0 until it is greater than 1.0.

USAGE: CALL OVUN (A, B, I)

A = Input number to be transformed.

B = Part of B which is greater than or equal to 1.0 and less than or equal to 10.0.

I = The power of 10 which is associated with B to form A.

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: None

86045

NAME: PFMULT

PURPOSE: To generate a point field matrix, relating the steady deflections of the center of a blade segment to those of the previous segment center.

METHOD: The point matrix is defined as W(5X5):

$$
\begin{array}{ccccccc}
\underline{\text{Column}} & 1 & 2 & 3 & 4 & 5 & \underline{\text{Row}} \\
W = & \multicolumn{5}{c}{\left[\begin{array}{ccccc}
1 & L/A & -L*J/2 & -L^2*J/6 & C*J*L/2 + L*\gamma*B \\
0 & 2/A-1 & -J & -A*J*L*(1/A+0.5)/3 & C*A*J*(B+2)+2*\gamma*B \\
0 & -2B/(A*J) & 1/A & L*(1/A+2)/3 & -C*(B+6)-2*\gamma*B/(A*J) \\
0 & 0 & 0 & 1 & -L*D \\
0 & 0 & 0 & 0 & 1
\end{array}\right]} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\end{array}
$$

Where:

1) D = Thrust or drag derivative/unit length between segment centers.

2) EI = Flatwise or edgewise segment stiffness.

3) L = Segment length.

4) M = Segment mass.

5) R = Segment radial position.

6) r = Blade lag or coning angle.

7) Ω = Rotor speed.

8) A = $\left[1-L^2*\Omega^2/(2*EI)\right] * \sum_i^N M_i R_i$

9) B = 1/A-1

10) C = L^2*D/12

11) J = L/(EI*A)

For inplane deflections, a point mass matrix is added to the fourth row whose elements then become

$$W(4,I) = W(4, I)-M*\Omega^2* \left[W(1,I)+R*\gamma*W(5,I)\right]$$

Conditions at the next center are obtained by pre-multiplying W by the matrix representing conditions at the previous center.

USAGE: CALL PFMULT (PFIN, PFOUT, LS, AS, EIS, DS, MOMG2S, GAMOS, RS, ISWTCH)

PFIN = 5x5 matrix corresponding to previous center.

PFOUT = 5x5 matrix corresponding to next outboard center.

LS = Distance between centers.

AS = $\left[1-L*^2\Omega^2/(2*EI)\right] * \sum_i^N M_i R_i$.

EIS = Flatwise or edgewise second moment of area between centers.

DS = Thrust or drag derivative/unit length between centers.

MOMG2S = $M*\Omega^2$ (centrifugal force).

GAMOS = γ - calculated lag or coning angle.

RS = Distance of next center from the blade root.

ISWTCH = 0 for inplane deflections.

= 1 for out-of-plane deflections.

SUBROUTINES CALLED: GMPRDD

ERROR RETURNS: None

RESTRICTIONS: None

86043

NAME: PICK

PURPOSE: Given 3 points on the curve $y=f(x)$, choose the best 2 to use for an estimate of the solution of $y=f(x)$.

METHOD: The 3 points are rearranged such that

$$y_1 - x_1 \geq y_2 - x_2 \geq y_3 - x_3$$

If $y_2 - x_2 \leq 0$, then points 1 and 2 in the rearranged order are used.

If $y_2 - x_2 > 0$, then points 2 and 3 in the rearranged order are used.

USAGE: CALL PICK (TT, CT)

TT = Array of argument values.

CT = Array of function values.

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: None

NAME:	PINT
PURPOSE:	To print the input data used in the blade natural frequency and mode shape calculations.
METHOD:	If the print option is set to 4, the input is printed. Otherwise, control is returned to MODES with no printing.
USAGE:	CALL PINT
SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	None

NAME:	POUT
PURPOSE:	To print the output of the blade mode subsegment of the program.
METHOD:	This routine is not used by the rotor stability program, but has been included for completeness.
USAGE:	CALL POUT
SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	None

NAME: PRELIM

PURPOSE: To prepare the data for use by the rotor aeroelastic analysis.

METHOD: The two main tasks of PRELIM are to standardize the data tables so that all the data tables are defined over the same segment lengths or at the same radial points, and to control the calculation of other quantities needed throughout the remainder of the program.

Transfer of data between the program segments is achieved through 3 COMMON blocks. Blank COMMON is used primarily to store input data. Quantities calculated by PRELIM from the input are stored in labelled COMMON/DYNINP. Printout control is maintained by switches set in labelled COMMON /PRNTSW. All other labelled COMMON blocks are used for the transfer of data within the PRELIM segment.

The following is a more detailed breakdown of the tasks performed.

1. The input data are read into blank COMMON via subroutine LOADIT and immediately stored on a temporary work file (Unit 11). Prior to each subsequent case in the run, this temporary file is read into blank COMMON; thus, only those variables which differ from the previous case need be input.

   It should be noted that PRELIM is the third computer dependent routine due to different requirements for reading input data. Coding for both IBM and CDC systems is retained in the program with the appropriate lines commented out.

2. The coupling matrix terms needed for the bifilar analysis portion are initialized to zero to start. For subsequent cases, the coupling matrices are retained and used again if the rotor is not changed or recalculated if a new rotor is employed.

3. Specific input locations are set for the coupling with the bifilar analysis portion.

4. Control switches are set and, where applicable, inputs are converted from generally accepted engineering units to standard units for program calculations.

5. The input blade segments, which form the basis of all radial distributions used by the program, are adjusted to include a 0.1 segment to accomodate pitch horn effects. They are then used to set up a radius vector representing the distance of the mid-points of each segment from the center of rotation. The offset is automatically included if it exists.

6. All input tables are extended or redefined over the distributions obtained in 3 above.

7. Pitch horn effects are added to the blade center of gravity and blade weight tables.

8. The thrust for the input pitch angle is calculated for hover or vertical flight.

9. An input vector used by MODES to calculate mode shapes and frequencies is set up. In multiple cases, if this input has not changed from the previous case, then no call is made to MODES and the program uses the values obtained in the previous case.

10. Pitch-lag and pitch-flap coupling effects are calculated using the blade bending modal components at the pitch horn.

11. Blade lag damper coupling terms are calculated from the input blade damper geometry.

12. The aerodynamic derivatives and steady deflections are calculated in hover.

13. Control system inputs are checked to eliminate contradictions.

14. Blade torsional properties are calculated including cross-beam blade characteristics.

15. Selected portions of the input and calculated data are printed out by subroutine PROUT.

USAGE: CALL PRELIM (NCASE, NMODE)

NCASE = 0 Input vector, INPUT (8100), is initialized to zero and the rotor analysis title (first one) is read in.

$\geq$ 1 Input vector is not set to zero, original input vector is read from Unit 11 and the first title card is not read in.

NMODE = Switch for calculating mode shapes and frequencies.

SUBROUTINES CALLED: LOADIT, SORTAB, EXTEND, FILL, SECAER, QTFG, PICK, MODES, REMOVE, STDEFL, FOLL, FULL, E159X, PROUT

ERROR RETURNS: None

RESTRICTIONS: None

86040

NAME:	PRODM
PURPOSE:	To calculate the matrix relating blade forces, moments and elastic deformations at the tip to the blade root and to apply the boundary conditions.
METHOD:	Since the elastic matrices and twist transformation matrices are independent of the frequency, they are calculated once for each segment and stored on a work file. For the given frequency, the mass transfer matrices are calculated and combined with the matrices on the work file to produce the tip-to-root relation matrix. Boundary conditions are applied, and the resulting 8 X 8 matrix is returned. Some data are stored in a temporary work file (Unit 8).
USAGE:	CALL PRODM (A, NEQ) A = The returned 8 X 8 matrix. NEQ = The dimensions of A.
SUBROUTINES CALLED:	ELO, MATEO, MATR, ELI, MATEI, GMPRDD, MATF
ERROR RETURNS:	None
RESTRICTIONS:	None

86039

NAME:	PROUT
PURPOSE:	To print selected input and calculated quantities.
METHOD:	All significant input and calculated quantities are printed out depending on the setting of the print switch.
USAGE:	CALL PROUT (NBLD, NSEG, IE, IF, GJ, T, BETOD, GAMOD)
	NBLD = Number of blades.
	NSEG = Number of blade segments.
	IE = Blade edgewise second moment of area.
	IF = Blade flatwise second moment of area.
	GJ = Blade torsional stiffness.
	T = Blade thrust.
	BETOD = Blade steady coning angle.
	GAMOD = Blade steady lag angle.
SUBROUTINES CALLED:	SKIPLN
ERROR RETURNS:	None
RESTRICTIONS:	None

86038

NAME:	QTFG
PURPOSE:	To compute the vector of integral values for a given table of argument and function values by the trapezoidal rule.
METHOD:	This routine is a copy of QTFG, and is obtained from IBM System/360 Scientific Subroutine Package.
USAGE:	CALL QTFG (X, Y, Z, NDIM) X = The input vector of argument values. Y = The input vector of function values. Z = The resulting vector of integral values. NDIM = The dimensions of X, Y, and Z.
SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	None

8603'1

NAME:	REMOVE
PURPOSE:	To delete a blade flatwise or edgewise mode from an array of mode shapes and frequencies.
METHOD:	The characteristics of each mode are examined until the required mode is found. Then, the mode shape and its frequency are deleted and the remaining modes are compressed in the array.
USAGE:	CALL REMOVE (SHAPE, MODE, TYPE, N) SHAPE = An array containing the type of each mode. MODE = Number of modes to be examined. TYPE = The type of mode to be removed. N = The occurrence of type which is removed, i.e., the 2nd flatwise.
SUBROUTINES CALLED:	None
ERROR RETURNS:	A message is printed if the requested mode could not be deleted.
RESTRICTIONS:	None

863977

NAME:	RHS
PURPOSE:	To shift the damping and stiffness contributions to the right-hand-side of the non-linear bifilar equations of motion.
METHOD:	The stiffness and damping matrices obtained at the completion of the calling sequence in SYSCTL are post-multiplied respectively by the initial values of the state variables displacements and velocities. The resulting vector, FRHS, is shifted to the r.h.s. by a change in sign and transferred through labelled COMMON/NLDAT2. It is dimensioned NRHS (see below).
USAGE:	CALL RHS (NRHS) NRHS = Total number of degrees-of-freedom not including number of non-linear bifilars (maximum values is 60).
SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	None

8G026

NAME: ROOTX

PURPOSE: To calculate the blade elastic torsional mode shape.

METHOD: The blade torsional mass moment of inertia and stiffness distributions are used to calculate the torsional mode shape using a given frequency trial and the input value of rotor speed.

The frequency and mode shape trials are printed out if location 119 is 6.

USAGE: CALL ROOTX (W2, XHI, O2, YTI, YTK)

W2 = Square of frequency trial, $(\text{rad/sec})^2$

XHI = Calculated blade torsional mode shape, non-dimensional

O2 = Square of rotor speed, $(\text{rad/sec})^2$

YTI = Blade torsional mass moment of inertia, in-lb-sec^2

YTK = Blade torsional stiffness, lb-in^2/in

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: None

86036

NAME: SECAER

PURPOSE: To calculate blade section coefficients and derivatives.

METHOD: Lift, drag, and pitching moment coefficients are input against angle of attack for various Mach numbers. For a given angle of attack and Mach number, the required coefficients and their derivatives with respect to Mach number and angle of attack are obtained from the corresponding table by linear interpolation.

The routine assumes airfoil data to be non-symmetric if the first input angle of attack value is negative.

For angles of attack greater than 30^{o}, a Mach number of 0.0001 is assumed.

USAGE: CALL SECAER (ALPHA, AMACH, I, M, FF, DFF, DM)

ALPHA = Angle of attack (radians).

AMACH = Mach number.

I = Not used by this program.

RFM = 1 for lift coefficient.
= 2 for drag coefficient.
= 3 for pitching moment coefficient.

FF = Returned coefficient.

DFF = Derivative with respect to angle of attack.

DM = Derivative with respect to Mach number.

SUBROUTINES CALLED: BLIN4

ERROR RETURNS: Any error termination from BLIN4 stops execution with the message 'TROUBLE IN BLIN4', followed by 6 numbers which represent angle of attack in radians, angle of attack in degrees, input Mach number, Mach number used, the error switch L from BLIN4 and the switch M respectively. See BLIN4 for description of errors.

RESTRICTIONS: See locations 1850-4548 of rotor stability input description and BLIN4.

80035

NAME:	SHAKIT
PURPOSE:	To control the principal logic flow of the rotor/bifilar coupled program.
METHOD:	MAIN first calls subroutine PRELIM, which controls the input, conversion and adjustment of the blade data. Then, the rotor blade dynamic and aerodynamic matrices are obtained from calls to DYNMAT and AERMAT respectively. The dynamic and aerodynamic matrices are cleaned out, added together, compressed and stored for coupling with the bifilar analysis portion in EIGER. The bifilar analysis is subsequently executed by calling MAINSV. After PRELIM is called, the rotor analysis calculations are bypassed if input location 110 is zero or -1. It should be added that SHAKIT is the fourth computer dependent routine. For CDC use, the first line of code is "PROGRAM SHAKIT (INPUT, OUTPUT, TAPE1, TAPE2, TAPE3)". For IBM use, this card is not needed; thus, the first line contains blank COMMON input data.
USAGE:	Program SHAKIT is never referenced in a CALL statement.
SUBPROGRAMS CALLED:	PRELIM, DYNMAT, AERMAT, EIGER, MAINSV
ERROR RETURNS:	None
RESTRICTIONS:	None

36034

NAME: SIMLIN

PURPOSE: To solve a system of linear simultaneous equations.

METHOD: The solution is obtained by elimination using largest pivotal division. The determinant of the coefficient matrix is produced during the process.

USAGE: CALL SIMLIN (A, B, X, N, NX, C)

A = Coefficient matrix (destroyed during computations) of dimensions XN by XN.

B = Vector of right-hand value, destroyed during the computations. On return, B(1) contains the determinant of the coefficient matrix.

X = Solution vector.

N = The number of rows in A.

NX = Dimensions of matrix A.

C = Work vector.

SUBROUTINES CALLED: None

ERROR RETURNS: None

RESTRICTIONS: None

8G033

NAME:	SKIPLN
PURPOSE:	To skip a given number of lines during printing.
METHOD:	In an effort to prevent the printout of output tables going over a page boundary, SKIPLN is used to center tables within a page or part of a page.
USAGE:	CALL SKIPLN(J) J = Number of lines to be skipped.
SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	None

86032

NAME:	SORTAB
PURPOSE:	To unscramble an array of alternating X and Y points into an array of X points and an array of Y points.
METHOD:	Even subscripted values in the input array are moved to another array. Both arrays are then compressed.
USAGE:	CALL SORTAB (G, M, KSTAR, NSTAR)

G	=	On input, G contains the alternating X and Y values. On output, G contains only X values.
M	=	The number of points in G on input.
KSTAR	=	The unscrambled Y points.
NSTAR	=	The number of points in G and KSTAR on output.

SUBROUTINES CALLED:	None
ERROR RETURNS:	None
RESTRICTIONS:	None

NAME: STDEFL

PURPOSE: To calculate the inplane and out-of-plane blade steady deflections and slopes.

METHOD: Blade deflections are calculated by successively calculating the deflection of each segment relative to the previous one. This is done by constructing a point matrix from the mass, stiffness, and geometric properties of the segment to relate the displacement of the center of a segment relative to the previous center. The process is started by assuming suitable conditions at the root of the blade. A correction is made to these deflections if the pitch bearing does not follow the blade root slope.

USAGE: CALL STDEFL (QEO, XEO, E, RB, MB, NB, RR, D, IE, IF, TH, OMEGA, EB, GAMOT, KGAMA, GAMO, KASE, IEF, ROTDEF)

QEO	=	Returned inplane or out-of-plane steady deflections.
XEO	=	Returned slope for inplane or out-of-plane deflections.
E	=	Blade offset.
RB	=	Array of distances of the blade segment centers from the center of rotation.
MB	=	Mass of each segment.
NB	=	Number of radii in RB.
RR	=	Blade segment lengths.
D	=	Drag derivative for inplane deflections or thrust derivative for out-of-plane deflections.
IE	=	Blade edgewise second moment of area.
IF	=	Blade flatwise second moment of area.
TH	=	Blade twist.
OMEGA	=	Rotational speed.

EB = Blade Young's modulus.

GAMOT = Calculated lag angle for inplane deflections or calculated coning angle for out-of-plane deflections.

KGAMA = Blade lag hinge spring constant for inplane deflections or blade flapping hinge spring constant for out-of-plane deflections.

GAMO = Blade prelag angle for inplane deflections or blade precone angle for out-of-plane deflections.

KASE = Blade pitch input control (derived from input location 115).

IEF = 0 for inplane deflections.

= 1 for out-of-plane deflections.

ROTDEF = Rotor definition (input location 114).

SUBROUTINES CALLED: PFMULT

ERROR RETURNS: None

RESTRICTIONS: None

2221

NAME:	SYSCTL
PURPOSE:	To include the contributions of the fixed system modes, rotor (if used), the fixed system absorbers and the linear inplane and vertical bifilars to the final system mass, damping and stiffness matrices in the bifilar analysis.
METHOD:	At first, SYSCTL calls FIXSYS to obtain the fixed system modes matrices. Then, if rotor contributions are desired, it restructures the rotor matrices and adds the rotor elements to the fixed system matrices using ADDOFR. Subsequently, it includes contributions due to fixed system absorbers from FIXABS, linear inplane bifilars from LINBIF and finally linear vertical bifilars from LVBIF. In all cases, the matrices are increased systematically by repeated calls to ADDOFR. Printout of the matrices is governed by the switches in locations 17 and 1490 through 1496.
USAGE:	CALL SYSCTL
SUBROUTINES CALLED:	FIXSYS, ADDOFR, FIXABS, LINBIF, LVBIF
ERROR RETURNS:	None
RESTRICTIONS:	None

REFERENCES

1. R. A. Johnston, "Helicopter Rotor Stability Analysis", USAAMRDL-TR-75-40, January 1976.

2. R. Piziali, "An Investigation of the Structural Dynamics of Helicopter Rotors", USAAVLABSTR-70-24, April 1970. AD872715.

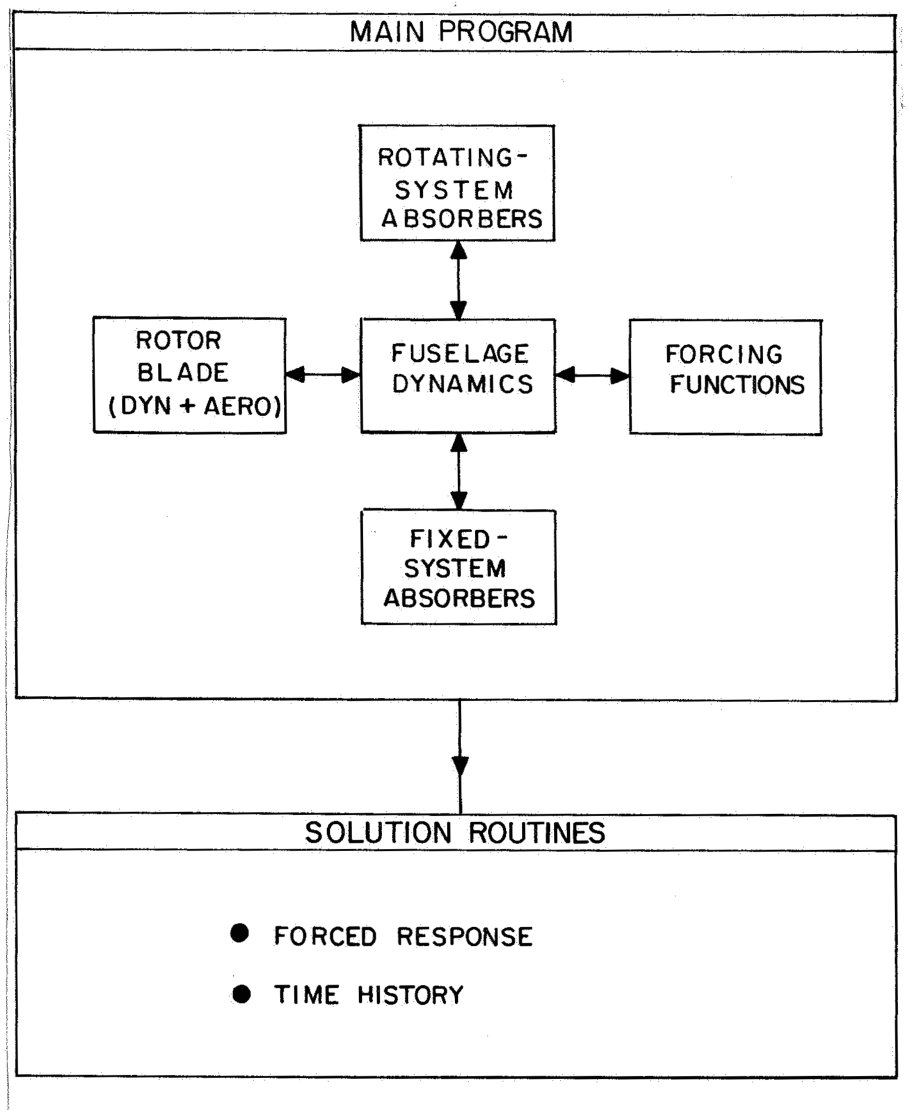


Figure 1. Block Diagram of Bifilar Analysis.

INPUT DATA

5	1	.23780D-02	1116.0	.99000	.0	258.00
3	6	26.833	1.2500	4.0000		
5	13	.0	.10000D+07	15.127	7.2500	.35000
2	19	258.00	1.0000			
3	107	1.0000	.0	1.0000		
1	111	1.0000				
3	113	.11000D+08	1.0000	11.000		
1	119	5.0000				
1	125	1.0000				
5	126	27.074	11.238	9.1780	6.8200	.35500
3	131	-1.0000	676.00	.0		
1	200	20.000				
5	201	17.500	17.500	25.000	16.800	16.800
5	206	16.800	16.800	16.800	17.490	17.490
5	211	17.490	17.490	15.900	15.900	15.900
5	216	12.970	12.970	7.7000	7.7000	4.0000
1	250	20.000				
4	251	.0	.0	15.000	.0	
4	255	15.010	8.3000	50.000	8.3000	
4	259	50.010	20.760	228.96	20.760	
4	263	228.97	22.317	276.66	22.317	
4	267	276.67	20.760	322.00	20.760	
1	350	14.000				
4	351	.0	-9.5000	50.000	-9.5000	
4	355	270.00	1.2000	290.00	2.8000	
4	359	305.00	3.8000	318.00	1.2000	
2	363	322.00	1.2000			
1	450	14.000				
4	451	.0	-9.5000	50.000	-9.5000	
4	455	270.00	1.2000	290.00	2.8000	
4	459	305.00	3.8000	318.00	1.2000	
2	463	322.00	1.2000			
1	550	58.000				
4	551	.0	.0	58.000	.0	
4	555	75.000	-.82200	159.00	-.82200	
4	559	159.01	-.66700	192.00	-.66700	
4	563	192.01	-.46850	210.00	-.46850	
4	567	210.01	-.67540	220.00	-.67540	
4	571	220.01	-.69920	224.00	-.68580	
4	575	228.96	.55690	228.97	.27880	
4	579	240.00	.29920	270.30	.31020	
4	583	270.31	.80000D-02	276.66	.12000D-01	
4	587	276.67	.36800	289.00	.36800	
4	591	290.00	1.2335	291.00	2.0990	
4	595	299.00	2.0990	299.50	2.1247	
4	599	301.25	1.7292	303.75	.89920	
4	603	306.69	.33950	318.00	-5.0000	
2	607	322.00	-6.4560			
1	650	18.000				
4	651	.0	.0	50.000	.0	
4	655	50.010	-.35000	228.96	-.35000	
4	659	228.97	.0	276.66	.0	
4	663	276.67	-.35000	302.60	-.35000	
2	667	322.00	-7.4110			
1	750	20.000				
4	751	.0	.0	15.000	.0	

IBM Z30...

FIGURE 2. Output Format - Rotor Blade Input Data.

TITLE 1 - TEST MAIN ROTOR DATA - COUPLED WITH BIFILAR ANALYSIS 000463

HOVER

MAIN ROTOR

PITCH ANGLE AT 75% RADIUS = 6.400 DEG

CALCULATED THRUST = 16364.911 LB

CALCULATED CONING ANGLE = 3.389 DEG

CALCULATED LAG ANGLE = 5.180 DEG

CALCULATED BLADE TORSIONAL FREQUENCY = 0.0 RAD/SEC

CALCULATED BLADE BENDING FREQUENCIES : MODE 1 = 77.5 RAD/SEC

MODE 2 = 126.7 RAD/SEC

RADIUS (IN)	STEADY DEFLECTIONS (IN) FLATWISE	EDGEWISE	ANGLE OF ATTACK (DEG)
0.0	0.0	0.0	-74.100
15.000	0.0	0.0	-33.210
23.750	-0.004	0.011	-20.206
41.250	-0.009	0.104	-6.880
62.500	0.041	0.285	-0.199
83.400	0.156	0.505	2.542
100.200	0.256	0.699	3.650
117.000	0.356	0.904	4.219
133.800	0.455	1.119	4.447
150.600	0.546	1.343	4.446
167.745	0.626	1.582	4.277
185.235	0.688	1.835	3.980
202.725	0.723	2.098	3.588
220.215	0.728	2.371	3.124
236.910	0.696	2.641	2.627
252.810	0.631	2.907	2.116
268.710	0.542	3.181	1.574
283.145	0.456	3.437	0.647
296.115	0.397	3.670	-0.156
306.450	0.368	3.857	-0.345
314.150	0.353	3.997	1.274
320.000	0.344	4.103	2.101

MODE NO.	PHIXPH	PHIZPH	PHELD	PHEPLD	PHFLD	PHFPLD
1	1.0000	-0.0009	0.0	0.0	0.0	0.0
2	-1.0000	0.0010	0.0	0.0	0.0	0.0

QEOLD	QEOPLD	QFOLD	QFOPLD	PHLD	THTLD	PHOS
0.0	0.0	0.0	0.0	0.0	0.0	0.0

(a)

FIGURE 3. Output Format - Rotor Blade Characteristics.

CASE DEFINITION

AIR DENSITY LB.SEC.-SQ/IN.4TH	SPEED OF SOUND FT./SEC.	TIP LOSS FACTOR	AXIAL VEL. KNOTS	ROTOR SPEED R.P.M.	BLADE RADIUS FEET
0.11468D-06	1116.00000	0.99000	0.0	258.00000	26.83333

OFFSET FEET	NUMBER OF BLADES	ROOT FLAP SPRING LB.IN./RAD.	ROOT LAG SPRING LB.IN./RAD.	PRELAG ANGLE RADIANS	PRECONE ANGLE RADIANS
1.25000	4	0.0	0.0	0.0	0.0

BLADE YOUNG'S MOD. LB/IN.-SQ	RADIUS OF PUSH ROD INCHES	LAG DAMPING FRACT CRITICAL	RIGID PITCH DAMP. FRACT CRITICAL	REF. ROTOR SPEED R.P.M.	BLADE BENDING MODES
0.10000D+07	15.12650	0.35000	0.0	258.00000	2

FIXED SYSTEM MODES	PITCH-LAG COUPLING DEG/DEG	WEIGHT AT PUSHROD, LB	PITCH BEAM STIFFNESS LB/IN.	ACTUATOR MOM. STIFFNESS LB-IN./RAD.	PITCH BEAM RADIUS INCHES
5	0.0	0.0	0.50000D+05	0.0	0.0

PITCH HORN LENGTH INCHES	FORWARD FLIGHT SPEED KNOTS	ELASTIC PITCH DAMP. FRACT. CRITICAL	LAG DAMPER COEFFICIENT LB-SEC/IN.	LAG DAMPER STIFFNESS LB/IN.	FLEXBEAM ROOT (XBR) INCHES
7.25000	0.0	0.0	676.00000	0.0	0.0

CONTROL SWITCHES

ROTEST	FTEST	SYSDEF	ROTDEF	ARTIC	PHASE
1.	1.	1000111.	1.	11.	0.

VECT	TRMASC	SUMASC	TSERVC	MRMASC	MSERVC
0.	1.	111.	1.	1.	111.

CIR	CIRN	LAGKII
1.	1.	1.

MODE NO.	ZETBLD	ZETG	MG	OMF	PHY	PHX	PHZ	PHTY	PHTX
1	0.0	0.0	0.0	0.0	0.0	1.0000	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	1.0000	0.0	0.0	0.0	0.0
3		0.0	0.0	0.0	0.0	0.0	1.0000	0.0	0.0
4		0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0000
5		0.0	0.0	0.0	0.0	0.0	0.0	1.0000	0.0

(b)

FIGURE 3.—Continued.

R	CHORD	STRUCTURAL TWIST	AERODYNAMIC TWIST	AC	CG	TORSIONAL MODE SHAPE	EA
0.0	0.0	-9.50000	-9.50000	0.0	0.0	0.0	0.0
15.00000	0.0	-9.50000	-9.50000	0.0	0.0	0.0	0.0
23.75000	8.30000	-9.50000	-9.50000	0.0	0.0	0.0	-2.07500
41.25000	8.30000	-9.50000	-9.50000	0.0	0.0	0.0	-2.07500
62.50000	20.76000	-8.89205	-8.89205	-0.35000	-0.21759	0.0	-5.19000
83.40000	20.76000	-7.87555	-7.87555	-0.35000	-0.82200	0.0	-5.19000
100.20000	20.76000	-7.05845	-7.05845	-0.35000	-0.82200	0.0	-5.19000
117.00000	20.76000	-6.24136	-6.24136	-0.35000	-0.82200	0.0	-5.19000
133.80000	20.76000	-5.42427	-5.42427	-0.35000	-0.82200	0.0	-5.19000
150.60000	20.76000	-4.60718	-4.60718	-0.35000	-0.82200	0.0	-5.19000
167.74500	20.76000	-3.77331	-3.77331	-0.35000	-0.66700	0.0	-5.19000
185.23500	20.76000	-2.92266	-2.92266	-0.35000	-0.66700	0.0	-5.19000
202.72500	20.76000	-2.07201	-2.07201	-0.35000	-0.46850	0.0	-5.19000
220.21500	20.76000	-1.22136	-1.22136	-0.35000	-0.69851	0.0	-5.19000
236.91000	22.31700	-0.40938	-0.40938	0.0	0.29349	0.0	-5.58000
252.81000	22.31700	0.36394	0.36394	0.0	0.30385	0.0	-5.58000
268.71000	22.31700	1.13726	1.13726	0.0	0.30962	0.0	-5.58000
283.14500	20.76000	2.25160	2.25160	-0.35000	0.36800	0.0	-5.19000
296.11500	20.76000	3.20767	3.20767	-0.35000	2.09900	0.0	-5.19000
306.45000	20.76000	3.51000	3.51000	-1.75128	0.38606	0.0	-5.19000
314.15000	20.76000	1.97000	1.97000	-4.55384	-3.18159	0.0	-5.19000
320.00000	20.76000	1.20000	1.20000	-6.68306	-5.72800	0.0	-5.19000

R	QEO	QFO	QEOP	QFOP
0.0	0.0	0.0	0.0	0.0
15.00000	0.0	0.0	0.0	0.0
23.75000	0.01108	-0.00402	0.00155	-0.00151
41.25000	0.10392	-0.00914	0.00659	-0.00127
62.50000	0.28471	0.04123	0.00918	0.00136
83.40000	0.50523	0.15567	0.01101	0.00444
100.20000	0.69939	0.25565	0.01170	0.00453
117.00000	0.90412	0.35611	0.01231	0.00450
133.80000	1.11881	1.45498	0.01290	0.00420
150.60000	1.34309	0.54645	0.01347	0.00351
167.74500	1.58162	0.62605	0.01404	0.00256
185.23500	1.83470	0.68764	0.01460	0.00133
202.72500	2.09751	0.72338	0.01516	-0.00032
220.21500	2.37055	0.72784	0.01577	-0.00215
236.91000	2.64091	0.69591	0.01635	-0.00418
252.81000	2.90709	0.63050	0.01689	-0.00600
268.71000	3.18148	0.54180	0.01741	-0.00672
283.14500	3.43658	0.45641	0.01779	-0.00611
296.11500	3.66970	0.39675	0.01806	-0.00370
306.45000	3.85694	0.36796	0.01815	-0.00217
314.15000	3.99675	0.35345	0.01816	-0.00171
320.00000	4.10300	0.34363	0.01816	-0.00167

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(c)

FIGURE 3. Continued.

R	D(DT)/DUT	D(DT)/DUP	D(DT)/DOT	D(DH)/DUT	D(DH)/DUP	D(DH)/DOT
0.0	0.0	0.0	0.0	0.0	0.0	0.0
15.00000	0.0	0.0	0.0	0.0	0.0	0.0
23.75000	-0.00040	-0.00073	0.33155	-0.00001	-0.00021	-0.16268
41.25000	0.00140	-0.00516	6.52470	0.00034	-0.00130	1.18437
62.50000	0.00414	-0.01307	23.85057	0.00109	-0.00327	6.46567
83.40000	0.00610	-0.01753	41.71577	0.00107	-0.00244	8.75691
100.20000	0.00745	-0.02118	59.83550	0.00103	-0.00180	10.47511
117.00000	0.00859	-0.02483	81.23457	0.00100	-0.00133	12.44987
133.80000	0.00954	-0.02851	105.97988	0.00096	-0.00090	14.26927
150.60000	0.01102	-0.03221	134.44925	0.00100	-0.00056	16.16053
167.74500	0.01195	-0.03662	169.58604	0.00098	-0.00031	18.40511
185.23500	0.01269	-0.04126	210.32202	0.00095	-0.00015	20.76438
202.72500	0.01218	-0.04602	255.52919	0.00083	-0.00014	23.23199
220.21500	0.01210	-0.05052	304.00663	0.00078	-0.00027	25.63134
236.91000	0.01266	-0.05901	381.23440	0.00078	-0.00053	29.98955
252.81000	0.01374	-0.06489	447.37186	0.00099	-0.00125	35.26863
268.71000	0.01276	-0.07412	541.85065	0.00084	-0.00173	38.29369
283.14500	0.00878	-0.07731	593.92714	0.00055	-0.00250	36.33645
296.11500	0.00549	-0.08701	697.52874	0.00039	-0.00370	38.61169
306.45000	0.01024	-0.09279	771.89581	0.00079	-0.00407	41.73612
314.15000	0.01896	-0.09562	817.74876	0.00168	-0.00494	74.68638
320.00000	0.02305	-0.08673	756.49209	0.00288	-0.00730	110.98781

R	D(DM)/DUT	D(DM)/DUP	D(DM)/DOT	DT	DH	DM
0.0	0.0	0.0	0.0	0.0	0.0	0.0
15.00000	0.0	0.0	0.0	0.0	0.0	0.0
23.75000	0.00044	0.00073	-0.26554	-0.29732	-0.05191	0.31267
41.25000	-0.00948	0.02382	-30.98563	-0.42541	-0.11705	0.28950
62.50000	0.00011	0.00040	-0.62797	0.43651	0.15563	0.19019
83.40000	0.00014	0.00051	-1.08320	2.76489	0.63633	0.27625
100.20000	0.00017	0.00060	-1.54369	5.12602	0.97658	0.36383
117.00000	0.00111	-0.00514	16.77181	7.77250	1.27528	0.54370
133.80000	0.00120	-0.00588	21.82340	10.56367	1.53142	0.79426
150.60000	0.00351	-0.00625	27.06373	13.41433	1.74900	1.17383
167.74500	0.00455	-0.00613	29.93084	16.53455	1.95691	2.61935
185.23500	0.00574	-0.00540	29.70216	19.55134	2.12254	4.59828
202.72500	-0.00021	-0.00558	30.45412	22.08643	2.22559	6.18703
220.21500	-0.00107	-0.00339	19.66104	23.68544	2.24479	5.50461
236.91000	-0.00210	-0.00102	5.55483	26.25026	2.37083	5.39755
252.81000	0.00324	-0.00014	2.46406	26.27911	2.30451	5.25127
268.71000	0.00322	-0.00493	37.29655	25.32963	2.20955	6.39222
283.14500	0.00180	-0.00851	65.92044	16.95756	1.59790	5.49944
296.11500	0.00052	-0.01268	101.65111	8.78590	1.13260	4.52887
306.45000	-0.00627	-0.01825	148.14500	7.69474	1.13389	2.42466
314.15000	-0.01425	-0.00596	43.94727	31.97936	2.69876	1.07474
320.00000	-0.02523	0.08714	-765.20454	46.48986	4.15806	-2.40927

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(d)

FIGURE 3. —Continued.

R	CL	CD	CM	D(CL)/DA	D(CD)/DA	D(CM)/DA	D(CL)/DM	D(CD)/DM	D(CM)/DM
0.0	-0.42808	1.88000	0.47020	-0.74305	0.0	-0.30558	0.0	0.0	0.0
15.00000	-0.95837	0.74466	0.17059	-0.74305	-2.04628	-0.47995	0.0	0.0	0.0
23.75000	-0.90206	0.44392	0.12548	0.57296	-1.08862	-0.10657	0.0	0.0	0.0
41.25000	-0.62922	0.08167	0.05016	9.31056	-2.06265	-5.36861	0.0	0.0	0.0
62.50000	0.12646	0.00913	0.00251	6.76090	-0.03820	-0.00828	0.0	0.0	0.0
83.40000	0.44991	0.00964	0.00211	6.76090	0.01432	-0.00828	0.0	0.0	0.0
100.20000	0.58070	0.00991	0.00195	6.76090	0.01432	-0.00828	0.0	0.0	0.0
117.00000	0.64789	0.01013	0.00215	6.76090	0.03438	0.06646	0.0	0.0	0.0
133.80000	0.67479	0.01027	0.00242	6.76090	0.03438	0.06646	0.0	0.0	0.0
150.60000	0.67739	0.01025	0.00283	6.78151	0.03460	0.06529	0.48248	-0.00304	0.07112
167.74500	0.67371	0.01004	0.00511	6.90300	0.03591	0.05834	0.47210	-0.00315	0.07172
185.23500	0.65383	0.00974	0.00736	7.02707	0.03627	0.04757	0.45377	-0.00342	0.07250
202.72500	0.61704	0.00937	0.00829	7.13287	0.03870	0.04078	0.06675	-0.00201	-0.05261
220.21500	0.56106	0.00897	0.00625	7.19546	0.04038	0.02234	0.05232	-0.00240	-0.04836
236.91000	0.49999	0.00854	0.00459	7.25523	0.04000	0.00472	0.03691	-0.00343	-0.04382
252.81000	0.43973	0.00838	0.00392	7.48130	0.07675	0.00184	0.35840	0.02610	0.01686
268.71000	0.37532	0.00850	0.00423	8.02034	0.04954	0.02468	0.19908	0.01148	0.01011
283.14500	0.24346	0.00800	0.00379	8.50984	0.0	0.04541	-0.07334	0.0	-0.00143
296.11500	0.11559	0.00808	0.00285	9.13770	-0.02865	0.06405	-0.31458	0.0	-0.01143
306.45000	0.09463	0.00852	0.00143	9.44286	-0.02314	0.08717	0.51367	0.01827	-0.06202
314.15000	0.37295	0.01086	0.00060	9.54040	0.34354	0.02461	0.96091	0.06649	-0.11108
320.00000	0.52272	0.01837	-0.00130	8.53200	0.78367	-0.41304	1.14977	0.16673	-0.14406

R	θ STRUCTURAL	θ AERODYNAMIC	U_P	U_T	U	ϕ	ALPHA	MACH NUMBER
0.0	15.90000	15.90000	468.02342	0.0	468.02342	90.00000	-74.10000	0.03495
15.00000	15.90000	15.90000	468.02342	405.26640	619.10159	49.11037	-33.21037	0.04623
23.75000	15.90000	15.90000	468.02342	641.67180	794.22202	36.10631	-20.20631	0.05931
41.25000	15.90000	15.90000	468.02342	1114.48260	1208.76689	22.77982	-6.87982	0.09026
62.50000	15.29205	15.29205	468.02342	1688.61000	1752.26986	15.49151	-0.19946	0.13084
83.40000	14.27555	14.27555	468.02342	2253.28118	2301.37394	11.73392	2.54163	0.17185
100.20000	13.45845	13.45845	468.02342	2707.17955	2747.33817	9.80847	3.64998	0.20515
117.00000	12.64136	12.64136	468.02342	3161.07792	3195.53744	8.42192	4.21944	0.23862
133.80000	11.82427	11.82427	468.02342	3614.97629	3645.14739	7.37693	4.44734	0.27219
150.60000	11.00718	11.00718	468.02342	4068.87466	4095.70347	6.56163	4.44556	0.30583
167.74500	10.17331	10.17331	468.02342	4532.09415	4556.19615	5.89596	4.27735	0.34022
185.23500	9.32266	9.32266	468.02342	5004.63477	5026.47144	5.34265	3.98001	0.37533
202.72500	8.47201	8.47201	468.02342	5477.17540	5497.13528	4.88405	3.58796	0.41048
220.21500	7.62136	7.62136	468.02342	5949.71602	5968.09573	4.49780	3.12356	0.44565
236.91000	6.80938	6.80938	468.02342	6400.77752	6417.86560	4.18201	2.62737	0.47923
252.81000	6.03606	6.03606	468.02342	6830.35991	6846.37586	3.91984	2.11622	0.51123
268.71000	5.26274	5.26274	468.02342	7259.94229	7275.01258	3.68856	1.57418	0.54324
283.14500	4.14840	4.14840	468.02342	7649.94366	7664.24712	3.50099	0.64741	0.57230
296.11500	3.19233	3.19233	468.02342	8000.36400	8014.04206	3.34800	-0.15567	0.59842
306.45000	2.89000	2.89000	468.02342	8279.59255	8292.81006	3.23534	-0.34534	0.61924
314.15000	4.43000	4.43000	468.02342	8487.62930	8500.52334	3.15620	1.27380	0.63475
320.00000	5.20000	5.20000	468.02342	8645.68320	8658.34187	3.09861	2.10139	0.64653

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(e)

FIGURE 3. Continued.

R	PHE(1,I)	PHF(1,I)	PHE(2,I)	PHF(2,I)	PHE(3,I)	PHF(3,I)	PHE(4,I)	PHF(4,I)
0.0	0.0	0.0	0.0	0.0				
15.00000	0.00000	0.00000	-0.00000	-0.00000				
23.75000	0.01443	-0.06550	-0.11533	0.06907				
41.25000	0.04265	-0.19433	-0.31780	0.20035				
62.50000	0.07436	-0.34180	-0.51612	0.32001				
83.40000	0.09919	-0.46391	-0.64142	0.33810				
100.20000	0.11385	-0.54193	-0.69542	0.28228				
117.00000	0.12353	-0.59983	-0.71296	0.17523				
133.80000	0.12788	-0.63489	-0.69571	0.02932				
150.60000	0.12652	-0.64399	-0.64607	-0.13980				
167.74500	0.11894	-0.62318	-0.56599	-0.31638				
185.23500	0.10502	-0.56974	-0.45975	-0.47719				
202.72500	0.08497	-0.48154	-0.33437	-0.59265				
220.21500	0.05895	-0.35641	-0.19443	-0.63587				
236.91000	0.02880	-0.20108	-0.05014	-0.58755				
252.81000	-0.00419	-0.02128	0.09481	-0.44739				
268.71000	-0.04047	0.18585	0.24475	-0.21509				
283.14500	-0.07552	0.39359	0.38317	0.06636				
296.11500	-0.10802	0.59156	0.50845	0.36215				
306.45000	-0.13423	0.75377	0.60848	0.61391				
314.15000	-0.15381	0.87562	0.68303	0.80485				
320.00000	-0.16870	0.96831	0.73966	0.95027				

R	PHEP(1,I)	PHFP(1,I)	PHEP(2,I)	PHFP(2,I)	PHEP(3,I)	PHFP(3,I)	PHEP(4,I)	PHFP(4,I)
0.0	0.0	0.0	0.0	0.0				
15.00000	0.00165	-0.00750	-0.01336	0.00793				
23.75000	0.00164	-0.00746	-0.01283	0.00781				
41.25000	0.00158	-0.00724	-0.01060	0.00697				
62.50000	0.00133	-0.00637	-0.00765	0.00298				
83.40000	0.00100	-0.00512	-0.00430	-0.00192				
100.20000	0.00070	-0.00396	-0.00201	-0.00538				
117.00000	0.00039	-0.00265	0.00012	-0.00816				
133.80000	0.00006	-0.00117	0.00213	-0.01009				
150.60000	-0.00030	0.00052	0.00396	-0.01093				
167.74500	-0.00066	0.00235	0.00551	-0.01046				
185.23500	-0.00101	0.00429	0.00674	-0.00855				
202.72500	-0.00136	0.00637	0.00767	-0.00508				
220.21500	-0.00169	0.00854	0.00839	-0.00009				
236.91000	-0.00198	0.01062	0.00892	0.00580				
252.81000	-0.00221	0.01246	0.00930	0.01181				
268.71000	-0.00239	0.01401	0.00953	0.01743				
283.14500	-0.00249	0.01507	0.00963	0.02156				
296.11500	-0.00253	0.01563	0.00967	0.02392				
306.45000	-0.00254	0.01581	0.00968	0.02471				
314.15000	-0.00255	0.01584	0.00968	0.02485				
320.00000	-0.00255	0.01585	0.00968	0.02486				

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(f)

FIGURE 3. Continued.

BLADE EDGEWISE M. OF I. ABOUT C.G. LB.IN.SEC-SQ/IN.	DELTA R IN.	BLADE FLATWISE M. OF I. ABOUT C.G. LB.IN.SEC-SQ/IN.	DELTA R IN.	BLADE TORSIONAL M. OF I. ABOUT C.G. LB.IN.SEC-SQ/IN.	DELTA R IN.
0.0	15.000			0.0	15.000
0.00269	8.300			0.00269	8.300
0.00479	2.800			0.00479	2.800
0.00823	3.900			0.00823	3.900
0.01842	15.830			0.01842	15.830
0.02398	29.170			0.02398	29.170
0.03180	84.000			0.03180	84.000
0.03474	61.000			0.03474	61.000
0.03971	8.960			0.03971	8.960
0.04650	11.040			0.04650	11.040
0.04354	36.660			0.04354	36.660
0.03381	12.340			0.03381	12.340
0.06848	2.000			0.06848	2.000
0.05861	8.000			0.05861	8.000
0.05723	9.500			0.05723	9.500
0.03805	13.500			0.03805	13.500

BLADE MASS LB.SEC-SQ/IN.-SQ	DELTA R IN.	BLADE EDGEWISE SECOND MOMENT OF AREA-IN.4TH	DELTA R IN.	BLADE FLATWISE SECOND MOMENT OF AREA-IN.4TH	DELTA R IN.
0.0	15.00000	0.88587E+02	17.50000	0.87885E+02	17.50000
0.00744	17.50000	0.37941E+03	17.50000	0.93696E+02	17.50000
0.00423	17.50000	0.46903E+03	25.00000	0.27593E+02	25.00000
0.00168	25.00000	0.71842E+03	16.80000	0.23430E+02	16.80000
0.00144	16.80000	0.85000E+03	16.80000	0.23430E+02	16.80000
0.00138	16.80000	0.85000E+03	16.80000	0.23430E+02	16.80000
0.00137	16.80000	0.85000E+03	16.80000	0.23430E+02	16.80000
0.00137	16.80000	0.85000E+03	16.80000	0.23430E+02	16.80000
0.00137	16.80000	0.90000E+03	17.49000	0.27065E+02	17.49000
0.00145	17.49000	0.90000E+03	17.49000	0.27430E+02	17.49000
0.00146	17.49000	0.84979E+03	17.49000	0.27272E+02	17.49000
0.00150	17.49000	0.73953E+03	17.49000	0.27190E+02	17.49000
0.00162	17.49000	0.64334E+03	15.90000	0.27116E+02	15.90000
0.00190	15.90000	0.59000E+03	15.90000	0.26950E+02	15.90000
0.00190	15.90000	0.56444E+03	15.90000	0.25433E+02	15.90000
0.00186	15.90000	0.53000E+03	12.97000	0.22950E+02	12.97000
0.00176	12.97000	0.53000E+03	12.97000	0.22950E+02	12.97000
0.00343	12.97000	0.53000E+03	7.70000	0.22950E+02	7.70000
0.00293	7.70000	0.53000E+03	7.70000	0.22950E+02	7.70000
0.00139	7.70000	0.53000E+03	4.00000	0.22950E+02	4.00000
0.00036	4.00000				

(g)

FIGURE 3. Continued.

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          BLADE RIGID BODY PROPERTIES

   FLAPPING MASS        =   .660383       LB SEC-SQ/IN
   1ST MOM ABOUT HINGE  =   86.1467       LB SEC-SQ
   FLAPPING INERTIA     =   18190.6       LB IN SEC-SQ

   LAG FREQUENCY        =   .266517       CYCLES/REV

            BLADE MODAL PROPERTIES

   BENDING MODE GENERALIZED MASSES

        MODE(1)   =   .125382        LB SEC-SQ/IN

        MODE(2)   =   .209446        LB SEC-SQ/IN


   BENDING MODE (M)(PHE)                            BENDING MODE  (M)(PHF)

        MODE(1)   =   .193489D-01    LB SEC-SQ/IN        MODE(1)   =  -.942375D-01   LB SEC-SQ/IN

        MODE(2)   =  -.115519        LB SEC-SQ/IN        MODE(2)   =   .385765D-02   LB SEC-SQ/IN

   BENDING MODE (M)(PHE)(R)                         BENDING MODE  (M)(PHF)(R)

        MODE(1)   =  -.805651D-01    LB SEC-SQ           MODE(1)   =   .621710       LB SEC-SQ

        MODE(2)   =   .683082        LB SEC-SQ           MODE(2)   =  -3.94313       LB SEC-SQ


         17 DEGREES OF FREEDOM ARE USED IN THIS RUN

     1  2  7  8  9 10 11 12 21 22 23 24 25 26 27 28 29
```

(h)

FIGURE 3. Concluded.

FINAL STIFFNESS MATRIX

17 17 1

1	714.170	-74.3204	-1252.21	760.385	.0	.0	.0	.0	.0	.0
	.0	.0	.0	.0	.0	.0	.0			
2	-1.41149	3330.19	-2304.16	-1473.63	.0	.0	.0	.0	.0	.0
	.0	.0	.0	.0	.0	.0	.0			
3	-1339.02	-1795.95	.118472D+08	.108102D+07	.0	.0	.0	.0	.0	.0
	.0	.0	.0	.0	.0	.0	.0			
4	319.047	-1228.43	240574.	909996.	.0	.0	.0	.0	.0	.0
	.0	.0	.0	.0	.0	.0	.0			
5	.0	.0	.0	.0	623.028	-93.0826	-74.1366	-43.8058	-1711.06	-7579.55
	830.526	-991.072	.0	.0	.0	-534.101	101.473			
6	.0	.0	.0	.0	93.0826	623.028	43.8058	-74.1366	7579.55	-1711.06
	991.072	830.526	.0	.0	.0	101.473	534.101			
7	.0	.0	.0	.0	-1.22765	-37.8535	3177.22	-65.1601	568.906	3285.78
	-2026.78	-1315.67	.0	.0	.0	-434.059	28.3808			
8	.0	.0	.0	.0	37.8535	-1.22765	65.1601	3177.22	-3285.78	568.906
	1315.67	-2026.78	.0	.0	.0	28.3808	434.059			
9	.0	.0	.0	.0	-1797.87	-7252.59	1077.11	1922.39	-.140193D+07	-.162945D+08
	.108281D+07	.135041D+07	.0	.0	.0	480170.	-187109.			
10	.0	.0	.0	.0	7252.59	-1797.87	-1922.39	1077.11	.162945D+08	-.140193D+07
	-.135041D+07	.108281D+07	.0	.0	.0	-187109.	-480170.			
11	.0	.0	.0	.0	389.188	859.306	-1781.58	391.228	242365.	.217900D+07
	-.123781D+08	-.272896D+07	.0	.0	.0	-208639.	-.161585D+07			

(a)

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FIGURE 4. Output Format - Rotor Blade Matrices.

12									
.0	.0	.0	.0	-859.306	389.188	-391.228	-1781.58	-.217900D+07	242365.
.272896D+07	-.123781D+08	.0	.0	.0	-.161585D+07	208639.			
13									
.0	.0	.0	.0	-33.0894	1.71156	-8.49605	-5.12357	-1732.55	-4083.65
-756.359	753.296	.0	.0	.0	125.043	-16335.3			
14									
.0	.0	.0	.0	1.71156	33.0894	-5.12357	8.49605	-4083.65	1732.55
753.296	756.359	.0	.0	.0	16335.3	125.043			
15									
-112.328	-112.808	-10129.7	3811.79	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0			
16									
.0	.0	.0	.0	7143.69	-32606.9	-3102.89	-35682.5	.174311D+08	-.415679D+07
-.235881D+07	.118284D+07	.0	.0	.0	-153513.	-515291.			
17									
.0	.0	.0	.0	-32606.9	-7143.69	-35682.5	3102.89	-.415679D+07	-.174311D+08
.118284D+07	.235881D+07	.0	.0	.0	130248.	-153513.			

(b)

FIGURE 4. Concluded.

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```
                BIFILAR  ANALYSIS  RESULTS

     ALL DISPLACEMENTS ARE IN G   AND   ALL ANGLES ARE IN DEGREES

     NUMBER OF FIXED SYSTEM MODES                 IS  9
     NUMBER OF FIXED SYSTEM ABSORBERS             IS  1
     NUMBER OF INPLANE BIFILARS                   IS  1
     NUMBER OF VERTICAL BIFILARS                  IS  1
     TOTAL NO. OF DEGREES-OF-FREEDOM
            (WITH NO ROTOR)                        IS 16
     NUMBER  OF A.C. STATIONS                      IS  4
     ROTOR COUPLING SWITCH       (0=NO,IOR-1=YES) IS  1
     ROTOR MATRICES         PRINTOUT(0=NO,1=YES)  IS  1
     FIXED SYSTEM MATRICES  PRINTOUT      "       IS  1
     ADD   ROTOR  MATRICES  PRINTOUT      "       IS  1
     ADD FIX.SYS. ABSORBER  PRINTOUT      "       IS  1
     ADD INPLANE  BIFILAR   PRINTOUT      "       IS  1
     ADD VERTICAL BIFILAR   PRINTOUT      "       IS  1
     INPLANE  BIFILAR (9X9) PRINTOUT      "       IS  1
     VERTICAL BIFILAR (9X9) PRINTOUT      "       IS  1
     GAMMAS                 PRINTOUT      "       IS  1
```

(a)

FIGURE 5. Output Format - Bifilar Linear Analysis Matrices.

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FIXED SYSTEM MATRICES OF ORDER 9

FIXED SYSTEM STIFFNESS MATRIX

1	1856.52	.0	.0	.0	.0	.0	.0	.0	.0
2	.0	4949.75	.0	.0	.0	.0	.0	.0	.0
3	.0	.0	47408.9	.0	.0	.0	.0	.0	.0
4	.0	.0	.0	18309.4	.0	.0	.0	.0	.0
5	.0	.0	.0	.0	20652.7	.0	.0	.0	.0
6	.0	.0	.0	.0	.0	10311.0	.0	.0	.0
7	.0	.0	.0	.0	.0	.0	11560.1	.0	.0
8	.0	.0	.0	.0	.0	.0	.0	56774.3	.0
9	.0	.0	.0	.0	.0	.0	.0	.0	110572.

(b)

FIGURE 5. Continued.

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ROTOR NO. OF DEGREES-OF-FREEDOM IS 12

TOTAL NO. OF DEGREES-OF-FREEDOM IS 28

ROTOR BLADE MATRICES
(INCLUDING 6 RIGID FIX.SYS. MODES) OF ORDER 18

ROTOR STIFFNESS MATRIX

1									
.0	.0	.0	125.043	-16335.3	.0	.0	.0	.0	.0
-33.0894	1.71156	-8.49605	-5.12357	-1732.55	-4083.65	-756.359	753.296		
2									
.0	.0	.0	16335.3	125.043	.0	.0	.0	.0	.0
1.71156	33.0894	-5.12357	8.49605	-4083.65	1732.55	753.296	756.359		
3									
.0	.0	.0	.0	.0	.0	-112.328	-112.808	-10129.7	3811.79
.0	.0	.0	.0	.0	.0	.0	.0		
4									
.0	.0	.0	-153513.	-515291.	.0	.0	.0	.0	.0
7143.69	-32606.9	-3102.89	-35682.5	.174311D+08	-.415679D+07	-.235881D+07	.118284D+07		
5									
.0	.0	.0	130248.	-153513.	.0	.0	.0	.0	.0
-32606.9	-7143.69	-35682.5	3102.89	-.415679D+07	-.174311D+08	.118284D+07	.235881D+07		
6									
.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0	.0		
7									
.0	.0	.0	.0	.0	.0	714.170	-74.3204	-1252.21	760.385
.0	.0	.0	.0	.0	.0	.0	.0		
8									
.0	.0	.0	.0	.0	.0	-1.41149	3330.19	-2304.16	-1473.63
.0	.0	.0	.0	.0	.0	.0	.0		
9									
.0	.0	.0	.0	.0	.0	-1339.02	-1795.95	.118472D+08	.108102D+07
.0	.0	.0	.0	.0	.0	.0	.0		

10

(c)

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FIGURE 5. Continued.

	.0	.0	.0	.0	.0	.0	319.047	-1228.43	240574.	909996.
	.0	.0	.0	.0	.0	.0	.0	.0		
11										
	.0	.0	.0	-534.101	101.473	.0	.0	.0	.0	.0
	623.028	-93.0826	-74.1366	-43.8058	-1711.06	-7579.55	830.526	-991.072		
12										
	.0	.0	.0	101.473	534.101	.0	.0	.0	.0	.0
	93.0826	623.028	43.8058	-74.1366	7579.55	-1711.06	991.072	830.526		
13										
	.0	.0	.0	-434.059	28.3808	.0	.0	.0	.0	.0
	-1.22765	-37.8535	3177.22	-65.1601	568.906	3285.78	-2026.78	-1315.67		
14										
	.0	.0	.0	28.3808	434.059	.0	.0	.0	.0	.0
	37.8535	-1.22765	65.1601	3177.22	-3285.78	568.906	1315.67	-2026.78		
15										
	.0	.0	.0	480170.	-187109.	.0	.0	.0	.0	.0
	-1797.87	-7252.59	1077.11	1922.39	-.140193D+07	-.162945D+08	.108281D+07	.135041D+07		
16										
	.0	.0	.0	-187109.	-480170.	.0	.0	.0	.0	.0
	7252.59	-1797.87	-1922.39	1077.11	.162945D+08	-.140193D+07	-.135041D+07	.108281D+07		
17										
	.0	.0	.0	-208639.	-.161585D+07	.0	.0	.0	.0	.0
	389.188	859.306	-1781.58	391.228	242365.	.217900D+07	-.123781D+08	-.272896D+07		
18										
	.0	.0	.0	-.161585D+07	208639.	.0	.0	.0	.0	.0
	-859.306	389.188	-391.228	-1781.58	-.217900D+07	242365.	.272896D+07	-.123781D+08		

(d)

FIGURE 5. Continued.

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```
PRESENT NO. OF DEGREES-OF-FREEDOM  (NP)    =  9
ADDITIONAL NO. DEGREES-OF-FREEDOM  (NA)    = 12
TOTAL  NO. OF  DEGREES-OF-FREEDOM  (NP+NA) = 21
ORDER OF MATRIX TO BE ADDED        (NL)    = 18
ORDER - D.O.F. OF ADDED MATRIX     (NL-NA) =  6
```

(e)

FIGURE 5. Continued.

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FIXED SYSTEM + ROTOR MATRICES OF ORDER 21

FIXED SYSTEM + ROTOR STIFFNESS MATRIX

1									
1840.85	-13.7054	29.0877	40.2377	-34.7691	-5.31752	-5.31752	59.0887	21.5081	-.112328
-.112808	-10.1297	3.81179	-51.8077	168.526	-1.80373	185.435	-91831.7	13276.2	12750.0
-4823.88									
2									
6.53520	4916.60	-43.4811	-7.79460	-15.3811	-2.18325	-2.18325	148.156	-2.92543	-12.3561
-12.4089	-1114.27	419.297	179.396	40.3897	188.198	-10.0499	20037.9	92234.3	-5589.71
-12623.0									
3									
2.57701	115.453	47495.4	-32.7940	92.7525	13.6929	13.6929	-513.012	-21.1438	34.8218
34.9706	3140.20	-1181.65	235.837	-384.257	143.606	-486.442	247642.	35837.3	-37848.8
3428.88									
4									
-38.6206	31.3830	124.255	18393.5	-35.5950	-5.72737	-5.72737	-144.084	42.8766	.112328
.112808	10.1297	-3.81179	85.0983	-378.529	-43.1991	-428.364	209086.	-50877.6	-28138.7
14951.0									
5									
-11.1006	-58.4978	-18.7467	39.7876	20590.5	-9.29177	-9.29177	258.778	22.8254	12.3561
12.4089	1114.27	-419.297	62.3753	288.688	143.541	279.060	-126021.	91883.3	15464.5
-17334.8									
6									
4.68930	.687792	-11.4586	-11.2524	7.78050	10312.2	1.20479	-2.50523	-5.90571	-2.47123
-2.48178	-222.853	83.8594	4.77058	41.8767	20.6461	42.1130	-19539.3	13099.2	2288.10
-2598.68									
7									
10.2006	16.9931	-12.4257	-28.0608	28.8399	4.37561	11564.4	-74.3508	-15.2570	-2.47123
-2.48178	-222.853	83.8594	-1.66189	39.3893	19.4673	40.4173	-19521.4	12176.0	2080.36
-2519.84									
8									
-66.4519	75.7296	231.326	139.692	-44.5378	-7.39395	-7.39395	56429.8	70.3950	-14.0411
-14.1010	-1266.21	476.474	-775.992	-144.332	-832.661	75.0061	-100578.	-404325.	27351.1
55326.4									
9									
-14.6906	12.4415	47.6708	31.8761	-13.1522	-2.12151	-2.12151	-57.0478	110588.	-.112328
-.112808	-10.1297	3.81179	40.2200	-201.431	-28.2559	-223.776	108961.	-29645.1	-14514.7
8158.73									

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(f)

FIGURE 5. Continued.

FIXED SYSTEM + ROTOR + FIXED ABSORBERS MATRICES OF ORDER 22

FIXED SYSTEM + ROTOR + FIXED ABSORBERS STIFFNESS MATRIX

1									
1840.85	-13.7054	29.0877	40.2377	-34.7691	-5.31752	-5.31752	59.0887	21.5081	-.112328
-.112808	-10.1297	3.81179	-51.8077	168.526	-1.80373	185.435	-91831.7	13276.2	12750.0
-4823.88	.0								
2									
6.53520	4916.60	-43.4811	-7.79460	-15.3811	-2.18325	-2.18325	148.156	-2.92543	-12.3561
-12.4089	-1114.27	419.297	179.396	40.3897	188.198	-10.0499	20037.9	92234.3	-5589.71
-12623.0	.0								
3									
2.57701	115.453	47495.4	-32.7940	92.7525	13.6929	13.6929	-513.012	-21.1438	34.8218
34.9706	3140.20	-1181.65	235.837	-384.257	143.606	-486.442	247642.	35837.3	-37848.8
3428.88	.0								
4									
-38.6206	31.3830	124.255	18393.5	-35.5950	-5.72737	-5.72737	-144.084	42.8766	.112328
.112808	10.1297	-3.81179	85.0983	-378.529	-43.1991	-428.364	209086.	-50877.6	-28138.7
14951.0	.0								
5									
-11.1006	-58.4978	-18.7467	39.7876	20590.5	-9.29177	-9.29177	258.778	22.8254	12.3561
12.4089	1114.27	-419.297	62.3753	288.688	143.541	279.060	-126021.	91883.3	15464.5
-17334.8	.0								
6									
4.68930	.687792	-11.4586	-11.2524	7.78050	10312.2	1.20479	-2.50523	-5.90571	-2.47123
-2.48178	-222.853	83.8594	4.77058	41.8767	20.6461	42.1130	-19539.3	13099.2	2288.10
-2598.68	.0								
7									
10.2006	16.9931	-12.4257	-28.0608	28.8399	4.37561	11564.4	-74.3508	-15.2570	-2.47123
-2.48178	-222.853	83.8594	-1.66189	39.3893	19.4673	40.4173	-19521.4	12176.0	2080.36
-2519.84	.0								
8									
-66.4519	75.7296	231.326	139.692	-44.5378	-7.39395	-7.39395	56429.8	70.3950	-14.0411
-14.1010	-1266.21	476.474	-775.992	-144.332	-832.661	75.0061	-100578.	-404325.	27351.1
55326.4	.0								
9									
-14.6906	12.4415	47.6708	31.8761	-13.1522	-2.12151	-2.12151	-57.0478	110588.	-.112328
-.112808	-10.1297	3.81179	40.2200	-201.431	-28.2559	-223.776	108961.	-29645.1	-14514.7
8158.73	.0								

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(g)

FIGURE 5. Continued.

INPLANE BIFILAR QGK MATRIX

1	.0	.0	.0	.0	.0	.0	.0	.0	.0
2	.0	.0	.0	.0	.0	.0	.0	.0	.0
3	.0	.0	.0	.0	.0	.0	.0	.0	.0
4	.0	.0	.0	.0	.0	.0	.0	.0	.0
5	.0	.0	.0	.0	.0	.0	.0	.0	.0
6	.0	.0	.0	.0	.0	.0	.0	.0	.0
7	.0	.0	.0	.0	.0	.0	6569.60	.0	.0
8	.0	.0	.0	.0	.0	.0	.0	5839.65	-43.7974
9	.0	.0	.0	.0	.0	.0	.0	43.7974	5839.65

(h)

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FIGURE 5. Continued

FIXED SYSTEM + ROTOR + FIXED ABSORBERS + INPLANE PENDULUMS MATRICES OF ORDER 25

FIXED SYSTEM + ROTOR + FIXED ABSORBERS + INPLANE PENDULUMS STIFFNESS MATRIX

1										
	1840.85	-13.7054	29.0877	40.2377	-34.7691	-5.31752	-5.31752	59.0887	21.5081	-.112328
	-.112808	-10.1297	3.81179	-51.8077	168.526	-1.80373	185.435	-91831.7	13276.2	12750.0
	-4823.88	.0	.0	.0	.0					
2										
	6.53520	4916.60	-43.4811	-7.79460	-15.3811	-2.18325	-2.18325	148.156	-2.92543	-12.3561
	-12.4089	-1114.27	419.297	179.396	40.3897	188.198	-10.0499	20037.9	92234.3	-5589.71
	-12623.0	.0	.0	.0	.0					
3										
	2.57701	115.453	47495.4	-32.7940	92.7525	13.6929	13.6929	-513.012	-21.1438	34.8218
	34.9706	3140.20	-1181.65	235.837	-384.257	143.606	-486.442	247642.	35837.3	-37848.8
	3428.88	.0	.0	.0	.0					
4										
	-38.6206	31.3830	124.255	18393.5	-35.5950	-5.72737	-5.72737	-144.084	42.8766	.112328
	.112808	10.1297	-3.81179	85.0983	-378.529	-43.1991	-428.364	209086.	-50877.6	-28138.7
	14951.0	.0	.0	.0	.0					
5										
	-11.1006	-58.4978	-18.7467	39.7876	20590.5	-9.29177	-9.29177	258.778	22.8254	12.3561
	12.4089	1114.27	-419.297	62.3753	288.688	143.541	279.060	-126021.	91883.3	15464.5
	-17334.8	.0	.0	.0	.0					
6										
	4.68930	.687792	-11.4586	-11.2524	7.78050	10312.2	1.20479	-2.50523	-5.90571	-2.47123
	-2.48178	-222.853	83.8594	4.77058	41.8767	20.6461	42.1130	-19539.3	13099.2	2288.10
	-2598.68	.0	.0	.0	.0					
7										
	10.2006	16.9931	-12.4257	-28.0608	28.8399	4.37561	11564.4	-74.3508	-15.2570	-2.47123
	-2.48178	-222.853	83.8594	-1.66189	39.3893	19.4673	40.4173	-19521.4	12176.0	2080.36
	-2519.84	.0	.0	.0	.0					
8										
	-66.4519	75.7296	231.326	139.692	-44.5378	-7.39395	-7.39395	56429.8	70.3950	-14.0411
	-14.1010	-1266.21	476.474	-775.992	-144.332	-832.661	75.0061	-100578.	-404325.	27351.1
	55326.4	.0	.0	.0	.0					
9										
	-14.6906	12.4415	47.6708	31.8761	-13.1522	-2.12151	-2.12151	-57.0478	110588.	-.112328
	-.112808	-10.1297	3.81179	40.2200	-201.431	-28.2559	-223.776	108961.	-29645.1	-14514.7
	8158.73	.0	.0	.0	.0					

(i)

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FIGURE 5. Continued.

VERTICAL BIFILAR QQK MATRIX

1	.0	.0	.0	.0	.0	.0	.0	.0	.0
2	.0	.0	.0	.0	.0	.0	.0	.0	.0
3	.0	.0	.0	.0	.0	.0	.0	.0	.0
4	.0	.0	.0	.0	.0	.0	.0	.0	.0
5	.0	.0	.0	.0	.0	.0	.0	.0	.0
6	.0	.0	.0	.0	.0	.0	.0	.0	.0
7	.0	.0	.0	.0	.0	.0	11679.3	.0	.0
8	.0	.0	.0	.0	.0	.0	.0	10949.3	-58.3965
9	.0	.0	.0	.0	.0	.0	.0	58.3965	10949.3

(j)

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FIGURE 5. Continued.

FIXED SYSTEM + ROTOR + FIXED ABSORBERS + INPLANE + VERTICAL PENDULUMS MATRICES OF ORDER 28

FIXED SYSTEM + ROTOR + FIXED ABSORBERS + INPLANE + VERTICAL PENDULUMS STIFFNESS MATRIX

1									
1840.85	-13.7054	29.0877	40.2377	-34.7691	-5.31752	-5.31752	59.0887	21.5081	-.112328
-.112808	-10.1297	3.81179	-51.8077	168.526	-1.80373	185.435	-91831.7	13276.2	12750.0
-4823.88	.0	.0	.0	.0	.0	.0	.0		
2									
6.53520	4916.60	-43.4811	-7.79460	-15.3811	-2.18325	-2.18325	148.156	-2.92543	-12.3561
-12.4089	-1114.27	419.297	179.396	40.3897	188.198	-10.0499	20037.9	92234.3	-5589.71
-12623.0	.0	.0	.0	.0	.0	.0	.0		
3									
2.57701	115.453	47495.4	-32.7940	92.7525	13.6929	13.6929	-513.012	-21.1438	34.8218
34.9706	3140.20	-1181.65	235.837	-384.257	143.606	-486.442	247642.	35837.3	-37848.8
3428.88	.0	.0	.0	.0	.0	.0	.0		
4									
-38.6206	31.3830	124.255	18393.5	-35.5950	-5.72737	-5.72737	-144.084	42.8766	.112328
.112808	10.1297	-3.81179	85.0983	-378.529	-43.1991	-428.364	209086.	-50877.6	-28138.7
14951.0	.0	.0	.0	.0	.0	.0	.0		
5									
-11.1006	-58.4978	-18.7467	39.7876	20590.5	-9.29177	-9.29177	258.778	22.8254	12.3561
12.4089	1114.27	-419.297	62.3753	288.688	143.541	279.060	-126021.	91883.3	15464.5
-17334.8	.0	.0	.0	.0	.0	.0	.0		
6									
4.68930	.687792	-11.4586	-11.2524	7.78050	10312.2	1.20479	-2.50523	-5.90571	-2.47123
-2.48178	-222.853	83.8594	4.77058	41.8767	20.6461	42.1130	-19539.3	13099.2	2288.10
-2598.68	.0	.0	.0	.0	.0	.0	.0		
7									
10.2006	16.9931	-12.4257	-28.0608	28.8399	4.37561	11564.4	-74.3508	-15.2570	-2.47123
-2.48178	-222.853	83.8594	-1.66189	39.3893	19.4673	40.4173	-19521.4	12176.0	2080.36
-2519.84	.0	.0	.0	.0	.0	.0	.0		
8									
-66.4519	75.7296	231.326	139.692	-44.5378	-7.39395	-7.39395	56429.8	70.3950	-14.0411
-14.1010	-1266.21	476.474	-775.992	-144.332	-832.661	75.0061	-100578.	-404325.	27351.1
55326.4	.0	.0	.0	.0	.0	.0	.0		
9									
-14.6906	12.4415	47.6708	31.8761	-13.1522	-2.12151	-2.12151	-57.0478	110588.	-.112328
-.112808	-10.1297	3.81179	40.2200	-201.431	-28.2559	-223.776	108961.	-29645.1	-14514.7
8158.73	.0	.0	.0	.0	.0	.0	.0		

(k)

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FIGURE 5. Concluded.

```
GENERALIZED FORCES - ORDER IS = 9

                    COSINE COMPONENT

 .500000   -170.000    500.000    .500000   -201.000    25.0000    120.000    350.000
-25.0000

                    SINE  COMPONENT

 75.0000    .500000    .500000    270.000    75.0000   -32.5000   -75.0000    260.000
 110.000

          (GAMMAS)

-.420508D-03 -.320688D-03  .257314D-05  .100956D-06 -.123203D-02  .397414D-02 -.915847D-02  .270569D-02  .129806D-03  .666964D-04
-.548605D-04 -.160107D-04 -.344875D-02  .705332D-03  .315306      .708454D-01 -.235034D-01  .510358D-02 -.122553D-02  .103080D-02
 .291653D-04 -.300397D-05  .705548D-07  .977919D-02 -.137627D-02 -.563780D-02  .193891D-02 -.147193D-03  .176337D-05  .228122D-05
 .750412D-04 -.150106D-02  .691230D-03  .718392D-01 -.317137     -.213538D-01  .326855D-02  .391058D-02

                    (GAMMAS)

          COSINE   -   SINE   -  AMPLITUDE - PHASE(DEG)

              ROTOR

         -.420508D-03  .103080D-02
         -.320688D-03  .291653D-04
          .257314D-05 -.300397D-05
          .100956D-06  .705548D-07
         -.123203D-02  .977919D-02
          .397414D-02 -.137627D-02
         -.915847D-02 -.563780D-02
          .270569D-02  .193891D-02
          .129806D-03 -.147193D-03
          .666964D-04  .176337D-05
         -.548605D-04  .228122D-05
         -.160107D-04  .750412D-04

                FIXED SYSTEM ABSORBER(S)

         -.344875D-02 -.150106D-02  .376126D-02 -156.479

                INPLANE BIFILAR PENDULUM(S)
        (EQT ORDER IS:0,SIN,COS)   (AMPL&PHASE ORDER IS:N,N-1,N+1 )
                                    DEG & DEG

          .705332D-03  .691230D-03  .141459D-01  44.4215
          .315306      .718392D-01  9.28677     -77.2864
```

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(a)

FIGURE 6. Output Format - Bifilar Linear Analysis Forced Response Results

```
      .708454D-01 -.317137      .298438D-01 -118.491

                 VERTICAL BIFILAR PENDULUM(S)
      (EQT ORDER IS:O,SIN,COS)   (AMPL&PHASE ORDER IS:N,N-1,N+1 )
                                  DEG & DEG

      -.235034D-01 -.213538D-01  .454860      -137.744
       .510358D-02  .326855D-02  .338881D-01  -30.2823
      -.122553D-02  .391058D-02  .144276       116.499

     INPUT FIXED SYSTEM FREQUENCIES (HZ)

 5.10000    6.40000    15.3000    14.3000    13.8000    11.6000    12.1000    17.4000    21.1000

     FORCING FREQUENCY (HZ)          =   17.2000

     THE CONVERSION  FACTOR TO G     =   30.2259

     FUSELAGE NO.OF DEGREES OF FREEDOM  =    9

                 COSINE  COMPONENT

-.214190D-03 -.448165D-04 -.349518D-03 -.295426D-02 -.567524D-03  .284549D-03  .641759D-03 -.256691D-02
 .214393D-03

                 SINE  COMPONENT

-.294794D-03 -.109048D-03  .122252D-02 -.224871D-02 -.105094D-02  .318558D-03  .889165D-03  .313195D-02
 .292297D-03

                  FIRST  A.C. STATION DISPLACEMENT IN G
             X                        Y                        Z
 COSINE        SINE        COSINE        SINE        COSINE        SINE
  .106454D-01  -.131066D-01  -.208498D-01  -.159455D-01  -.143570D-01  .182978D-02

                  FIRST  A.C. STATION TOTAL DISPLACEMENT IN G
                    X            Y            Z
                  .168852D-01  .262483D-01  .144732D-01

                  FIRST  A.C. STATION PHASE ANGLE IN DEG
```

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(b)

FIGURE 6. Continued.

-50.9158	-142.592	172.737

SECOND A.C. STATION DISPLACEMENT IN G

X		Y		Z	
COSINE	SINE	COSINE	SINE	COSINE	SINE
.112625D-01	-.152813D-01	-.881726D-02	.884320D-02	-.978861D-02	-.282865D-01

SECOND A.C. STATION TOTAL DISPLACEMENT IN G

X	Y	Z
.189832D-01	.124878D-01	.299323D-01

SECOND A.C. STATION PHASE ANGLE IN DEG

-53.6093	134.916	-109.088

THIRD A.C. STATION DISPLACEMENT IN G

X		Y		Z	
COSINE	SINE	COSINE	SINE	COSINE	SINE
.566726D-02	-.669938D-02	-.130642D-02	.746452D-02	.160931D-01	.584755D-02

THIRD A.C. STATION TOTAL DISPLACEMENT IN G

X	Y	Z
.877494D-02	.757798D-02	.171225D-01

THIRD A.C. STATION PHASE ANGLE IN DEG

-49.7708	99.9272	19.9690

FOURTH A.C. STATION DISPLACEMENT IN G

X		Y		Z	
COSINE	SINE	COSINE	SINE	COSINE	SINE
.175254D-01	.101499D-01	.224440D-01	.275827D-01	-.131746D-01	.320180D-02

FOURTH A.C. STATION TOTAL DISPLACEMENT IN G

X	Y	Z
.202524D-01	.355603D-01	.135581D-01

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(c)
FIGURE 6. Continued.

FOURTH A.C. STATION PHASE ANGLE IN DEG

30.0775	50.8648	166.340

ROTOR HEAD DISPLACEMENT IN G

X		Y		Z	
COSINE	SINE	COSINE	SINE	COSINE	SINE
-.528534D-01	.123521	-.941640D-01	.374169D-02	-.398021D-02	.438087D-02

ROTOR HEAD TOTAL DISPLACEMENT IN G

X	Y	Z
.134354	.942383D-01	.591896D-02

ROTOR HEAD DISPLACEMENT PHASE ANGLES IN DEG

113.166	177.725	132.256

(d)
FIGURE 6. Concluded.

D.O.F.= 29,NF= 9,KROTOR= 12,NFABS= 1,KINDS OF INPL. BIF.= 0,KINDS OF VERT. BIF.= 1,NUMB. OF N.L. INPL. BIF= 4

FIXED SYSTEM + ROTOR + FIXED ABSORBERS (R.H.S.) OF ORDER 25

.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0					

THE MASS (L.H.S.) MATRIX OF ORDER 10

.24565	.0	.0	.0	.0	.34977D-07	.0	-.12433	.34977D-08	.12433
.0	.24565	.0	.0	.0	.34977D-07	.12433	-.17488D-08	-.12433	.52465D-08
.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.34977D-07	.34977D-07	.0	.0	.0	100.68	2.5169	2.5169	2.5169	2.5169
.0	.49396	.0	.0	.0	10.000	1.0000	.0	.0	.0
-.49396	-.69483D-08	.0	.0	.0	10.000	.0	1.0000	.0	.0
.13897D-07	-.49396	.0	.0	.0	10.000	.0	.0	1.0000	.0
.49396	.20845D-07	.0	.0	.0	10.000	.0	.0	.0	1.0000

THE BIFILAR FORCE VECTOR OF ORDER 10

.25528D-04	-.25528D-04	.0	.0	.0	.0	.0	.0	.0	.0

(a)

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FIGURE 7. Output Format - Bifilar Nonlinear Analysis Matrices.

THE ROTOR HEAD MODE SHAPES,TRANSPOSE(PHI), OF ORDER 9 X 6

.10000D-02	.15000	.10000D-02	-.51200D-02	.47400D-03	.15680D-02
-.34000	.10000D-02	.11000	-.12000D-03	-.51830D-02	.28000D-03
1.0000	.10000D-02	-.31000	.13020D-01	-.53950D-02	.15000D-03
.10000D-02	.54000	-.10000D-02	.12140D-01	.77200D-04	.36830D-03
-.40200	.15000	-.11000	-.80100D-02	-.32520D-02	-.43820D-03
.50000D-01	-.65000D-01	.22000D-01	-.12440D-02	-.47300D-03	-.60000D-02
.24000	-.15000	.22000D-01	-.12440D-02	-.47300D-03	-.60000D-02
.70000	.52000	.12500	-.70000D-04	.23100D-01	.15000D-03
-.50000D-01	.22000	.10000D-02	.63500D-02	.22000D-03	-.12700D-02

THE EXPANDED BIFILAR MASS MATRIX OF ORDER = 13

.57749D-02	-.24725D-05	.30618D-03	.19956D-01	.53592D-02	-.33300D-02	-.64154D-02	.19357D-01	.78938D-02	.22596D-01	.38222D-02	-.14703D-01	.40709D-02
-.24725D-05	.28406D-01	-.83517D-01	.59513D-04	.33600D-01	-.43612D-02	-.20251D-01	-.58333D-01	.41943D-02	.82907D-03	.42976D-01	.58041D-03	-.41567D-01
.30618D-03	-.83517D-01	.24565	.38387D-03	-.98722D-01	.12176D-01	.58829D-01	.17209	-.12248D-01	.50187D-03	-.12395	.25322D-03	.12470
.19956D-01	.59513D-04	.38387D-03	.71646D-01	.19783D-01	-.88326D-02	-.20061D-01	.69157D-01	.29124D-01	.68064D-01	.80266D-03	-.66210D-01	.10513D-02
.53592D-02	.33600D-01	-.98722D-01	.19783D-01	.45245D-01	-.70680D-02	-.28963D-01	-.49972D-01	.13100D-01	.17546D-01	.48877D-01	-.19752D-01	-.51082D-01
-.33300D-02	-.43612D-02	.12176D-01	-.88326D-02	-.70680D-02	.52764D-02	.89673D-02	.20417D-03	-.33598D-02	-.23183D-01	-.21318D-01	-.70203D-02	-.88853D-02
-.64154D-02	-.20251D-01	.58829D-01	-.20061D-01	-.28963D-01	.89673D-02	.23301D-01	.22018D-01	-.10287D-01	-.33751D-01	-.44940D-01	.35475D-02	.14737D-01
.19357D-01	-.58333D-01	.17209	.69157D-01	-.49972D-01	.20417D-03	.22018D-01	.18680	.19486D-01	.65028D-01	-.86652D-01	-.64273D-01	.87407D-01
.78938D-02	.41943D-02	-.12248D-01	.29124D-01	.13100D-01	-.33598D-02	-.10287D-01	.19486D-01	.12666D-01	.24155D-01	.30199D-02	-.30549D-01	-.94129D-02
.89774D-01	.32940D-02	.19940D-02	.27042	.69712D-01	-.92108D-01	-.13409	.25836	.95972D-01	1.0000	.0	.0	.0
.15186D-01	.17075	-.49246	.31890D-02	.19419	-.84698D-01	-.17855	-.34427	.11998D-01	.0	1.0000	.0	.0
-.58414D-01	.23060D-02	.10061D-02	-.26306	-.78476D-01	-.27892D-01	.14094D-01	-.25536	-.12137	.0	.0	1.0000	.0
.16174D-01	-.16515	.49546	.41770D-02	-.20295	-.35302D-01	.58551D-01	.34727	-.37398D-01	.0	.0	.0	1.0000

(b)

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FIGURE 7. Continued.

THE EXPANDED BIFILAR FORCE VECTOR OF ORDER = 13

```
 -.38036D-05  -.87050D-05   .25502D-04  -.13759D-04  -.14091D-04   .29357D-05   .99558D-05   .45950D-05  -.68925D-05   .0
  .0           .0           .0
```

(c)
FIGURE 7. Continued.

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FINAL COMBINED MASS MATRIX OF ORDER 29 FOR PSI= .0

1									
3.0344	-.81765D-01	-2.9687	-2.3651	1.7839	.22634	.17970	.79122	-1.2707	-.94238D-04
.38577D-05	.86147D-01	.0	-.34010D-02	.46788D-02	.17445D-01	.19518D-01	-.33621	-101.32	-18.629
.73043	.10999D-02	.74987D-04	-.75763D-02	-.70140D-03	.22596D-01	.38222D-02	-.14703D-01	.40709D-02	
2									
-.81765D-01	4.6785	-.11174	-.77927D-01	1.2008	.60633D-01	-.16217	-6.0335	-.21990D-01	-.10366D-01
.42434D-03	9.4761	.0	.54962D-02	.68219D-02	.18770D-01	-.40917D-01	-102.89	7.3431	2.8095
35.048	-.17236D-01	.82485D-02	-.17757D-03	.76695D-02	.82907D-03	.42976D-01	-.58041D-03	-.41567D-01	
3									
-2.9687	-.11174	16.746	6.5549	-4.9101	-.44102	.15529	-2.9867	3.2545	.29214D-01
-.11959D-02	-26.705	.0	.54139D-03	-.29834D-01	.20923D-01	.63568D-01	-75.389	263.46	5.5910
-80.063	-.19410D-02	-.23246D-01	.19266D-01	.79832D-02	.50187D-03	-.12395	.25322D-03	.12470	
4									
-2.3651	-.77927D-01	6.5549	9.1646	-3.8371	-.72238	-.84336	.78956	3.5180	.94238D-04
-.38577D-05	-.86147D-01	.0	-.10372D-01	-.75966D-02	.58028D-01	-.49529D-01	24.190	233.26	-32.934
4.5767	-.64700D-03	-.74987D-04	.17964D-01	-.11424D-03	.68064D-01	.80266D-03	-.66210D-01	.10513D-02	
5									
1.7839	1.2008	-4.9101	-3.8371	6.7034	.38545	.76440D-01	-3.9865	-2.0075	.10366D-01
-.42434D-03	-9.4761	.0	.11480D-02	.14920D-01	.31152D-01	-.17411D-01	-80.374	-150.60	-18.270
39.574	.64700D-01	-.82485D-02	-.11853D-01	.48121D-02	.17546D-01	.48877D-01	-.19752D-01	-.51082D-01	
6									
.22634	.60633D-01	-.44102	-.72238	.38545	2.0415	.14325	-.41731	-.37584	-.20732D-02
.84868D-04	1.8952	.0	.14237D-02	-.22847D-03	-.50321D-02	.10771D-01	-11.265	-22.983	3.7638
-4.3471	.32350D-02	.16497D-02	-.18408D-02	.69992D-03	-.23183D-01	-.21318D-01	-.70203D-02	-.88853D-02	
7									
.17970	-.16217	.15529	-.84336	.76440D-01	.14325	2.3217	-.47330D-01	-.45179	-.20732D-02
.84868D-04	1.8952	.0	.23202D-02	-.42724D-02	-.14042D-01	.33230D-01	-10.263	-22.534	9.3940
-21.450	.64700D-03	.16497D-02	-.18408D-02	.69992D-03	-.33751D-01	-.44940D-01	.35475D-02	.14737D-01	
8									
.79122	-6.0335	-2.9867	.78956	-3.9865	-.41731	-.47330D-01	30.528	.35510	-.11780D-01
.48221D-03	10.768	.0	-.31467D-01	-.11927D-01	-.26221D-01	.87611D-01	453.11	-47.266	-51.099
-81.393	.12940D-01	.93733D-02	-.10358D-03	-.34182D-01	.65028D-01	-.86652D-01	-.64273D-01	.87407D-01	
9									
-1.2707	-.21990D-01	3.2545	3.5180	-2.0075	-.37584	-.45179	.35510	8.1061	-.94238D-04
.38577D-05	.86147D-01	.0	-.41496D-02	-.32419D-02	.22168D-01	-.31427D-01	15.882	122.01	-11.405
5.9875	-.12940D-02	.74987D-04	.93963D-02	-.32554D-03	.24155D-01	.30199D-02	-.30549D-01	-.94129D-02	
10									
-.37695D-03	-.41465D-01	.11685	.37695D-03	.41465D-01	-.82929D-02	-.82929D-02	-.47119D-01	-.37695D-03	.12486
-.25186D-03	.62860	-.96089D-01	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0	.0	.0	
11									
.15431D-04	.16974D-02	-.47835D-02	-.15431D-04	-.16974D-02	.33947D-03	.33947D-03	.19288D-02	.15431D-04	-.25186D-03
.20956	-3.9359	.75778	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0	.0	.0	
12									

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(d)
FIGURE 7. Continued.

```
FINAL COMBINED FORCE VECTOR OF ORDER 29  FOR PSI=  .0

 -.38036D-05  -.87050D-05   .25502D-04  -.13759D-04  -.14091D-04   .29357D-05   .99558D-05   .45950D-05  -.68925D-05   .0
  .0           .0           .0           .0           .0           .0           .0           .0           .0           .0
  .0           .0           .0           .0           .0           .0           .0           .0           .0

STATE VARIABLES (DISP&VEL) FOR PSI =   .0

 -.25286D-11  -.19571D-08
 -.26472D-11  -.20489D-08
  .46451D-11   .35953D-08
 -.71434D-11  -.55290D-08
 -.52201D-11  -.40403D-08
  .16185D-11   .12527D-08
  .51322D-11   .39723D-08
  .23660D-12   .18313D-09
 -.12391D-11  -.95909D-09
 -.30384D-11  -.23517D-08
  .41981D-12   .32493D-09
  .18716D-13   .14486D-10
 -.30990D-16  -.23987D-13
 -.11559D-11  -.89465D-09
  .36247D-11   .28055D-08
  .63828D-11   .49403D-08
 -.10919D-10  -.84510D-08
 -.19122D-13  -.14801D-10
 -.31698D-13  -.24534D-10
 -.50001D-13  -.38701D-10
 -.93811D-13   .72610D-10
  .44214D-11   .34222D-08
  .31637D-11   .24487D-08
 -.40165D-12  -.31088D-09
  .18834D-12   .14578D-09

BIFILAR INITIAL DISP & VEL FOR PSI =   .0

  .34172D-11   .26449D-08
  .49642D-11   .38423D-08
 -.25522D-11  -.19754D-08
 -.40992D-11  -.31728D-08

FIXED SYSTEM + ROTOR + FIXED ABSORBERS (R.H.S.) OF ORDER 25        (PSI = 2 DEG)

 -.13624D-07   .44860D-07  -.22938D-06   .23800D-06   .13370D-06  -.28845D-07  -.92330D-07  -.10942D-06   .20289D-06   .41828D-08
  .27024D-09  -.50329D-06   .84167D-06   .94019D-08   .19417D-07  -.12205D-06  -.45731D-08   .14607D-05  -.36007D-05   .73651D-04
  .30834D-04  -.70800D-07  -.42243D-07   .50807D-08  -.23538D-08
```

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(e)
FIGURE 7. Continued

```
NREV=  5, G1= .462D-02, G2= .344D-01, XHUB= .333D-02, YHUB= .592D-02, THZH= .122D-04, DXHUB= -.770 , DYHUB= .486D-01
NREV=  6, G1= .188D-02, G2= .301D-01, XHUB= .321D-02, YHUB= .601D-02, THZH= .131D-04, DXHUB= -.812 , DYHUB= .176D-01
NREV=  7, G1= .437D-02, G2= .219D-01, XHUB= .109D-02, YHUB= .526D-02, THZH= .146D-04, DXHUB= -.749 , DYHUB= -.140
NREV=  8, G1= .128D-01, G2= .984D-02, XHUB= -.271D-03, YHUB= .371D-02, THZH= .128D-04, DXHUB= -.614 , DYHUB= -.183
NREV=  9, G1= .124D-01, G2= .213D-02, XHUB= -.247D-03, YHUB= .261D-02, THZH= .103D-04, DXHUB= -.468 , DYHUB= -.148
NREV= 10, G1= .653D-02, G2= .821D-02, XHUB= .454D-03, YHUB= .226D-02, THZH= .844D-05, DXHUB= -.382 , DYHUB= -.608D-01
NREV= 11, G1= .454D-03, G2= .813D-02, XHUB= .141D-02, YHUB= .257D-02, THZH= .723D-05, DXHUB= -.361 , DYHUB= .176D-01
NREV= 12, G1= .452D-02, G2= .419D-02, XHUB= .191D-02, YHUB= .312D-02, THZH= .781D-05, DXHUB= -.399 , DYHUB= .573D-01
NREV= 13, G1= .481D-02, G2= .310D-03, XHUB= .201D-02, YHUB= .356D-02, THZH= .846D-05, DXHUB= -.450 , DYHUB= .470D-01
NREV= 14, G1= .252D-02, G2= .298D-02, XHUB= .174D-02, YHUB= .374D-02, THZH= .932D-05, DXHUB= -.483 , DYHUB= .131D-01
NREV= 15, G1= .193D-03, G2= .309D-02, XHUB= .147D-02, YHUB= .371D-02, THZH= .989D-05, DXHUB= -.486 , DYHUB= -.195D-01
NREV= 16, G1= .179D-02, G2= .148D-02, XHUB= .135D-02, YHUB= .361D-02, THZH= .104D-04, DXHUB= -.464 , DYHUB= -.327D-01
      THE NUMBER OF REVOLUTIONS REQUIRED TO CONVERGE = 16

      INPUT FIXED SYSTEM MODES FREQUENCIES IN HZ
  5.10   6.40   15.3   14.3   13.8   11.6   12.1   17.4   21.1

      THE CONVERGENCE FACTOR TO G =  30.2
```

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(f)
FIGURE 7. Concluded.

BIFILAR HARMONIC OUTPUT - AMPLITUDE AND PHASE

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	1 P	2 P	3 P	4 P	5 P	6 P	7 P	8 P	9 P
1	.72570D-02	.20305D-01	9.7569	.55991D-01	.18446D-01	.29965D-02	.51515D-02	.39280D-02	.27998D-02
	52.592	53.478	96.204	-124.06	29.256	-124.17	-103.32	-100.26	-132.76
2	.13026D-01	.27397D-01	9.7675	.36192D-01	.37197D-01	.33446D-02	.41663D-02	.29031D-02	.31304D-02
	167.32	-177.75	-173.99	-49.115	-46.272	30.184	-45.499	-31.080	-18.104
3	.33807D-02	.87900D-02	9.7962	.34774D-01	.21669D-01	.13022D-01	.67488D-02	.60291D-02	.32922D-02
	-107.27	-66.614	-83.966	83.398	-154.00	18.479	79.690	75.105	63.689
4	.12278D-01	.12959D-01	9.7978	.47673D-01	.35955D-01	.13860D-01	.31602D-02	.23191D-02	.32495D-02
	5.5568	-31.531	6.2263	167.95	139.47	-166.74	154.27	124.83	170.57

HARMONIC HUB OUTPUT - COSINE , SINE AND TOTAL RESPONSE AND PHASE ANGLE IN DEGREES

	1 P	2 P	3 P	4 P	5 P	6 P	7 P	8 P	9 P
1	-.35387D-03	-.71332D-03	-.10783D-02	.13513D-01	.11137D-01	.43362D-03	.23521D-03	.59209D-03	.93772D-04
	.71784D-04	.15737D-03	.13435D-03	-.13798	.19954D-02	-.88499D-03	-.48685D-03	-.21467D-03	-.27708D-03
	.36108D-03	.73048D-03	.10866D-02	.13864	.11314D-01	.98551D-03	.54069D-03	.62981D-03	.29251D-03
	168.53	167.56	172.90	-84.407	10.158	-63.896	-64.214	-19.929	-71.303
2	.84053D-04	-.17284D-03	-.40974D-03	.94763D-01	.98815D-02	.10481D-03	.62883D-04	.22187D-03	.38672D-04
	.12153D-03	-.11495D-03	-.54539D-03	-.31934D-01	.16442D-02	-.19251D-03	-.16556D-04	-.22391D-03	.30311D-04
	.14776D-03	.20757D-03	.68215D-03	.99998D-01	.10017D-01	.21919D-03	.65026D-04	.31522D-03	.49135D-04
	55.331	-146.37	-126.92	-18.623	9.4468	-61.433	-14.750	-45.263	38.090
3	-.18544D-03	-.12456D-03	-.20801D-03	.49658D-02	.29709D-02	.36247D-03	.16057D-03	.73182D-04	.68470D-04
	-.20083D-04	-.20787D-04	-.10326D-03	-.65451D-02	.67828D-02	-.10717D-03	-.29909D-04	-.29406D-04	-.36232D-05
	.18653D-03	.12629D-03	.23223D-03	.82157D-02	.74049D-02	.37798D-03	.16333D-03	.78869D-04	.68566D-04
	-173.82	-170.53	-153.60	-52.813	66.346	-16.471	-10.552	-21.891	-3.0290
4	.11692D-04	.18351D-04	.20934D-04	.10088D-02	.25708D-03	-.40229D-04	-.18089D-04	-.63736D-05	-.77610D-05
	-.34145D-05	.35567D-05	-.17642D-05	-.27056D-03	-.65194D-03	-.96333D-05	-.43971D-05	-.42516D-05	-.22284D-05
	.12181D-04	.18692D-04	.21008D-04	.10444D-02	.70080D-03	.41367D-04	.18615D-04	.76615D-05	.80745D-05
	-16.279	10.969	-4.8173	-15.013	-68.479	-166.53	-166.34	-146.29	-163.98
5	-.20246D-04	-.28300D-04	-.39290D-04	.11809D-02	.32143D-03	.43543D-04	.20206D-04	.17703D-04	.86785D-05
	.57440D-06	-.19054D-05	-.77411D-05	-.25992D-02	.67686D-03	-.17481D-04	-.75718D-05	-.61179D-05	-.33195D-05
	.20254D-04	.28364D-04	.40045D-04	.28549D-02	.74930D-03	.46921D-04	.21578D-04	.18731D-04	.92917D-05
	178.37	-176.15	-168.85	-65.566	64.598	-21.874	-20.543	-19.064	-20.932
6	.12641D-05	-.79781D-06	.28154D-05	.29220D-03	.44537D-04	.11795D-05	.42791D-06	-.17881D-06	.21827D-06
	.17520D-05	-.63593D-06	.17588D-06	.17726D-03	.33771D-04	-.23400D-05	-.10164D-05	-.19328D-05	-.46859D-06
	.21604D-05	.10202D-05	.28209D-05	.34176D-03	.55893D-04	.26205D-05	.11028D-05	.19411D-05	.51694D-06
	54.189	-141.44	3.5747	31.243	37.173	-63.248	-67.169	-95.285	-65.024

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(a)

FIGURE 8. Output Format - Bifilar Nonlinear Analysis Time History Results.

VIBRATION LEVELS AT 4 A/C LOCATIONS

COSINE,SINE,TOTAL AMPLITUDE IN G AND PHASE ANGLE IN DEGREES OF POINT 1

	1 P	2 P	3 P	4 P	5 P	6 P	7 P	8 P	9 P
1	.74646D-04	.10540D-03	.16495D-03	-.68218D-02	-.17800D-03	-.13784D-03	-.69643D-04	-.77459D-04	-.32567D-04
	-.17197D-04	.10159D-04	.45374D-04	.16037D-01	-.15142D-02	-.87238D-05	-.90956D-05	.72264D-05	-.73596D-05
	.76601D-04	.10589D-03	.17108D-03	.17428D-01	.15246D-02	.13811D-03	.70234D-04	.77795D-04	.33389D-04
	-12.973	5.5052	15.381	113.04	-96.705	-176.38	-172.56	174.67	-167.27
2	-.44641D-03	-.23631D-03	-.40666D-03	.25074D-01	-.26798D-02	.55354D-03	.25294D-03	.10032D-03	.11363D-03
	-.20551D-03	-.23326D-04	.38511D-05	.11033D-01	.88283D-02	.15384D-03	.87506D-04	-.12696D-04	.54312D-04
	.49145D-03	.23746D-03	.40668D-03	.27394D-01	.92260D-02	.57452D-03	.26765D-03	.10112D-03	.12594D-03
	-155.28	-174.36	179.46	23.751	106.89	15.532	19.083	-7.2126	25.547
3	-.21825D-03	-.39448D-03	-.56827D-03	.13519D-01	-.38145D-02	.62622D-03	.29079D-03	.18752D-03	.12778D-03
	-.27545D-04	-.80199D-04	-.10098D-03	-.51186D-02	.92417D-02	.19182D-03	.96591D-04	.33801D-04	.54140D-04
	.21998D-03	.40255D-03	.57717D-03	.14456D-01	.99980D-02	.65494D-03	.30642D-03	.19054D-03	.13878D-03
	-172.81	-168.51	-169.92	-20.737	112.43	17.031	18.375	10.218	22.962

COSINE,SINE,TOTAL AMPLITUDE IN G AND PHASE ANGLE IN DEGREES OF POINT 2

	1 P	2 P	3 P	4 P	5 P	6 P	7 P	8 P	9 P
1	.10408D-03	.14141D-03	.21754D-03	-.67874D-02	-.71084D-03	-.18505D-03	-.90701D-04	-.99222D-04	-.40508D-04
	-.70780D-05	.37864D-05	.37375D-04	.18411D-01	-.22860D-02	.37913D-04	.16089D-04	.17450D-04	.65888D-05
	.10432D-03	.14146D-03	.22073D-03	.19622D-01	.23939D-02	.18889D-03	.92117D-04	.10074D-03	.41041D-04
	-3.8905	1.5338	9.7485	110.24	-107.27	168.42	169.94	170.03	170.76
2	-.79428D-03	-.27576D-03	-.32357D-03	.60930D-02	.96273D-02	.59969D-03	.25702D-03	.14243D-03	.99368D-04
	-.39016D-03	.16613D-03	.40307D-03	-.10963D-01	.11193D-01	-.76927D-03	-.39836D-03	-.23445D-03	-.21426D-03
	.88493D-03	.32194D-03	.51688D-03	.12543D-01	.14763D-01	.97539D-03	.47408D-03	.27432D-03	.23618D-03
	-153.84	148.93	128.76	-60.936	49.299	-52.061	-57.171	-58.721	-65.120
3	.11722D-03	-.18740D-03	-.82776D-04	.15241D-01	-.12303D-01	.15987D-03	.82942D-04	-.57137D-05	.47371D-04
	.22443D-04	-.15717D-03	-.76443D-04	.28371D-01	.86053D-03	.69282D-03	.33292D-03	.12336D-03	.17070D-03
	.11935D-03	.24459D-03	.11267D-03	.32206D-01	.12333D-01	.71102D-03	.34310D-03	.12350D-03	.17715D-03
	10.839	-140.01	-137.28	61.755	176.00	77.007	76.010	92.652	74.490

COSINE,SINE,TOTAL AMPLITUDE IN G AND PHASE ANGLE IN DEGREES OF POINT 3

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FIGURE 8.(b) Continued.

INITIAL BIFILAR DISPLACEMENTS (LC.1720-1739)

```
-.34889D-01  -.16560   .34955D-01  .16588
```

INITIAL BIFILAR VELOCITIES (LC.1740-1759)

```
13.328  -3.5460  -13.417  3.5655
```

INITIAL HUB DISPLACEMENTS (LC.1768-1773)

```
.15065D-02  .35604D-02  .25974D-03  .45858D-04  .59105D-04  .10595D-04
```

INITIAL HUB VELOCITIES (LC.1774-1779)

```
-.43641  -.27048D-01  .16799D-01  -.33085D-02  -.43242D-02  .10283D-02
```

INITIAL STATE VARIABLES DISPL.(LC.1780-1859)

```
-.41930D-04   .83752D-04   .47216D-04   .38650D-02   .19628D-03  -.44489D-03  -.97618D-03   .25778D-02  -.29884D-03   .10835D-02
-.47164D-04  -.53035D-05   .31296D-07   .38200D-03  -.56627D-02   .10109D-01  -.13128D-02  -.13922D-03  -.92779D-04   .58519D-04
 .13830D-04   .56357D-02   .20740D-01  -.11073      .51973D-01
```

INITIAL STATE VARIABLES VELOC.(LC.1860-1939)

```
 .47650D-01  -.11799D-01  -.21021       .15078       .14798      -.33559D-01  -.99727D-01  -.22058      -.18196       .20168D-01
-.10169      -.33443D-03  -.11013D-04  -1.2027       .30764D-01   .67974      -.27664       .92194D-02   .76840D-02   .44570D-03
-.79445D-02  -.29740      2.7402       5.2717      14.502
```

(c)

FIGURE 8. Concluded.

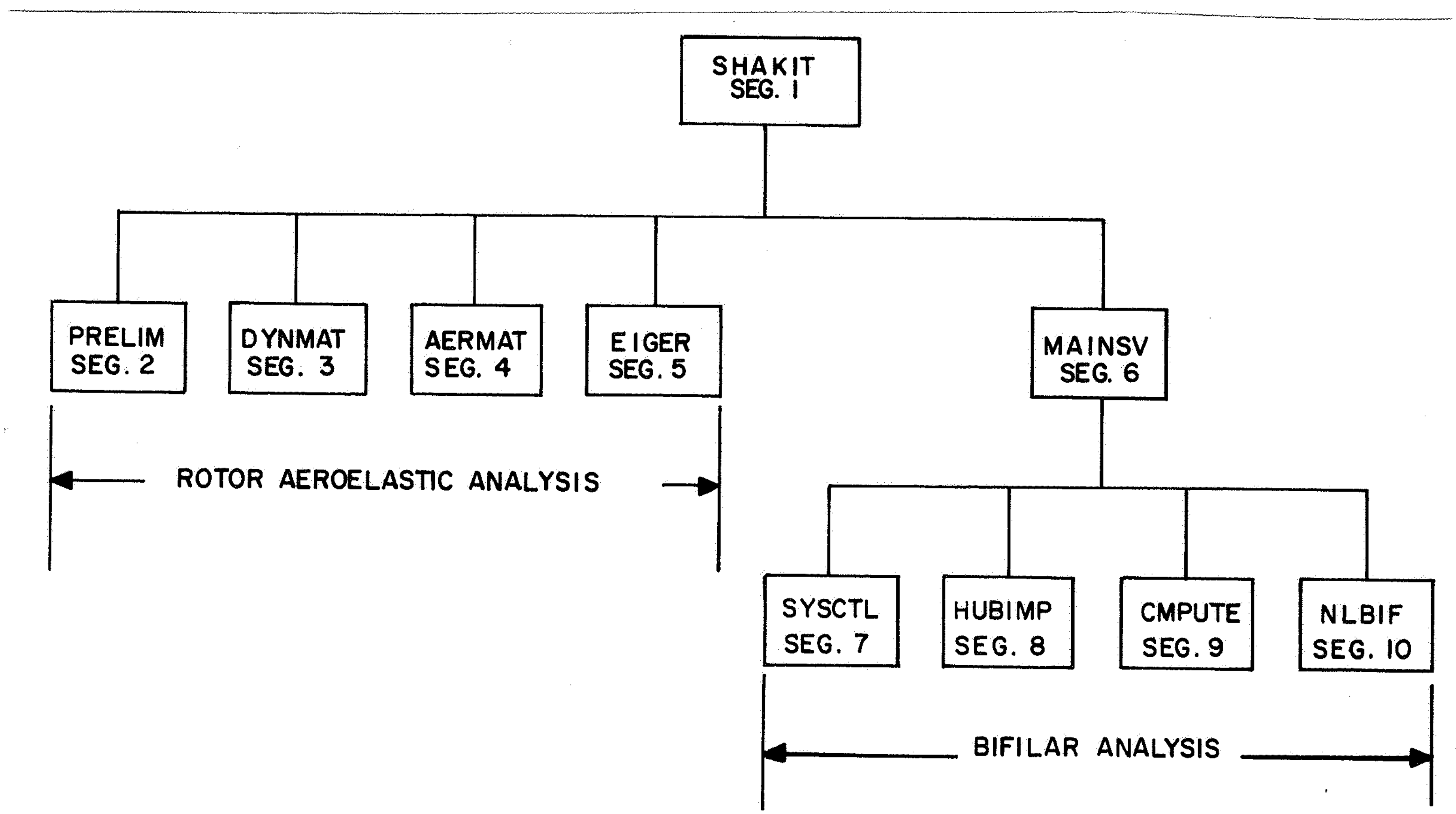


Figure 9. Bifilar Analysis Segmentation Structure.

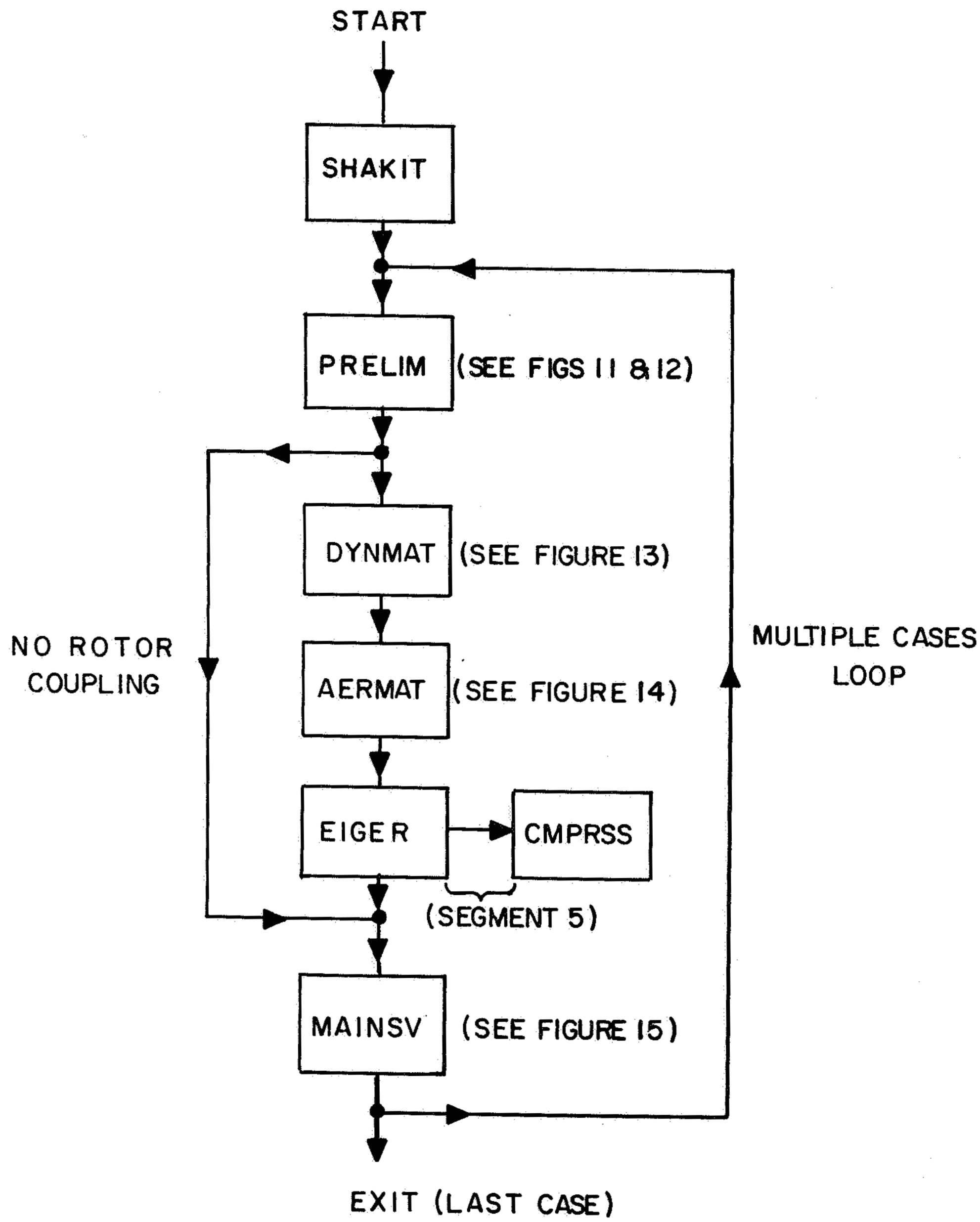


Figure 10. Flow Chart for the Main Program.

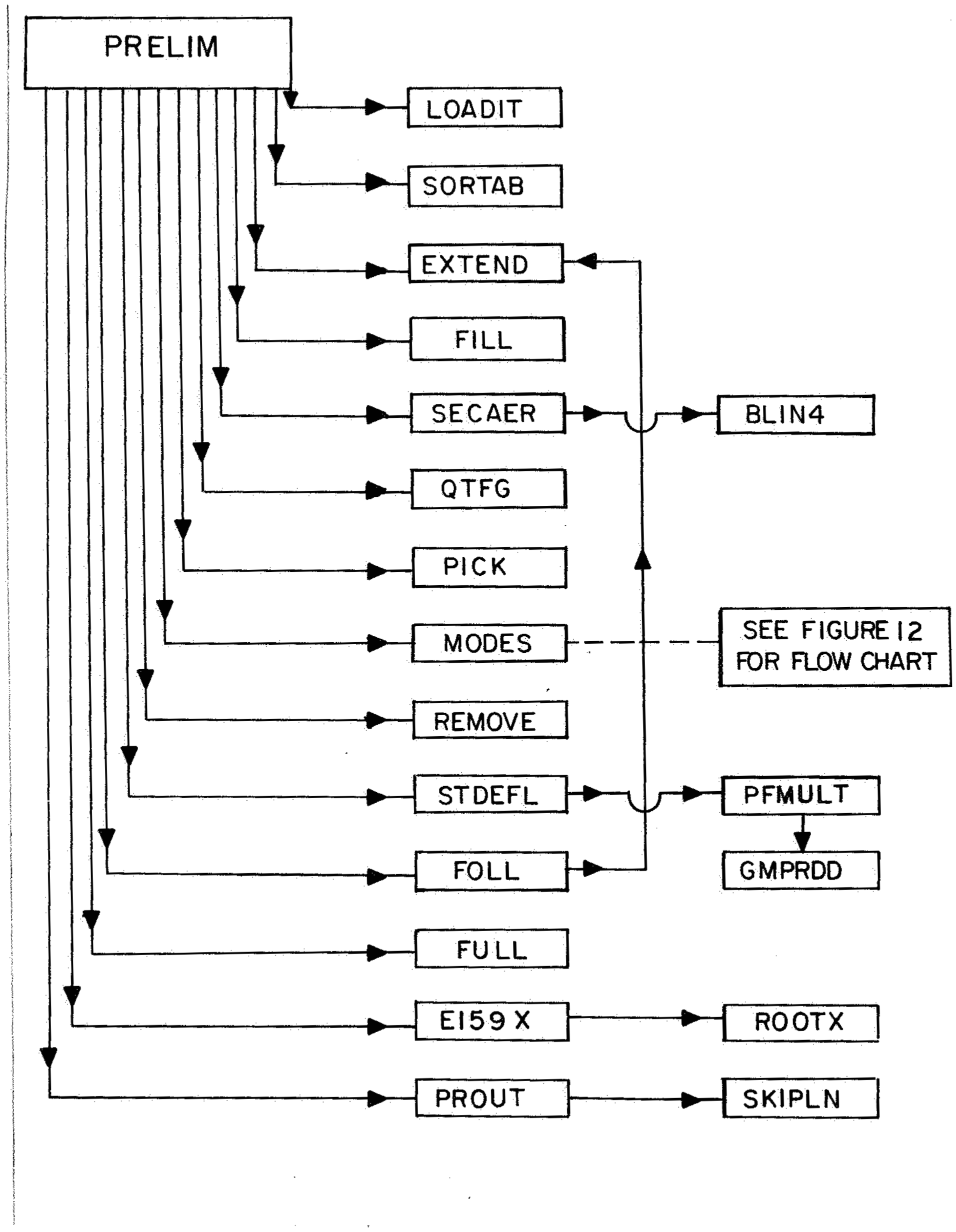


Figure 11. Flow Chart for Subroutine PRELIM.

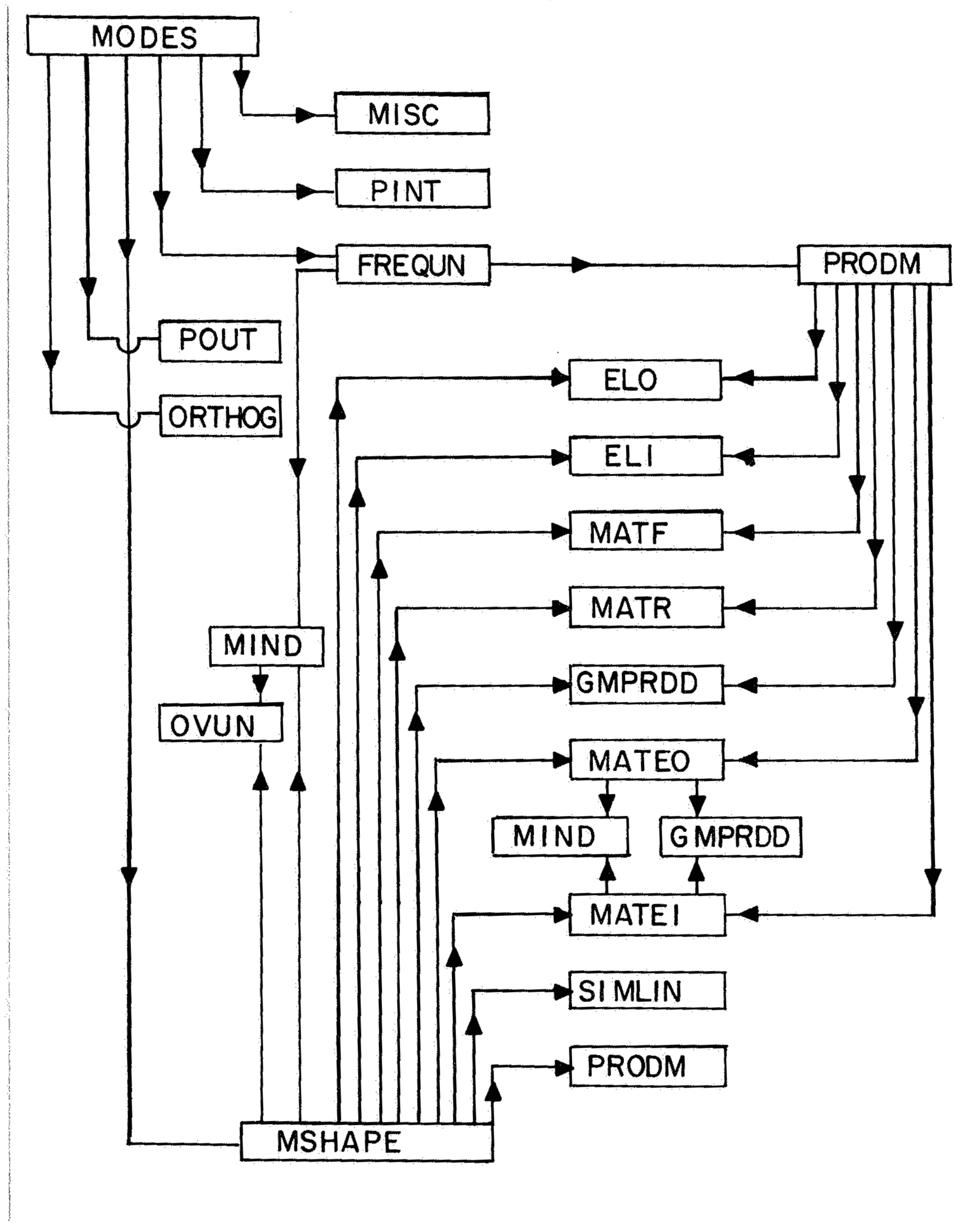


Figure 12. Flow Chart for Subroutine MODES.

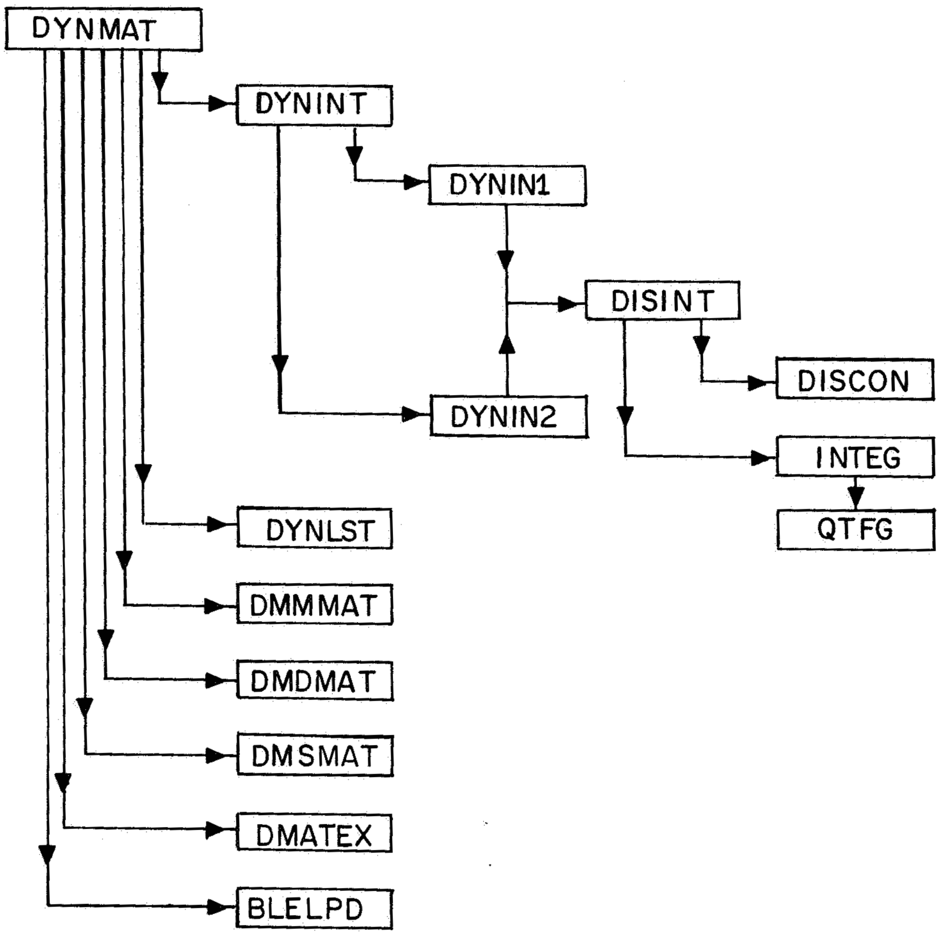


Figure 13. Flow Chart for Subroutine DYNMAT.

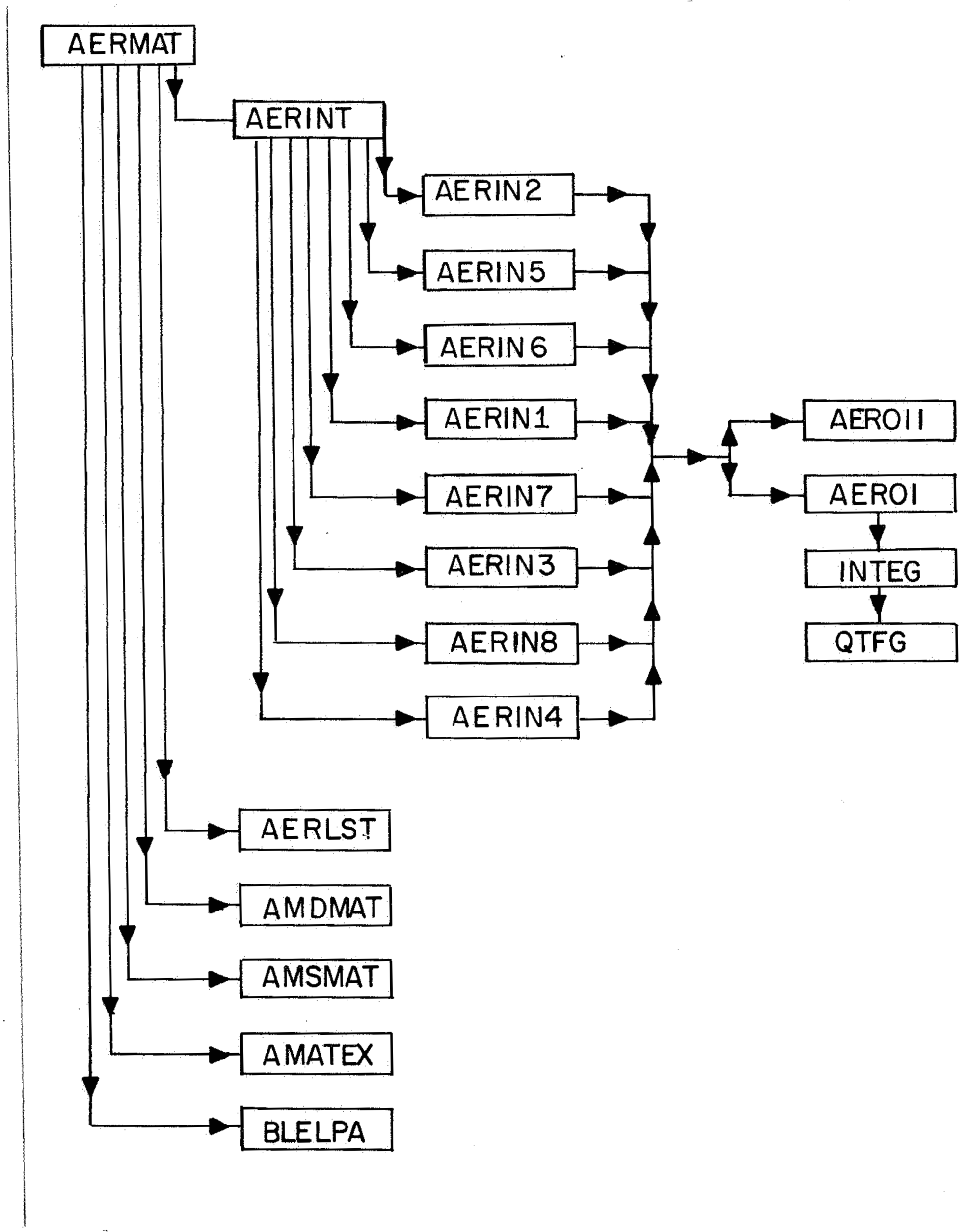


Figure 14. Flow Chart for Subroutine AERMAT.

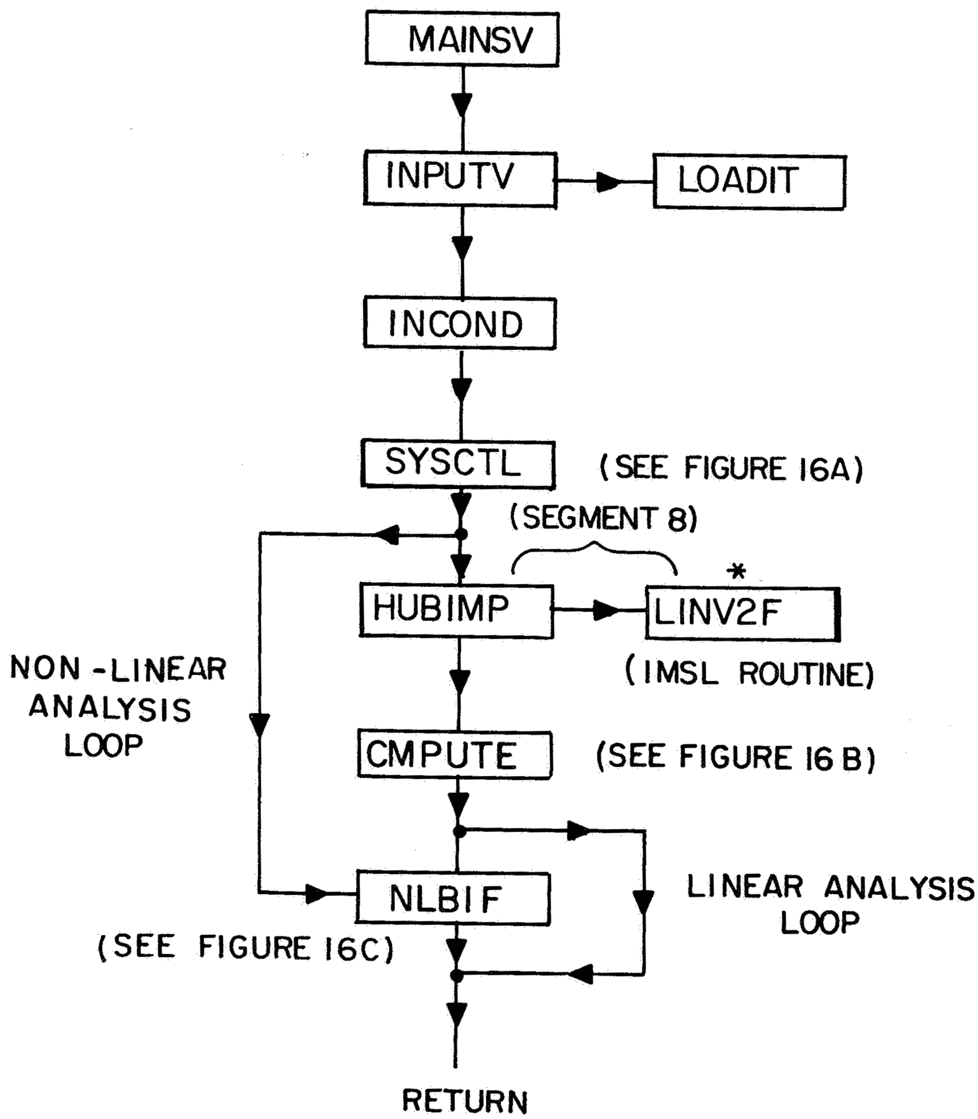
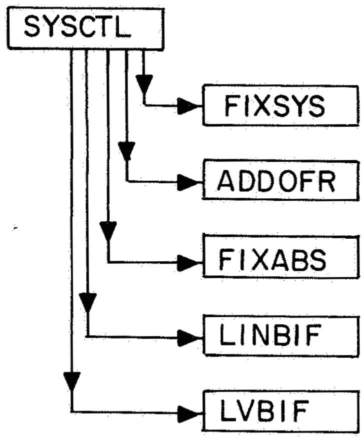
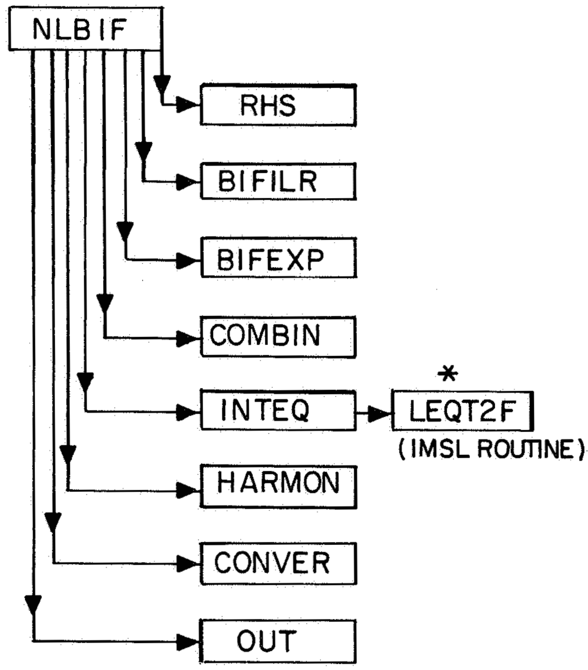


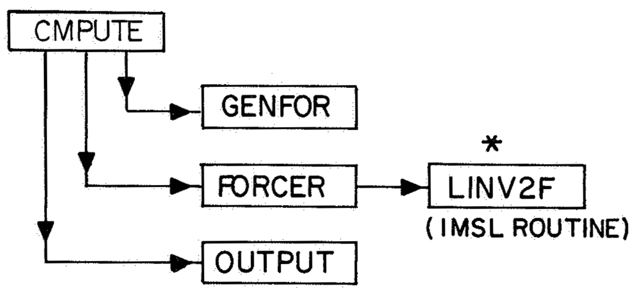
Figure 15. Flow Chart for the Bifilar Analysis.



16a. SYSCTL Flow Chart.



16c. NLBIF Flow Chart.



16b. CMPUTE Flow Chart.

Figure 16. Flow Charts for Subroutines SYSCTL, CMPUTE, NLBIF.

COMMON BLOCK \ SUBROUTINE	46 FOLL	47 FORCER	48 FREQUN	49 FULL	50 GENFOR	51 GMPRDD	52 HARMON	53 HUBIMP	54 INCOND	55 INPUTV	56 INTEG	57 INTEQ	58 LINBIF	59 LOADIT	60 LVBIF	61 MAINSV	62 MATEI	63 MATEO	64 MATF	65 MATR	66 MIND	67 MISC	68 MODES	69 MSHAPE
1 BLANK																								
2 AERO1																								
3 AERO2																								
4 AERO3																								
5 AERO4																								
6 AERO5																								
7 CDCAER																								
8 CONT																							X	
9 DAT			X														X	X	X	X		X		X
10 DYNINP																								
11 DYNOUT																								
12 EMATI																	X							X
13 EMATO																		X						X
14 EOF6																X								
15 FMAT																			X					X
16 FREQ			X																					X
17 HARM							X																	
18 INDAT		X			X		X	X		X		X	X		X	X								
19 INEIG																								
20 INEIGN													X		X									
21 LAGDAM																								
22 NAMIC																								
23 NDOF		X			X			X					X		X	X								
24 NIMIC																								
25 NLDAT1																X								
26 NLDAT2							X					X												
27 PHTNO																								
28 PMAT			X																					X
29 PRAM1																	X	X	X					X
30 PRNTSW																							X	
31 PRSWTH																X								
32 TMDS																								
33 TORFIN																								
34 TOTMAT								X	X															
35 TWISTM																				X				X
36 WORKA			X														X	X	X			X	X	X
37 XFRDAT		X			X			X				X	X		X									

COMMON BLOCK \ SUBROUTINE	70 NLBIF	71 ORTHOG	72 OUT	73 OUTPUT	74 OVUN	75 PFMULT	76 PICK	77 PINT	78 POUT	79 PRELIM	80 PRODM	81 PROUT	82 QTFG	83 REMOVE	84 RHS	35 ROOTX	86 SECAER	87 SHAKIT	88 SIMLIN	89 SKIPLN	90 SORTAB	91 STDEFL	92 SYSCTL
BLANK										X		X		X			X	X					
AERO1																							
AERO2																							
AERO3																							
AERO4																							
AERO5																							
CDCAER																							
CONT											X												
DAT									X		X												
DYNINP										X		X		X				X					
DYNOUT																							
EMATI											X												
EMATO											X												
EOF6										X								X					X
FMAT											X												
FREQ									X	X													
HARM			X																				
INDAT	X		X	X											X								X
INEIG																		X					X
INEIGN																		X					X
LAGDAM										X		X											
NAMIC																							
NDOF	X			X																			X
NIMIC										X													
NLDAT1	X																						X
NLDAT2	X														X								
PHTNO										X													
PMAT											X												
PRAM1																							
PRNTSW						X				X						X		X				X	
PRSWTH	X																						X
TMDS										X		X											
TORFIN										X													
TOTMAT															X								X
TWISTM											X												
WORKA		X						X	X	X	X	X										X	
XFRDAT				X																			X

(a)

Figure 17. COMMON Blocks.

No.	Common block	1 ADDOFR	2 AERINT	3 AERIN1	4 AERIN2	5 AERIN3	6 AERIN4	7 AERIN5	8 AERIN6	9 AERIN7	10 AERIN8	11 AERLST	12 AERMAT	13 AEROI	14 AEROII	15 AMATEX
1	BLANK		X	X	X	X	X	X	X	X	X	X				
2	AERO1		X	X	X	X	X	X	X	X	X	X				
3	AERO2		X	X	X	X	X	X	X	X	X	X				
4	AERO3		X	X	X	X	X	X	X	X	X	X				
5	AERO4		X	X	X	X	X	X	X	X	X	X				
6	AERO5		X	X	X	X	X	X	X	X	X	X				
7	CDCAER													X	X	
8	CONT															
9	DAT															
10	DYNINP		X	X	X	X	X	X	X	X	X	X	X			
11	DYNOUT															
12	EMAT1															
13	EMATO															
14	EOF6															
15	FMAT															
16	FREQ															
17	HARM															
18	INDAT															
19	INEIG												X			X
20	INEIGN	X											X			X
21	LAGDAM															
22	NAMIC															
23	NDOF	X														
24	NIMIC			X	X	X	X	X	X	X	X					
25	NLDAT1															
26	NLDAT2															
27	PHTNO															
28	PMAT															
29	PRAM1															
30	PRNTSW											X				
31	PRSWTH															
32	TMDS			X	X	X	X	X	X	X	X					
33	TORFIN															
34	TOTMAT	X														
35	TWISTM															
36	WORKA															
37	XFRDAT	X														

No.	Common block	16 AMDMAT	17 AMSMAT	18 BIFEXP	19 BIFILR	20 BLELPA	21 BLELPD	22 BLIN4	23 CMPRSS	24 CMPUTE	25 COMBIN	26 CONVER	27 DISCON	28 DISINT	29 DMATEX	30 DMDMAT
1	BLANK	X	X			X	X									X
2	AERO1	X	X			X										
3	AERO2	X	X			X										
4	AERO3	X	X			X										
5	AERO4	X	X			X										
6	AERO5	X	X			X										
7	CDCAER															
8	CONT															
9	DAT															
10	DYNINP	X	X			X	X									X
11	DYNOUT						X									
12	EMAT1															
13	EMATO															
14	EOF6															
15	FMAT															
16	FREQ															
17	HARM															
18	INDAT			X	X					X	X	X				
19	INEIG	X	X												X	X
20	INEIGN			X	X	X	X								X	
21	LAGDAM						X									X
22	NAMIC															
23	NDOF															
24	NIMIC															
25	NLDAT1															
26	NLDAT2			X	X						X					
27	PHTNO															
28	PMAT															
29	PRAM1															
30	PRNTSW															
31	PRSWTH															
32	TMDS						X									
33	TORFIN															
34	TOTMAT										X					
35	TWISTM															
36	WORKA															
37	XFRDAT			X												

No.	Common block	31 DMMMAT	32 DMSMAT	33 DYNINT	34 DYNIN1	35 DYNIN2	36 DYNLST	37 DYNMAT	38 EIGER	39 ELI	40 ELO	41 EXTEND	42 E159X	43 FILL	44 FIXABS	45 FIXSYS
1	BLANK	X	X	X	X	X	X	X	X				X			
2	AERO1															
3	AERO2															
4	AERO3															
5	AERO4															
6	AERO5															
7	CDCAER															
8	CONT															
9	DAT									X	X					
10	DYNINP	X	X	X	X	X	X	X	X				X			
11	DYNOUT	X	X	X	X	X	X	X								
12	EMAT1															
13	EMATO															
14	EOF6															
15	FMAT															
16	FREQ															
17	HARM															
18	INDAT														X	X
19	INEIG	X	X					X								
20	INEIGN							X	X						X	
21	LAGDAM		X													
22	NAMIC				X	X										
23	NDOF														X	X
24	NIMIC															
25	NLDAT1															
26	NLDAT2															
27	PHTNO															
28	PMAT															
29	PRAM1									X	X					
30	PRNTSW					X										
31	PRSWTH															
32	TMDS				X	X							X			
33	TORFIN												X			
34	TOTMAT															X
35	TWISTM															
36	WORKA									X	X					
37	XFRDAT														X	

(b)

Figure 17. Concluded.

APPENDIX A. JOB CONTROL LANGUAGE

```
CARD
NO.
     *---------------------------------------------------------------*
     *     JOB  CONTROL  LANGUAGE TO RUN ROTOR/BIFILAR COUPLED PROGRAM *
     *---------------------------------------------------------------*
     *                                                               *
1    //E90BCRUN JOB (4045,STAL,02,25),'CASSARINO - X-6565',MSGCLASS=T,
     //            REGION=2000K,MSGLEVEL=1,TIME=02,CLASS=B,NOTIFY=ET473
     //*-------------------------------------------------------------
     //*           ROTOR/BIFILAR COUPLED PROGRAM
     //*   USES OVERLAY STRUCTURE - DOUBLE PRECISION VERSION
     //*   MODULE IS "E90BCFIN" & IS FOUND IN "ET473.SEBBY.LOAD"
     //*   INPUT COMES FROM "ET473.BIFILAR.DATA(NASARUN)"
     //*-------------------------------------------------------------
2    //STEP01    EXEC    PGM=E90BCFIN
3    //STEPLIB   DD      DSN=ET473.SEBBY.LOAD,DISP=SHR
     //*-------------------------------------------------------------
4    //FT05F001  DD      DSN=ET473.BIFILAR.DATA(NASARUN),DISP=SHR
     //*-------------------------------------------------------------
5    //FT06F001  DD      SYSOUT=T
6    //FT07F001  DD      DUMMY
7    //FT08F001  DD UNIT=WORK,DISP=(NEW,DELETE),SPACE=(TRK,(5,1)),
     //            DCB=(RECFM=VSB,LRECL=680,BLKSIZE=2050)
8    //FT11F001  DD UNIT=WORK,DISP=(NEW,DELETE),SPACE=(TRK,(13,1)),
     //            DCB=(RECFM=VSB,LRECL=404,BLKSIZE=2050)
9    //
     *---------------------------------------------------------------
     *           END OF JOB CONTROL LANGUAGE
     *---------------------------------------------------------------
```

APPENDIX B. TEST CASES INPUT DATA

```
-----------------------------------------------------------------------
 NASA  TEST  CASES :
             CASE 1.  WITH ROTOR - LINEAR     ANALYSYS OPTION
             CASE 2.  WITH ROTOR - NON-LINEAR ANALYSYS OPTION
             CASE 3.  NO   ROTOR - LINEAR     ANALYSYS OPTION
             CASE 4.  NO   ROTOR - NON-LINEAR ANALYSYS OPTION
------------------------------------------------------------------------
             INPUT TO ROTOR/BIFILAR COUPLED PROGRAM
------------------------------------------------------------------------
         CARD FORMAT = A,B,C,D E,F,G
  WHERE :
         A = COLUMN  2    , NO. OF ITEMS, INTEGER  (5 MAXIMUM)
         B = COLUMNS 2 - 6, LOCATION NO., INTEGER, (RIGHT ADJUSTED)
         C = COLUMNS 7 -18,   QUANTITY ,     REAL
         D = COLUMNS 19-30,   QUANTITY ,     REAL
         E = COLUMNS 31-42,   QUANTITY ,     REAL
         F = COLUMNS 43-54,   QUANTITY ,     REAL
         G = COLUMNS 55-66,   QUANTITY ,     REAL

------------------------------------------------------------------------
                        COLUMN NUMBER
123456789012345678901234567890123456789012345678901234567890123456789
0
--------------------- BEGINNING OF INPUT -----------------------------

 5    1 0.002378    1116.       .99         .0          258.
 3    6 26.833333   1.25        4.
 5   13 6.4         1000000.    15.1265     7.25        .35
 2   19 258.0       2.
 3  107 1.          0.          1.
 1  111 1.
 3  113 01000111.   1.          11.
 1  119 5.
 1  125 1.
 5  126 27.074      11.238      9.178       6.82        .355
 3  131 -1.         676.        0.
 1  200 20.
 5  201 17.5        17.5        25.         16.8        16.8
 5  206 16.8        16.8        16.8        17.49       17.49
 5  211 17.49       17.49       15.9        15.9        15.9
 5  216 12.97       12.97       7.7         7.7         4.
 1  250 20.
 4  251 .0          .0          15.         .0
 4  255 15.01       8.3         50.         8.3
 4  259 50.01       20.76       228.96      20.76
 4  263 228.97      22.317      276.66      22.317
 4  267 276.67      20.76       322.        20.76
 1  350 14.
 4  351 0.          -9.5        50.         -9.5
 4  355 270.        1.2         290.        2.8
 4  359 305.        3.8         318.        1.2
 2  363 322.        1.2
 1  450 14.
 4  451 0.          -9.5        50.         -9.5
 4  455 270.        1.2         290.        2.8
 4  459 305.        3.8         318.        1.2
 2  463 322.        1.2
```

1 550	58.			
4 551	.0	.0	58.	.0
4 555	75.	-.822	159.	-.822
4 559	159.01	-.667	192.	-.667
4 563	192.01	-.4685	210.	-.4685
4 567	210.01	-.6754	220.	-.6754
4 571	220.01	-.6992	224.	-.6858
4 575	228.96	.5569	228.97	.2788
4 579	240.	.2992	270.3	.3102
4 583	270.31	.008	276.66	.0120
4 587	276.67	.368	289.	.368
4 591	290.	1.2335	291.	2.099
4 595	299.	2.099	299.5	2.1247
4 599	301.25	1.7292	303.75	.8992
4 603	306.695	.3395	318.	-5.
2 607	322.	-6.456		
1 650	18.			
4 651	.0	.0	50.	.0
4 655	50.01	-.35	228.96	-.35
4 659	228.97	.0	276.66	.0
4 663	276.67	-.35	302.6	-.35
2 667	322.	-7.411		
1 750	20.			
4 751	.0	.0	15.	.0
4 755	15.01	-2.075	50.	-2.075
4 759	50.01	-5.19	228.96	-5.19
4 763	228.97	-5.58	276.66	-5.58
4 767	276.67	-5.19	322.	-5.19
1 950	52.			
4 951	.0	15.	3.0097	15.
4 955	2.07	5.	2.17	7.5
4 959	2.10	.25	.9	4.75
4 963	.93	10.	.53	17.5
4 967	.56	10.	.55	10.
4 971	.53	64.	.5615	33.
4 975	.5812	32.	.7354	46.3
4 979	.69551	6.36	.68	13.34
4 983	1.51	2.5	1.3	6.35
4 987	1.61447	1.15	1.16447	2.5
4 991	1.46447	2.5	1.06447	3.39
4 995	.75	1.61	1.25	2.5
4 999	.25	5.5	.14	4.
11050	34.			
21051	0.	15.		
41053	55.02	8.3	98.04	2.8
41057	644.2	3.9	207.	5.1
41061	532.5	4.9	390.	5.83
41065	443.	11.17	480.	18.
41069	650.	10.	850.	74.
41073	900.	41.	820.	5.
41077	830.	15.	670.	20.
41081	590.	30.3	530.	51.7
11150	34.			
21151	0.	15.		
41153	55.02	8.3	98.04	2.8
41157	305.5	3.9	360.	5.1
41161	237.	7.5	57.	1.5
41165	54.	3.8	41.	9.3
41169	24.37	17.8	23.43	84.
41173	25.43	3.	27.43	38.

41177	27.19	40.	26.95	30.3
41181	24.95	1.7	22.95	50.
11350	32.			
41351	.0	15.	.00269	8.3
41355	.00479	2.8	.00823	3.9
41359	.01842	15.83	.02398	29.17
41363	.0318	84.	.03474	61.
41367	.03971	8.96	.0465	11.04
41371	.04354	36.66	.03381	12.34
41375	.06848	2.	.05861	8.
41379	.05723	9.5	.03805	13.5
11450	32.			
41451	0.	15.	.00269	8.3
41455	.00479	2.8	.00823	3.9
41459	.01842	15.83	.02398	29.17
41463	.0318	84.	.03474	61.
41467	.03971	8.96	.0465	11.04
41471	.04354	36.66	.03381	12.34
41475	.06848	2.	.05861	8.
41479	.05723	9.5	.03805	13.5
11550	12.			
41551	0.	15.	70320000.	31.
41555	47660000.	10.	25600000.	9.5
41559	27050000.	9.5	27870000.	247.
11773	50000.			
21779	17.4	10.75		
41850	32.0000	.0000	-180.0000	.0200
41854	-179.0000	.0250	-175.0000	.0650
41858	-172.0000	.1100	-150.0000	.6420
41862	-115.0000	1.8800	-65.0000	1.8800
41866	-30.0000	.6300	-30.0000	.6300
41870	-10.0000	.2500	-7.0000	.0860
41874	-6.0000	.0500	-5.6000	.0390
41878	-4.8000	.0280	-4.0000	.0180
41882	-3.0000	.0110	.0000	.0090
41886	4.0000	.0100	9.0000	.0130
41890	10.0000	.0140	11.0000	.0180
41894	12.0000	.0220	13.0000	.0300
41898	14.0000	.0640	16.3000	.1780
41902	29.9000	.6300	30.0000	.6300
41906	65.0000	1.8800	150.0000	.6420
41910	172.0000	.1100	175.0000	.0650
41914	180.0000	.0200	.0000	.0000
41925	32.0000	.3000	-180.0000	.0200
41929	-179.0000	.0250	-175.0000	.0650
41933	-172.0000	.1100	-150.0000	.6420
41937	-115.0000	1.8800	-65.0000	1.8800
41941	-30.0000	.6300	-30.0000	.6300
41945	-10.0000	.2500	-7.0000	.0860
41949	-6.0000	.0500	-5.6000	.0390
41953	-4.8000	.0280	-4.0000	.0180
41957	-3.0000	.0110	.0000	.0090
41961	4.0000	.0100	9.0000	.0130
41965	10.0000	.0140	11.0000	.0180
41969	12.0000	.0220	13.0000	.0300
41973	14.0000	.0640	16.3000	.1780
41977	29.9000	.6300	30.0000	.6300
41981	65.0000	1.8800	150.0000	.6420
41985	172.0000	.1100	175.0000	.0650
41989	180.0000	.0200	.0000	.0000

42000	19.0000	.4000	-30.0000	.6300
42004	-10.0000	.2600	-7.0000	.1010
42008	-6.0000	.0620	-5.0000	.0340
42012	-4.5000	.0200	-4.0000	.0130
42016	-3.0000	.0100	-2.0000	.0080
42020	1.0000	.0080	3.0000	.0090
42024	6.0000	.0110	8.0000	.0150
42028	9.0000	.0175	10.0000	.0270
42032	11.0000	.0500	12.8000	.1360
42036	15.0000	.2300	30.0000	.6300
42075	19.0000	.5000	-30.0000	.6300
42079	-10.0000	.2700	-7.0000	.1060
42083	-6.0000	.0700	-5.0000	.0380
42087	-4.0000	.0240	-3.0000	.0150
42091	-2.0000	.0100	-1.0000	.0085
42095	.0000	.0080	2.0000	.0080
42099	4.0000	.0095	5.0000	.0110
42103	6.0000	.0180	7.0000	.0270
42107	8.0000	.0440	12.0000	.1780
42111	15.0000	.2800	30.0000	.6300
42150	20.0000	.6000	-30.0000	.6300
42154	-10.0000	.2880	-8.0000	.1370
42158	-6.0000	.0810	-5.0000	.0450
42162	-4.6000	.0350	-4.0000	.0250
42166	-3.0000	.0170	-2.0000	.0120
42170	-1.0000	.0085	.0000	.0080
42174	1.0000	.0080	2.0000	.0100
42178	3.0000	.0160	4.0000	.0250
42182	5.0000	.0380	6.0000	.0600
42186	10.5000	.1760	15.0000	.3000
42190	30.0000	.6300	11.0000	.0180
42225	14.0000	.7000	-30.0000	.6300
42229	-10.0000	.3100	-7.0000	.1550
42233	-6.0000	.0940	-5.0000	.0600
42237	-3.0000	.0270	-2.0000	.0130
42241	-1.0000	.0100	.0000	.0100
42245	1.0000	.0115	2.0000	.0250
42249	8.0000	.1600	15.0000	.3200
42253	30.0000	.6300	4.0000	.0250
42300	15.0000	.7500	-30.0000	.6300
42304	-10.0000	.3260	-7.0000	.1680
42308	-6.0000	.1090	-5.0000	.0850
42312	-2.4000	.0200	-2.0000	.0150
42316	-1.0000	.0120	.0000	.0135
42320	1.0000	.0240	4.0000	.0950
42324	6.0000	.1340	7.2000	.1550
42328	15.0000	.3300	30.0000	.6300
42375	20.0000	.8000	-30.0000	.6300
42379	-12.0000	.2900	-10.0000	.2250
42383	-8.0000	.1700	-6.0000	.1220
42387	-4.0000	.0750	-3.0000	.0420
42391	-2.0000	.0280	-1.0000	.0260
42395	-.5000	.0255	.0000	.0250
42399	.5000	.0350	1.0000	.0420
42403	2.0000	.0700	4.0000	.1080
42407	6.0000	.1480	8.0000	.1850
42411	10.0000	.2300	12.0000	.2850
42415	30.0000	.6300	11.0000	.0180
42450	17.0000	.9000	-30.0000	.6300
42454	-12.0000	.3300	-10.0000	.2620

42458	-8.0000	.2100	-6.0000	.1630
42462	-4.0000	.1150	-2.0000	.0660
42466	-1.0000	.0630	.0000	.0600
42470	1.0000	.0780	2.0000	.1000
42474	4.0000	.1380	6.0000	.1820
42478	8.0000	.2210	10.0000	.2620
42482	12.0000	.3225	30.0000	.6300
42525	15.0000	1.0000	-30.0000	.6300
42529	-12.0000	.3700	-10.0000	.2970
42533	-8.0000	.2480	-6.0000	.2020
42537	-4.0000	.1520	-2.0000	.1170
42541	.0000	.1000	2.0000	.1360
42545	4.0000	.1700	6.0000	.2150
42549	8.0000	.2550	10.0000	.2980
42553	12.0000	.3630	30.0000	.6300
42600	15.0000	2.0000	-30.0000	.6300
42604	-12.0000	.3620	-10.0000	.2970
42608	-8.0000	.2480	-6.0000	.2020
42612	-4.0000	.1520	-2.0000	.1170
42616	.0000	.1000	2.0000	.1360
42620	4.0000	.1700	6.0000	.2150
42624	8.0000	.2550	10.0000	.2980
42628	12.0000	.3425	30.0000	.6300
42750	26.0000	.0000	-180.0000	.0000
42754	-172.0000	.7800	-160.0000	.6400
42758	-158.0000	.6600	-30.0000	-1.0000
42762	-10.0000	-.8000	-7.5000	-.7300
42766	-6.7000	-.6000	-5.0000	-.4400
42770	5.0000	.7400	10.0000	1.3000
42774	11.0000	1.3800	12.0000	1.4400
42778	13.0000	1.4900	14.0000	1.5300
42782	15.2000	1.2100	19.0000	1.0800
42786	30.0000	1.0000	30.1000	1.0000
42790	149.9000	-.9500	150.0000	-.9500
42794	156.0000	-.7000	158.0000	-.6600
42798	160.0000	-.6400	172.0000	-.7800
42802	180.0000	.0000	.0000	.0000
42825	26.0000	.3000	-180.0000	.0000
42829	-172.0000	.7800	-160.0000	.6400
42833	-158.0000	.6600	-30.0000	-1.0000
42837	-10.0000	-.8000	-7.5000	-.7300
42841	-6.7000	-.6000	-5.0000	-.4400
42845	5.0000	.7400	10.0000	1.3000
42849	11.0000	1.3800	12.0000	1.4400
42853	13.0000	1.4900	14.0000	1.5300
42857	15.2000	1.2100	19.0000	1.0800
42861	30.0000	1.0000	30.1000	1.0000
42865	149.9000	-.9500	150.0000	-.9500
42869	156.0000	-.7000	158.0000	-.6600
42873	160.0000	-.6400	172.0000	-.7800
42877	180.0000	.0000	.0000	.0000
42900	13.0000	.4000	-30.0000	-1.0000
42904	-10.0000	-.7400	-8.6000	-.7100
42908	-7.0000	-.6400	-5.0000	-.4500
42912	7.0000	1.0400	8.0000	1.1500
42916	9.0000	1.2200	10.0000	1.2700
42920	11.2000	1.2900	12.0000	1.1300
42924	18.0000	1.1200	30.0000	1.0000
42975	14.0000	.5000	-30.0000	-1.0000
42979	-10.0000	-.6000	-8.5000	-.6600

42983	-7.0000	-.6500	-5.0000	-.4700
42987	6.0000	.9300	7.0000	1.0000
42991	8.0000	1.0400	9.0000	1.0600
42995	10.0000	1.0800	11.0000	1.0900
42999	12.0000	1.1100	16.0000	1.1100
43003	30.0000	1.0000	14.0000	1.5300
43050	18.0000	.6000	-30.0000	-1.0000
43054	-10.0000	-.6000	-7.0000	-.6000
43058	-6.0000	-.5800	-5.0000	-.5000
43062	-4.0000	-.3600	-3.0000	-.2400
43066	-2.0000	-.1200	-1.0000	-.0200
43070	.0000	.1400	3.0000	.6100
43074	4.0000	.7500	5.0000	.8400
43078	6.0000	.9000	7.0000	.9200
43082	14.0000	1.0400	15.0000	1.0700
43086	30.0000	1.0000	30.1000	1.0000
43125	15.0000	.7000	-30.0000	-1.0000
43129	-10.0000	-.6000	-7.0000	-.6000
43133	-5.8000	-.5900	-5.0000	-.5500
43137	-4.0000	-.4400	-3.0000	-.3100
43141	-2.0000	-.1700	2.0000	.5700
43145	3.0000	.7100	4.0000	.8100
43149	5.0000	.8500	9.4000	.9200
43153	15.0000	.9800	30.0000	1.0000
43200	15.0000	.7500	-30.0000	-1.0000
43204	-10.0000	-.7000	-6.5000	-.7000
43208	-5.7000	-.6900	-5.0000	-.6500
43212	-4.0000	-.5400	-3.0000	-.3800
43216	-2.0000	-.2000	1.4000	.5500
43220	2.0000	.6300	3.0000	.7000
43224	4.0000	.7400	7.0000	.8300
43228	15.0000	.9500	30.0000	1.0000
43275	14.0000	.8000	-30.0000	-.9500
43279	-14.0000	-.8000	-12.0000	-.7900
43283	-10.0000	-.8100	-6.0000	-.6900
43287	-2.0000	-.2500	.0000	.0700
43291	2.0000	.3500	4.0000	.5600
43295	6.0000	.7050	8.0000	.8050
43299	9.0000	.8400	15.0000	.8500
43303	30.0000	1.0000	30.0000	1.0000
43350	14.0000	.8500	-30.0000	-.9500
43354	-16.0000	-.8030	-13.0000	-.7720
43358	-10.0000	-.7400	-6.0000	-.6800
43362	-2.0000	-.2900	.0000	-.0450
43366	2.0000	.2300	4.0000	.4600
43370	6.0000	.6400	8.0000	.7600
43374	9.0000	.8020	15.0000	.8500
43378	30.0000	1.0000	30.0000	1.0000
43425	14.0000	.9000	-30.0000	-.9500
43429	-16.0000	-.7540	-13.0000	-.7120
43433	-10.0000	-.6700	-6.0000	-.6630
43437	-2.0000	-.3100	.0000	-.1500
43441	1.0000	.0000	2.0000	.1380
43445	4.0000	.3900	6.0000	.6400
43449	8.0000	.7650	10.0000	.8100
43453	30.0000	1.0000	30.0000	1.0000
43500	13.0000	.9500	-30.0000	-.9500
43504	-16.0000	-.7410	-13.0000	-.6960
43508	-10.0000	-.6510	-6.0000	-.6410
43512	-2.0000	-.2700	.0000	-.0900

43516	2.0000	.1800	4.0000	.4350
43520	6.0000	.6800	8.0000	.7950
43524	10.0000	.8100	30.0000	1.0000
43575	13.0000	2.0000	-30.0000	-.9500
43579	-16.0000	-.7260	-13.0000	-.6780
43583	-10.0000	-.6300	-6.0000	-.6150
43587	-2.0000	-.2400	.0000	-.0500
43591	2.0000	.2000	4.0000	.4490
43595	6.0000	.7000	8.0000	.8060
43599	10.0000	.8500	30.0000	1.0000
43650	33.0000	.0000	-180.0000	-.0130
43654	-174.0000	.3590	-160.0000	.3000
43658	-145.0000	.4810	-125.0000	.5570
43662	-90.0000	.5550	-60.0000	.3950
43666	-30.0000	.1437	-30.0000	.1437
43670	-10.0000	.1065	-7.4000	.0989
43674	-6.4000	.0052	-5.0000	.0032
43678	4.0000	.0019	14.0000	.0135
43682	15.2000	-.0932	19.0000	-.1303
43686	30.0000	-.1437	30.1000	-.1437
43690	34.9000	-.2220	35.0000	-.2220
43694	45.0000	-.2950	60.0000	-.3950
43698	80.0000	-.5000	95.0000	-.5550
43702	110.0000	-.5600	125.0000	-.5570
43706	135.0000	-.5380	145.0000	-.4810
43710	150.0000	-.4380	160.0000	-.3000
43714	174.0000	-.3590	180.0000	-.0130
43725	33.0000	.3000	-180.0000	-.0130
43729	-174.0000	.3590	-160.0000	.3000
43733	-145.0000	.4810	-125.0000	.5570
43737	-90.0000	.5550	-60.0000	.3950
43741	-30.0000	.1437	-30.0000	.1437
43745	-10.0000	.1065	-7.4000	.0989
43749	-6.4000	.0052	-5.0000	.0032
43753	4.0000	.0019	14.0000	.0135
43757	15.2000	-.0932	19.0000	-.1303
43761	30.0000	-.1437	30.1000	-.1437
43765	34.9000	-.2220	35.0000	-.2220
43769	45.0000	-.2950	60.0000	-.3950
43773	80.0000	-.5000	95.0000	-.5550
43777	110.0000	-.5600	125.0000	-.5570
43781	135.0000	-.5380	145.0000	-.4810
43785	150.0000	-.4380	160.0000	-.3000
43789	174.0000	-.3590	180.0000	-.0130
43800	10.0000	.4000	-30.0000	.1437
43804	-10.0000	.1427	-7.0000	.1356
43808	-6.0000	.0038	-5.0000	.0019
43812	8.0000	.0124	11.2000	.0115
43816	12.2000	-.1299	18.0000	-.1341
43820	30.0000	-.1437	-7.4000	.0989
43875	9.0000	.5000	-30.0000	.1437
43879	-10.0000	.1108	-9.0000	.0952
43883	-7.0000	.0483	-5.0000	.0045
43887	8.0000	.0031	12.0000	-.0800
43891	16.0000	-.1293	30.0000	-.1437
43950	11.0000	.6000	-30.0000	.1437
43954	-25.0000	.1267	-20.0000	.1047
43958	-15.0000	.0878	-10.0000	.0707
43962	-3.0000	-.0004	5.0000	.0087
43966	8.0000	-.0490	13.0000	-.1415

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43970     15.0000      -.1352     30.0000      -.1437
44025     15.0000       .7000    -30.0000       .1437
44029    -25.0000       .1416    -20.0000       .1397
44033    -15.0000       .1327    -10.0000       .1306
44037     -3.0000      -.0119       .0000      -.0025
44041      1.0000      -.0064      2.0000      -.0073
44045      3.0000      -.0241      4.0000      -.0569
44049      6.0000      -.1105      8.0000      -.1347
44053     15.0000      -.1470     30.0000      -.1437
44100     18.0000       .7500    -30.0000       .1437
44104    -25.0000       .1361    -20.0000       .1335
44108    -15.0000       .1260    -10.0000       .1234
44112     -8.0000       .1039     -6.0000       .0544
44116     -4.0000      -.0291     -3.0000      -.0335
44120     -2.0000      -.0245       .0000      -.0146
44124      1.0000      -.0197      2.0000      -.0459
44128      3.0000      -.0943      4.0000      -.1154
44132      5.0000      -.1177     15.0000      -.1526
44136     30.0000      -.1437     30.1000      -.1437
44175     15.0000       .8000    -30.0000       .1500
44179     -8.0000       .0750     -6.0000       .0600
44183     -4.0000       .0350     -2.0000      -.0120
44187       .0000      -.0200       .5000      -.0150
44191      1.0000      -.0120      1.5000      -.0170
44195      2.0000      -.0290      4.0000      -.0750
44199      6.0000      -.1000      8.0000      -.1150
44203     18.0000      -.1300     30.0000      -.1500
44250     17.0000       .9000    -30.0000       .1400
44254     -8.0000       .1200     -6.0000       .0970
44258     -4.0000       .0430     -2.0000      -.0120
44262       .0000      -.0200       .1000      -.0010
44266       .2500       .0120       .5000       .0170
44270       .7500       .0090      1.0000      -.0070
44274      1.5000      -.0300      2.0000      -.0350
44278      4.0000      -.0830      6.0000      -.1370
44282      8.0000      -.1600     30.0000      -.1900
44325     17.0000      1.0000    -30.0000       .1400
44329     -8.0000       .1200     -6.0000       .0970
44333     -4.0000       .0430     -2.0000      -.0120
44337       .0000      -.0200       .1000      -.0010
44341       .2500       .0120       .5000       .0170
44345       .7500       .0090      1.0000      -.0070
44349      1.5000      -.0300      2.0000      -.0350
44353      4.0000      -.0830      6.0000      -.1370
44357      8.0000      -.1600     30.0000      -.1900
44400     17.0000      2.0000    -30.0000       .1400
44404     -8.0000       .1200     -6.0000       .0970
44408     -4.0000       .0430     -2.0000      -.0120
44412       .0000      -.0200       .1000      -.0010
44416       .2500       .0120       .5000       .0170
44420       .7500       .0090      1.0000      -.0070
44424      1.5000      -.0300      2.0000      -.0350
44428      4.0000      -.0830      6.0000      -.1370
44432      8.0000      -.1600     30.0000      -.1900
-1 110 1.0
1 TITLE 1 - ROTOR ANALYSYS INPUT - TEST MAIN ROTOR - 4 TEST CASES
  TITLE 2 - BIFILAR ANALYSIS INPUT
 4   1 1.        4.         4.0          30.
 5   6 4.        258.       2.           15.         0.
 3  12 1.        4.         0.0
```

```
1  17 0.
1  18 0.
2  20 .0647       .0647
2  30 1172.       1172.
2  40 .005        .005
5  50 43.5        43.5        43.5        10.66       88.2
5  55 1276.3      1.808       3.061       5.13        2.268
5  60 2.747       1.941       2.0         4.75        6.291
5  70 0.0         0.0         0.0         0.0         0.0
5  75 0.0         306.0       384.0       918.0       858.0
5  80 828.0       696.0       726.0       1044.0      1266.0
5  90 0.0         0.0         0.0         0.0         0.0
5  95 0.0         .025        .025        .025        .025
5 100 .025        .025        .025        .025        .025
1 110 500.0
1 121 500.0
1 190 .0614
1 200 18.22
1 210 3.0
1 220 .01
1 230 .0608
1 240 18.5
1 250 4.0
1 260 .01
5 450 1.          0.0         0.0         0.0         0.0
5 456 0.          1.0         0.0         0.0         0.0
5 462 0.0         0.0         1.0         0.0         0.0
5 468 .0          1.          .0          -.01381     0.0
5 474 1.          .0          .25691      .0          .01381
5 480 .0          1.          .0          .0          .0
1 485 -.05376
5 486 .001        .15         .001        -.00512     .000474
1 491 .001568
5 492 -.34        .001        .11         -.00012     -.005183
1 497 .00028
5 498 1.0         .001        -.31        .01302      -.005395
1 503 .00015
5 504 .001        .54         -.001       .01214      .0000772
1 509 .0003683
5 510 -.402       .15         -.11        -.00801     -.003252
1 515 -.0004382
5 516 .05         -.065       .022        -.001244    -.000473
1 521 -.006
5 522 .24         -.15        .022        -.001244    -.000473
1 527 -.006
5 528 .7          .52         .125        -.00007     .0231
1 533 .00015
5 534 -.05        .22         .001        .00635      .00022
1 539 -.00127
5 750 0.          0.          1.          .333        -.054
5 755 .00323      .017        -.2664      -.03        -.01
5 760 1.0         .05         .01         .20         -.02
5 770 0.0         .0          1.0         .0356644    -.482
5 775 .014        -.0644      .02315      -.213       -.04
5 780 -.01175     -.02        .0057       -.164       -.03446
51000 1.0         0.0         0.0         0.0         0.0
51006 0.0         1.0         0.0         0.0         0.0
51012 0.0         0.0         1.0         0.0         0.0
51018 0.0         -.416       -.180       -.0138      0.0
51024 -.416       0.0         1.787       0.0         0.0138
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51030	0.180	6.950	0.0	0.0	0.0
11035	-.054				
31036	-.02632	-.2362	-.035		
31042	-.032	-.04	-.2664		
31048	-.08	-.01855	-.03		
31054	.015	.03522	-.01		
31060	-.06	.6	.44		
31066	.020665	-.22	.142		
31072	.020665	-.02	.04		
31078	-.1239	.08	.135		
31084	-.04551	-.10875	-.02		
51100	1.0	0.0	0.0	0.0	0.0
51106	0.0	1.0	0.0	0.0	0.0
51112	0.0	0.0	1.0	0.0	0.0
51118	0.0	-.566	0.0	-.0138	0.0
51124	-.566	0.0	2.44	0.0	0.0138
51130	0.0	9.50	0.0	0.0	0.0
11135	-.054				
31136	-.00972	-.52	.00357		
31142	.001847	-.05478	-.42		
31148	-.0795	.006274	.25		
31154	.0102	-.031	-.0712		
31160	-.04347	-.08173	.91		
31166	.001914	-.65	.01		
31172	.001914	-.015	-.3		
31178	-.137	.1	-.08374		
31184	-.02403	-.42	.1227		
51200	1.0	0.0	0.0	.0	0.0
51206	0.0	1.0	0.0	.0	0.0
51212	0.0	0.0	1.0	.0	0.0
51218	0.0	-.1464	0.0	-.0138	0.0
51224	-.1464	0.0	-4.37	0.0	0.0138
51230	0.0	-17.0	0.0	0.0	0.0
11235	-.054				
31236	-.002634	-.01	.04		
31242	.02794	-.03884	-.5556		
31248	.007521	.14	-.3323		
31254	-.000371	-.07	-.1		
31260	.0003	-.402	-.21956		
31266	.000374	-1.5	.05		
31272	.000374	-.05	.01		
31278	-.07358	-.00364	.019		
31284	.003116	.07674	.03752		
51300	1.0	0.0	0.0	0.0	0.0
51306	0.0	1.0	0.0	0.0	0.0
51312	0.0	0.0	1.0	0.0	0.0
51318	0.0	-.905	0.0	-.0138	0.0
51324	-.905	0.0	-5.226	0.0	0.0138
51330	0.0	-20.34	0.0	0.0	0.0
11335	-.054				
31336	-.04543	-1.75	.07506		
31342	1.6	-.15914	-1.		
31348	-.01	-.01234	.4804		
31354	-.00436	.011	-.0402		
31360	.6	.015	.18		
31366	.05	1.1	-1.		
31372	1.33	.1	-.1		
31378	-.030532	-.0048	-.016637		
31384	.0923	.03625	-.034		
51490	0.	0.	0.	0.	0.

41495	0.	0.	0.	.0	
41500	23.73	23.73	23.73	23.73	
41520	.01	.01	.01	.01	
41540	3.	3.	3.	3.	
41560	18.22	18.22	18.22	18.22	
41580	0.	90.	180.	270.	
11606	500.				
11627	500.				
51760	5.	2.	10801.0	4.	4.
11765	9.				
1 1	1.				
1 4	18.				
1 8	1.				
1 9	9.				
1 12	1.				
1 18	0.				
4 50	1.808	3.061	5.13	2.268	
5 54	2.747	1.941	2.0	4.75	6.291
4 70	306.0	384.0	918.0	858.0	
5 74	828.0	696.0	726.0	1044.0	1266.0
4 90	.025	.025	.025	.025	
5 94	.025	.025	.025	.025	.025
5 450	.001	.15	.001	-.00512	.000474
1 455	.001568				
5 456	-.34	.001	.11	-.00012	-.005183
1 461	.00028				
5 462	1.0	.001	-.31	.01302	-.005395
1 467	.00015				
5 468	.001	.54	-.001	.01214	.0000772
1 473	.0003683				
5 474	-.402	.15	-.11	-.00801	-.003252
1 479	-.0004382				
5 480	.05	-.065	.022	-.001244	-.000473
1 485	-.006				
5 486	.24	-.15	.022	-.001244	-.000473
1 491	-.006				
5 492	.7	.52	.125	-.00007	.0231
1 497	.00015				
5 498	-.05	.22	.001	.00635	.00022
1 503	-.00127				
4 750	.017	-.2664	-.03	-.01	
5 754	1.0	.05	.01	.20	-.02
4 770	-.0644	.02315	-.213	-.04	
5 774	-.01175	-.02	.0057	-.164	-.03446
31000	-.02632	-.2362	-.035		
31006	-.032	-.04	-.2664		
31012	-.08	-.01855	-.03		
31018	.015	.03522	-.01		
31024	-.06	.6	.44		
31030	.020665	-.22	.142		
31036	.020665	-.02	.04		
31042	-.1239	.08	.135		
31048	-.04551	-.10875	-.02		
31100	-.00972	-.52	.00357		
31106	.001847	-.05478	-.42		
31112	-.0795	.006274	.25		
31118	.0102	-.031	-.0712		
31124	-.04347	-.08173	.91		
31130	.001914	-.65	.01		
31136	.001914	-.015	-.3		

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 31142 -.137       .1          -.08374
 31148 -.02403     -.42        .1227
 31200 -.002634    -.01        .04
 31206 .02794      -.03884     -.5556
 31212 .007521     .14         -.3323
 31218 -.000371    -.07        -.1
 31224 .0003       -.402       -.21956
 31230 .000374     -1.5        .05
 31236 .000374     -.05        .01
 31242 -.07358     -.00364     .019
 31248 .003116     .07674      .03752
 31300 -.04543     -1.75       .07506
 31306 1.6         -.15914     -1.
 31312 -.01        -.01234     .4804
 31318 -.00436     .011        -.0402
 31324 .6          .015        .18
 31330 .05         1.1         -1.
 31336 1.33        .1          -.1
 31342 -.030532    -.0048      -.016637
 31348 .0923       .03625      -.034
 1  17 .0
 51490 .0          .0          .0          .0          .0
 31495 .0          .0          .0
 21492 .0          .0
-12200 0.
-1 110 -1.
 1   1 0.
 1  18 1.
 21492 1.          1.
-12200 0.
-1 110 0.
 1   1 1.
 1  18 0.
 21492 0.          0.
-12200 0.
-1 110 0.
 1   1 0.
 1  18 1.
 21492 1.          1.
-12200 1.
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                         END  OF  INPUT
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APPENDIX C. TEST CASES RESULTS
CASE 1. ROTOR RESULTS

TITLE 1 - TEST MAIN ROTOR DATA - COUPLED WITH BIFILAR ANALYSIS 000463

HOVER

MAIN ROTOR

PITCH ANGLE AT 75% RADIUS = 6.400 DEG

CALCULATED THRUST = 16364.911 LB

CALCULATED CONING ANGLE = 3.389 DEG

CALCULATED LAG ANGLE = 5.180 DEG

CALCULATED BLADE TORSIONAL FREQUENCY = 0.0 RAD/SEC

CALCULATED BLADE BENDING FREQUENCIES : MODE 1 = 77.5 RAD/SEC

MODE 2 = 126.7 RAD/SEC

RADIUS (IN)	STEADY DEFLECTIONS (IN) FLATWISE	EDGEWISE	ANGLE OF ATTACK (DEG)
0.0	0.0	0.0	-74.100
15.000	0.0	0.0	-33.210
23.750	-0.004	0.011	-20.206
41.250	-0.009	0.104	-6.880
62.500	0.041	0.285	-0.199
83.400	0.156	0.505	2.542
100.200	0.256	0.699	3.650
117.000	0.356	0.904	4.219
133.800	0.455	1.119	4.447
150.600	0.546	1.343	4.446
167.745	0.626	1.582	4.277
185.235	0.688	1.835	3.980
202.725	0.723	2.098	3.588
220.215	0.728	2.371	3.124
236.910	0.696	2.641	2.627
252.810	0.631	2.907	2.116
268.710	0.542	3.181	1.574
283.145	0.456	3.437	0.647
296.115	0.397	3.670	-0.156
306.450	0.368	3.857	-0.345
314.150	0.353	3.997	1.274
320.000	0.344	4.103	2.101

MODE NO.	PHIXPH	PHIZPH	PHELD	PHEPLD	PHFLD	PHFPLD
1	1.0000	-0.0009	0.0	0.0	0.0	0.0
2	-1.0000	0.0010	0.0	0.0	0.0	0.0

QEOLD	QEOPLD	QFOLD	QFOPLD	PHLD	THTLD	PHOS
0.0	0.0	0.0	0.0	0.0	0.0	0.0

CASE DEFINITION

AIR DENSITY LB.SEC.-SQ/IN.4TH	SPEED OF SOUND FT./SEC.	TIP LOSS FACTOR	AXIAL VEL. KNOTS	ROTOR SPEED R.P.M.	BLADE RADIUS FEET
0.11468D-06	1116.00000	0.99000	0.0	258.00000	26.83333

OFFSET FEET	NUMBER OF BLADES	ROOT FLAP SPRING LB.IN./RAD.	ROOT LAG SPRING LB.IN./RAD.	PRELAG ANGLE RADIANS	PRECONE ANGLE RADIANS
1.25000	4	0.0	0.0	0.0	0.0

BLADE YOUNG'S MOD. LB/IN.-SQ	RADIUS OF PUSH ROD INCHES	LAG DAMPING FRACT CRITICAL	RIGID PITCH DAMP. FRACT CRITICAL	REF. ROTOR SPEED R.P.M.	BLADE BENDING MODES
0.10000D+07	15.12650	0.35000	0.0	258.00000	2

FIXED SYSTEM MODES	PITCH-LAG COUPLING DEG/DEG	WEIGHT AT PUSHROD, LB	PITCH BEAM STIFFNESS LB/IN.	ACTUATOR MOM. STIFFNESS LB-IN./RAD.	PITCH BEAM RADIUS INCHES
5	0.0	0.0	0.50000D+05	0.0	0.0

PITCH HORN LENGTH INCHES	FORWARD FLIGHT SPEED KNOTS	ELASTIC PITCH DAMP. FRACT. CRITICAL	LAG DAMPER COEFFICIENT LB-SEC/IN.	LAG DAMPER STIFFNESS LB/IN.	FLEXBEAM ROOT (XBR) INCHES
7.25000	0.0	0.0	676.00000	0.0	0.0

CONTROL SWITCHES

ROTEST	FTEST	SYSDEF	ROTDEF	ARTIC	PHASE
1.	1.	01000111.	1.	11.	0.

VECT	TRMASC	SUMASC	TSERVC	MRMASC	MSERVC
0.	1.	111.	1.	1.	111.

CIR	CIRN	LAGKII
1.	1.	1.

MODE NO.	ZETBLD	ZETG	MG	OMF	PHY	PHX	PHZ	PHTY	PHTX
1	0.0	0.0	0.0	0.0	0.0	1.0000	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	1.0000	0.0	0.0	0.0	0.0
3		0.0	0.0	0.0	0.0	0.0	1.0000	0.0	0.0
4		0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0000
5		0.0	0.0	0.0	0.0	0.0	0.0	1.0000	0.0

R	CHORD	STRUCTURAL TWIST	AERODYNAMIC TWIST	AC	CG	TORSIONAL MODE SHAPE	EA
0.0	0.0	-9.50000	-9.50000	0.0	0.0	0.0	0.0
15.00000	0.0	-9.50000	-9.50000	0.0	0.0	0.0	0.0
23.75000	8.30000	-9.50000	-9.50000	0.0	0.0	0.0	-2.07500
41.25000	8.30000	-9.50000	-9.50000	0.0	0.0	0.0	-2.07500
62.50000	20.76000	-8.89205	-8.89205	-0.35000	-0.21759	0.0	-5.19000
83.40000	20.76000	-7.87555	-7.87555	-0.35000	-0.82200	0.0	-5.19000
100.20000	20.76000	-7.05845	-7.05845	-0.35000	-0.82200	0.0	-5.19000
117.00000	20.76000	-6.24136	-6.24136	-0.35000	-0.82200	0.0	-5.19000
133.80000	20.76000	-5.42427	-5.42427	-0.35000	-0.82200	0.0	-5.19000
150.60000	20.76000	-4.60718	-4.60718	-0.35000	-0.82200	0.0	-5.19000
167.74500	20.76000	-3.77331	-3.77331	-0.35000	-0.66700	0.0	-5.19000
185.23500	20.76000	-2.92266	-2.92266	-0.35000	-0.66700	0.0	-5.19000
202.72500	20.76000	-2.07201	-2.07201	-0.35000	-0.46850	0.0	-5.19000
220.21500	20.76000	-1.22136	-1.22136	-0.35000	-0.69851	0.0	-5.19000
236.91000	22.31700	-0.40938	-0.40938	0.0	0.29349	0.0	-5.58000
252.81000	22.31700	0.36394	0.36394	0.0	0.30385	0.0	-5.58000
268.71000	22.31700	1.13726	1.13726	0.0	0.30962	0.0	-5.58000
283.14500	20.76000	2.25160	2.25160	-0.35000	0.36800	0.0	-5.19000
296.11500	20.76000	3.20767	3.20767	-0.35000	2.09900	0.0	-5.19000
306.45000	20.76000	3.51000	3.51000	-1.75128	0.38606	0.0	-5.19000
314.15000	20.76000	1.97000	1.97000	-4.55384	-3.18159	0.0	-5.19000
320.00000	20.76000	1.20000	1.20000	-6.68306	-5.72800	0.0	-5.19000

R	QEO	QFO	QEOP	QFOP
0.0	0.0	0.0	0.0	0.0
15.00000	0.0	0.0	0.0	0.0
23.75000	0.01108	-0.00402	0.00155	-0.00151
41.25000	0.10392	-0.00914	0.00659	-0.00127
62.50000	0.28471	0.04123	0.00918	0.00136
83.40000	0.50523	0.15567	0.01101	0.00444
100.20000	0.69939	0.25565	0.01170	0.00453
117.00000	0.90412	0.35611	0.01231	0.00450
133.80000	1.11881	0.45498	0.01290	0.00420
150.60000	1.34309	0.54645	0.01347	0.00351
167.74500	1.58162	0.62605	0.01404	0.00256
185.23500	1.83470	0.68764	0.01460	0.00133
202.72500	2.09751	0.72338	0.01516	-0.00032
220.21500	2.37055	0.72784	0.01577	-0.00215
236.91000	2.64091	0.69591	0.01635	-0.00418
252.81000	2.90709	0.63050	0.01689	-0.00600
268.71000	3.18148	0.54180	0.01741	-0.00672
283.14500	3.43658	0.45641	0.01779	-0.00611
296.11500	3.66970	0.39675	0.01806	-0.00370
306.45000	3.85694	0.36796	0.01815	-0.00217
314.15000	3.99675	0.35345	0.01816	-0.00171
320.00000	4.10300	0.34363	0.01816	-0.00167

R	D(DT)/DUT	D(DT)/DUP	D(DT)/DOT	D(DH)/DUT	D(DH)/DUP	D(DH)/DOT
0.0	0.0	0.0	0.0	0.0	0.0	0.0
15.00000	0.0	0.0	0.0	0.0	0.0	0.0
23.75000	-0.00040	-0.00073	0.33155	-0.00001	-0.00021	-0.16268
41.25000	0.00140	-0.00516	6.52470	0.00034	-0.00130	1.18437
62.50000	0.00414	-0.01307	23.85057	0.00109	-0.00327	6.46567
83.40000	0.00610	-0.01753	41.71577	0.00107	-0.00244	8.75691
100.20000	0.00745	-0.02118	59.83550	0.00103	-0.00180	10.47511
117.00000	0.00859	-0.02483	81.23457	0.00100	-0.00133	12.44987
133.80000	0.00954	-0.02851	105.97988	0.00096	-0.00090	14.26927
150.60000	0.01102	-0.03221	134.44925	0.00100	-0.00056	16.16053
167.74500	0.01195	-0.03662	169.58604	0.00098	-0.00031	18.40511
185.23500	0.01269	-0.04126	210.32202	0.00095	-0.00015	20.76438
202.72500	0.01218	-0.04602	255.52919	0.00083	-0.00014	23.23199
220.21500	0.01210	-0.05052	304.00663	0.00078	-0.00027	25.63134
236.91000	0.01266	-0.05901	381.23440	0.00078	-0.00053	29.98955
252.81000	0.01374	-0.06489	447.37186	0.00099	-0.00125	35.26863
268.71000	0.01276	-0.07412	541.85065	0.00084	-0.00173	38.29369
283.14500	0.00878	-0.07731	593.92714	0.00055	-0.00250	36.33645
296.11500	0.00549	-0.08701	697.52874	0.00039	-0.00370	38.61169
306.45000	0.01024	-0.09279	771.89581	0.00079	-0.00407	41.73612
314.15000	0.01896	-0.09562	817.74876	0.00168	-0.00494	74.68638
320.00000	0.02305	-0.08673	756.49209	0.00288	-0.00730	110.98781

R	D(DM)/DUT	D(DM)/DUP	D(DM)/DOT	DT	DH	DM
0.0	0.0	0.0	0.0	0.0	0.0	0.0
15.00000	0.0	0.0	0.0	0.0	0.0	0.0
23.75000	0.00044	0.00073	-0.26554	-0.29732	-0.05191	0.31267
41.25000	-0.00948	0.02382	-30.98563	-0.42541	-0.11705	0.28950
62.50000	0.00011	0.00040	-0.62797	0.43651	0.15563	0.19019
83.40000	0.00014	0.00051	-1.08320	2.76489	0.63633	0.27625
100.20000	0.00017	0.00060	-1.54369	5.12602	0.97658	0.36383
117.00000	0.00111	-0.00514	16.77181	7.77250	1.27528	0.54370
133.80000	0.00120	-0.00588	21.82340	10.56367	1.53142	0.79426
150.60000	0.00351	-0.00625	27.06373	13.41433	1.74900	1.17383
167.74500	0.00455	-0.00613	29.93084	16.53455	1.95691	2.61935
185.23500	0.00574	-0.00540	29.70216	19.55134	2.12254	4.59828
202.72500	-0.00021	-0.00558	30.45412	22.08643	2.22559	6.18703
220.21500	-0.00107	-0.00339	19.66104	23.68544	2.24479	5.50461
236.91000	-0.00210	-0.00102	5.55483	26.25026	2.37083	5.39755
252.81000	0.00324	-0.00014	2.46406	26.27911	2.30451	5.25127
268.71000	0.00322	-0.00493	37.29655	25.32963	2.20955	6.39222
283.14500	0.00180	-0.00851	65.92044	16.95756	1.59790	5.49944
296.11500	0.00052	-0.01268	101.65111	8.78590	1.13260	4.52887
306.45000	-0.00627	-0.01825	148.14500	7.69474	1.13389	2.42466
314.15000	-0.01425	-0.00596	43.94727	31.97936	2.69876	1.07474
320.00000	-0.02523	0.08714	-765.20454	46.48986	4.15806	-2.40927

R	CL	CD	CM	D(CL)/DA	D(CD)/DA	D(CM)/DA	D(CL)/DM	D(CD)/DM	D(CM)/DM
0.0	-0.42808	1.88000	0.47020	-0.74305	0.0	-0.30558	0.0	0.0	0.0
15.00000	-0.95837	0.74466	0.17059	-0.74305	-2.04628	-0.47995	0.0	0.0	0.0
23.75000	-0.90206	0.44392	0.12548	0.57296	-1.08862	-0.10657	0.0	0.0	0.0
41.25000	-0.62922	0.08167	0.05016	9.31056	-2.06265	-5.36861	0.0	0.0	0.0
62.50000	0.12646	0.00913	0.00251	6.76090	-0.03820	-0.00828	0.0	0.0	0.0
83.40000	0.44991	0.00964	0.00211	6.76090	0.01432	-0.00828	0.0	0.0	0.0
100.20000	0.58070	0.00991	0.00195	6.76090	0.01432	-0.00828	0.0	0.0	0.0
117.00000	0.64789	0.01013	0.00215	6.76090	0.03438	0.06646	0.0	0.0	0.0
133.80000	0.67479	0.01027	0.00242	6.76090	0.03438	0.06646	0.0	0.0	0.0
150.60000	0.67739	0.01025	0.00283	6.78151	0.03460	0.06529	0.48248	-0.00304	0.07112
167.74500	0.67371	0.01004	0.00511	6.90300	0.03591	0.05834	0.47210	-0.00315	0.07172
185.23500	0.65383	0.00974	0.00736	7.02707	0.03627	0.04757	0.45377	-0.00342	0.07250
202.72500	0.61704	0.00937	0.00829	7.13287	0.03870	0.04078	0.06675	-0.00201	-0.05261
220.21500	0.56106	0.00897	0.00625	7.19546	0.04038	0.02234	0.05232	-0.00240	-0.04836
236.91000	0.49999	0.00854	0.00459	7.25523	0.04000	0.00472	0.03691	-0.00343	-0.04382
252.81000	0.43973	0.00838	0.00392	7.48130	0.07675	0.00184	0.35840	0.02610	0.01686
268.71000	0.37532	0.00850	0.00423	8.02034	0.04954	0.02468	0.19908	0.01148	0.01011
283.14500	0.24346	0.00800	0.00379	8.50984	0.0	0.04541	-0.07334	0.0	-0.00143
296.11500	0.11559	0.00808	0.00285	9.13770	-0.02865	0.06405	-0.31458	0.0	-0.01143
306.45000	0.09463	0.00852	0.00143	9.44286	-0.02314	0.08717	0.51367	0.01827	-0.06202
314.15000	0.37295	0.01086	0.00060	9.54040	0.34354	0.02461	0.96091	0.06649	-0.11108
320.00000	0.52272	0.01837	-0.00130	8.53200	0.78367	-0.41304	1.14977	0.16673	-0.14406

R	O STRUCTURAL	O AERODYNAMIC	U P	U T	U	O	ALPHA	MACH NUMBER
0.0	15.90000	15.90000	468.02342	0.0	468.02342	90.00000	-74.10000	0.03495
15.00000	15.90000	15.90000	468.02342	405.26640	619.10159	49.11037	-33.21037	0.04623
23.75000	15.90000	15.90000	468.02342	641.67180	794.22202	36.10631	-20.20631	0.05931
41.25000	15.90000	15.90000	468.02342	1114.48260	1208.76689	22.77982	-6.87982	0.09026
62.50000	15.29205	15.29205	468.02342	1688.61000	1752.26986	15.49151	-0.19946	0.13084
83.40000	14.27555	14.27555	468.02342	2253.28118	2301.37394	11.73392	2.54163	0.17185
100.20000	13.45845	13.45845	468.02342	2707.17955	2747.33817	9.80847	3.64998	0.20515
117.00000	12.64136	12.64136	468.02342	3161.07792	3195.53744	8.42192	4.21944	0.23862
133.80000	11.82427	11.82427	468.02342	3614.97629	3645.14739	7.37693	4.44734	0.27219
150.60000	11.00718	11.00718	468.02342	4068.87466	4095.70347	6.56163	4.44556	0.30583
167.74500	10.17331	10.17331	468.02342	4532.09415	4556.19615	5.89596	4.27735	0.34022
185.23500	9.32266	9.32266	468.02342	5004.63477	5026.47144	5.34265	3.98001	0.37533
202.72500	8.47201	8.47201	468.02342	5477.17540	5497.13528	4.88405	3.58796	0.41048
220.21500	7.62136	7.62136	468.02342	5949.71602	5968.09573	4.49780	3.12356	0.44565
236.91000	6.80938	6.80938	468.02342	6400.77752	6417.86560	4.18201	2.62737	0.47923
252.81000	6.03606	6.03606	468.02342	6830.35991	6846.37586	3.91984	2.11622	0.51123
268.71000	5.26274	5.26274	468.02342	7259.94229	7275.01258	3.68856	1.57418	0.54324
283.14500	4.14840	4.14840	468.02342	7649.94366	7664.24712	3.50099	0.64741	0.57230
296.11500	3.19233	3.19233	468.02342	8000.36400	8014.04206	3.34800	-0.15567	0.59842
306.45000	2.89000	2.89000	468.02342	8279.59255	8292.81006	3.23534	-0.34534	0.61924
314.15000	4.43000	4.43000	468.02342	8487.62930	8500.52334	3.15620	1.27380	0.63475
320.00000	5.20000	5.20000	468.02342	8645.68320	8658.34187	3.09861	2.10139	0.64653

R	PHE(1,I)	PHF(1,I)	PHE(2,I)	PHF(2,I)	PHE(3,I)	PHF(3,I)	PHE(4,I)	PHF(4,I)
0.0	0.0	0.0	0.0	0.0				
15.00000	0.00000	0.00000	-0.00000	-0.00000				
23.75000	0.01443	-0.06550	-0.11533	0.06907				
41.25000	0.04265	-0.19433	-0.31780	0.20035				
62.50000	0.07436	-0.34180	-0.51612	0.32001				
83.40000	0.09919	-0.46391	-0.64142	0.33810				
100.20000	0.11385	-0.54193	-0.69542	0.28228				
117.00000	0.12353	-0.59983	-0.71296	0.17523				
133.80000	0.12788	-0.63489	-0.69571	0.02932				
150.60000	0.12652	-0.64399	-0.64607	-0.13980				
167.74500	0.11894	-0.62318	-0.56599	-0.31638				
185.23500	0.10502	-0.56974	-0.45975	-0.47719				
202.72500	0.08497	-0.48154	-0.33437	-0.59265				
220.21500	0.05895	-0.35641	-0.19443	-0.63587				
236.91000	0.02880	-0.20108	-0.05014	-0.58755				
252.81000	-0.00419	-0.02128	0.09481	-0.44739				
268.71000	-0.04047	0.18585	0.24475	-0.21509				
283.14500	-0.07552	0.39359	0.38317	0.06636				
296.11500	-0.10802	0.59156	0.50845	0.36215				
306.45000	-0.13423	0.75377	0.60848	0.61391				
314.15000	-0.15381	0.87562	0.68303	0.80485				
320.00000	-0.16870	0.96831	0.73966	0.95027				

R	PHEP(1,I)	PHFP(1,I)	PHEP(2,I)	PHFP(2,I)	PHEP(3,I)	PHFP(3,I)	PHEP(4,I)	PHFP(4,I)
0.0	0.0	0.0	0.0	0.0				
15.00000	0.00165	-0.00750	-0.01336	0.00793				
23.75000	0.00164	-0.00746	-0.01283	0.00781				
41.25000	0.00158	-0.00724	-0.01060	0.00697				
62.50000	0.00133	-0.00637	-0.00765	0.00298				
83.40000	0.00100	-0.00512	-0.00430	-0.00192				
100.20000	0.00070	-0.00396	-0.00201	-0.00538				
117.00000	0.00039	-0.00265	0.00012	-0.00816				
133.80000	0.00006	-0.00117	0.00213	-0.01009				
150.60000	-0.00030	0.00052	0.00396	-0.01093				
167.74500	-0.00066	0.00235	0.00551	-0.01046				
185.23500	-0.00101	0.00429	0.00674	-0.00855				
202.72500	-0.00136	0.00637	C.00767	-0.00508				
220.21500	-0.00169	0.00854	0.00839	-0.00009				
236.91000	-0.00198	0.01062	0.00892	0.00580				
252.81000	-0.00221	0.01246	0.00930	0.01181				
268.71000	-0.00239	0.01401	0.00953	0.01743				
283.14500	-0.00249	0.01507	0.00963	0.02156				
296.11500	-0.00253	0.01563	0.00967	0.02392				
306.45000	-0.00254	0.01581	0.00968	0.02471				
314.15000	-0.00255	0.01584	0.00968	0.02485				
320.00000	-0.00255	0.01585	0.00968	0.02486				

BLADE EDGEWISE M. OF I. ABOUT C.G. LB.IN.SEC-SQ/IN.	DELTA R IN.	BLADE FLATWISE M. OF I. ABOUT C.G. LB.IN.SEC-SQ/IN.	DELTA R IN.	BLADE TORSIONAL M. OF I. ABOUT C.G. LB.IN.SEC-SQ/IN.	DELTA R IN.
0.0	15.000			0.0	15.000
0.00269	8.300			0.00269	8.300
0.00479	2.800			0.00479	2.800
0.00823	3.900			0.00823	3.900
0.01842	15.830			0.01842	15.830
0.02398	29.170			0.02398	29.170
0.03180	84.000			0.03180	84.000
0.03474	61.000			0.03474	61.000
0.03971	8.960			0.03971	8.960
0.04650	11.040			0.04650	11.040
0.04354	36.660			0.04354	36.660
0.03381	12.340			0.03381	12.340
0.06848	2.000			0.06848	2.000
0.05861	8.000			0.05861	8.000
0.05723	9.500			0.05723	9.500
0.03805	13.500			0.03805	13.500

BLADE MASS LB.SEC-SQ/IN.-SQ	DELTA R IN.	BLADE EDGEWISE SECOND MOMENT OF AREA-IN.4TH	DELTA R IN.	BLADE FLATWISE SECOND MOMENT OF AREA-IN.4TH	DELTA R IN.
0.0	15.00000	0.88587E+02	17.50000	0.87885E+02	17.50000
0.00744	17.50000	0.37941E+03	17.50000	0.93696E+02	17.50000
0.00423	17.50000	0.46903E+03	25.00000	0.27593E+02	25.00000
0.00168	25.00000	0.71842E+03	16.80000	0.23430E+02	16.80000
0.00144	16.80000	0.85000E+03	16.80000	0.23430E+02	16.80000
0.00138	16.80000	0.85000E+03	16.80000	0.23430E+02	16.80000
0.00137	16.80000	0.85000E+03	16.80000	0.23430E+02	16.80000
0.00137	16.80000	0.85000E+03	16.80000	0.23430E+02	16.80000
0.00137	16.80000	0.90000E+03	17.49000	0.27065E+02	17.49000
0.00145	17.49000	0.90000E+03	17.49000	0.27430E+02	17.49000
0.00146	17.49000	0.84979E+03	17.49000	0.27272E+02	17.49000
0.00150	17.49000	0.73953E+03	17.49000	0.27190E+02	17.49000
0.00162	17.49000	0.64334E+03	15.90000	0.27116E+02	15.90000
0.00190	15.90000	0.59000E+03	15.90000	0.26950E+02	15.90000
0.00190	15.90000	0.56444E+03	15.90000	0.25433E+02	15.90000
0.00186	15.90000	0.53000E+03	12.97000	0.22950E+02	12.97000
0.00176	12.97000	0.53000E+03	12.97000	0.22950E+02	12.97000
0.00343	12.97000	0.53000E+03	7.70000	0.22950E+02	7.70000
0.00293	7.70000	0.53000E+03	7.70000	0.22950E+02	7.70000
0.00139	7.70000	0.53000E+03	4.00000	0.22950E+02	4.00000
0.00036	4.00000				

BLADE RIGID BODY PROPERTIES

FLAPPING MASS	=	.660383	LB SEC-SQ/IN
1ST MOM ABOUT HINGE	=	86.1467	LB SEC-SQ
FLAPPING INERTIA	=	18190.6	LB IN SEC-SQ
LAG FREQUENCY	=	.266517	CYCLES/REV

BLADE MODAL PROPERTIES

BENDING MODE GENERALIZED MASSES

MODE(1)	=	.125382	LB SEC-SQ/IN
MODE(2)	=	.209446	LB SEC-SQ/IN

BENDING MODE (M)(PHE)				BENDING MODE (M)(PHF)			
MODE(1)	=	.193489D-01	LB SEC-SQ/IN	MODE(1)	=	-.942375D-01	LB SEC-SQ/IN
MODE(2)	=	-.115519	LB SEC-SQ/IN	MODE(2)	=	.385765D-02	LB SEC-SQ/IN
BENDING MODE (M)(PHE)(R)				BENDING MODE (M)(PHF)(R)			
MODE(1)	=	-.805651D-01	LB SEC-SQ	MODE(1)	=	.621710	LB SEC-SQ
MODE(2)	=	.683082	LB SEC-SQ	MODE(2)	=	-3.94313	LB SEC-SQ

17 DEGREES OF FREEDOM ARE USED IN THIS RUN

1 2 7 8 9 10 11 12 21 22 23 24 25 26 27 28 29

FINAL STIFFNESS MATRIX

17 17 1

1	714.170	-74.3204	-1252.21	760.385	.0	.0	.0	.0	.0	.0
	.0	.0	.0	.0	.0	.0	.0			
2	-1.41149	3330.19	-2304.16	-1473.63	.0	.0	.0	.0	.0	.0
	.0	.0	.0	.0	.0	.0	.0			
3	-1339.02	-1795.95	.118472D+08	.108102D+07	.0	.0	.0	.0	.0	.0
	.0	.0	.0	.0	.0	.0	.0			
4	319.047	-1228.43	240574.	909996.	.0	.0	.0	.0	.0	.0
	.0	.0	.0	.0	.0	.0	.0			
5	.0	.0	.0	.0	623.028	-93.0826	-74.1366	-43.8058	-1711.06	-7579.55
	830.526	-991.072	.0	.0	.0	-534.101	101.473			
6	.0	.0	.0	.0	93.0826	623.028	43.8058	-74.1366	7579.55	-1711.06
	991.072	830.526	.0	.0	.0	101.473	534.101			
7	.0	.0	.0	.0	-1.22765	-37.8535	3177.22	-65.1601	568.906	3285.78
	-2026.78	-1315.67	.0	.0	.0	-434.059	28.3808			
8	.0	.0	.0	.0	37.8535	-1.22765	65.1601	3177.22	-3285.78	568.906
	1315.67	-2026.78	.0	.0	.0	28.3808	434.059			
9	.0	.0	.0	.0	-1797.87	-7252.59	1077.11	1922.39	-.140193D+07	-.162945D+08
	.108281D+07	.135041D+07	.0	.0	.0	480170.	-187109.			
10	.0	.0	.0	.0	7252.59	-1797.87	-1922.39	1077.11	.162945D+08	-.140193D+07
	-.135041D+07	.108281D+07	.0	.0	.0	-187109.	-480170.			
11	.0	.0	.0	.0	389.188	859.306	-1781.58	391.228	242365.	.217900D+07
	-.123781D+08	-.272896D+07	.0	.0	.0	-208639.	-.161585D+07			

12									
.0	.0	.0	.0	-859.306	389.188	-391.228	-1781.58	-.217900D+07	242365.
.272896D+07	-.123781D+08	.0	.0	.0	-.161585D+07	208639.			
13									
.0	.0	.0	.0	-33.0894	1.71156	-8.49605	-5.12357	-1732.55	-4083.65
-756.359	753.296	.0	.0	.0	125.043	-16335.3			
14									
.0	.0	.0	.0	1.71156	33.0894	-5.12357	8.49605	-4083.65	1732.55
753.296	756.359	.0	.0	.0	16335.3	125.043			
15									
-112.328	-112.808	-10129.7	3811.79	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0			
16									
.0	.0	.0	.0	7143.69	-32606.9	-3102.89	-35682.5	.174311D+08	-.415679D+07
-.235881D+07	.118284D+07	.0	.0	.0	-153513.	-515291.			
17									
.0	.0	.0	.0	-32606.9	-7143.69	-35682.5	3102.89	-.415679D+07	-.174311D+08
.118284D+07	.235881D+07	.0	.0	.0	130248.	-153513.			

FINAL DAMPING MATRIX

17 17 2

1	3.44524	1.62137	280.539	36.6822	.0	.0	.0	.0	.0	.0
	.0	.0	.0	.0	-2.05853	.0	.0			
2	1.40106	2.41175	-121.616	48.6965	.0	.0	.0	.0	.0	.0
	.0	.0	.0	.0	-3.50275	.0	.0			
3	268.438	-71.1529	603103.	-49982.4	.0	.0	.0	.0	.0	.0
	.0	.0	.0	.0	10337.5	.0	.0			
4	-31.8052	-14.4804	-80650.8	101006.	.0	.0	.0	.0	.0	.0
	.0	.0	.0	.0	-339.039	.0	.0			
5	.0	.0	.0	.0	3.44524	-6.74680	1.62137	.136094D-01	280.539	-33.9665
	36.6822	5.19222	-.212816	-1.63738	.0	139.377	547.568			
6	.0	.0	.0	.0	6.74680	3.44524	-.136094D-01	1.62137	33.9665	280.539
	-5.19222	36.6822	-1.63738	.212816	.0	547.568	-139.377			
7	.0	.0	.0	.0	1.40106	.136094D-01	2.41175	-11.3237	-121.616	212.679
	48.6965	-40.9471	-.154362	-.423867	.0	384.608	-308.591			
8	.0	.0	.0	.0	-.136094D-01	1.40106	11.3237	2.41175	-212.679	-121.616
	40.9471	48.6965	-.423867	.154362	.0	-308.591	-384.608			
9	.0	.0	.0	.0	268.438	-33.9665	-71.1529	212.679	603103.	-980771.
	-49982.4	132.618	399.786	1588.63	.0	-.193681D+07	.148565D+07			
10	.0	.0	.0	.0	33.9665	268.438	-212.679	-71.1529	980771.	603103.
	-132.618	-49982.4	1588.63	-399.786	.0	.148565D+07	.193681D+07			
11	.0	.0	.0	.0	-31.8052	5.19222	-14.4804	-40.9471	-80650.8	132.618
	101006.	-983657.	-19.0341	-88.5144	.0	-17771.1	-43258.5			

12										
	.0	.0	.0	.0	-5.19222	-31.8052	40.9471	-14.4804	-132.618	-80650.8
	983657.	101006.	-88.5144	19.0341	.0	-43258.5	17771.1			
13										
	.0	.0	.0	.0	-.547969D-01	.221337D-01	-.147067D-01	.195906	154.845	84.7249
	-33.9177	-44.5914	.531518	.267172	.0	183.720	310.191			
14										
	.0	.0	.0	.0	.221337D-01	.547969D-01	.195906	.147067D-01	84.7249	-154.845
	-44.5914	33.9177	-.267172	.531518	.0	-310.191	183.720			
15										
	-.619499	-.386813	2584.08	-518.957	.0	.0	.0	.0	.0	.0
	.0	.0	.0	.0	49.4180	.0	.0			
16										
	.0	.0	.0	.0	-8.16080	261.645	-211.864	-73.6610	.111417D+07	540907.
	-10154.6	-117905.	1100.99	-328.002	.0	.136682D+07	.229132D+07			
17										
	.0	.0	.0	.0	261.645	8.16080	-73.6610	211.864	540907.	-.111417D+07
	-117905.	10154.6	328.002	1100.99	.0	-.229132D+07	.136682D+07			

FINAL MASS MATRIX

17 17 3

1	.124859	-.251860D-03	.628595	-.960889D-01	.0	.0	.0	.0	.0	.0
	.0	.0	.0	.0	-.376950	.0	.0			
2	-.251860D-03	.209560	-3.93592	.757781	.0	.0	.0	.0	.0	.0
	.0	.0	.0	.0	.154306D-01	.0	.0			
3	.628595	-3.93592	18150.5	-2.45428	.0	.0	.0	.0	.0	.0
	.0	.0	.0	.0	344.587	.0	.0			
4	-.960889D-01	.757781	-2.45428	18203.9	.0	.0	.0	.0	.0	.0
	.0	.0	.0	.0	.0	.0	.0			
5	.0	.0	.0	.0	.124859	.0	-.251860D-03	.0	.628595	.0
	-.960889D-01	.0	-.800491D-02	-.389862D-01	.0	.362875D-01	-1.60412			
6	.0	.0	.0	.0	.0	.124859	.0	-.251860D-03	.0	.628595
	.0	-.960889D-01	-.389862D-01	.800491D-02	.0	-1.60412	-.362875D-01			
7	.0	.0	.0	.0	-.251860D-03	.0	.209560	.0	-3.93592	.0
	.757781	.0	.910438D-02	.232338	.0	-.726197	-7.77846			
8	.0	.0	.0	.0	.0	-.251860D-03	.0	.209560	.0	-3.93592
	.0	.757781	.232338	-.910438D-02	.0	-7.77846	.726197			
9	.0	.0	.0	.0	.628595	.0	-3.93592	.0	18150.5	.0
	-2.45428	.0	10.5538	.0	.0	3736.74	38922.2			
10	.0	.0	.0	.0	.0	.628595	.0	-3.93592	.0	18150.5
	.0	-2.45428	.0	-10.5538	.0	38922.2	-3736.74			
11	.0	.0	.0	.0	-.960889D-01	.0	.757781	.0	-2.45428	.0
	18203.9	.0	-17.7265	-172.099	.0	2230.99	-6.14894			

12									
.0	.0	.0	.0	.0	-.960889D-01	.0	.757781	.0	-2.45428
.0	18203.9	-172.099	17.7265	.0	-6.14894	-2230.99			
13									
.0	.0	.0	.0	-.400246D-02	-.194931D-01	.455219D-02	.116169	5.27690	.0
-8.86324	-86.0494	2.64153	.0	.0	.0	21.1076			
14									
.0	.0	.0	.0	-.194931D-01	.400246D-02	.116169	-.455219D-02	.0	-5.27690
-86.0494	8.86324	.0	2.64153	.0	-21.1076	.0			
15									
-.942375D-01	.385765D-02	86.1467	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	2.64153	.0	.0			
16									
.0	.0	.0	.0	.181437D-01	-.802058	-.363098	-3.88923	1868.37	19461.1
1115.49	-3.07447	.0	-21.1076	.0	42429.5	.0			
17									
.0	.0	.0	.0	-.802058	-.181437D-01	-3.88923	.363098	19461.1	-1868.37
-3.07447	-1115.49	21.1076	.0	.0	.0	42429.5			

CASE 1: BIFILAR RESULTS

TITLE 2: BIFILAR DATA - WITH ROTOR - 9 FIX. SYS-3P FORCES-LINEAR CASE

BIFILAR ANALYSIS RESULTS

ALL DISPLACEMENTS ARE IN G AND ALL ANGLES ARE IN DEGREES

```
NUMBER OF FIXED SYSTEM MODES                     IS  9
NUMBER OF FIXED SYSTEM ABSORBERS                 IS  1
NUMBER OF INPLANE BIFILARS                       IS  1
NUMBER OF VERTICAL BIFILARS                      IS  1
TOTAL NO. OF DEGREES-OF-FREEDOM
       (WITH NO ROTOR)                           IS 16
NUMBER  OF A.C. STATIONS                         IS  4
ROTOR COUPLING SWITCH     (0=NO,1OR-1=YES)       IS  1
ROTOR MATRICES          PRINTOUT(0=NO,1=YES)     IS  0
FIXED SYSTEM MATRICES   PRINTOUT      "          IS  0
ADD   ROTOR  MATRICES   PRINTOUT      "          IS  0
ADD FIX.SYS. ABSORBER   PRINTOUT      "          IS  0
ADD INPLANE  BIFILAR    PRINTOUT      "          IS  0
ADD VERTICAL BIFILAR    PRINTOUT      "          IS  0
INPLANE  BIFILAR (9X9)  PRINTOUT      "          IS  0
VERTICAL BIFILAR (9X9)  PRINTOUT      "          IS  0
GAMMAS                  PRINTOUT      "          IS  0
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ROTOR NO. OF DEGREES-OF-FREEDOM IS 12
TOTAL NO. OF DEGREES-OF-FREEDOM IS 28

PRESENT NO. OF DEGREES-OF-FREEDOM   (NP)    =  9
ADDITIONAL NO. DEGREES-OF-FREEDOM   (NA)    = 12
TOTAL  NO. OF  DEGREES-OF-FREEDOM (NP+NA)   = 21
ORDER OF MATRIX TO BE ADDED         (NL)    = 18
ORDER - D.O.F. OF ADDED MATRIX    (NL-NA)   =  6

PRESENT NO. OF DEGREES-OF-FREEDOM   (NP)    = 21
ADDITIONAL NO. DEGREES-OF-FREEDOM   (NA)    =  1
TOTAL  NO. OF  DEGREES-OF-FREEDOM (NP+NA)   = 22
ORDER OF MATRIX TO BE ADDED         (NL)    =  2
ORDER - D.O.F. OF ADDED MATRIX    (NL-NA)   =  1

INPLANE  BIFILAR  PARAMETERS

TOTAL NO. OF BIFILARS (N)  =  4.00000
BIFILAR MASS          (M)  =  .614000D-01
BIFILAR ARM         (RCR)  =  18.2200
BIFILAR FREQUENCY     (W)  =  3.00000
BIFILAR RADIUS (R=RCR/W/W) =  2.02444

N*M*R*R                    =  1.00656
1+W*W                      =  10.0000
N*M*R*R*(1.+W*W)           =  10.0656
N*M*R*R*(1.+W*W)**2        =  100.656
```

```
PRESENT NO. OF DEGREES-OF-FREEDOM (NP)     = 22
ADDITIONAL NO. DEGREES-OF-FREEDOM (NA)     = 3
TOTAL NO. OF DEGREES-OF-FREEDOM (NP+NA)    = 25
ORDER OF MATRIX TO BE ADDED       (NL)     = 9
ORDER = D.O.F. OF ADDED MATRIX    (NL-NA)  = 6


VERTICAL BIFILAR PARAMETERS

TOTAL NO. OF BIFILARS (N)  =  4.00000
BIFILAR MASS          (M)  =  .608000D-01
DISTANCE FROM C.R.    (R)  =  18.5000
BIFILAR FREQUENCY     (W)  =  4.00000

R1=R/(W*W-1.)              =  1.23333
R1*R1                      =  1.52111
R+R1                       =  19.7333
(R+R1)**2                  =  389.404
M*N*(R+R1)                 =  4.79915
M*N*(R+R1)**2              =  94.7032
M*N*R1*(R+R1)              =  5.91895


PRESENT NO. OF DEGREES-OF-FREEDOM (NP)     = 25
ADDITIONAL NO. DEGREES-OF-FREEDOM (NA)     = 3
TOTAL NO. OF DEGREES-OF-FREEDOM (NP+NA)    = 28
ORDER OF MATRIX TO BE ADDED       (NL)     = 9
ORDER = D.O.F. OF ADDED MATRIX    (NL-NA)  = 6
```

GENERALIZED FORCES - ORDER IS = 9

COSINE COMPONENT

```
  .500000    -170.000    500.000    .500000    -201.000    25.0000    120.000    350.000
 -25.0000
```

SINE COMPONENT

```
  75.0000    .500000     .500000    270.000    75.0000    -32.5000   -75.0000    260.000
 110.000
```

(GAMMAS)

COSINE - SINE - AMPLITUDE - PHASE(DEG)

ROTOR

```
-.420508D-03  .103080D-02
-.320688D-03  .291653D-04
 .257314D-05 -.300397D-05
 .100956D-06  .705548D-07
-.123203D-02  .977919D-02
 .397414D-02 -.137627D-02
-.915847D-02 -.563780D-02
 .270569D-02  .193891D-02
 .129806D-03 -.147193D-03
 .666964D-04  .176337D-05
-.548605D-04  .228122D-05
-.160107D-04  .750412D-04
```

FIXED SYSTEM ABSORBER(S)

```
-.344875D-02 -.150106D-02  .376126D-02 -156.479
```

INPLANE BIFILAR PENDULUM(S)
(EQT ORDER IS:O,SIN,COS)   (AMPL&PHASE ORDER IS:N,N-1,N+1)
DEG & DEG

```
.705332D-03  .691230D-03  .141459D-01   44.4215
.315306      .718392D-01  9.28677      -77.2864
.708454D-01 -.317137      .298438D-01 -118.491
```

VERTICAL BIFILAR PENDULUM(S)
(EQT ORDER IS:O,SIN,COS)   (AMPL&PHASE ORDER IS:N,N-1,N+1)
DEG & DEG

-.235034D-01	-.213538D-01	.454860	-137.744
.510358D-02	.326855D-02	.338881D-01	-30.2823
-.122553D-02	.391058D-02	.144276	116.499

INPUT FIXED SYSTEM FREQUENCIES (HZ)

5.10000 6.40000 15.3000 14.3000 13.8000 11.6000 12.1000 17.4000 21.1000

FORCING FREQUENCY (HZ) = 17.2000

THE CONVERSION FACTOR TO G = 30.2259

FUSELAGE NO.OF DEGREES OF FREEDOM = 9

COSINE COMPONENT

-.214190D-03 -.448165D-04 -.349518D-03 -.295426D-02 -.567524D-03 .284549D-03 .641759D-03 -.256691D-02
.214393D-03

SINE COMPONENT

-.294794D-03 -.109048D-03 .122252D-02 -.224871D-02 -.105094D-02 .318558D-03 .889165D-03 .313195D-02
.292297D-03

FIRST A.C. STATION DISPLACEMENT IN G

X		Y		Z	
COSINE	SINE	COSINE	SINE	COSINE	SINE
.106454D-01	-.131066D-01	-.208498D-01	-.159455D-01	-.143570D-01	.182978D-02

FIRST A.C. STATION TOTAL DISPLACEMENT IN G

X	Y	Z
.168852D-01	.262483D-01	.144732D-01

FIRST A.C. STATION PHASE ANGLE IN DEG

-50.9158 -142.592 172.737

SECOND A.C. STATION DISPLACEMENT IN G

X		Y		Z	
COSINE	SINE	COSINE	SINE	COSINE	SINE

.112625D-01	-.152813D-01	-.881726D-02	.884320D-02	-.978861D-02	-.282865D-01

SECOND A.C. STATION TOTAL DISPLACEMENT IN G

X	Y	Z
.189832D-01	.124878D-01	.299323D-01

SECOND A.C. STATION PHASE ANGLE IN DEG

-53.6093	134.916	-109.088

THIRD A.C. STATION DISPLACEMENT IN G

X		Y		Z	
COSINE	SINE	COSINE	SINE	COSINE	SINE
.566726D-02	-.669938D-02	-.130642D-02	.746452D-02	.160931D-01	.584755D-02

THIRD A.C. STATION TOTAL DISPLACEMENT IN G

X	Y	Z
.877494D-02	.757798D-02	.171225D-01

THIRD A.C. STATION PHASE ANGLE IN DEG

-49.7708	99.9272	19.9690

FOURTH A.C. STATION DISPLACEMENT IN G

X		Y		Z	
COSINE	SINE	COSINE	SINE	COSINE	SINE
.175254D-01	.101499D-01	.224440D-01	.275827D-01	-.131746D-01	.320180D-02

FOURTH A.C. STATION TOTAL DISPLACEMENT IN G

X	Y	Z
.202524D-01	.355603D-01	.135581D-01

FOURTH A.C. STATION PHASE ANGLE IN DEG

30.0775	50.8648	166.340

ROTOR HEAD DISPLACEMENT IN G

X		Y		Z	
COSINE	SINE	COSINE	SINE	COSINE	SINE
-.528534D-01	.123521	-.941640D-01	.374169D-02	-.398021D-02	.438087D-02

ROTOR HEAD TOTAL DISPLACEMENT IN G

X	Y	Z
.134354	.942383D-01	.591896D-02

ROTOR HEAD DISPLACEMENT PHASE ANGLES IN DEG

113.166	177.725	132.256

CASE 2. BIFILAR RESULTS

BIFILAR ANALYSIS RESULTS

ALL DISPLACEMENTS ARE IN G AND ALL ANGLES ARE IN DEGREES

NUMBER OF FIXED SYSTEM MODES			IS 9
NUMBER OF FIXED SYSTEM ABSORBERS			IS 1
NUMBER OF INPLANE BIFILARS			IS 0
NUMBER OF VERTICAL BIFILARS			IS 1
TOTAL NO. OF DEGREES-OF-FREEDOM (WITH NO ROTOR)			IS 13
NUMBER OF A.C. STATIONS			IS 4
ROTOR COUPLING SWITCH	(0=NO,1OR-1=YES)		IS -1
ROTOR MATRICES	PRINTOUT(0=NO,1=YES)		IS 0
FIXED SYSTEM MATRICES	PRINTOUT	"	IS 0
ADD ROTOR MATRICES	PRINTOUT	"	IS 0
ADD FIX.SYS. ABSORBER	PRINTOUT	"	IS 1
ADD INPLANE BIFILAR	PRINTOUT	"	IS 1
ADD VERTICAL BIFILAR	PRINTOUT	"	IS 0
INPLANE BIFILAR (9X9)	PRINTOUT	"	IS 0
VERTICAL BIFILAR (9X9)	PRINTOUT	"	IS 0
GAMMAS	PRINTOUT	"	IS 0

```
ROTOR NO. OF DEGREES-OF-FREEDOM IS 12
TOTAL NO. OF DEGREES-OF-FREEDOM IS 25

PRESENT NO. OF DEGREES-OF-FREEDOM    (NP)    =  9
ADDITIONAL NO. DEGREES-OF-FREEDOM    (NA)    = 12
TOTAL  NO. OF  DEGREES-OF-FREEDOM (NP+NA)    = 21
ORDER OF MATRIX TO BE ADDED          (NL)    = 18
ORDER -  D.O.F. OF ADDED MATRIX   (NL-NA)    =  6

PRESENT NO. OF DEGREES-OF-FREEDOM    (NP)    = 21
ADDITIONAL NO. DEGREES-OF-FREEDOM    (NA)    =  1
TOTAL  NO. OF  DEGREES-OF-FREEDOM (NP+NA)    = 22
ORDER OF MATRIX TO BE ADDED          (NL)    =  2
ORDER -  D.O.F. OF ADDED MATRIX   (NL-NA)    =  1
```

```
VERTICAL BIFILAR PARAMETERS

TOTAL NO. OF BIFILARS (N)  =   4.00000
BIFILAR MASS           (M)  =   .608000D-01
DISTANCE FROM C.R.     (R)  =   18.5000
BIFILAR FREQUENCY      (W)  =   4.00000

R1=R/(W*W-1.)               =   1.23333
R1*R1                       =   1.52111
R+R1                        =   19.7333
(R+R1)**2                   =   389.404
M*N*(R+R1)                  =   4.79915
M*N*(R+R1)**2               =   94.7032
M*N*R1*(R+R1)               =   5.91895


PRESENT NO. OF DEGREES-OF-FREEDOM  (NP)    = 22
ADDITIONAL NO. DEGREES-OF-FREEDOM  (NA)    = 3
TOTAL NO. OF DEGREES-OF-FREEDOM (NP+NA)    = 25
ORDER OF MATRIX TO BE ADDED        (NL)    = 9
ORDER - D.O.F. OF ADDED MATRIX  (NL-NA)    = 6
```

D.O.F.= 29,NF= 9,KROTOR= 12,NFABS= 1,KINDS OF INPL. BIF.= 0,KINDS OF VERT. BIF.= 1,NUMB. OF N.L. INPL. BIF= 4

FIXED SYSTEM + ROTOR + FIXED ABSORBERS (R.H.S.) OF ORDER 25

.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0					

THE MASS (L.H.S.) MATRIX OF ORDER 10

.24565	.0	.0	.0	.0	.34977D-07	.0	-.12433	.34977D-08	.12433
.0	.24565	.0	.0	.0	.34977D-07	.12433	-.17488D-08	-.12433	.52465D-08
.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.34977D-07	.34977D-07	.0	.0	.0	100.68	2.5169	2.5169	2.5169	2.5169
.0	.49396	.0	.0	.0	10.000	1.0000	.0	.0	.0
-.49396	-.69483D-08	.0	.0	.0	10.000	.0	1.0000	.0	.0
.13897D-07	-.49396	.0	.0	.0	10.000	.0	.0	1.0000	.0
.49396	.20845D-07	.0	.0	.0	10.000	.0	.0	.0	1.0000

THE BIFILAR FORCE VECTOR OF ORDER 10

.25528D-04	-.25528D-04	.0	.0	.0	.0	.0	.0	.0	.0

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THE ROTOR HEAD MODE SHAPES,TRANSPOSE(PHI), OF ORDER 9 X 6

.10000D-02	.15000	.10000D-02	-.51200D-02	.47400D-03	.15680D-02
-.34000	.10000D-02	.11000	-.12000D-03	-.51830D-02	.28000D-03
1.0000	.10000D-02	-.31000	.13020D-01	-.53950D-02	.15000D-03
.10000D-02	.54000	-.10000D-02	.12140D-01	.77200D-04	.36830D-03
-.40200	.15000	-.11000	-.80100D-02	-.32520D-02	-.43820D-03
.50000D-01	-.65000D-01	.22000D-01	-.12440D-02	-.47300D-03	-.60000D-02
.24000	-.15000	.22000D-01	-.12440D-02	-.47300D-03	-.60000D-02
.70000	.52000	.12500	-.70000D-04	.23100D-01	.15000D-03
-.50000D-01	.22000	.10000D-02	.63500D-02	.22000D-03	-.12700D-02

THE EXPANDED BIFILAR MASS MATRIX OF ORDER = 13

.57749D-02	-.24725D-05	.30618D-03	.19956D-01	.53592D-02	-.33300D-02	-.64154D-02	.19357D-01	.78938D-02	.22596D-01	.38222D-02	-.14703D-01	.40709D-02
-.24725D-05	.28406D-01	-.83517D-01	.59513D-04	.33600D-01	-.43612D-02	-.20251D-01	-.58333D-01	.41943D-02	.82907D-03	.42976D-01	.58041D-03	-.41567D-01
.30618D-03	-.83517D-01	.24565	.38387D-03	-.98722D-01	.12176D-01	.58829D-01	.17209	-.12248D-01	.50187D-03	-.12395	.25322D-03	.12470
.19956D-01	.59513D-04	.38387D-03	.71646D-01	.19783D-01	-.88326D-02	-.20061D-01	.69157D-01	.29124D-01	.68064D-01	.80266D-03	-.66210D-01	.10513D-02
.53592D-02	.33600D-01	-.98722D-01	.19783D-01	.45245D-01	-.70680D-02	-.28963D-01	-.49972D-01	.13100D-01	.17546D-01	.48877D-01	-.19752D-01	-.51082D-01
-.33300D-02	-.43612D-02	.12176D-01	-.88326D-02	-.70680D-02	.52764D-02	.89673D-02	.20417D-03	-.33598D-02	-.23183D-01	-.21318D-01	-.70203D-02	-.88853D-02
-.64154D-02	-.20251D-01	.58829D-01	-.20061D-01	-.28963D-01	.89673D-02	.23301D-01	.22018D-01	-.10287D-01	-.33751D-01	-.44940D-01	.35475D-02	.14737D-01
.19357D-01	-.58333D-01	.17209	.69157D-01	-.49972D-01	.20417D-03	.22018D-01	.18680	.19486D-01	.65028D-01	-.86652D-01	-.64273D-01	.87407D-01
.78938D-02	.41943D-02	-.12248D-01	.29124D-01	.13100D-01	-.33598D-02	-.10287D-01	.19486D-01	.12666D-01	.24155D-01	.30199D-02	-.30549D-01	-.94129D-02
.89774D-01	.32940D-02	.19940D-02	.27042	.69712D-01	-.92108D-01	-.13409	.25836	.95972D-01	1.0000	.0	.0	.0
.15186D-01	.17075	-.49246	.31890D-02	.19419	-.84698D-01	-.17855	-.34427	.11998D-01	.0	1.0000	.0	.0
-.58414D-01	.23060D-02	.10061D-02	-.26306	-.78476D-01	-.27892D-01	.14094D-01	-.25536	-.12137	.0	.0	1.0000	.0
.16174D-01	-.16515	.49546	.41770D-02	-.20295	-.35302D-01	.58551D-01	.34727	-.37398D-01	.0	.0	.0	1.0000

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THE EXPANDED BIFILAR FORCE VECTOR OF ORDER = 13

```
-.38036D-05  -.87050D-05   .25502D-04  -.13759D-04  -.14091D-04   .29357D-05   .99558D-05   .45950D-05  -.68925D-05   .0
  .0           .0           .0
```

FINAL COMBINED MASS MATRIX OF ORDER 29 FOR PSI= .0

1									
3.0344	-.81765D-01	-2.9687	-2.3651	1.7839	.22634	.17970	.79122	-1.2707	-.94238D-04
.38577D-05	.86147D-01	.0	-.34010D-02	.46788D-02	.17445D-01	.19518D-01	-.33621	-101.32	-18.629
.73043	.10999D-02	.74987D-04	-.75763D-02	-.70140D-03	.22596D-01	.38222D-02	-.14703D-01	.40709D-02	
2									
-.81765D-01	4.6785	-.11174	-.77927D-01	1.2008	.60633D-01	-.16217	-6.0335	-.21990D-01	-.10366D-01
.42434D-03	9.4761	.0	.54962D-02	.68219D-02	.18770D-01	-.40917D-01	-102.89	7.3431	2.8095
35.048	-.17236D-01	.82485D-02	-.17757D-03	.76695D-02	.82907D-03	.42976D-01	.58041D-03	-.41567D-01	
3									
-2.9687	-.11174	16.746	6.5549	-4.9101	-.44102	.15529	-2.9867	3.2545	.29214D-01
-.11959D-02	-26.705	.0	.54139D-03	-.29834D-01	.20923D-01	.63568D-01	-75.389	263.46	5.5910
-80.063	-.19410D-02	-.23246D-01	.19266D-01	.79832D-02	.50187D-03	-.12395	.25322D-03	.12470	
4									
-2.3651	-.77927D-01	6.5549	9.1646	-3.8371	-.72238	-.84336	.78956	3.5180	.94238D-04
-.38577D-05	-.86147D-01	.0	-.10372D-01	-.75966D-02	.58028D-01	-.49529D-01	24.190	233.26	-32.934
4.5767	-.64700D-03	-.74987D-04	.17964D-01	-.11424D-03	.68064D-01	.80266D-03	-.66210D-01	.10513D-02	
5									
1.7839	1.2008	-4.9101	-3.8371	6.7034	.38545	.76440D-01	-3.9865	-2.0075	.10366D-01
-.42434D-03	-9.4761	.0	.11480D-02	.14920D-01	.31152D-01	-.17411D-01	-80.374	-150.60	-18.270
39.574	.64700D-01	-.82485D-02	-.11853D-01	.48121D-02	.17546D-01	.48877D-01	-.19752D-01	-.51082D-01	
6									
.22634	.60633D-01	-.44102	-.72238	.38545	2.0415	.14325	-.41731	-.37584	-.20732D-02
.84868D-04	1.8952	.0	.14237D-02	-.22847D-03	-.50321D-02	.10771D-01	-11.265	-22.983	3.7638
-4.3471	.32350D-02	.16497D-02	-.18408D-02	.69992D-03	-.23183D-01	-.21318D-01	-.70203D-02	-.88853D-02	
7									
.17970	-.16217	.15529	-.84336	.76440D-01	.14325	2.3217	-.47330D-01	-.45179	-.20732D-02
.84868D-04	1.8952	.0	.23202D-02	-.42724D-02	-.14042D-01	.33230D-01	-10.263	-22.534	9.3940
-21.450	.64700D-03	.16497D-02	-.18408D-02	.69992D-03	-.33751D-01	-.44940D-01	.35475D-02	.14737D-01	
8									
.79122	-6.0335	-2.9867	.78956	-3.9865	-.41731	-.47330D-01	30.528	.35510	-.11780D-01
.48221D-03	10.768	.0	-.31467D-01	-.11927D-01	-.26221D-01	.87611D-01	453.11	-47.266	-51.099
-81.393	.12940D-01	.93733D-02	-.10358D-03	-.34182D-01	.65028D-01	-.86652D-01	-.64273D-01	.87407D-01	
9									
-1.2707	-.21990D-01	3.2545	3.5180	-2.0075	-.37584	-.45179	.35510	8.1061	-.94238D-04
.38577D-05	.86147D-01	.0	-.41496D-02	-.32419D-02	.22168D-01	-.31427D-01	15.882	122.01	-11.405
5.9875	-.12940D-02	.74987D-04	.93963D-02	-.32554D-03	.24155D-01	.30199D-02	-.30549D-01	-.94129D-02	
10									
-.37695D-03	-.41465D-01	.11685	.37695D-03	.41465D-01	-.82929D-02	-.82929D-02	-.47119D-01	-.37695D-03	.12486
-.25186D-03	.62860	-.96089D-01	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0	.0	.0	
11									
.15431D-04	.16974D-02	-.47835D-02	-.15431D-04	-.16974D-02	.33947D-03	.33947D-03	.19288D-02	.15431D-04	-.25186D-03
.20956	-3.9359	.75778	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0	.0	.0	
12									

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.34459	37.905	-106.82	-.34459	-37.905	7.5809	7.5809	43.073	.34459	.62860
-3.9359	18150.	-2.4543	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0	.0	.0	
13									
.0	.0	.0	.0	.0	.0	.0	.0	.0	-.96089D-01
.75778	-2.4543	18204.	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0	.0	.0	
14									
-.68021D-02	.10992D-01	.10828D-02	-.20744D-01	.22960D-02	.28475D-02	.46404D-02	-.62934D-01	-.82992D-02	.0
.0	.0	.0	.12486	.0	-.25186D-03	.0	.62860	.0	-.96089D-01
.0	.0	.0	.0	.0	.0	.0	.0	.0	
15									
.93576D-02	.13644D-01	-.59668D-01	-.15193D-01	.29840D-01	-.45695D-03	-.85447D-02	-.23854D-01	-.64837D-02	.0
.0	.0	.0	.0	.12486	.0	-.25186D-03	.0	.62860	.0
-.96089D-01	.0	.0	.0	.0	.0	.0	.0	.0	
16									
.34891D-01	.37540D-01	.41846D-01	.11606	.62303D-01	-.10064D-01	-.28083D-01	-.52443D-01	.44337D-01	.0
.0	.0	.0	-.25186D-03	.0	.20956	.0	-3.9359	.0	.75778
.0	.0	.0	.0	.0	.0	.0	.0	.0	
17									
.39037D-01	-.81835D-01	.12714	-.99059D-01	-.34822D-01	.21542D-01	.66460D-01	.17522	-.62853D-01	.0
.0	.0	.0	.0	-.25186D-03	.0	.20956	.0	-3.9359	.0
.75778	.0	.0	.0	.0	.0	.0	.0	.0	
18									
-.67243	-205.77	-150.78	48.379	-160.75	-22.531	-20.526	906.23	31.763	.0
.0	.0	.0	.62860	.0	-3.9359	.0	18150.	.0	-2.4543
.0	.0	.0	.0	.0	.0	.0	.0	.0	
19									
-202.64	14.686	526.92	466.53	-301.20	-45.966	-45.069	-94.531	244.01	.0
.0	.0	.0	.0	.62860	.0	-3.9359	.0	18150.	.0
-2.4543	.0	.0	.0	.0	.0	.0	.0	.0	
20									
-37.258	5.6191	11.182	-65.867	-36.539	7.5277	18.788	-102.20	-22.810	.0
.0	.0	.0	-.96089D-01	.0	.75778	.0	-2.4543	.0	18204.
.0	.0	.0	.0	.0	.0	.0	.0	.0	
21									
1.4609	70.095	-160.13	9.1533	79.147	-8.6943	-42.900	-162.79	11.975	.0
.0	.0	.0	.0	-.96089D-01	.0	.75778	.0	-2.4543	.0
18204.	.0	.0	.0	.0	.0	.0	.0	.0	
22									
.17000D-01	-.26640	-.30000D-01	-.10000D-01	1.0000	.50000D-01	.10000D-01	.20000	-.20000D-01	.0
.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.0	1.0000	.0	.0	.0	.0	.0	.0	.0	
23									
.32432D-02	.35676	-1.0054	-.32432D-02	-.35676	.71351D-01	.71351D-01	.40541	.32432D-02	.0
.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.0	.0	1.0000	.0	.0	.0	.0	.0	.0	
24									

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 -.16384     -.38400D-02  .41664       .38848      -.25632      -.39808D-01 -.39808D-01 -.22400D-02  .20320      .0
  .0          .0          .0           .0           .0           .0          .0          .0          .0          .0
  .0          .0          .0          1.0000        .0           .0          .0          .0          .0

25
 -.15168D-01  .16586      .17264      -.24704D-02   .10406       .15136D-01  .15136D-01 -.73920     -.70400D-02  .0
  .0          .0          .0           .0           .0           .0          .0          .0          .0          .0
  .0          .0          .0           .0          1.0000        .0          .0          .0          .0

26
  .89774D-01  .32940D-02  .19940D-02   .27042       .69712D-01  -.92108D-01 -.13409      .25836      .95972D-01  .0
  .0          .0          .0           .0           .0           .0          .0          .0          .0          .0
  .0          .0          .0           .0           .0          1.0000       .0          .0          .0

27
  .15186D-01  .17075     -.49246       .31890D-02   .19419      -.84698D-01 -.17855     -.34427      .11998D-01  .0
  .0          .0          .0           .0           .0           .0          .0          .0          .0          .0
  .0          .0          .0           .0           .0           .0         1.0000       .0          .0

28
 -.58414D-01  .23060D-02  .10061D-02  -.26306      -.78476D-01  -.27892D-01  .14094D-01 -.25536     -.12137      .0
  .0          .0          .0           .0           .0           .0          .0          .0          .0          .0
  .0          .0          .0           .0           .0           .0          .0         1.0000       .0

29
  .16174D-01 -.16515      .49546       .41770D-02  -.20295      -.35302D-01  .58551D-01  .34727     -.37398D-01  .0
  .0          .0          .0           .0           .0           .0          .0          .0          .0          .0
  .0          .0          .0           .0           .0           .0          .0          .0         1.0000

          FINAL COMBINED FORCE VECTOR OF ORDER 29  FOR PSI=  .0

 -.38036D-05 -.87050D-05  .25502D-04  -.13759D-04  -.14091D-04   .29357D-05  .99558D-05  .45950D-05 -.68925D-05  .0
  .0          .0          .0           .0           .0           .0          .0          .0          .0          .0
  .0          .0          .0           .0           .0           .0          .0          .0          .0

       STATE VARIABLES (DISP&VEL) FOR PSI =    .0

       -.25286D-11   -.19571D-08
       -.26472D-11   -.20489D-08
        .46451D-11    .35953D-08
       -.71434D-11   -.55290D-08
       -.52201D-11   -.40403D-08
        .16185D-11    .12527D-08
        .51322D-11    .39723D-08
        .23660D-12    .18313D-09
       -.12391D-11   -.95909D-09
       -.30384D-11   -.23517D-08
        .41981D-12    .32493D-09
        .18716D-13    .14486D-10
       -.30990D-16   -.23987D-13
       -.11559D-11   -.89465D-09
        .36247D-11    .28055D-08
        .63828D-11    .49403D-08
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-.10919D-10	-.84510D-08
-.19122D-13	-.14801D-10
-.31698D-13	-.24534D-10
-.50001D-13	-.38701D-10
-.93811D-13	-.72610D-10
.44214D-11	.34222D-08
.31637D-11	.24487D-08
-.40165D-12	-.31088D-09
.18834D-12	.14578D-09

BIFILAR INITIAL DISP & VEL FOR PSI = .0

.34172D-11	.26449D-08
.49642D-11	.38423D-08
-.25522D-11	-.19754D-08
-.40992D-11	-.31728D-08

FIXED SYSTEM + ROTOR + FIXED ABSORBERS (R.H.S.) OF ORDER 25 (Ψ=2 DEC)

-.13624D-07	.44860D-07	-.22938D-06	.23800D-06	.13370D-06	-.28845D-07	-.92330D-07	-.10942D-06	.20289D-06	.41828D-08
.27024D-09	-.50329D-06	.84167D-06	.94019D-08	.19417D-07	-.12205D-06	-.45731D-08	.14607D-05	-.36007D-05	.73651D-04
.30834D-04	-.70800D-07	-.42243D-07	.50807D-08	-.23538D-08					

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STATE VARIABLES (DISP&VEL) FOR PSI =   30.000

 -.25048D-04   -.38094D-02
  .11111D-04    .29375D-03
 -.12945D-04    .60366D-03
 -.61756D-04   -.87113D-02
 -.34311D-05   -.23132D-02
  .69925D-05    .13406D-02
  .10632D-04    .30400D-02
 -.37044D-04   -.32374D-02
 -.73536D-05   -.10282D-02
  .24983D-05   -.90190D-03
 -.12617D-06    .16874D-03
 -.25183D-07    .41743D-05
 -.94880D-09   -.10185D-06
 -.41310D-04   -.35801D-02
 -.20113D-04    .32268D-03
  .59878D-04    .48574D-02
  .44100D-05   -.61698D-02
  .20980D-05    .18932D-03
  .15160D-05    .12076D-03
 -.61764D-06   -.77497D-04
 -.22019D-06    .32475D-04
  .12211D-04    .25428D-02
 -.12016D-05    .11571D-02
  .20469D-04    .11880D-02
 -.19166D-04   -.81023D-03

BIFILAR INITIAL DISP & VEL FOR PSI =   30.000

  .25266D-04    .38448D-02
 -.20070D-04   -.16933D-03
 -.22174D-04   -.32296D-02
  .23163D-04    .78467D-03

NREV=  2, G1=  .183D-01, G2=  .160D-01, XHUB=  .412D-02, YHUB=  .317D-02, THZH=  .301D-05, DXHUB= -.315    , DYHUB=  .224

NREV=  3, G1=  .183D-01, G2=  .288D-01, XHUB=  .439D-02, YHUB=  .484D-02, THZH=  .698D-05, DXHUB= -.534    , DYHUB=  .197

NREV=  4, G1=  .116D-01, G2=  .351D-01, XHUB=  .397D-02, YHUB=  .569D-02, THZH=  .976D-05, DXHUB= -.686    , DYHUB=  .117
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NREV=  5, G1= .462D-02, G2= .344D-01, XHUB= .333D-02, YHUB= .592D-02, THZH= .122D-04, DXHUB= -.770 , DYHUB= .486D-01
NREV=  6, G1= .188D-02, G2= .301D-01, XHUB= .321D-02, YHUB= .601D-02, THZH= .131D-04, DXHUB= -.812 , DYHUB= .176D-01
NREV=  7, G1= .437D-02, G2= .219D-01, XHUB= .109D-02, YHUB= .526D-02, THZH= .146D-04, DXHUB= -.749 , DYHUB= -.140
NREV=  8, G1= .128D-01, G2= .984D-02, XHUB= -.271D-03, YHUB= .371D-02, THZH= .128D-04, DXHUB= -.614 , DYHUB= -.183
NREV=  9, G1= .124D-01, G2= .213D-02, XHUB= -.247D-03, YHUB= .261D-02, THZH= .103D-04, DXHUB= -.468 , DYHUB= -.148
NREV= 10, G1= .653D-02, G2= .821D-02, XHUB= .454D-03, YHUB= .226D-02, THZH= .844D-05, DXHUB= -.382 , DYHUB= -.608D-01
NREV= 11, G1= .454D-03, G2= .813D-02, XHUB= .141D-02, YHUB= .257D-02, THZH= .723D-05, DXHUB= -.361 , DYHUB= .176D-01
NREV= 12, G1= .452D-02, G2= .419D-02, XHUB= .191D-02, YHUB= .312D-02, THZH= .781D-05, DXHUB= -.399 , DYHUB= .573D-01
NREV= 13, G1= .481D-02, G2= .310D-03, XHUB= .201D-02, YHUB= .356D-02, THZH= .846D-05, DXHUB= -.450 , DYHUB= .470D-01
NREV= 14, G1= .252D-02, G2= .298D-02, XHUB= .174D-02, YHUB= .374D-02, THZH= .932D-05, DXHUB= -.483 , DYHUB= .131D-01
NREV= 15, G1= .193D-03, G2= .309D-02, XHUB= .147D-02, YHUB= .371D-02, THZH= .989D-05, DXHUB= -.486 , DYHUB= -.195D-01
NREV= 16, G1= .179D-02, G2= .148D-02, XHUB= .135D-02, YHUB= .361D-02, THZH= .104D-04, DXHUB= -.464 , DYHUB= -.327D-01
     THE NUMBER OF REVOLUTIONS REQUIRED TO CONVERGE = 16

      INPUT FIXED SYSTEM MODES FREQUENCIES IN HZ
  5.10    6.40    15.3    14.3    13.8    11.6    12.1    17.4    21.1

      THE CONVERGENCE FACTOR TO G =  30.2

      BIFILAR HARMONIC OUTPUT - AMPLITUDE AND PHASE
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	1 P	2 P	3 P	4 P	5 P	6 P	7 P	8 P	9 P
1	.72570D-02	.20305D-01	9.7569	.55991D-01	.18446D-01	.29965D-02	.51515D-02	.39280D-02	.27998D-02
	52.592	53.478	96.204	-124.06	29.256	-124.17	-103.32	-100.26	-132.76
2	.13026D-01	.27397D-01	9.7675	.36192D-01	.37197D-01	.33446D-02	.41663D-02	.29031D-02	.31304D-02
	167.32	-177.75	-173.99	-49.115	-46.272	30.184	-45.499	-31.080	-18.104
3	.33807D-02	.87900D-02	9.7962	.34774D-01	.21669D-01	.13022D-01	.67488D-02	.60291D-02	.32922D-02
	-107.27	-66.614	-83.966	83.398	-154.00	18.479	79.690	75.105	63.689
4	.12278D-01	.12959D-01	9.7978	.47673D-01	.35955D-01	.13860D-01	.31602D-02	.23191D-02	.32495D-02
	5.5568	-31.531	6.2263	167.95	139.47	-166.74	154.27	124.83	170.57

HARMONIC HUB OUTPUT - COSINE , SINE AND TOTAL RESPONSE AND PHASE ANGLE IN DEGREES

	1 P	2 P	3 P	4 P	5 P	6 P	7 P	8 P	9 P
1	-.35387D-03	-.71332D-03	-.10783D-02	.13513D-01	.11137D-01	.43362D-03	.23521D-03	.59209D-03	.93772D-04
	.71784D-04	.15737D-03	.13435D-03	-.13798	.19954D-02	-.88499D-03	-.48685D-03	-.21467D-03	-.27708D-03
	.36108D-03	.73048D-03	.10866D-02	.13864	.11314D-01	.98551D-03	.54069D-03	.62981D-03	.29251D-03
	168.53	167.56	172.90	-84.407	10.158	-63.896	-64.214	-19.929	-71.303
2	.84053D-04	-.17284D-03	-.40974D-03	.94763D-01	.98815D-02	.10481D-03	.62883D-04	.22187D-03	.38672D-04
	.12153D-03	-.11495D-03	-.54539D-03	-.31934D-01	.16442D-02	-.19251D-03	-.16556D-04	-.22391D-03	.30311D-04
	.14776D-03	.20757D-03	.68215D-03	.99998D-01	.10017D-01	.21919D-03	.65026D-04	.31522D-03	.49135D-04
	55.331	-146.37	-126.92	-18.623	9.4468	-61.433	-14.750	-45.263	38.090
3	-.18544D-03	-.12456D-03	-.20801D-03	.49658D-02	.29709D-02	.36247D-03	.16057D-03	.73182D-04	.68470D-04
	-.20083D-04	-.20787D-04	-.10326D-03	-.65451D-02	.67828D-02	-.10717D-03	-.29909D-04	-.29406D-04	-.36232D-05
	.18653D-03	.12629D-03	.23223D-03	.82157D-02	.74049D-02	.37798D-03	.16333D-03	.78869D-04	.68566D-04
	-173.82	-170.53	-153.60	-52.813	66.346	-16.471	-10.552	-21.891	-3.0290
4	.11692D-04	.18351D-04	.20934D-04	.10088D-02	.25708D-03	-.40229D-04	-.18089D-04	-.63736D-05	-.77610D-05
	-.34145D-05	.35567D-05	-.17642D-05	-.27056D-03	-.65194D-03	-.96333D-05	-.43971D-05	-.42516D-05	-.22284D-05
	.12181D-04	.18692D-04	.21008D-04	.10444D-02	.70080D-03	.41367D-04	.18615D-04	.76615D-05	.80745D-05
	-16.279	10.969	-4.8173	-15.013	-68.479	-166.53	-166.34	-146.29	-163.98
5	-.20246D-04	-.28300D-04	-.39290D-04	.11809D-02	.32143D-03	.43543D-04	.20206D-04	.17703D-04	.86785D-05
	.57440D-06	-.19054D-05	-.77411D-05	-.25992D-02	.67686D-03	-.17481D-04	-.75718D-05	-.61179D-05	-.33195D-05
	.20254D-04	.28364D-04	.40045D-04	.28549D-02	.74930D-03	.46921D-04	.21578D-04	.18731D-04	.92917D-05
	178.37	-176.15	-168.85	-65.566	64.598	-21.874	-20.543	-19.064	-20.932
6	.12641D-05	-.79781D-06	.28154D-05	.29220D-03	.44537D-04	.11795D-05	.42791D-06	-.17881D-06	.21827D-06
	.17520D-05	-.63593D-06	.17588D-06	.17726D-03	.33771D-04	-.23400D-05	-.10164D-05	-.19328D-05	-.46859D-06
	.21604D-05	.10202D-05	.28209D-05	.34176D-03	.55893D-04	.26205D-05	.11028D-05	.19411D-05	.51694D-06
	54.189	-141.44	3.5747	31.243	37.173	-63.248	-67.169	-95.285	-65.024

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VIBRATION LEVELS AT 4 A/C LOCATIONS.

COSINE,SINE,TOTAL AMPLITUDE IN G AND PHASE ANGLE IN DEGREES OF POINT 1

	1 P	2 P	3 P	4 P	5 P	6 P	7 P	8 P	9 P
1	.74646D-04	.10540D-03	.16495D-03	-.68218D-02	-.17800D-03	-.13784D-03	-.69643D-04	-.77459D-04	-.32567D-04
	-.17197D-04	.10159D-04	.45374D-04	.16037D-01	-.15142D-02	-.87238D-05	-.90956D-05	.72264D-05	-.73596D-05
	.76601D-04	.10589D-03	.17108D-03	.17428D-01	.15246D-02	.13811D-03	.70234D-04	.77795D-04	.33389D-04
	-12.973	5.5052	15.381	113.04	-96.705	-176.38	-172.56	174.67	-167.27
2	-.44641D-03	-.23631D-03	-.40666D-03	.25074D-01	-.26798D-02	.55354D-03	.25294D-03	.10032D-03	.11363D-03
	-.20551D-03	-.23326D-04	.38511D-05	.11033D-01	.88283D-02	.15384D-03	.87506D-04	-.12696D-04	.54312D-04
	.49145D-03	.23746D-03	.40668D-03	.27394D-01	.92260D-02	.57452D-03	.26765D-03	.10112D-03	.12594D-03
	-155.28	-174.36	179.46	23.751	106.89	15.532	19.083	-7.2126	25.547
3	-.21825D-03	-.39448D-03	-.56827D-03	.13519D-01	-.38145D-02	.62622D-03	.29079D-03	.18752D-03	.12778D-03
	-.27545D-04	-.80199D-04	-.10098D-03	-.51186D-02	.92417D-02	.19182D-03	.96591D-04	.33801D-04	.54140D-04
	.21998D-03	.40255D-03	.57717D-03	.14456D-01	.99980D-02	.65494D-03	.30642D-03	.19054D-03	.13878D-03
	-172.81	-168.51	-169.92	-20.737	112.43	17.031	18.375	10.218	22.962

COSINE,SINE,TOTAL AMPLITUDE IN G AND PHASE ANGLE IN DEGREES OF POINT 2

	1 P	2 P	3 P	4 P	5 P	6 P	7 P	8 P	9 P
1	.10408D-03	.14141D-03	.21754D-03	-.67874D-02	-.71084D-03	-.18505D-03	-.90701D-04	-.99222D-04	-.40508D-04
	-.70780D-05	.37864D-05	.37375D-04	.18411D-01	-.22860D-02	.37913D-04	.16089D-04	.17450D-04	.65888D-05
	.10432D-03	.14146D-03	.22073D-03	.19622D-01	.23939D-02	.18889D-03	.92117D-04	.10074D-03	.41041D-04
	-3.8905	1.5338	9.7485	110.24	-107.27	168.42	169.94	170.03	170.76
2	-.79428D-03	-.27576D-03	-.32357D-03	.60930D-02	.96273D-02	.59969D-03	.25702D-03	.14243D-03	.99368D-04
	-.39016D-03	.16613D-03	.40307D-03	-.10963D-01	.11193D-01	-.76927D-03	-.39836D-03	-.23445D-03	-.21426D-03
	.88493D-03	.32194D-03	.51688D-03	.12543D-01	.14763D-01	.97539D-03	.47408D-03	.27432D-03	.23618D-03
	-153.84	148.93	128.76	-60.936	49.299	-52.061	-57.171	-58.721	-65.120
3	.11722D-03	-.18740D-03	-.82776D-04	.15241D-01	-.12303D-01	.15987D-03	.82942D-04	-.57137D-05	.47371D-04
	.22443D-04	-.15717D-03	-.76443D-04	.28371D-01	.86053D-03	.69282D-03	.33292D-03	.12336D-03	.17070D-03
	.11935D-03	.24459D-03	.11267D-03	.32206D-01	.12333D-01	.71102D-03	.34310D-03	.12350D-03	.17715D-03
	10.839	-140.01	-137.28	61.755	176.00	77.007	76.010	92.652	74.490

COSINE,SINE,TOTAL AMPLITUDE IN G AND PHASE ANGLE IN DEGREES OF POINT 3

	1 P	2 P	3 P	4 P	5 P	6 P	7 P	8 P	9 P
1	.63665D-04	.99252D-04	.13585D-03	-.39269D-02	-.89703D-03	-.14680D-03	-.68260D-04	-.60697D-04	-.29305D-04
	-.39469D-05	.71563D-05	.22363D-04	.83731D-02	-.22399D-02	.52829D-04	.23639D-04	.19217D-04	.10721D-04
	.63787D-04	.99510D-04	.13767D-03	.92482D-02	.24129D-02	.15602D-03	.72237D-04	.63666D-04	.31205D-04
	-3.5476	4.1240	9.3481	115.13	-111.82	160.21	160.90	162.43	159.90
2	.22796D-03	.21018D-03	.60564D-03	-.11864D-02	.74275D-02	-.52364D-03	-.24430D-03	-.12237D-03	-.10347D-03
	.38921D-04	-.24978D-04	.49249D-03	-.72463D-02	-.75659D-02	-.42657D-03	-.21032D-03	-.16250D-03	-.11460D-03
	.23126D-03	.21166D-03	.78061D-03	.73428D-02	.10602D-01	.67540D-03	.32236D-03	.20342D-03	.15440D-03
	9.6888	-6.7775	39.117	-99.298	-45.529	-140.83	-139.27	-126.98	-132.08
3	.12023D-03	-.29805D-03	-.15678D-03	-.16435D-01	.14920D-02	.32953D-03	.13912D-03	.87638D-04	.52740D-04
	.73060D-04	-.10535D-03	.31328D-04	-.13471D-02	.62943D-02	-.16450D-03	-.89878D-04	-.15848D-04	-.51079D-04
	.14069D-03	.31612D-03	.15988D-03	.16490D-01	.64687D-02	.36831D-03	.16562D-03	.89059D-04	.73421D-04
	31.285	-160.53	168.70	-175.31	76.664	-26.528	-32.865	-10.250	-44.083

COSINE,SINE,TOTAL AMPLITUDE IN G AND PHASE ANGLE IN DEGREES OF POINT 4

	1 P	2 P	3 P	4 P	5 P	6 P	7 P	8 P	9 P
1	-.60456D-03	.58779D-03	-.43829D-03	-.19349D-01	-.13883D-01	-.80049D-04	-.39968D-05	-.99326D-04	.15687D-04
	-.27274D-03	.15103D-03	-.21240D-04	-.70285D-02	-.42247D-02	.10613D-02	.56597D-03	.44778D-03	.31678D-03
	.66323D-03	.60688D-03	.43881D-03	.20586D-01	.14512D-01	.10643D-02	.56599D-03	.45866D-03	.31717D-03
	-155.72	14.410	-177.23	-160.04	-163.07	94.313	90.405	102.51	87.165
2	-.14888D-02	.50814D-03	.18117D-03	-.28815D-01	.11071D-01	-.61440D-03	-.29150D-03	-.21725D-03	-.14729D-03
	-.14651D-02	.53793D-03	.16710D-03	-.21429D-01	-.84907D-02	-.77973D-03	-.41105D-03	-.61673D-04	-.22418D-03
	.20888D-02	.73999D-03	.24647D-03	.35910D-01	.13952D-01	.99270D-03	.50392D-03	.22584D-03	.26824D-03
	-135.46	46.631	42.687	-143.36	-37.485	-128.24	-125.34	-164.15	-123.31
3	.42239D-03	-.38248D-03	-.12887D-04	.79289D-02	-.66885D-03	-.24120D-04	-.93473D-05	.11982D-03	-.49814D-05
	.18882D-03	-.15571D-03	.43586D-03	-.38277D-02	-.16426D-02	-.18224D-03	-.13732D-03	-.12142D-03	-.96211D-04
	.46267D-03	.41296D-03	.43605D-03	.88044D-02	.17735D-02	.18383D-03	.13764D-03	.17058D-03	.96340D-04
	24.085	-157.85	91.694	-25.769	-112.16	-97.539	-93.894	-45.380	-92.964

INITIAL BIFILAR DISPLACEMENTS (LC.1720-1739)

-.34889D-01 -.16560 .34955D-01 .16588

INITIAL BIFILAR VELOCITIES (LC.1740-1759)

13.328 -3.5460 -13.417 3.5655

```
INITIAL HUB DISPLACEMENTS      (LC.1768-1773)

   .15065D-02  .35604D-02  .25974D-03  .45858D-04  .59105D-04  .10595D-04

INITIAL HUB VELOCITIES         (LC.1774-1779)

  -.43641     -.27048D-01  .16799D-01 -.33085D-02 -.43242D-02  .10283D-02

INITIAL STATE VARIABLES DISPL.(LC.1780-1859)

  -.41930D-04  .83752D-04  .47216D-04  .38650D-02  .19628D-03 -.44489D-03 -.97618D-03  .25778D-02 -.29884D-03  .10835D-02
  -.47164D-04 -.53035D-05  .31296D-07  .38200D-03 -.56627D-02  .10109D-01 -.13128D-02 -.13922D-03 -.92779D-04  .58519D-04
   .13830D-04  .56357D-02  .20740D-01 -.11073      .51973D-01

INITIAL STATE VARIABLES VELOC.(LC.1860-1939)

   .47650D-01 -.11799D-01 -.21021      .15078      .14798     -.33559D-01 -.99727D-01 -.22058     -.18196      .20168D-01
  -.10169     -.33443D-03 -.11013D-04 -1.2027      .30764D-01  .67974     -.27664      .92194D-02  .76840D-02  .44570D-03
  -.79445D-02 -.29740      2.7402      5.2717      14.502
```

CASE 3. BIFILAR RESULTS

```
                    BIFILAR  ANALYSIS  RESULTS

   ALL DISPLACEMENTS ARE IN G   AND   ALL ANGLES ARE IN DEGREES

   NUMBER OF FIXED SYSTEM MODES                  IS  9
   NUMBER OF FIXED SYSTEM ABSORBERS              IS  1
   NUMBER OF INPLANE BIFILARS                    IS  1
   NUMBER OF VERTICAL BIFILARS                   IS  1
   TOTAL NO. OF DEGREES-OF-FREEDOM
         (WITH NO ROTOR)                          IS 16
   NUMBER  OF A.C. STATIONS                       IS  4
   ROTOR COUPLING SWITCH     (0=NO,1OR-1=YES)     IS  0
   ROTOR MATRICES        PRINTOUT(0=NO,1=YES)     IS  0
   FIXED SYSTEM MATRICES  PRINTOUT       "        IS  0
   ADD   ROTOR  MATRICES  PRINTOUT       "        IS  0
   ADD FIX.SYS. ABSORBER  PRINTOUT       "        IS  0
   ADD INPLANE  BIFILAR   PRINTOUT       "        IS  0
   ADD VERTICAL BIFILAR   PRINTOUT       "        IS  0
   INPLANE  BIFILAR (9X9) PRINTOUT       "        IS  0
   VERTICAL BIFILAR (9X9) PRINTOUT       "        IS  0
   GAMMAS                 PRINTOUT       "        IS  0
```

```
PRESENT NO. OF DEGREES-OF-FREEDOM  (NP)    =  9
ADDITIONAL NO. DEGREES-OF-FREEDOM  (NA)    =  1
TOTAL  NO. OF  DEGREES-OF-FREEDOM (NP+NA) =  10
ORDER OF MATRIX TO BE ADDED        (NL)    =  2
ORDER -  D.O.F. OF ADDED MATRIX    (NL-NA) =  1


INPLANE  BIFILAR  PARAMETERS

TOTAL NO. OF BIFILARS (N)  =   4.00000
BIFILAR MASS         (M)   =   .614000D-01
BIFILAR ARM        (RCR)   =   18.2200
BIFILAR FREQUENCY    (W)   =   3.00000
BIFILAR RADIUS (R=RCR/W/W) =   2.02444

N*M*R*R                    =   1.00656
1+W*W                      =   10.0000
N*M*R*R*(1.+W*W)           =   10.0656
N*M*R*R*(1.+W*W)**2        =   100.656


PRESENT NO. OF DEGREES-OF-FREEDOM  (NP)    =  10
ADDITIONAL NO. DEGREES-OF-FREEDOM  (NA)    =  3
TOTAL  NO. OF  DEGREES-OF-FREEDOM (NP+NA) =  13
ORDER OF MATRIX TO BE ADDED        (NL)    =  9
ORDER -  D.O.F. OF ADDED MATRIX    (NL-NA) =  6
```

```
VERTICAL BIFILAR PARAMETERS

TOTAL NO. OF BIFILARS (N)  =  4.00000
BIFILAR MASS           (M)  =  .608000D-01
DISTANCE FROM C.R.     (R)  =  18.5000
BIFILAR FREQUENCY      (W)  =  4.00000

R1=R/(W*W-1.)               =  1.23333
R1*R1                       =  1.52111
R+R1                        =  19.7333
(R+R1)**2                   =  389.404
M*N*(R+R1)                  =  4.79915
M*N*(R+R1)**2               =  94.7032
M*N*R1*(R+R1)               =  5.91895


PRESENT NO. OF DEGREES-OF-FREEDOM (NP)     = 13
ADDITIONAL NO. DEGREES-OF-FREEDOM (NA)     =  3
TOTAL NO. OF DEGREES-OF-FREEDOM (NP+NA)    = 16
ORDER OF MATRIX TO BE ADDED          (NL)  =  9
ORDER - D.O.F. OF ADDED MATRIX    (NL-NA)  =  6
```

```
GENERALIZED FORCES - ORDER IS = 9

                    COSINE COMPONENT
 .500000   -170.000   500.000   .500000   -201.000   25.0000   120.000   350.000
-25.0000

                    SINE  COMPONENT
 75.0000   .500000   .500000   270.000   75.0000   -32.5000   -75.0000   260.000
 110.000

                    (GAMMAS)

          COSINE  -   SINE  -  AMPLITUDE - PHASE(DEG)

                FIXED SYSTEM ABSORBER(S)

        -.166206D-02  .216659D-02  .273066D-02  127.493

                 INPLANE BIFILAR PENDULUM(S)
       (EQT ORDER IS:0,SIN,COS)   (AMPL&PHASE ORDER IS:N,N-1,N+1 )
                                        DEG & DEG

         .346150D-03  .243546D-03  .606250D-02  35.1296
         .329375      .265358D-01  9.52124      -85.7398
         .228424D-01 -.333496      .792694D-01 -131.867

                 VERTICAL BIFILAR PENDULUM(S)
       (EQT ORDER IS:0,SIN,COS)   (AMPL&PHASE ORDER IS:N,N-1,N+1 )
                                        DEG & DEG

        -.373718D-01  .363299D-01  .746566      135.810
         .804502D-02  .162357D-02  .468243D-01 -61.7531
        -.764629D-04  .516534D-02  .190785      97.3330

       INPUT FIXED SYSTEM FREQUENCIES (HZ)
 5.10000   6.40000   15.3000   14.3000   13.8000   11.6000   12.1000   17.4000   21.1000

       FORCING FREQUENCY (HZ)            =   17.2000

       THE CONVERSION  FACTOR TO G       =   30.2259
```

FUSELAGE NO.OF DEGREES OF FREEDOM = 9

COSINE COMPONENT

-.137705D-03 .185885D-03 -.125943D-02 -.186186D-02 -.131067D-03 .166214D-03 .218335D-03 -.169248D-02
.120032D-03

SINE COMPONENT

-.198076D-03 -.188549D-03 .153812D-02 -.151517D-02 -.441784D-03 .878125D-04 .389724D-03 .533932D-02
.182804D-03

FIRST A.C. STATION DISPLACEMENT IN G

X		Y		Z	
COSINE	SINE	COSINE	SINE	COSINE	SINE
.878210D-02	-.232140D-01	-.861884D-02	.264515D-02	-.739075D-02	.174402D-01

FIRST A.C. STATION TOTAL DISPLACEMENT IN G

X	Y	Z
.248196D-01	.901561D-02	.189416D-01

FIRST A.C. STATION PHASE ANGLE IN DEG

-69.2778 162.939 112.966

SECOND A.C. STATION DISPLACEMENT IN G

X		Y		Z	
COSINE	SINE	COSINE	SINE	COSINE	SINE
.961892D-02	-.257500D-01	-.631791D-02	.181442D-01	-.869028D-02	-.112396D-01

SECOND A.C. STATION TOTAL DISPLACEMENT IN G

X	Y	Z
.274879D-01	.192127D-01	.142074D-01

SECOND A.C. STATION PHASE ANGLE IN DEG

-69.5168 109.198 -127.711

THIRD A.C. STATION DISPLACEMENT IN G

X		Y		Z	
COSINE	SINE	COSINE	SINE	COSINE	SINE
.368109D-02	-.116330D-01	-.737542D-02	.106301D-01	.153405D-01	-.148632D-02

THIRD A.C. STATION TOTAL DISPLACEMENT IN G

X	Y	Z
.122015D-01	.129381D-01	.154124D-01

THIRD A.C. STATION PHASE ANGLE IN DEG

-72.4408	124.754	-5.53402

FOURTH A.C. STATION DISPLACEMENT IN G

X		Y		Z	
COSINE	SINE	COSINE	SINE	COSINE	SINE
.183530D-01	-.574138D-02	.127445D-01	.136297D-01	-.276256D-01	.203164D-01

FOURTH A.C. STATION TOTAL DISPLACEMENT IN G

X	Y	Z
.192300D-01	.186599D-01	.342918D-01

FOURTH A.C. STATION PHASE ANGLE IN DEG

-17.3713	46.9224	143.668

ROTOR HEAD DISPLACEMENT IN G

X		Y		Z	
COSINE	SINE	COSINE	SINE	COSINE	SINE
-.726017D-01	.169399	-.587601D-01	.556059D-01	.677164D-02	.696585D-02

ROTOR HEAD TOTAL DISPLACEMENT IN G

X	Y	Z
.184301	.808997D-01	.971484D-02

ROTOR HEAD DISPLACEMENT PHASE ANGLES IN DEG

113.199 136.580 45.8099

CASE 4. BIFILAR RESULTS

BIFILAR ANALYSIS RESULTS

ALL DISPLACEMENTS ARE IN G AND ALL ANGLES ARE IN DEGREES

```
NUMBER OF FIXED SYSTEM MODES                   IS  9
NUMBER OF FIXED SYSTEM ABSORBERS               IS  1
NUMBER OF INPLANE BIFILARS                     IS  0
NUMBER OF VERTICAL BIFILARS                    IS  1
TOTAL NO. OF DEGREES-OF-FREEDOM
      (WITH NO ROTOR)                           IS 13
NUMBER  OF A.C. STATIONS                        IS  4
ROTOR COUPLING SWITCH      (0=NO,1OR-1=YES)     IS  0
ROTOR MATRICES          PRINTOUT(0=NO,1=YES)    IS  0
FIXED SYSTEM MATRICES   PRINTOUT      "         IS  0
ADD   ROTOR  MATRICES   PRINTOUT      "         IS  0
ADD FIX.SYS. ABSORBER   PRINTOUT      "         IS  1
ADD INPLANE  BIFILAR    PRINTOUT      "         IS  1
ADD VERTICAL BIFILAR    PRINTOUT      "         IS  0
INPLANE  BIFILAR (9X9)  PRINTOUT      "         IS  0
VERTICAL BIFILAR (9X9)  PRINTOUT      "         IS  0
GAMMAS                  PRINTOUT      "         IS  0
```

```
PRESENT NO. OF DEGREES-OF-FREEDOM (NP)     = 9
ADDITIONAL NO. DEGREES-OF-FREEDOM (NA)     = 1
TOTAL NO. OF DEGREES-OF-FREEDOM (NP+NA)    = 10
ORDER OF MATRIX TO BE ADDED        (NL)    = 2
ORDER - D.O.F. OF ADDED MATRIX  (NL-NA)    = 1


VERTICAL BIFILAR PARAMETERS

 TOTAL NO. OF BIFILARS (N)  =  4.00000
 BIFILAR MASS          (M)  =  .608000D-01
 DISTANCE FROM C.R.    (R)  =  18.5000
 BIFILAR FREQUENCY     (W)  =  4.00000

 R1=R/(W*W-1.)              =  1.23333
 R1*R1                      =  1.52111
 R+R1                       =  19.7333
 (R+R1)**2                  =  389.404
 M*N*(R+R1)                 =  4.79915
 M*N*(R+R1)**2              =  94.7032
 M*N*R1*(R+R1)              =  5.91895


PRESENT NO. OF DEGREES-OF-FREEDOM (NP)     = 10
ADDITIONAL NO. DEGREES-OF-FREEDOM (NA)     = 3
TOTAL NO. OF DEGREES-OF-FREEDOM (NP+NA)    = 13
ORDER OF MATRIX TO BE ADDED        (NL)    = 9
ORDER - D.O.F. OF ADDED MATRIX  (NL-NA)    = 6
```

D.O.F.= 17,NF= 9,KROTOR= 0,NFABS= 1,KINDS OF INPL. BIF.= 0,KINDS OF VERT. BIF.= 1,NUMB. OF N.L. INPL. BIF= 4

FIXED SYSTEM + ROTOR + FIXED ABSORBERS (R.H.S.) OF ORDER 13

.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0							

THE MASS (L.H.S.) MATRIX OF ORDER 10

.24565	.0	.0	.0	.0	.34977D-07	.0	-.12433	.34977D-08	.12433
.0	.24565	.0	.0	.0	.34977D-07	.12433	-.17488D-08	-.12433	.52465D-08
.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
.34977D-07	.34977D-07	.0	.0	.0	100.68	2.5169	2.5169	2.5169	2.5169
.0	.49396	.0	.0	.0	10.000	1.0000	.0	.0	.0
-.49396	-.69483D-08	.0	.0	.0	10.000	.0	1.0000	.0	.0
.13897D-07	-.49396	.0	.0	.0	10.000	.0	.0	1.0000	.0
.49396	.20845D-07	.0	.0	.0	10.000	.0	.0	.0	1.0000

THE BIFILAR FORCE VECTOR OF ORDER 10

.25528D-04	-.25528D-04	.0	.0	.0	.0	.0	.0	.0	.0

THE ROTOR HEAD MODE SHAPES,TRANSPOSE(PHI), OF ORDER 9 X 6

.10000D-02	.15000	.10000D-02	-.51200D-02	.47400D-03	.15680D-02
-.34000	.10000D-02	.11000	-.12000D-03	-.51830D-02	.28000D-03
1.0000	.10000D-02	-.31000	.13020D-01	-.53950D-02	.15000D-03
.10000D-02	.54000	-.10000D-02	.12140D-01	.77200D-04	.36830D-03
-.40200	.15000	-.11000	-.80100D-02	-.32520D-02	-.43820D-03
.50000D-01	-.65000D-01	.22000D-01	-.12440D-02	-.47300D-03	-.60000D-02
.24000	-.15000	.22000D-01	-.12440D-02	-.47300D-03	-.60000D-02
.70000	.52000	.12500	-.70000D-04	.23100D-01	.15000D-03
-.50000D-01	.22000	.10000D-02	.63500D-02	.22000D-03	-.12700D-02

THE EXPANDED BIFILAR MASS MATRIX OF ORDER = 13

.57749D-02	-.24725D-05	.30618D-03	.19956D-01	.53592D-02	-.33300D-02	-.64154D-02	.19357D-01	.78938D-02	.22596D-01	.38222D-02	-.14703D-01	.40709D-02
-.24725D-05	.28406D-01	-.83517D-01	.59513D-04	.33600D-01	-.43612D-02	-.20251D-01	-.58333D-01	.41943D-02	.82907D-03	.42976D-01	.58041D-03	-.41567D-01
.30618D-03	-.83517D-01	.24565	.38387D-03	-.98722D-01	.12176D-01	.58829D-01	.17209	-.12248D-01	.50187D-03	-.12395	.25322D-03	.12470
.19956D-01	.59513D-04	.38387D-03	.71646D-01	.19783D-01	-.88326D-02	-.20061D-01	.69157D-01	.29124D-01	.68064D-01	.80266D-03	-.66210D-01	.10513D-02
.53592D-02	.33600D-01	-.98722D-01	.19783D-01	.45245D-01	-.70680D-02	-.28963D-01	-.49972D-01	.13100D-01	.17546D-01	.48877D-01	-.19752D-01	-.51082D-01
-.33300D-02	-.43612D-02	.12176D-01	-.88326D-02	-.70680D-02	.52764D-02	.89673D-02	.20417D-03	-.33598D-02	-.23183D-01	-.21318D-01	-.70203D-02	-.88853D-02
-.64154D-02	-.20251D-01	.58829D-01	-.20061D-01	-.28963D-01	.89673D-02	.23301D-01	.22018D-01	-.10287D-01	-.33751D-01	-.44940D-01	.35475D-02	.14737D-01
.19357D-01	-.58333D-01	.17209	.69157D-01	-.49972D-01	.20417D-03	.22018D-01	.18680	.19486D-01	.65028D-01	-.86652D-01	-.64273D-01	.87407D-01
.78938D-02	.41943D-02	-.12248D-01	.29124D-01	.13100D-01	-.33598D-02	-.10287D-01	.19486D-01	.12666D-01	.24155D-01	.30199D-02	-.30549D-01	-.94129D-02
.89774D-01	.32940D-02	.19940D-02	.27042	.69712D-01	-.92108D-01	-.13409	.25836	.95972D-01	1.0000	.0	.0	.0
.15186D-01	.17075	-.49246	.31890D-02	.19419	-.84698D-01	-.17855	-.34427	.11998D-01	.0	1.0000	.0	.0
-.58414D-01	.23060D-02	.10061D-02	-.26306	-.78476D-01	-.27892D-01	.14094D-01	-.25536	-.12137	.0	.0	1.0000	.0
.16174D-01	-.16515	.49546	.41770D-02	-.20295	-.35302D-01	.58551D-01	.34727	-.37398D-01	.0	.0	.0	1.0000

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THE EXPANDED BIFILAR FORCE VECTOR OF ORDER = 13

```
-.38036D-05  -.87050D-05   .25502D-04  -.13759D-04  -.14091D-04   .29357D-05   .99558D-05   .45950D-05  -.68925D-05   .0
 .0           .0           .0
```

FINAL COMBINED MASS MATRIX OF ORDER 17 FOR PSI= .0

1										
	1.8208	-.36060D-03	-.27779D-02	.36758D-01	.13611D-01	-.62287D-02	-.12413D-01	.39304D-01	.14162D-01	.10999D-02
	.74987D-04	-.75763D-02	-.70140D-03	.22596D-01	.38222D-02	-.14703D-01	.40709D-02			
2										
	-.36060D-03	3.1263	-.17273	.17560D-03	.47531D-01	-.88206D-02	-.39753D-01	-.12186	.86300D-02	-.17236D-01
	.82485D-02	-.17757D-03	.76695D-02	.82907D-03	.42976D-01	.58041D-03	-.41567D-01			
3										
	-.27779D-02	-.17273	5.6517	.83232D-02	-.19421	.21833D-01	.11475	.32670	-.20550D-01	-.19410D-02
	-.23246D-01	.19266D-01	.79832D-02	.50187D-03	-.12395	.25322D-03	.12470			
4										
	.36758D-01	.17560D-03	.83232D-02	2.4176	.34132D-01	-.18321D-01	-.40640D-01	.13751	.61624D-01	-.64700D-03
	-.74987D-04	.17964D-01	-.11424D-03	.68064D-01	.80266D-03	-.66210D-01	.10513D-02			
5										
	.13611D-01	.47531D-01	-.19421	.34132D-01	2.9082	-.10887D-01	-.57047D-01	-.93380D-01	.22304D-01	.64700D-01
	-.82485D-02	-.11853D-01	.48121D-02	.17546D-01	.48877D-01	-.19752D-01	-.51082D-01			
6										
	-.62287D-02	-.88206D-02	.21833D-01	-.18321D-01	-.10887D-01	1.9517	.17900D-01	.12133D-02	-.71622D-02	.32350D-02
	.16497D-02	-.18408D-02	.69992D-03	-.23183D-01	-.21318D-01	-.70203D-02	-.88853D-02			
7										
	-.12413D-01	-.39753D-01	.11475	-.40640D-01	-.57047D-01	.17900D-01	2.0464	.44106D-01	-.20896D-01	.64700D-03
	.16497D-02	-.18408D-02	.69992D-03	-.33751D-01	-.44940D-01	.35475D-02	.14737D-01			
8										
	.39304D-01	-.12186	.32670	.13751	-.93380D-01	.12133D-02	.44106D-01	5.1534	.38769D-01	.12940D-01
	.93733D-02	-.10358D-03	-.34182D-01	.65028D-01	-.86652D-01	-.64273D-01	.87407D-01			
9										
	.14162D-01	.86300D-02	-.20550D-01	.61624D-01	.22304D-01	-.71622D-02	-.20896D-01	.38769D-01	6.3181	-.12940D-02
	.74987D-04	.93963D-02	-.32554D-03	.24155D-01	.30199D-02	-.30549D-01	-.94129D-02			
10										
	.17000D-01	-.26640	-.30000D-01	-.10000D-01	1.0000	.50000D-01	.10000D-01	.20000	-.20000D-01	1.0000
	.0	.0	.0	.0	.0	.0	.0			
11										
	.32432D-02	.35676	-1.0054	-.32432D-02	-.35676	.71351D-01	.71351D-01	.40541	.32432D-02	.0
	1.0000	.0	.0	.0	.0	.0	.0			
12										
	-.16384	-.38400D-02	.41664	.38848	-.25632	-.39808D-01	-.39808D-01	-.22400D-02	.20320	.0
	.0	1.0000	.0	.0	.0	.0	.0			
13										
	-.15168D-01	.16586	.17264	-.24704D-02	.10406	.15136D-01	.15136D-01	-.73920	-.70400D-02	.0
	.0	.0	1.0000	.0	.0	.0	.0			
14										
	.89774D-01	.32940D-02	.19940D-02	.27042	.69712D-01	-.92108D-01	-.13409	.25836	.95972D-01	.0
	.0	.0	.0	1.0000	.0	.0	.0			

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 15
 .15186D-01  .17075     -.49246     .31890D-02  .19419     -.84698D-01 -.17855     -.34427      .11998D-01  .0
 .0          .0          .0         .0          1.0000      .0          .0

 16
-.58414D-01  .23060D-02  .10061D-02 -.26306     -.78476D-01 -.27892D-01  .14094D-01 -.25536     -.12137      .0
 .0          .0          .0         .0          .0          1.0000      .0

 17
 .16174D-01 -.16515      .49546     .41770D-02 -.20295     -.35302D-01  .58551D-01  .34727     -.37398D-01  .0
 .0          .0          .0         .0          .0          .0          1.0000

          FINAL COMBINED FORCE VECTOR OF ORDER 17  FOR PSI=  .0

-.38036D-05 -.87050D-05  .25502D-04 -.13759D-04 -.14091D-04  .29357D-05  .99558D-05  .45950D-05 -.68925D-05  .0
 .0          .0          .0          .0          .0          .0          .0

      STATE VARIABLES (DISP&VEL) FOR PSI =   .0

      -.32735D-11    -.25337D-08
      -.41554D-11    -.32163D-08
       .72632D-11     .56217D-08
      -.94506D-11    -.73148D-08
      -.76307D-11    -.59062D-08
       .22745D-11     .17605D-08
       .74409D-11     .57592D-08
       .11436D-11     .88514D-09
      -.16882D-11    -.13067D-08
       .62521D-11     .48391D-08
       .48912D-11     .37858D-08
      -.11306D-11    -.87507D-09
       .84278D-12     .65231D-09

      BIFILAR INITIAL DISP & VEL FOR PSI =   .0

       .44545D-11     .34478D-08
       .77832D-11     .60242D-08
      -.32281D-11    -.24986D-08
      -.65568D-11    -.50750D-08

          FIXED SYSTEM + ROTOR + FIXED ABSORBERS (R.H.S.) OF ORDER 13  (Ψ=2 DEG)

 .13417D-07  .40363D-07 -.48296D-06  .24756D-06  .22793D-06 -.35905D-07 -.12980D-06 -.87910D-07  .24116D-06 -.10012D-06
-.65309D-07  .14320D-07 -.10572D-07
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STATE VARIABLES (DISP&VEL) FOR PSI =   30.000

 -.31667D-04    -.47168D-02
  .18040D-04     .44988D-03
 -.23050D-04     .68643D-03
 -.79735D-04    -.11020D-01
  .13172D-06    -.28642D-02
  .80914D-05     .16604D-02
  .95903D-05     .37338D-02
 -.49609D-04    -.36824D-02
 -.93070D-05    -.13032D-02
  .13845D-04     .33312D-02
 -.47531D-05     .15521D-02
  .28783D-04     .13234D-02
 -.24599D-04    -.46604D-03

BIFILAR INITIAL DISP & VEL FOR PSI =   30.000

  .31746D-04     .49550D-02
 -.30011D-04     .17144D-03
 -.28218D-04    -.41782D-02
  .33540D-04     .60549D-03

NREV=  2, G1=  .634D-02, G2=  .327D-01, XHUB=  .310D-02, YHUB=  .263D-02, THZH=  .462D-05, DXHUB= -.863    , DYHUB= -.249

NREV=  3, G1=  .513D-02, G2=  .320D-01, XHUB=  .281D-02, YHUB=  .192D-02, THZH=  .337D-05, DXHUB= -.837    , DYHUB= -.330

NREV=  4, G1=  .480D-03, G2=  .238D-01, XHUB=  .411D-02, YHUB=  .248D-02, THZH=  .308D-05, DXHUB= -.829    , DYHUB= -.256

NREV=  5, G1=  .530D-02, G2=  .273D-01, XHUB=  .482D-02, YHUB=  .350D-02, THZH=  .533D-05, DXHUB= -1.01    , DYHUB= -.286

NREV=  6, G1=  .233D-02, G2=  .310D-01, XHUB=  .429D-02, YHUB=  .363D-02, THZH=  .699D-05, DXHUB= -1.16    , DYHUB= -.368
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NREV= 7, G1= .743D-02, G2= .172D-01, XHUB= .161D-03, YHUB= .161D-02, THZH= .475D-05, DXHUB= -.790 , DYHUB= -.416

NREV= 8, G1= .607D-02, G2= .605D-02, XHUB= .152D-02, YHUB= .109D-02, THZH= .268D-05, DXHUB= -.439 , DYHUB= -.163

NREV= 9, G1= .601D-02, G2= .596D-02, XHUB= .266D-02, YHUB= .230D-02, THZH= .421D-05, DXHUB= -.554 , DYHUB= -.976D-01

NREV= 10, G1= .264D-02, G2= .390D-02, XHUB= .187D-02, YHUB= .230D-02, THZH= .567D-05, DXHUB= -.688 , DYHUB= -.216

NREV= 11, G1= .376D-02, G2= .145D-02, XHUB= .134D-02, YHUB= .160D-02, THZH= .452D-05, DXHUB= -.623 , DYHUB= -.234

NREV= 12, G1= .106D-02, G2= .314D-02, XHUB= .217D-02, YHUB= .178D-02, THZH= .371D-05, DXHUB= -.563 , DYHUB= -.186

NREV= 13, G1= .198D-02, G2= .108D-02, XHUB= .272D-02, YHUB= .227D-02, THZH= .459D-05, DXHUB= -.610 , DYHUB= -.195

THE NUMBER OF REVOLUTIONS REQUIRED TO CONVERGE = 13

INPUT FIXED SYSTEM MODES FREQUENCIES IN HZ

5.10 6.40 15.3 14.3 13.8 11.6 12.1 17.4 21.1

THE CONVERGENCE FACTOR TO G = 30.2

BIFILAR HARMONIC OUTPUT - AMPLITUDE AND PHASE

	1 P	2 P	3 P	4 P	5 P	6 P	7 P	8 P	9 P
1	.67049D-02 167.18	.12416D-01 52.740	9.5710 87.610	.27991D-01 -124.05	.87991D-01 35.044	.11213D-01 70.857	.35755D-02 41.773	.44847D-02 39.427	.38823D-02 70.457
2	.41905D-02 9.1424	.24610D-01 -127.52	9.5752 177.26	.23692D-01 -29.191	.76112D-01 -55.124	.10635D-01 76.653	.26310D-02 98.182	.41733D-02 66.658	.30317D-02 54.734
3	.11155D-01 8.6491	.33980D-01 25.261	9.6299 -92.716	.22214D-01 82.317	.89253D-01 -146.67	.13026D-01 -57.202	.23000D-02 -173.77	.36585D-03 94.511	.28055D-02 -126.83
4	.61452D-02 129.49	.24692D-01 -168.30	9.6216 -2.4178	.28710D-01 179.05	.73405D-01 127.26	.16500D-01 -141.71	.50300D-02 -88.468	.36349D-02 -53.766	.36387D-02 -121.85

IBM Z30687

HARMONIC HUB OUTPUT - COSINE , SINE AND TOTAL RESPONSE AND PHASE ANGLE IN DEGREES

	1 P	2 P	3 P	4 P	5 P	6 P	7 P	8 P	9 P
1	.65430D-03	.58575D-03	.11206D-02	.35998D-01	.17899D-01	-.60414D-03	-.30850D-03	.56521D-03	-.14283D-03
	-.18117D-04	-.47222D-03	-.95128D-03	-.18974	.23803D-03	-.13125D-03	.96796D-04	.31234D-04	.12379D-03
	.65455D-03	.75240D-03	.14700D-02	.19312	.17901D-01	.61823D-03	.32333D-03	.56607D-03	.18901D-03
	-1.5860	-38.875	-40.327	-79.257	.76189	-167.74	162.58	3.1630	139.08
2	.60527D-03	.78265D-03	.12837D-02	.48626D-01	.11867D-01	-.78696D-03	-.42662D-03	-.23171D-03	-.20902D-03
	.45630D-04	.93018D-04	.37411D-03	-.71377D-01	-.16023D-02	-.62218D-03	-.28682D-03	-.61975D-03	-.14193D-03
	.60699D-03	.78816D-03	.13371D-02	.86366D-01	.11975D-01	.10032D-02	.51408D-03	.66165D-03	.25266D-03
	4.3112	6.7779	16.248	-55.735	-7.6898	-141.67	-146.09	-110.50	-145.82
3	-.84736D-04	-.36787D-04	-.87203D-04	-.66114D-02	.28069D-02	.15570D-03	.56732D-04	-.26395D-04	.17736D-04
	-.12085D-04	.10367D-03	.19798D-03	-.69261D-02	.42874D-02	-.25734D-03	-.13637D-03	-.10526D-03	-.74383D-04
	.85594D-04	.11000D-03	.21633D-03	.95751D-02	.51245D-02	.30077D-03	.14770D-03	.10852D-03	.76468D-04
	-171.88	109.54	113.77	-133.67	56.787	-58.825	-67.411	-104.08	-76.589
4	.23402D-04	.25569D-04	.42118D-04	.10883D-02	.15794D-03	-.39211D-04	-.18225D-04	-.53425D-05	-.78922D-05
	-.63601D-06	-.28530D-05	-.31023D-05	-.39214D-03	-.52730D-03	.29290D-05	.29562D-05	-.22754D-05	.21201D-05
	.23411D-04	.25728D-04	.42232D-04	.11568D-02	.55045D-03	.39321D-04	.18463D-04	.58069D-05	.81720D-05
	-1.5567	-6.3667	-4.2126	-19.816	-73.326	175.73	170.79	-156.93	164.96
5	.45204D-05	.31240D-05	.85702D-05	.32059D-03	.47932D-03	.66731D-05	.11688D-05	.61159D-05	-.27882D-06
	.21109D-05	-.15778D-05	-.99350D-06	-.39548D-02	.42771D-03	-.24576D-04	-.10858D-04	-.12090D-04	-.50560D-05
	.49890D-05	.34999D-05	.86276D-05	.39678D-02	.64241D-03	.25466D-04	.10921D-04	.13549D-04	.50636D-05
	25.032	-26.797	-6.6125	-85.366	41.744	-74.809	-83.856	-63.167	-93.156
6	.46929D-06	.14676D-05	.37223D-05	.12394D-03	.31970D-04	-.60135D-06	-.50591D-06	-.23038D-05	-.28873D-06
	.61710D-06	.13014D-05	.34197D-06	.63666D-04	.15021D-04	-.39849D-05	-.22633D-05	-.33538D-05	-.13859D-05
	.77527D-06	.19615D-05	.37380D-05	.13934D-03	.35323D-04	.40300D-05	.23191D-05	.40689D-05	.14157D-05
	52.748	41.564	5.2491	27.189	25.166	-98.582	-102.60	-124.49	-101.77

VIBRATION LEVELS AT 4 A/C LOCATIONS

COSINE,SINE,TOTAL AMPLITUDE IN G AND PHASE ANGLE IN DEGREES OF POINT 1

	1 P	2 P	3 P	4 P	5 P	6 P	7 P	8 P	9 P
1	-.27695D-04	-.59704D-04	-.90025D-04	-.36910D-02	-.13996D-02	.17316D-04	.12665D-04	-.25113D-04	.60542D-05
	.21009D-04	.16851D-04	.91842D-04	.26162D-01	-.10014D-02	.25028D-05	-.15837D-04	.23570D-04	-.15682D-04
	.34762D-04	.62037D-04	.12861D-03	.26421D-01	.17210D-02	.17496D-04	.20278D-04	.34442D-04	.16810D-04
	142.82	164.24	134.43	98.030	-144.42	8.2244	-51.351	136.82	-68.890

	1 P	2 P	3 P	4 P	5 P	6 P	7 P	8 P	9 P
2	-.72084D-04	-.16597D-03	-.37226D-03	.63784D-02	-.10120D-02	.34856D-03	.14439D-03	-.66094D-04	.54919D-04
	.62235D-04	.65610D-04	.28222D-03	-.51076D-02	.65640D-02	-.22498D-03	-.14968D-03	-.18375D-03	-.94960D-04
	.95233D-04	.17846D-03	.46714D-03	.81713D-02	.66416D-02	.41486D-03	.20797D-03	.19527D-03	.10970D-03
	139.19	158.43	142.83	-38.686	98.765	-32.840	-46.031	-109.78	-59.957
3	-.94080D-04	-.26315D-03	-.37530D-03	.25661D-02	-.53434D-03	.38477D-03	.16573D-03	.83107D-04	.65015D-04
	.99993D-04	-.62958D-04	.75563D-04	-.20262D-01	.71374D-02	-.13114D-03	-.85545D-04	-.10625D-03	-.51631D-04
	.13729D-03	.27057D-03	.38283D-03	.20424D-01	.71573D-02	.40650D-03	.18650D-03	.13489D-03	.83022D-04
	133.26	-166.54	168.62	-82.782	94.281	-18.821	-27.302	-51.968	-38.454

COSINE,SINE,TOTAL AMPLITUDE IN G AND PHASE ANGLE IN DEGREES OF POINT 2

	1 P	2 P	3 P	4 P	5 P	6 P	7 P	8 P	9 P
1	-.39484D-04	-.49805D-04	-.87375D-04	-.40499D-02	-.20487D-02	.45642D-05	.88678D-05	-.46729D-04	.57362D-05
	.95415D-05	.29269D-04	.88404D-04	.28955D-01	-.15027D-02	.40698D-04	.29137D-05	.31377D-04	-.62740D-05
	.40620D-04	.57769D-04	.12430D-03	.29237D-01	.25407D-02	.40953D-04	.93343D-05	.56286D-04	.85010D-05
	166.41	149.56	134.66	97.962	-143.74	83.601	18.189	146.12	-47.564
2	.11862D-03	-.29671D-04	-.12265D-03	.22750D-02	.82010D-02	.21690D-03	.68740D-04	.37644D-04	.14387D-04
	.83251D-04	.34504D-04	.20731D-03	-.20037D-01	.68821D-02	-.65465D-03	-.34417D-03	-.19868D-03	-.19107D-03
	.14492D-03	.45507D-04	.24087D-03	.20166D-01	.10706D-01	.68965D-03	.35096D-03	.20222D-03	.19161D-03
	35.062	130.69	120.61	-83.522	40.002	-71.669	-78.705	-79.271	-85.694
3	-.11616D-03	-.36398D-03	-.45786D-03	.69180D-02	-.89367D-02	.33506D-03	.16816D-03	-.61372D-04	.78882D-04
	.14958D-03	-.13546D-03	-.16147D-03	.10018D-01	.22078D-02	.28392D-03	.98974D-04	-.38464D-04	.31437D-04
	.18939D-03	.38837D-03	.48549D-03	.12175D-01	.92054D-02	.43918D-03	.19512D-03	.72430D-04	.84915D-04
	127.83	-159.59	-160.57	55.373	166.12	40.277	30.480	-147.92	21.729

COSINE,SINE,TOTAL AMPLITUDE IN G AND PHASE ANGLE IN DEGREES OF POINT 3

	1 P	2 P	3 P	4 P	5 P	6 P	7 P	8 P	9 P
1	-.15004D-04	-.50245D-05	-.22113D-04	-.13559D-02	-.14941D-02	-.26787D-04	-.62453D-05	-.24176D-04	-.14351D-06
	-.10084D-04	.10199D-04	.73033D-05	.13049D-01	-.14495D-02	.77287D-04	.34237D-04	.38962D-04	.15954D-04
	.18078D-04	.11369D-04	.23288D-04	.13119D-01	.20817D-02	.81797D-04	.34802D-04	.45853D-04	.15955D-04
	-146.10	116.23	161.72	95.933	-135.87	109.12	100.34	121.82	90.515
2	.38069D-03	.48545D-03	.38884D-03	.52029D-02	.45986D-02	-.51517D-03	-.24187D-03	-.15102D-03	-.10098D-03
	.26912D-04	-.83176D-04	-.29612D-04	-.12030D-01	-.67643D-02	-.73314D-04	-.80477D-05	-.75078D-04	-.35946D-05
	.38164D-03	.49253D-03	.38997D-03	.13107D-01	.81794D-02	.52036D-03	.24201D-03	.16865D-03	.10104D-03
	4.0437	-9.7225	-4.3549	-66.612	-55.791	-171.90	-178.09	-153.57	-177.96
3	-.22441D-04	-.31298D-03	-.33762D-03	-.13867D-01	.18382D-02	.26617D-03	.11676D-03	.11170D-03	.44076D-04
	.19563D-03	-.16403D-03	.34856D-05	.40963D-02	.42725D-02	-.22817D-03	-.13753D-03	-.43835D-04	-.83262D-04

	.19691D-03	.35336D-03	.33764D-03	.14460D-01	.46512D-02	.35058D-03	.18041D-03	.11999D-03	.94208D-04
	96.544	-152.34	179.41	163.54	66.720	-40.605	-49.670	-21.427	-62.105

COSINE,SINE,TOTAL AMPLITUDE IN G AND PHASE ANGLE IN DEGREES OF POINT 4

	1 P	2 P	3 P	4 P	5 P	6 P	7 P	8 P	9 P
1	-.55953D-03	.20750D-03	-.87139D-03	-.16897D-01	-.12049D-01	.27150D-03	.16528D-03	.29958D-04	.94838D-04
	-.61893D-03	.44420D-03	.65862D-03	.92566D-02	-.44070D-03	.95565D-03	.53245D-03	.55112D-03	.31675D-03
	.83436D-03	.49027D-03	.10923D-02	.19266D-01	.12057D-01	.99347D-03	.55751D-03	.55193D-03	.33065D-03
	-132.11	64.961	142.92	151.28	-177.91	74.140	72.755	86.889	73.332
2	.52005D-03	-.13840D-03	.15258D-03	-.14447D-01	.45731D-02	-.42795D-03	-.18518D-03	.55066D-04	-.83739D-04
	.53036D-04	-.16442D-03	-.22733D-03	-.11566D-01	-.80284D-02	-.10840D-03	-.47682D-04	.30537D-03	-.15255D-04
	.52275D-03	.21492D-03	.27379D-03	.18506D-01	.92395D-02	.44147D-03	.19122D-03	.31029D-03	.85117D-04
	5.8231	-130.09	-56.130	-141.32	-60.334	-165.79	-165.56	79.778	-169.68
3	.40576D-03	.12381D-04	.29263D-04	.19887D-01	.19456D-02	-.13007D-03	-.66639D-04	.11750D-03	-.31806D-04
	.32976D-03	-.44204D-03	-.27504D-03	-.25319D-01	-.10863D-02	-.78699D-04	-.42304D-04	-.14298D-03	-.32329D-04
	.52286D-03	.44221D-03	.27660D-03	.32195D-01	.22283D-02	.15203D-03	.78932D-04	.18507D-03	.45352D-04
	39.100	-88.396	-83.927	-51.852	-29.176	-148.82	-147.59	-50.585	-134.53

INITIAL BIFILAR DISPLACEMENTS (LC.1720-1739)

-.95150D-02 -.16587 .96961D-02 .16657

INITIAL BIFILAR VELOCITIES (LC.1740-1759)

13.629 -1.6505 -13.697 1.7188

INITIAL HUB DISPLACEMENTS (LC.1768-1773)

.26291D-02 .23980D-02 -.90123D-04 .44863D-04 .44689D-04 .55148D-05

INITIAL HUB VELOCITIES (LC.1774-1779)

-.63632 -.22496 -.48864D-02 -.33717D-02 -.11282D-01 .39509D-03

INITIAL STATE VARIABLES DISPL.(LC.1780-1859)

.48442D-05 -.24740D-03 .10737D-02 .25081D-02 -.23990D-03 -.24822D-03 -.36078D-03 .20783D-02 -.33130D-03 .20011D-02
.27121D-01 -.75610D-01 .55942D-01

INITIAL STATE VARIABLES VELOC.(LC.1860-1939)

```
 .43979D-01   .54850D-02  -.22847      .10464       .81171D-01  -.44101D-02  -.38748D-01  -.52990      -.13892      -.63792
-4.4349       6.5571      9.2277
```

Appendix D. Details of Coupled Rotor/Bifilar/Airframe Analysis

TABLE OF CONTENTS

D.1 Inplane Bifilar Equations of Motion

The inplane bifilar rotor head absorber is a single degree-of-freedom system. It can be represented as a one degree-of-freedom pendulum as shown in Figure D.1-1. The rotating coordinate system shown here is located on the main rotor hub. The equivalent pendulum arm, r, and the distance from the center of rotation, R, are related by equation (1) (see Figure D.1-2), often referred to as tuning equation. The location of the c.g. relative to the inertia frame system is given by equation (2) shown in Figure D.1-2. Substituting equation (2) into the Lagrange's equations, the non-linear equations of motion are derived, equations (3)-(6), and shown in Figure D.1-3.

To obtain the linear set of equations of motion, equations (7)-(10) (see Figure D.1-4), small angle assumptions were made, e.g. $\sin\gamma_k \sim \gamma k$, and $\cos\gamma_k \sim 1$. Also, the second order terms were neglected. For identical bifilars, further simplification of these equations can be made by transferring the rotating bifilar coordinate, γ_k, into the fixed system coordinates. The equation for the coordinate transformation used in shown in Figure D.1-5, equation (11). Thus the transformed equations of motion are derived, equations (12)-(17) shown in Figure D.1-5.

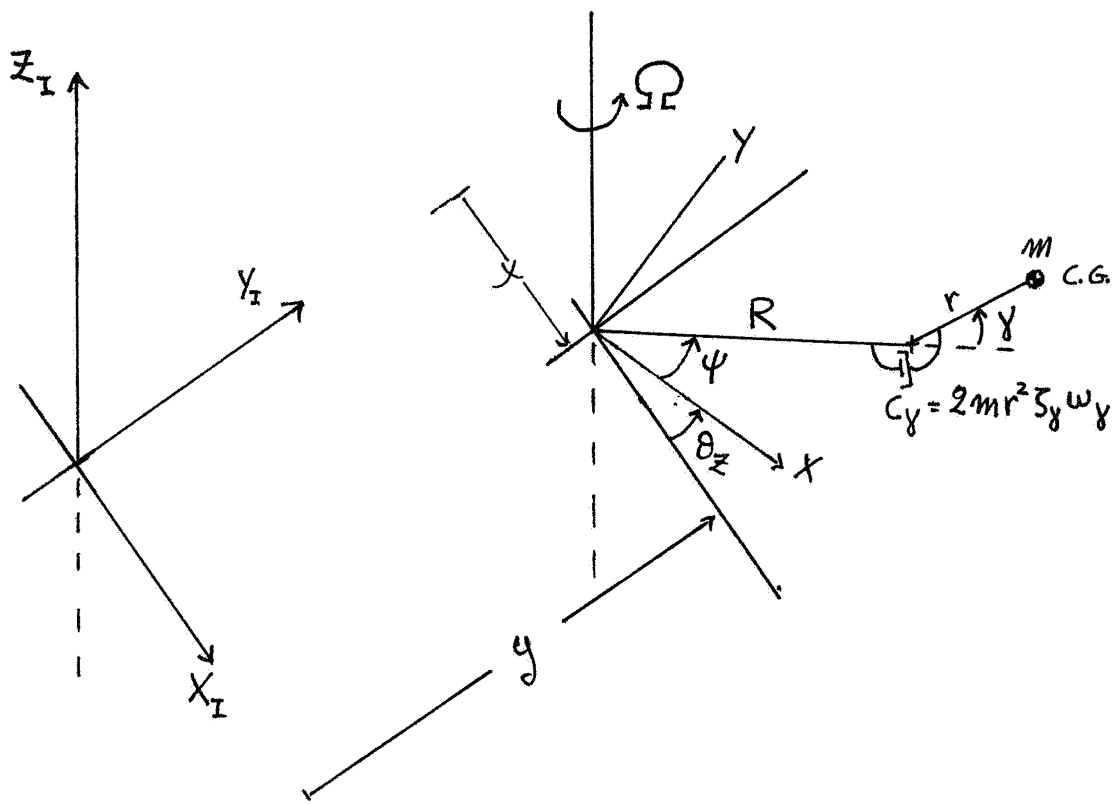

FIGURE D.1-1: INPLANE BIFILAR MATH MODEL

$$\eta = \sqrt{\frac{R}{r}} \quad (\text{TUNING} \sim \text{PER REV}) \qquad (1)$$

$$\{X_I\} = \{X\} + [\Theta_Z]\cdot[\Psi]\cdot\{R\{V_1\} + r[\Gamma]\cdot\{V_1\}\} \qquad (2)$$

WHERE:

$$[\Theta_Z] = \begin{bmatrix} \cos\theta_Z & -\sin\theta_Z & 0 \\ \sin\theta_Z & \cos\theta_Z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[\Psi] = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad [\Gamma] = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\{V_1\} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

FIGURE D.1-2: POSITION VECTOR FOR INPLANE BIFILAR MASS

X- EQUATION

$$(m_G + M_T)\ddot{X} + 2 m_G \omega_x \zeta_x \dot{X} + m_G \omega_x^2 X$$

$$+\sum_{K=1}^{N} m_K \Big\{ -\big[r_K \sin(\psi_K + \theta_z + \gamma_K) + R_K \sin(\psi_K + \theta_z)\big]\ddot{\theta}_z - \big[r_K \sin(\psi_K + \theta_z + \gamma_K)\big]\ddot{\gamma}_K$$

$$-2\Omega\big[r_K \cos(\psi_K + \theta_z + \gamma_K)\big]\dot{\gamma}_K - \big[r_K \cos(\psi_K + \theta_z + \gamma_K)\big]\dot{\gamma}_K^2$$

$$-2\big[r_K \cos(\psi_K + \theta_z + \gamma_K)\big]\dot{\theta}_z \dot{\gamma}_K - 2\Omega\big[r_K \cos(\psi_K + \theta_z + \gamma_K) + R_K \cos(\psi_K + \theta_z)\big]\dot{\theta}_z$$

$$-\big[r_K \cos(\psi_K + \theta_z + \gamma_K) + R_K \cos(\psi_K + \theta_z)\big]\dot{\theta}_z^2$$

$$-\Omega^2\big[r_K \cos(\psi_K + \theta_z + \gamma_K) + R_K \cos(\psi_K + \theta_z)\big]\Big\} = F_X \qquad (3)$$

FIGURE D.1-3: NON-LINEAR EQUATIONS OF MOTION FOR INPLANE BIFILAR

Y- EQUATION

$$(m_G+M_T)\ddot{y} + 2m_G\omega_y\zeta_y\dot{y} + m_G\omega_y^2 y$$

$$+\sum_{K=1}^{N} m_K\Big\{\big[r_K\cos(\psi_K+\theta_z+\gamma_K)+R_K\cos(\psi_K+\theta_z)\big]\ddot{\theta}_z+\big[r_K\cos(\psi_K+\theta_z+\gamma_K)\big]\ddot{\gamma}_K$$

$$-2\Omega\big[r_K\sin(\psi_K+\theta_z+\gamma_K)\big]\dot{\gamma}_K-\big[r_K\sin(\psi_K+\theta_z+\gamma_K)\big]\dot{\gamma}_K^2$$

$$-2\big[r_K\sin(\psi_K+\theta_z+\gamma_K)\big]\dot{\theta}_z\dot{\gamma}_K-2\Omega\big[r_K\sin(\psi_K+\theta_z+\gamma_K)+R_K\sin(\psi_K+\theta_z)\big]\dot{\theta}_z$$

$$-\big[r_K\sin(\psi_K+\theta_z+\dot{\gamma}_K)+R_K\sin(\psi_K+\theta_z)\big]\dot{\theta}_z^2$$

$$-\Omega^2\big[r_K\sin(\psi_K+\theta_z+\gamma_K)+R\sin(\psi_K+\theta_z)\big]\Big\} = F_y \qquad (4)$$

FIGURE D.1-3: NON-LINEAR EQUATIONS OF MOTION FOR INPLANE BIFILAR (CONTINUED)

θ_z - EQUATION

$$\left\{ m_G + \left[\sum_{K=1}^{N} m_K \left(r_K + R_K + 2 r_K R_K \cos\gamma_K \right) \right] \right\} \ddot{\theta}_z$$

$$+ 2 m_G \omega_{\theta z} \zeta_{\theta z} \dot{\theta}_z + m_G \omega_{\theta z}^2 \theta_z$$

$$+ \sum_{K=1}^{N} m_K \Big\{ - \left[r_K \sin\left(\psi_K + \theta_z + \gamma_K\right) + R_K \sin\left(\psi_K + \theta_z\right) \right] \ddot{X}$$

$$+ \left[r_K \cos\left(\psi_K + \theta_z + \gamma_K\right) + R_K \cos\left(\psi_K + \theta_z\right) \right] \ddot{Y} + r_K \left[r_K + R_K \cos\gamma_K \right] \ddot{\gamma}_K$$

$$- \left[2 r_K R_K \Omega \sin\gamma_K \right] \dot{\gamma}_K - \left[2 r_K R_K \sin\gamma_K \right] \dot{\theta}_z \dot{\gamma}_K$$

$$+ \left[r_K \sin\gamma_K \left(2 r_K \sin\gamma_K + R_K \right) \right] \dot{\gamma}_K^2 \Big\} = M_z \qquad (5)$$

FIGURE D.1-3: NON-LINEAR EQUATIONS OF MOTION FOR INPLANE BIFILAR (CONTINUED)

γ_K - EQUATION

$$m_K r_K \Big\{ -\left[\sin(\psi_K + \theta_z + \gamma_K)\right]\ddot{x} + \left[\cos(\psi_K + \theta_z + \gamma_K)\right]\ddot{y}$$

$$+ \left[r_K + R_K \cos\gamma_K\right]\ddot{\theta}_z + r_K \ddot{\gamma}_K + \left[2\, r_K\, \zeta_{\gamma_K}\, \omega_{\gamma_K}\right]\dot{\gamma}_K$$

$$+ \left[2\Omega R_K \sin\gamma_K\right]\dot{\theta}_z + \left[R_K \sin\gamma_K\right]\dot{\theta}_z^2$$

$$+ R_K \Omega^2 \sin\gamma_K \Big\} = 0.0 \qquad (6)$$

FIGURE D.1-3: NON-LINEAR EQUATIONS OF MOTION FOR INPLANE BIFILAR (CONCLUDED)

X- EQUATION

$$(m_G + M_T)\ddot{X} + 2m_G\omega_X\zeta_X\dot{X} + m_G\omega_X^2 X$$

$$+\sum_{K=1}^{N} m_K\Big\{-\big[(r_K+R_K)\sin\psi_K\big]\ddot{\theta}_Z - \big[r_K\sin\psi_K\big]\ddot{\gamma}_K$$

$$-\big[2\Omega(r_K+R_K)\cos\psi_K\big]\dot{\theta}_Z - \big[2\Omega r_K\cos\psi_K\big]\dot{\gamma}_K$$

$$-\Omega^2\big[(r_K+R_K)\cos\psi_K - r_K\gamma_K\sin\psi_K - (r_K+R_K)\theta_Z\sin\psi_K\big]\Big\} = F_X \qquad (7)$$

FIGURE D.1-4: LINEAR EQUATIONS OF MOTION FOR INPLANE BIFILAR

Y- EQUATION

$$(m_G + M_T)\ddot{y} + 2 m_G \omega_y \zeta_y \dot{y} + m_G \omega_y^2 y$$

$$+ \sum_{K=1}^{N} m_K \Big\{ \big[(r_K + R_K)\cos\psi_K\big]\ddot{\theta}_z + \big[r_K \cos\psi_K\big]\ddot{\gamma}_K$$

$$- \big[2\Omega(r_K + R_K)\sin\psi_K\big]\dot{\theta}_z - \big[2\Omega r_K \sin\psi_K\big]\dot{\gamma}_K$$

$$- \Omega^2\big[(r_K + R_K)\sin\psi_K + r_K \gamma_K \cos\psi_K + (r_K + R_K)\theta_z \cos\psi_K\big]\Big\} = F_y \qquad (8)$$

FIGURE D.1-4: LINEAR EQUATIONS OF MOTION FOR INPLANE BIFILAR (CONTINUED)

θ_z - EQUATION

$$\left\{ m_G + \left[\sum_{k=1}^{N} m_k (r_k + R_k)^2 \right] \right\} \ddot{\theta}_z$$

$$+ 2 m_G \omega_{\theta z} \zeta_{\theta z} \dot{\theta}_z \quad + \quad m_G \omega_{\theta z}^2 \theta_z$$

$$+ \sum_{k=1}^{N} m_k \left\{ - \left[(r_k + R_k) \sin \psi_k \right] \ddot{X} \; + \left[(r_k + R_k) \cos \psi_k \right] \ddot{Y} \right.$$

$$\left. + \left[r_k (r_k + R_k) \right] \ddot{\gamma}_k \right\} = M_{\theta z} \qquad (9)$$

FIGURE D.1-4: LINEAR EQUATIONS OF MOTION FOR INPLANE BIFILAR (CONTINUED)

γ_k - EQUATION

$$m_k r_k \Big\{ [-\sin\psi_k]\ddot{x} + [\cos\psi_k]\ddot{y}$$

$$+[r_k + R_k]\ddot{\theta}_z + r_k\ddot{\gamma}_k$$

$$+[2 r_k \zeta_k \omega_{\gamma_k}]\dot{\gamma}_k + R_k\Omega^2\gamma_k \Big\} = 0.0 \qquad (10)$$

FIGURE D.1-4: LINEAR EQUATIONS OF MOTION FOR INPLANE BIFILAR (CONCLUDED)

$$\gamma = \frac{1}{N}\gamma_o + \frac{2}{N}\gamma_s \sin\psi + \frac{2}{N}\gamma_c \cos\psi \qquad (11)$$

X- EQUATION

$$(m_G + M_T)\ddot{X} + (2m_G\omega_x\zeta_x)\dot{X} + (m_G\omega_x^2)X - mr\ddot{\gamma}_s = F_x \qquad (12)$$

Y- EQUATION

$$(m_G + M_T)\ddot{Y} + (2m_G\omega_y\zeta_y)\dot{Y} + (m_G\omega_y^2)Y + mr\ddot{\gamma}_c = F_y \qquad (13)$$

θ_z - EQUATION

$$\left[m_G + M_T(r+R)^2\right]\ddot{\theta}_z + (2m_G\omega_{\theta_z}\zeta_{\theta_z})\dot{\theta}_z + (m_G\omega_{\theta_z}^2)\theta_z$$

$$+ \left[m(r+R)r\right]\ddot{\gamma}_o = M_z \qquad (14)$$

FIGURE D.1-5: INPLANE BIFILAR EQUATIONS IN FIXED SYSTEM COORDINATES

γ_0 - EQUATION

$$N\left(1+\frac{R}{r}\right)\ddot{\theta}_z + \ddot{\gamma}_0 + \frac{R}{r}\Omega^2\gamma_0 + 2\zeta_\gamma\omega_\gamma\dot{\gamma}_0 = 0 \qquad (15)$$

γ_S - EQUATION

$$\left(-\frac{N}{2r}\right)\ddot{x} + \ddot{\gamma}_s + 2\zeta_\gamma\omega_\gamma\dot{\gamma}_s - 2\Omega\dot{\gamma}_c + \Omega^2\gamma_s\left(\frac{R}{r}-1\right) - 2\zeta_\gamma\omega_\gamma\Omega\gamma_c = 0 \qquad (16)$$

γ_C - EQUATION

$$\left(\frac{N}{2r}\right)\ddot{y} + \ddot{\gamma}_c + 2\zeta_\gamma\omega_\gamma\dot{\gamma}_c + 2\Omega\dot{\gamma}_s + \Omega^2\left(\frac{R}{r}-1\right)\gamma_c + 2\zeta_\gamma\omega_\gamma\Omega\gamma_s = 0 \qquad (17)$$

FIGURE D.1-5 INPLANE BIFILAR EQUATIONS IN FIXED SYSTEM COORDINATES (CONCLUDED)

D.2 Vertical Bifilar Equations of Motion

The vertical bifilar rotor head absorber is a single degree of freedom system. The math model is presented in Figure D.2-1. The equivalent pendulum arm, r, and the distance from the center of rotation, R, are related by the tuning equation (18), shown in Figure D.2-2. The vector location of the dynamic mass (c.g.) is given by equation (19), shown in Figure D.2-2. Substituting equation (19) into the Lagrange's equation, also assuming small motions and neglecting second order terms, the linear set of equations of motion with six hub degrees-of-freedom are derived. They are shown in Figure D.2-3, equations (20)-(26).

For identical vertical bifilars, further simplification of these equations can be made by transferring the rotating coordinate, β_k, into the fixed system coordinates. The equation for the coordinate transformation used is shown in Figure D.2-4, equation (27). Thus the transformed equations of motions are derived, equations (28)-(36), shown in Figure D.2-4.

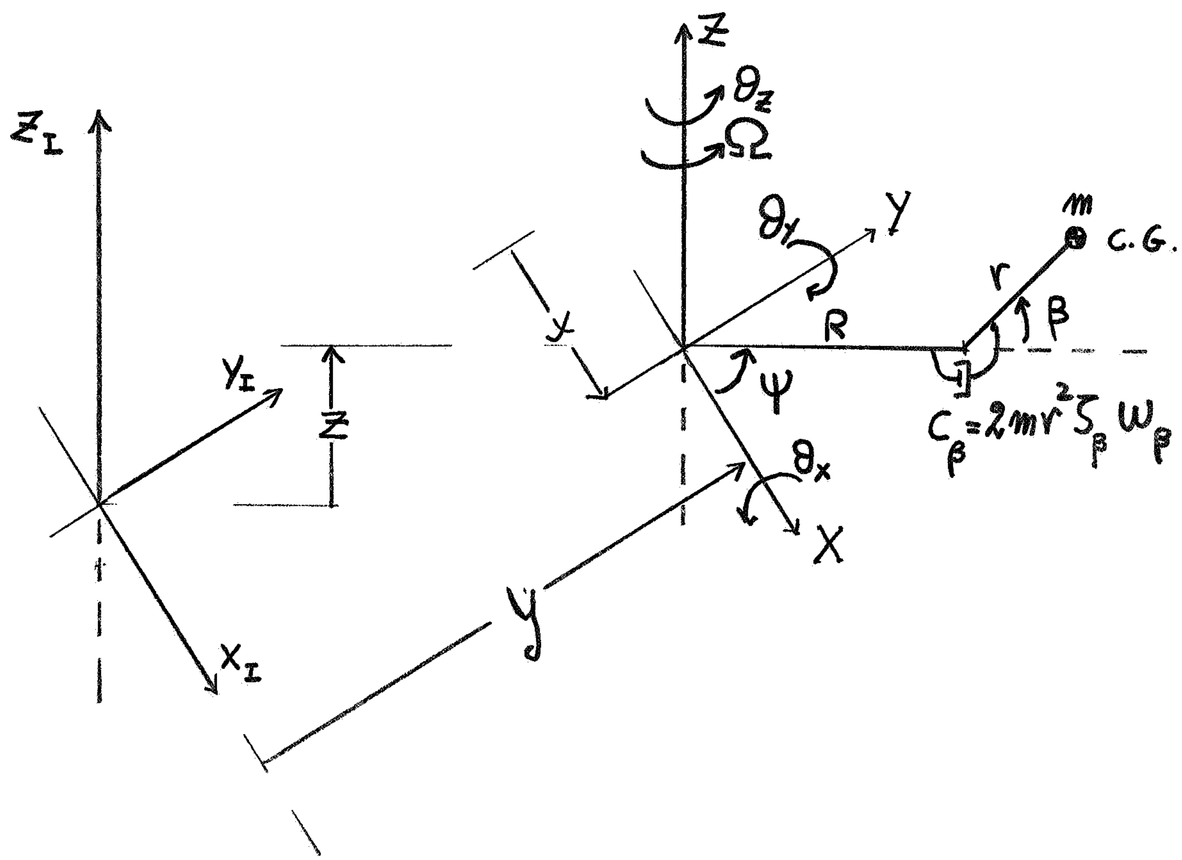

FIGURE D.2-1: VERTICAL BIFILAR MATH MODEL

$$\eta = \sqrt{\frac{R+r}{r}} \quad (\text{TUNING} \sim \text{PER REV}) \qquad (18)$$

$$\{X_I\} = \{X\} + [\Theta_z]\cdot[\Theta_x]\cdot[\Theta_y]\cdot[\Psi]\cdot\{R\{V_1\} + r[B]\cdot\{V_1\}\} \qquad (19)$$

WHERE:

$$[\Theta_z] = \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad [\Theta_x] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{bmatrix}$$

$$[\Theta_y] = \begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y \\ 0 & 1 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y \end{bmatrix} \qquad [\Psi] = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[B] = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \qquad \{V_1\} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

FIGURE D.2-2: POSITION VECTOR FOR VERTICAL BIFILAR MASS

X- EQUATION

$$
\begin{aligned}
&(m_G + M_T)\ddot{x} + (2 m_G \omega_x \zeta_x)\dot{x} + (m_G \omega_x^2)\, x \\
&+ \sum_{k=1}^{N} m_k \Big\{ -\left[(r_k + R_k)\sin\psi_k\right]\ddot{\theta}_z - \left[2\Omega (r_k + R_k)\cos\psi_k\right]\dot{\theta}_z \\
&\qquad + \left[\Omega^2 (r_k + R_k)\sin\psi_k\right]\theta_z - \Omega^2 (r_k + R_k)\cos\psi_k \Big\} = F_x
\end{aligned}
\tag{20}
$$

Y- EQUATION

$$
\begin{aligned}
&(m_G + M_T)\ddot{y} + (2 m_G \omega_y \zeta_y)\dot{y} + (m_G \omega_y^2)\, y \\
&+ \sum_{k=1}^{N} m_k \Big\{ \left[(r_k + R_k)\cos\psi_k\right]\ddot{\theta}_z - \left[2\Omega (r_k + R_k)\sin\psi_k\right]\dot{\theta}_z \\
&\qquad - \left[\Omega^2 (r_k + R_k)\cos\psi_k\right]\theta_z - \Omega^2 (r_k + R_k)\sin\psi_k \Big\} = F_y
\end{aligned}
\tag{21}
$$

FIGURE D.2-3: LINEAR EQUATIONS OF MOTION FOR VERTICAL BIFILAR

Z - EQUATION

$$
\begin{aligned}
&(m_G+M_T)\ddot{Z} + (2m_G\omega_Z\zeta_Z)\dot{Z} + (m_G\omega_Z^2)Z \\
&+\sum_{K=1}^{N} m_K\Big\{(r_K)\ddot{\beta}_K + [(r_K+R_K)\sin\psi_K]\ddot{\theta}_X + [-(r_K+R_K)\cos\psi_K]\ddot{\theta}_Y \\
&+[2\Omega(r_K+R_K)\cos\psi_K]\dot{\theta}_X + [2\Omega(r_K+R_K)\sin\psi_K]\dot{\theta}_Y \\
&-[\Omega^2(r_K+R_K)\sin\psi_K]\theta_X + [\Omega^2(r_K+R_K)\cos\psi_K]\theta_Y\Big\} = F_Z
\end{aligned}
\tag{22}
$$

FIGURE D.2-3: LINEAR EQUATIONS OF MOTION FOR VERTICAL BIFILAR (CONTINUED)

θ_x - EQUATION

$$
\begin{aligned}
&\left\{ m_G + \left[\sum_{k=1}^{N} m_k (r_k + R_k)^2 \sin^2 \psi_k \right] \right\} \ddot{\theta}_x \\
&+ \left\{ 2 m_G \omega_{\theta x} \zeta_{\theta x} + \left[\sum_{k=1}^{N} 2 m_k \Omega (r_k + R_k)^2 \cos\psi_k \sin\psi_k \right] \right\} \dot{\theta}_x \\
&+ \left(m_G \omega_{\theta_x}^2 \right) \theta_x \\
&+ \sum_{k=1}^{N} m_k \Big\{ \left[r_k (r_k + R_k) \sin\psi_k \right] \ddot{\beta}_k + \left[(r_k + R_k) \sin\psi_k \right] \ddot{z} \\
&\quad + \left[-(r_k + R_k)^2 \cos\psi_k \sin\psi_k \right] \ddot{\theta}_y + \left[2\Omega (r_k + R_k) \cos\psi_k \right] \dot{z} \\
&\quad + \left[2\Omega (r_k + R_k)^2 \sin^2\psi_k \right] \dot{\theta}_y + \left[r_k \Omega^2 (r_k + R_k) \sin\psi_k \right] \beta_k \\
&\quad + \left[\Omega^2 (r_k + R_k)^2 \sin\psi_k \cos\psi_k \right] \theta_y \Big\} = M_x \qquad (23)
\end{aligned}
$$

FIGURE D.2-3: LINEAR EQUATIONS OF MOTION FOR VERTICAL BIFILAR (CONTINUED)

θ_y - EQUATION

$$\left\{m_G + \left[\sum_{K=1}^{N} m_K (r_K + R_K)^2 \cos^2\psi_K\right]\right\} \ddot{\theta}_y \quad + \left(m_G\, \omega_{\theta y}^2\right)\theta_y$$

$$+\left\{2 m_G\, \omega_{\theta y} \zeta_{\theta y} - 2\Omega\left[\sum_{K=1}^{N} m_K (r_K + R_K)^2 \sin\psi_K \cos\psi_K\right]\right\} \dot{\theta}_y$$

$$+\sum_{K=1}^{N} m_K \Big\{ -\left[r_K (r_K + R_K)\cos\psi_K\right]\ddot{\beta}_K - \left[(r_K + R_K)\cos\psi_K\right]\ddot{z}$$

$$-\left[(r_K + R_K)^2 \cos\psi_K \sin\psi_K\right]\ddot{\theta}_x + \left[2\Omega(r_K + R_K)\sin\psi_K\right]\dot{z}$$

$$-\left[2\Omega (r_K + R_K)^2 \cos^2\psi_K\right]\dot{\theta}_x + \left[r_K \Omega^2 (r_K + R_K)\cos\psi_K\right]\beta_K = M_y$$

$$(24)$$

FIGURE D.2-3: LINEAR EQUATIONS OF MOTION FOR VERTICAL BIFILAR (CONTINUED)

ϑ_z - EQUATION

$$\left[m_G + \sum_{k=1}^{N} m_k (r_k + R_k)^2\right] \ddot{\vartheta}_z$$

$$+ \left[2 m_G \omega_{\theta_z} \zeta_{\theta_z}\right] \dot{\vartheta}_z + \left(m_G \omega_{\theta_z}^2\right) \vartheta_z$$

$$+ \sum_{k=1}^{N} m_k \Big\{ -\left[(r_k + R_k) \sin \psi_k\right] \ddot{X} + \left[(r_k + R_k) \cos \psi_k\right] \ddot{Y}$$

$$- \left[2\Omega (r_k + R_k) \cos \psi_k\right] \dot{X} - \left[2\Omega (r_k + R_k) \sin \psi_k\right] \dot{Y} = M_z$$

(25)

FIGURE D.2-3: LINEAR EQUATIONS OF MOTION FOR VERTICAL BIFILAR (CONTINUED)

β_k - EQUATION

$$(m_k r_k)\ddot{z} + \left[m_k r_k (r_k + R_k)\sin\psi_k\right]\ddot{\theta}_x + \left(m_k r_k^2\right)\ddot{\beta}$$

$$-\left[m_k r_k (r_k + R_k)\cos\psi_k\right]\ddot{\theta}_y + \left[2 m_k r_k^2 \zeta_{\beta_k} \omega_{\beta_k}\right]\dot{\beta}_k$$

$$+\left[2 m_k r_k \Omega (r_k + R_k)\cos\psi_k\right]\dot{\theta}_x$$

$$+\left[2 m_k r_k \Omega (r_k + R_k)\sin\psi_k\right]\dot{\theta}_y + \left[m_k r_k \Omega^2 (r_k + R_k)\right]\beta_k = 0 \qquad (26)$$

FIGURE D.2-3: LINEAR EQUATIONS OF MOTION FOR VERTICAL BIFILAR (CONCLUDED)

$$\beta = \frac{1}{N}\beta_0 + \frac{2}{N}\beta_s \sin\psi + \frac{2}{N}\beta_c \cos\psi \qquad (27)$$

X- EQUATION

$$\left(m_G + M_T\right)\ddot{X} + \left(2 m_G \omega_X \zeta_X\right)\dot{X} + \left(m_G \omega_X^2\right) X = F_X \qquad (28)$$

Y- EQUATION

$$\left(m_G + M_T\right)\ddot{Y} + \left(2 m_G \omega_Y \zeta_Y\right)\dot{Y} + \left(m_G \omega_Y^2\right) Y = F_Y \qquad (29)$$

Z- EQUATION

$$\left(m_G + M_T\right)\ddot{Z} + \left(2 m_G \omega_Z \zeta_Z\right)\dot{Z} + \left(m_G \omega_Z^2\right) Z + m r \ddot{\beta}_0 = F_Z \qquad (30)$$

FIGURE D.2-4: VERTICAL BIFILAR EQUATIONS IN FIXED SYSTEM COORDINATES

θ_x - EQUATION

$$\left[m_G + \tfrac{1}{2} M_T (r+R)^2\right] \ddot{\theta}_x + \left(2 m_G \omega_{\theta x} \zeta_{\theta x}\right) \dot{\theta}_x + \left(m_G \omega_{\theta x}^2\right) \theta_x$$

$$+ \left[m(r+R)r\right] \ddot{\beta}_s - \left[2 m r \Omega (r+R)\right] \dot{\beta}_c + \left[M_T \Omega (r+R)^2\right] \dot{\theta}_y = M_x \qquad (31)$$

θ_y - EQUATION

$$\left[m_G + \tfrac{1}{2} M_T (r+R)^2\right] \ddot{\theta}_y + \left(2 m_G \omega_{\theta y} \zeta_{\theta y}\right) \dot{\theta}_y + \left(m_G \omega_{\theta y}^2\right) \theta_y$$

$$+ \left[- m r (r+R)\right] \ddot{\beta}_c - \left[2 m r \Omega (r+R)\right] \dot{\beta}_s - \left[M_T \Omega (r+R)^2\right] \dot{\theta}_x = M_y \qquad (32)$$

θ_z - EQUATION

$$\left[m_G + M_T (r+R)^2\right] \ddot{\theta}_z + \left(2 m_G \omega_{\theta z} \zeta_{\theta z}\right) \dot{\theta}_z + \left(m_G \omega_{\theta z}^2\right) \theta_z = M_z \qquad (33)$$

FIGURE D.2-4: VERTICAL BIFILAR EQUATIONS IN FIXED SYSTEM COORDINATES (CONTINUED)

β_0 - EQUATION

$$(M_T r)\ddot{Z} + (mr^2)\ddot{\beta}_0 + (2mr^2\zeta_\beta\omega_\beta)\dot{\beta}_0 + [mr\Omega^2(r+R)]\beta_0 = 0 \tag{34}$$

β_s - EQUATION

$$\left[\frac{M_T}{2} r(r+R)\right]\ddot{\theta}_x + (mr^2)\ddot{\beta}_s + (2mr^2\zeta_\beta\omega_\beta)\dot{\beta}_s$$
$$-(2mr^2\Omega)\dot{\beta}_c + [M_T r\Omega(r+R)]\dot{\theta}_y + (mrR\Omega^2)\beta_s$$
$$-(2mr^2\zeta_\beta\omega_\beta\Omega)\beta_c = 0 \tag{35}$$

β_c - EQUATION

$$-\left[\frac{M_T}{2} r(r+R)\right]\ddot{\theta}_y + (mr^2)\ddot{\beta}_c + (2mr^2\zeta_\beta\omega_\beta)\dot{\beta}_c + (2mr^2\Omega)\dot{\beta}_s$$
$$+[M_T r\Omega(r+R)]\dot{\theta}_x + (mrR\Omega^2)\beta_c + (2mr^2\zeta_\beta\omega_\beta\Omega)\beta_s = 0 \tag{36}$$

FIGURE D.2-4: VERTICAL BIFILAR EQUATIONS IN FIXED SYSTEM COORDINATES (CONCLUDED)

D.3 Rotor Equations With Hover Aerodynamics

The rotor equations of motion have been derived with five hub degrees of freedom (X, Y, Z, θ_x, θ_y), as well as with four coupled flap-lag flexible blade bending, two torsional, a rigid flapping and a rigid lead-lag degrees of freedom. Definition of the coordinate system is shown in Figure D.3-1. A detailed derivation of the rotor equations of motion are presented in Reference 1. For completeness sake, these equations are included herewith.

The equations of motion are shown on pp 305 to 331. The airframe generalized coordinates have been utilized for the hub flexibilities. The first equation is for the fixed system, and the other four are rotor equations, i.e. bending, torsion, rigid flapping and rigid lead-lag equations (37)-(41) respectively. These equations consist of the acceleration, velocity and displacement coefficients which account for the left-hand-side of the Lagrange's equations of motion. The right-hand-side of the Lagrange's equations of motion, the generalized forces, are shown by equations (42)-(46). In every equation, all generalized coordinates and physical properties associated with the rotor system have a subscript n, the blade number. The definition of all parameters used in all rotor equations are described in the list of symbols immediately following the equations (pp 332 to 338).

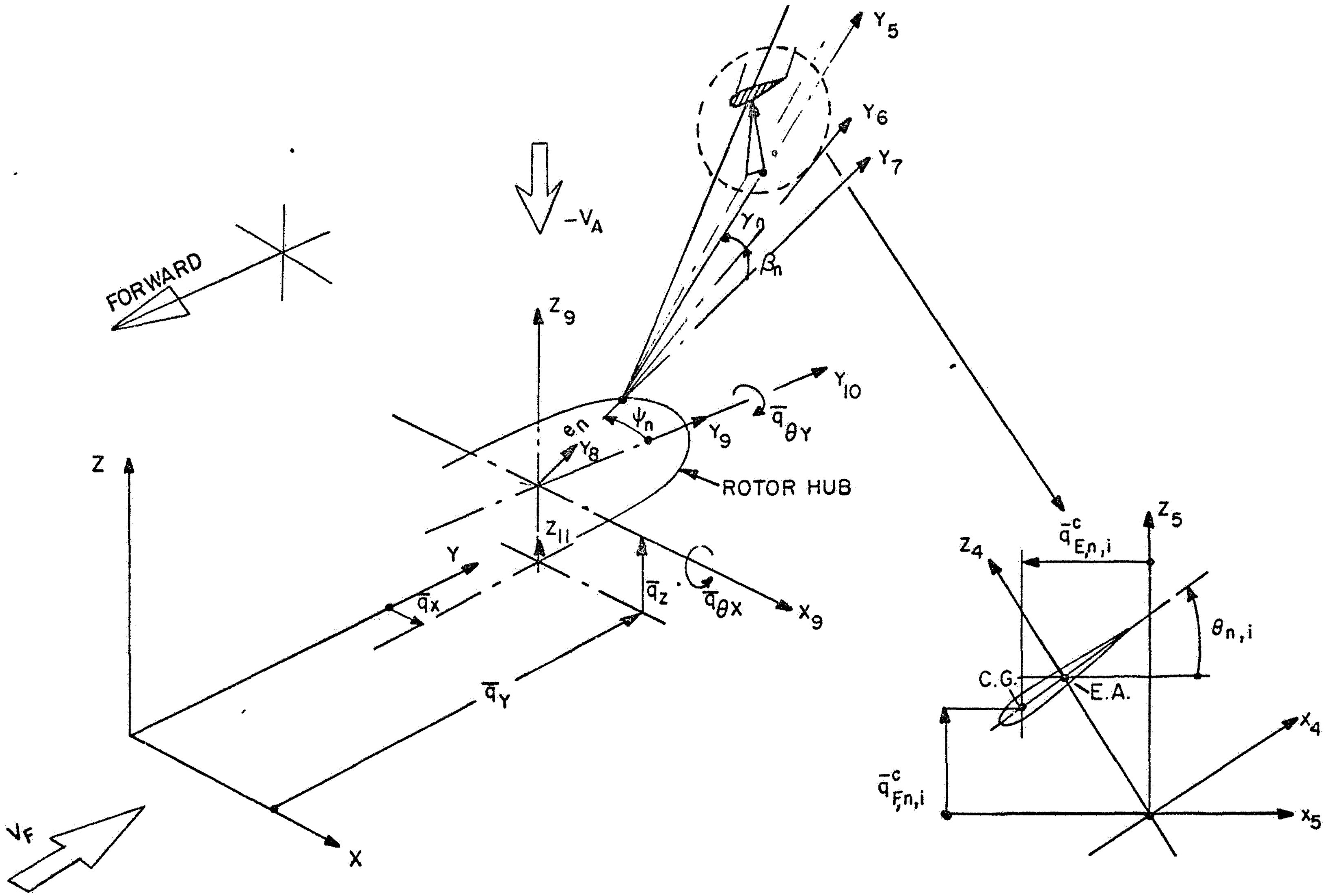

Figure D.3-1: Aeroelastic Rotor Stability Analysis Coordinate System

Airframe Mode Equations

$$\Big\{\sum_{n=1}^{N}\sum_{i=1}^{NA}\Big[\int_0^{R-e} mdr\{\phi_{X,j}(\phi_{X,i} + \phi_{\theta Y,i}[b_2 + r\beta_o]) + \phi_{Y,j}(\phi_{Y,i} - \phi_{\theta X,i}[b_2 + r\beta_o])$$

$$+ \phi_{Z,j}(\phi_{Z,i} + \phi_{\theta X,i}[(a_2 - r\gamma_o)\sin\psi + (e + r + a_2\gamma_o - b_2\beta_o)\cos\psi]$$

$$+ \phi_{\theta Y,i}[(e + r + a_2\gamma_o - b_2\beta_o)\sin\psi - (a_2 - r\gamma_o)\cos\psi]) + \phi_{\theta Y,j}((b_2 + r\beta_o)\phi_{X,i}$$

$$+ \phi_{Z,i}[(e + r + a_2\gamma_o - b_2\beta_o)\sin\psi - (a_2 - r\gamma_o)\cos\psi]) + \phi_{\theta X,j}(-(b_2 + r\beta_o)\phi_{Y,i}$$

$$+ \phi_{Z,i}[(e + r + a_2\gamma_o - b_2\beta_o)\cos\psi + (a_2 - r\gamma_o)\sin\psi])$$

$$+ \phi_{\theta Y,j}(\phi_{\theta Y,i}[(-a_2(a_2\gamma_o - b_2\beta_o) - (r + e)(a_2 - r\gamma_o))\sin 2\psi + ((r + e)^2$$

$$+ 2(r + e)(a_2\gamma_o - b_2\beta_o))\sin^2\psi + a_2(a_2 - 2r\gamma_o)\cos^2\psi + b_2(b_2 + 2r\beta_o)]$$

$$+ \phi_{\theta X,i}[\tfrac{1}{2}(-a_2(a_2 - 2r\gamma_o) + (r + e)^2 + 2(r + e)(a_2\gamma_o - b_2\beta_o))\sin 2\psi$$

$$+ (-(r + e)(a_2 - r\gamma_o) - a_2(a_2\gamma_o - b_2\beta_o))\cos 2\psi]) + \phi_{\theta X,j}(\phi_{\theta Y,i}[\tfrac{1}{2}(-a_2(a_2 - 2r\gamma_o)$$

$$+ (r + e)^2 + 2(r + e)(a_2\gamma_o - b_2\beta_o))\sin 2\psi + (-(r + e)(a_2 - r\gamma_o)$$

$$- a_2(a_2\gamma_o - b_2\beta_o))\cos 2\psi] + \phi_{\theta X,i}[(a_2(a_2\gamma_o - b_2\beta_o) + (r + e)(a_2 - r\gamma_o))\sin 2\psi$$

$$+ ((r + e)^2 + 2(r + e)(a_2\gamma_o - b_2\beta_o))\cos^2\psi + a_2(a_2 - 2r\gamma_o)\sin^2\psi$$

$$+ b_2(b_2 + 2r\beta_o)]\}\Big] + I_X dr\ \{\ \phi_{\theta Y,j}q'_{EO}(-\ \phi_{\theta Y,i}\cos\theta_o\sin 2\psi - \phi_{\theta X,i}\cos\theta_o\cos 2\psi)$$

$$+ \phi_{\theta X,j}q'_{EO}(-\ \phi_{\theta Y,i}\cos\theta_o\cos 2\psi + \phi_{\theta X,i}\cos\theta_o\sin 2\psi) + \phi_{\theta Y,j}(\phi_{\theta Y,i}(\cos^2\theta_o\sin^2\psi$$

$$+ (\gamma_o\cos^2\theta_o - \tfrac{1}{2}\beta_o\sin 2\theta_o)\sin 2\psi) + \phi_{\theta X,i}(\tfrac{1}{2}\cos^2\theta_o\sin 2\psi$$

$$+ (\gamma_o\cos^2\theta_o - \tfrac{1}{2}\beta_o\sin 2\theta_o)\cos 2\psi)) + \phi_{\theta X,j}(\phi_{\theta Y,i}(\tfrac{1}{2}\cos^2\theta_o\sin 2\psi + (\gamma_o\cos^2\theta_o$$

$$- \tfrac{1}{2}\beta_o\sin 2\theta_o)\cos 2\psi) + \phi_{\theta X,i}(\cos^2\theta_o\cos^2\psi - (\gamma_o\cos^2\theta_o - \tfrac{1}{2}\beta_o\sin 2\theta_o)\sin 2\psi))\}$$

$$+ I_Y dr\ \{\ \phi_{\theta Y,j}(\phi_{\theta Y,i}(q'_{EO}\cos\theta_o - q'_{FO}\sin\theta_o)\sin 2\psi + \phi_{\theta X,i}(q'_{EO}\cos\theta_o$$

$$- q'_{FO}\sin\theta_o)\cos 2\psi) + \phi_{\theta X,j}(\phi_{\theta Y,i}(q'_{EO}\cos\theta_o - q'_{FO}\sin\theta_o)\cos 2\psi$$

$$+ \phi_{\theta X,i}(-q'_{EO}\cos\theta_o + q'_{FO}\sin\theta_o)\sin 2\psi) + \phi_{\theta Y,j}(\phi_{\theta Y,i}(\cos^2\psi - \gamma_o\sin 2\psi)$$

$$+ \phi_{\theta X,i}(-\gamma_o\cos 2\psi - \tfrac{1}{2}\sin 2\psi)) + \phi_{\theta X,j}(\phi_{\theta Y,i}(-\gamma_o\cos 2\psi - \tfrac{1}{2}\sin 2\psi)$$

$$+ \phi_{\theta X,i}(\sin^2\psi + \gamma_o\sin 2\psi))\} + I_Z dr\{\phi_{\theta Y,j}q'_{FO}(\phi_{\theta Y,i}\sin\theta_o\sin 2\psi$$

$$+ \phi_{\theta X,i}\sin\theta_o\cos 2\psi) + \phi_{\theta X,j}q'_{FO}(\phi_{\theta Y,i}\sin\theta_o\cos 2\psi - \phi_{\theta X,i}\sin\theta_o\sin 2\psi)$$

$$+ \phi_{\theta Y,j}(\phi_{\theta Y,i}(\sin^2\theta_o\sin^2\psi + (\gamma_o\sin^2\theta_o + \tfrac{1}{2}\beta_o\sin 2\theta_o)\sin 2\psi)$$

$$+ \phi_{\theta X,i}(\tfrac{1}{2}\sin^2\theta_o\sin 2\psi + (\gamma_o\sin^2\theta_o + \tfrac{1}{2}\beta_o\sin 2\theta_o)\cos 2\psi))$$

$$+ \phi_{\theta X,j}(\phi_{\theta Y,i}(\tfrac{1}{2}\sin^2\theta_o\sin 2\psi + (\gamma_o\sin^2\theta_o + \tfrac{1}{2}\beta_o\sin 2\theta_o)\cos 2\psi)$$

$$+ \phi_{\theta X,i}(\sin^2\theta_o\cos^2\psi - (\gamma_o\sin^2\theta_o + \tfrac{1}{2}\beta_o\sin 2\theta_o)\sin 2\psi))\}\Big]\ddot{\bar{q}}_i\Big\}$$

$$+ \Big\{\sum_{i=1}^{NA}[\phi_{\theta X,i}\phi_{\theta X,j}I_{FA} + \phi_{\theta Y,i}\phi_{\theta Y,j}I_L + \phi_{Z,i}\phi_{Z,j}M_S]\ddot{\bar{q}}_i\Big\} + \Big\{M_{A,j}\ddot{\bar{q}}_j\Big\}$$

$$+ \Big\{\sum_{n=1}^{N}\Big[\int_0^{R-e} mdr\{\phi_{X,j}((b_2\gamma_o + a_2\beta_o)\sin\psi - b_2\cos\psi) + \phi_{Y,j}(-(b_2\gamma_o$$

$$+ a_2\beta_o)\cos\psi - b_2\sin\psi) + \phi_{Z,j}a_2 + \phi_{\theta Y,j}[((a_2^2 + b_2^2)\gamma_o + a_2(r + e))\sin\psi$$

$$- (a_2(a_2 - r\gamma_o) + b_2(b_2 + r\beta_o))\cos\psi] + \phi_{\theta X,j}[(a_2(a_2 - r\gamma_o)$$

$$+ b_2(b_2 + r\beta_o))\sin\psi + ((a_2^2 + b_2^2)\gamma_o + a_2(r + e))\cos\psi]\}$$

$$+ I_X dr\{\phi_{\theta Y,j}q'_{EO}\cos\theta_o\sin\psi + \phi_{\theta X,j}q'_{EO}\cos\theta_o\cos\psi\} + I_Y dr\{\phi_{\theta Y,j}(-q'_{EO}\cos\theta_o$$

$$+ q'_{FO}\sin\theta_o)\sin\psi + \phi_{\theta X,j}(-q'_{EO}\cos\theta_o + q'_{FO}\sin\theta_o)\cos\psi + \phi_{\theta Y,j}(\gamma_o\sin\psi$$

$$- \cos\psi) + \phi_{\theta X,j}(\gamma_o\cos\psi + \sin\psi)\} + I_Z dr\{-\phi_{\theta Y,j}q'_{FO}\sin\theta_o\sin\psi$$

$$- \phi_{\theta X,j}q'_{FO}\sin\theta_o\cos\psi\}\Big]\phi_\theta\ddot{\theta}_T\Big\} + \Big\{\sum_{n=1}^{N}\Big[\int_0^{R-e} mdr\{\phi_{X,j}(b_2 + r\beta_o)\sin\psi - \phi_{Y,j}(b_2$$

$$+ r\beta_o)\cos\psi + \phi_{Z,j}(r + a_2\gamma_o - b_2\beta_o) + \phi_{\theta Y,j}[(e(a_2\gamma_o - b_2\beta_o) + r(r + e + 2a_2\gamma_o)$$

$$+ b_2^2)\sin\psi - (a_2(a_2\gamma_o - b_2\beta_o) + r(a_2 - r\gamma_o))\cos\psi] + \phi_{\theta X,j}[(a_2(a_2\gamma_o - b_2\beta_o)$$

$$+ r(a_2 - r\gamma_o))\sin\psi + (e(a_2\gamma_o - b_2\beta_o) + r(r + e + 2a_2\gamma_o) + b_2^2)\cos\psi]\}$$

$$+ I_X dr\{-\phi_{\theta Y,j}q'_{EO}\cos\theta_o\cos\psi + \phi_{\theta X,j}q'_{EO}\cos\theta_o\sin\psi + \phi_{\theta Y,j}(\cos^2\theta_o\sin\psi$$

$$+ (\gamma_o\cos^2\theta_o - \tfrac{1}{2}\beta_o\sin 2\theta_o)\cos\psi) + \phi_{\theta X,j}(\cos^2\theta_o\cos\psi - (\gamma_o\cos^2\theta_o$$

$$- \tfrac{1}{2}\beta_o\sin 2\theta_o)\sin\psi)\} + I_Y dr\{\phi_{\theta Y,j}(q'_{EO}\cos\theta_o - q'_{FO}\sin\theta_o)\cos\psi$$

$$+ \phi_{\theta X,j}(- q'_{EO}\cos\theta_o + q'_{FO}\sin\theta_o)\sin\psi + \phi_{\theta X,j}\gamma_o\sin\psi - \phi_{\theta Y,j}\gamma_o\cos\psi\}$$

$$+ I_Z dr\{\phi_{\theta Y,j}q'_{FO}\sin\theta_o\cos\psi - \phi_{\theta X,j}q'_{FO}\sin\theta_o\sin\psi + \phi_{\theta Y,j}(\sin^2\theta_o\sin\psi$$

$$+ (\gamma_o\sin^2\theta_o + \tfrac{1}{2}\beta_o\sin 2\theta_o)\cos\psi) + \phi_{\theta X,j}(\sin^2\theta_o\cos\psi - (\gamma_o\sin^2\theta_o$$

$$+ \tfrac{1}{2}\beta_o\sin 2\theta_o)\sin\psi)\}\Big]\ddot{\beta}\Big\} + \Big\{\sum_{n=1}^{N}\Big[\int_0^{R-e} m dr\{\phi_{X,j}(-(a_2 - r\gamma_o)\sin\psi - (a_2\gamma_o$$

$$+ r)\cos\psi) + \phi_{Y,j}(-(a_2\gamma_o + r)\sin\psi + (a_2 - r\gamma_o)\cos\psi) + \phi_{Z,j}a_2\beta_o$$

$$+ \phi_{\theta Y,j}[(-b_2(a_2 - r\gamma_o) + a_2 e\beta_o)\sin\psi + (-(a_2^2 + r^2)\beta_o - b_2(a_2\gamma_o + r))\cos\psi]$$

$$+ \phi_{\theta X,j}[((a_2^2 + r^2)\beta_o + b_2(a_2\gamma_o + r))\sin\psi + (-b_2(a_2 - r\gamma_o) + a_2 e\beta_o)\cos\psi]\}$$

$$+ I_X dr\{- \phi_{\theta Y,j}q'_{EO}\sin\theta_o\cos\psi + \phi_{\theta X,j}q'_{EO}\sin\theta_o\sin\psi + \phi_{\theta Y,j}[\tfrac{1}{2}\sin 2\theta_o\sin\psi$$

$$+ (\tfrac{1}{2}\gamma_o\sin 2\theta_o - \beta_o\sin^2\theta_o)\cos\psi] + \phi_{\theta X,j}[\tfrac{1}{2}\sin 2\theta_o\cos\psi - (\tfrac{1}{2}\gamma_o\sin 2\theta_o - \beta_o\sin^2\theta_o)\sin\psi]\}$$

$$+ I_Y dr\{\phi_{\theta Y,j}(q'_{EO}\sin\theta_o + q'_{FO}\cos\theta_o)\cos\psi + \phi_{\theta X,j}(- q'_{EO}\sin\theta_o$$

$$- q'_{FO}\cos\theta_o)\sin\psi\} + I_Z dr\{- \phi_{\theta Y,j}q'_{FO}\cos\theta_o\cos\psi + \phi_{\theta X,j}q'_{FO}\cos\theta_o\sin\psi$$

$$+ \phi_{\theta Y,j}[- \tfrac{1}{2}\sin 2\theta_o\sin\psi - (\tfrac{1}{2}\gamma_o\sin 2\theta_o + \beta_o\cos^2\theta_o)\cos\psi] + \phi_{\theta X,j}[- \tfrac{1}{2}\sin 2\theta_o\cos\psi$$

$$+ (\tfrac{1}{2}\gamma_o\sin 2\theta_o + \beta_o\cos^2\theta_o)\sin\psi]\}\Big]\ddot{\gamma}\Big\} + \Big\{\sum_{n=1}^{N}\sum_{i=1}^{NE}\Big[\int_0^{R-e} m dr\{\phi_{X,j}(\phi_{E,i}(\gamma_o\sin\psi - \cos\psi)$$

$$+ \nu_{1,i}\sin\psi + \phi_{F,i}\beta_o\sin\psi) + \phi_{Y,j}(\phi_{E,i}(- \sin\psi - \gamma_o\cos\psi) - \nu_{1,i}\cos\psi$$

$$- \phi_{F,i}\beta_o\cos\psi) + \phi_{Z,j}\phi_{F,i} + \phi_{\theta Y,j}[(\phi_{F,i}(a_2\gamma_o + r + e) + \phi_{E,i}b_2\gamma_o)\sin\psi$$

$$+ (- \phi_{F,i}(a_2 - r\gamma_o) - \phi_{E,i}(b_2 + r\beta_o))\cos\psi] + \phi_{\theta X,j}[(\phi_{F,i}(a_2 - r\gamma_o) + \phi_{E,i}(b_2$$

$$+ r\beta_o))\sin\psi + (\phi_{E,i}b_2\gamma_o + \phi_{F,i}(a_2\gamma_o + r + e))\cos\psi] + \phi_{\theta Y,j}\nu_{1,i}b_2\sin\psi$$

$$+ \phi_{\theta X,j}\nu_{1,i}b_2\cos\psi\} + I_X dr\{\phi_{\theta Y,j}[\cos\theta_o\sin\psi + (\gamma_o\cos\theta_o - \beta_o\sin\theta_o$$

$$- q'_{EO})\cos\psi]\phi'_{F,i} + \phi_{\theta X,j}[\cos\theta_o\cos\psi - (\gamma_o\cos\theta_o - \beta_o\sin\theta_o - q'_{EO})\sin\psi]\phi'_{F,i}\}$$ 3

$$+ I_Y dr\{(\phi_{\theta Y,j} q'_{FO}\cos\psi - \phi_{\theta X,j} q'_{FO}\sin\psi)\phi'_{E,i}\}$$

$$+ I_Z dr\{\phi_{\theta Y,j}[- \sin\theta_o \sin\psi - (\gamma_o \sin\theta_o + \beta_o \cos\theta_o + q'_{FO})\cos\psi]\phi'_{E,i}$$

$$+ \phi_{\theta X,j}[- \sin\theta_o \cos\psi + (\gamma_o \sin\theta_o + \beta_o \cos\theta_o + q'_{FO})\sin\psi]\phi'_{E,i}\}\Big]\ddot{q}_{T,i}\Big\}$$

$$+ \Big\{ \tfrac{1}{2}[\frac{1}{R_S}(\phi_{\theta X,j} I_{FA} - \phi_{\theta Y,j} I_L) + \phi_{Z,j} M_S] \ddot{X}_A \Big\} + \Big\{ \tfrac{1}{2}[\frac{1}{R_S}(- \phi_{\theta X,j} I_{FA} - \phi_{\theta Y,j} I_L)$$

$$+ \phi_{Z,j} M_S] \ddot{X}_F \Big\} + \Big\{ \frac{1}{R_S}\phi_{\theta Y,j} I_L \ddot{X}_L \Big\} + \Big\{ \sum_{n=1}^{N} \sum_{i=1}^{NA} \Big[\int_0^{R-e} m dr\{2\Omega\phi_{Z,j}[\phi_{\theta X,i}((a_2 - r\gamma_o)\cos\psi$$

$$- (e + r + a_2\gamma_o - b_2\beta_o)\sin\psi) + \phi_{\theta Y,i}((e + r + a_2\gamma_o - b_2\beta_o)\cos\psi$$

$$+ (a_2 - r\gamma_o)\sin\psi)] + \Omega\phi_{\theta Y,j}[\phi_{\theta Y,i}([(r + e)^2 + 2(r + e)(a_2\gamma_o - b_2\beta_o)$$

$$- a_2(a_2 - r\gamma_o)]\sin 2\psi + [-(r + e)(a_2 - r\gamma_o) - a_2(a_2\gamma_o - b_2\beta_o)]2\cos 2\psi)$$

$$+ \phi_{\theta X,i}([a_2(a_2\gamma_o - b_2\beta_o) + (r + e)(a_2 - r\gamma_o)]2\sin 2\psi + [(r + e)^2$$

$$- a_2(a_2 - 2r\gamma_o) + 2(r + e)(a_2\gamma_o - b_2\beta_o)]\cos 2\psi - a_2(a_2 - 2r\gamma_o) - (r + e)^2$$

$$- 2(r + e)(a_2\gamma_o - b_2\beta_o))] + \Omega\phi_{\theta X,j}[\phi_{\theta Y,i}([a_2(a_2\gamma_o - b_2\beta_o) + (r + e)(a_2$$

$$- r\gamma_o)]2\sin 2\psi + [- a_2(a_2 - 2r\gamma_o) + (r + e)^2 + 2(r + e)(a_2\gamma_o - b_2\beta_o)]\cos 2\psi$$

$$+ a_2(a_2 - 2r\gamma_o) + (r + e)^2 + 2(r + e)(a_2\gamma_o - b_2\beta_o)) + \phi_{\theta X,i}([- (r + e)^2$$

$$- 2(r + e)(a_2\gamma_o - b_2\beta_o) + a_2(a_2 - r\gamma_o)]\sin 2\psi + [(r + e)(a_2 - r\gamma_o)$$

$$+ a_2(a_2\gamma_o - b_2\beta_o)]2\cos 2\psi)]\} + I_X dr\{2\Omega q'_{EO}(\phi_{\theta Y,j}[- \phi_{\theta Y,i}\cos\theta_o \cos 2\psi$$

$$+ \phi_{\theta X,i}\cos\theta_o \sin 2\psi] + \phi_{\theta X,j}[\phi_{\theta Y,i}\cos\theta_o \sin 2\psi + \phi_{\theta X,i}\cos\theta_o \cos 2\psi])$$

$$+ \Omega(\phi_{\theta Y,j}[\phi_{\theta Y,i}(\cos^2\theta_o \sin 2\psi + (2\gamma_o \cos^2\theta_o - \beta_o \sin 2\theta_o)\cos 2\psi)$$

$$+ \phi_{\theta X,i}(\cos^2\theta_o \cos 2\psi - (2\gamma_o \cos^2\theta_o - \beta_o \sin 2\theta_o)\sin 2\psi) - \phi_{\theta X,i}\sin^2\theta_o]$$

$$+ \phi_{\theta X,j}[\phi_{\theta Y,i}(\cos^2\theta_o \cos 2\psi - (2\gamma_o \cos^2\theta_o - \beta_o \sin 2\theta_o)\sin 2\psi)$$

$$+ \phi_{\theta X,i}(- \cos^2\theta_o \sin 2\psi - (2\gamma_o \cos^2\theta_o - \beta_o \sin 2\theta_o)\cos 2\psi) + \phi_{\theta Y,i}\sin^2\theta_o])\}$$

$$+ I_Y dr\{2\Omega(\phi_{\theta Y,j}[\phi_{\theta Y,i}(q'_{EO}\cos\theta_o - q'_{FO}\sin\theta_o)\cos 2\psi + \phi_{\theta X,i}(- q'_{EO}\cos\theta_o$$

$$+ q'_{FO}\sin\theta_o)\sin2\psi] + \phi_{\theta X,j}[\phi_{\theta Y,i}(- q'_{EO}\cos\theta_o$$ 4

$$+ q'_{FO}\sin\theta_o)\sin2\psi + \phi_{\theta X,i}(- q'_{EO}\cos\theta_o + q'_{FO}\sin\theta_o)\cos2\psi])$$

$$+ \Omega(\phi_{\theta Y,j}[\phi_{\theta Y,i}(- 2\gamma_o\cos2\psi - \sin2\psi) + \phi_{\theta X,i}(- \cos2\psi + 2\gamma_o\sin2\psi)]$$

$$+ \phi_{\theta X,j}[\phi_{\theta Y,i}(- \cos2\psi + 2\gamma_o\sin2\psi) + \phi_{\theta X,i}(2\gamma_o\cos2\psi + \sin2\psi)]))\}$$

$$+ I_Z dr\{2\Omega q'_{FO}(\phi_{\theta Y,j}[\phi_{\theta Y,i}\sin\theta_o\cos2\psi - \phi_{\theta X,i}\sin\theta_o\cos2\psi] + \phi_{\theta X,j}[$$

$$- \phi_{\theta Y,i}\sin\theta_o\sin2\psi - \phi_{\theta X,i}\sin\theta_o\cos2\psi]) + \Omega(\phi_{\theta Y,j}[\phi_{\theta Y,i}(\sin^2\theta_o\sin2\psi + (2\gamma_o\sin^2\theta_o$$

$$+ \beta_o\sin2\theta_o)\cos2\psi) + \phi_{\theta X,i}(\sin^2\theta_o\cos2\psi - (2\gamma_o\sin^2\theta_o + \beta_o\sin2\theta_o)\sin2\psi)$$

$$- \phi_{\theta X,i}\cos^2\theta_o\ ] + \phi_{\theta X,j}\ [\phi_{\theta Y,i}(\sin^2\theta_o\cos2\psi -(2\gamma_o\sin^2\theta_o + \beta_o\sin2\theta_o)\sin2\psi)$$

$$+ \phi_{\theta X,i}(-\sin^2\theta_o\sin2\psi - (2\gamma_o\sin^2\theta_o + \beta_o\sin2\theta_o)\cos2\psi)$$

$$+ \phi_{\theta Y,i}\cos^2\theta_o\ ])\}\Big]\dot{q}_j\Big\} + \Big\{2\zeta_{A,j}M_{A,j}\omega_{A,j}\ \dot{q}_j\Big\}$$

$$+ \Big\{\sum_{n=1}^{N}\Big[\int_0^{R-e} m dr\{2\Omega\ (\phi_{X,j}\ ((b_2\gamma_o + a_2\beta_o)\cos\psi + b_2\sin\psi)$$

$$+ \phi_{Y,j}((b_2\gamma_o + a_2\beta_o)\sin\psi - b_2\cos\psi) + \phi_{\theta Y,j}(b_2(b_2$$

$$+ r\beta_o)\sin\psi + b_2(a_2\beta_o + b_2\gamma_o)\cos\psi) + \phi_{\theta X,j}(-b_2(b_2\gamma_o + a_2\beta_o)\sin\psi$$

$$+ b_2(b_2 + r\beta_o)\cos\psi))\} + I_X dr\{\Omega(\phi_{\theta Y,j}[\cos2\theta_o\sin\psi + (\gamma_o\cos2\theta_o$$

$$- \beta_o\sin2\theta_o)\cos\psi] + \phi_{\theta X,j}[\cos2\theta_o\cos\psi - (\gamma_o\cos2\theta_o - \beta_o\sin2\theta_o)\sin\psi\ ])\}$$

$$+ I_Y dr\{\Omega(\phi_{\theta Y,j}(\gamma_o\cos\psi + \sin\psi) + \phi_{\theta X,j}(\cos\psi - \gamma_o\sin\psi))\}$$

$$+ I_Z dr\{\Omega\ (\phi_{\theta Y,j}[-\cos2\theta_o\sin\psi - (\gamma_o\cos2\theta_o - \beta_o\sin2\theta_o)\cos\psi]$$

$$+ \phi_{\theta X,j}[\ - \cos2\theta_o\cos\psi + (\gamma_o\cos2\theta_o - \beta_o\sin2\theta_o)\sin\psi])\}\Big]\phi_\theta\dot{\theta}_T\Big\}$$

$$+ \Big\{\sum_{n=1}^{N}\Big[\ \int_0^{R-e} m dr\{2\Omega(\phi_{X,j}(b_2 + r\beta_o)\cos\psi + \phi_{Y,j}(b_2 + r\beta_o)\sin\psi$$

$$+ \phi_{\theta Y,j}b_2(b_2 + 2r\beta_o)\cos\psi - \phi_{\theta X,j}b_2(b_2 + 2r\beta_o)\sin\psi)\}$$

$$+ I_X dr\{\Omega(\phi_{\theta Y,j}\cos2\theta_o\cos\psi - \phi_{\theta X,j}\cos2\theta_o\sin\psi)\} + I_Y dr\{\Omega(\phi_{\theta Y,j}\cos\psi$$ 5

$$
\begin{aligned}
& - \phi_{\theta X,j}\sin\psi)\} + I_Z dr\{\Omega(- \phi_{\theta Y,j}\cos2\theta_o\cos\psi + \phi_{\theta X,j}\cos2\theta_o\sin\psi)\}\Big] \dot{\beta} \Big\} \\
& + \Big\{ \sum_{n=1}^{N} \Big[\int_0^{R-e} m dr\{2\Omega(\phi_{X,j}[- (a_2 - r\gamma_o)\cos\psi + (a_2\gamma_o + r)\sin\psi] \\
& + \phi_{Y,j}[- (a_2\gamma_o + r)\cos\psi - (a_2 - r\gamma_o)\sin\psi] + \phi_{\theta Y,j}[(r^2\beta_o + b_2(a_2\gamma_o \\
& + r))\sin\psi - (b_2(a_2 - r\gamma_o) + a_2 r\beta_o)\cos\psi] + \phi_{\theta X,j}[(r^2\beta_o + b_2(a_2\gamma_o + r))\cos\psi \\
& + (b_2(a_2 - r\gamma_o) + a_2 r\beta_o)\sin\psi])\} + I_X dr\{2\Omega q'_{EO}(\phi_{\theta Y,j}\sin\theta_o\sin\psi \\
& + \phi_{\theta X,j}\sin\theta_o\cos\psi) + \Omega(\phi_{\theta Y,j}[\sin2\theta_o\cos\psi - (\gamma_o\sin2\theta_o - \beta_o)\sin\psi] \\
& + \phi_{\theta X,j}[- \sin2\theta_o\sin\psi - (\gamma_o\sin2\theta_o - \beta_o)\cos\psi])\} \\
& + I_Y dr\{2\Omega(\phi_{\theta Y,j}(- q'_{EO}\sin\theta_o - q'_{FO}\cos\theta_o)\sin\psi + \phi_{\theta X,j}(- q'_{EO}\sin\theta_o \\
& - q'_{FO}\cos\theta_o)\cos\psi) + \Omega(-\phi_{\theta Y,j}\beta_o\sin\psi - \phi_{\theta X,j}\beta_o\cos\psi)\} \\
& + I_Z dr\{2\Omega q'_{FO}(\phi_{\theta Y,j}\cos\theta_o\sin\psi + \phi_{\theta X,j}\cos\theta_o\cos\psi) + \Omega(\phi_{\theta Y,j}(- \sin2\theta_o\cos\psi \\
& + (\gamma_o\sin2\theta_o + \beta_o)\sin\psi) + \phi_{\theta X,j}(\sin2\theta_o\sin\psi + (\gamma_o\sin2\theta_o + \beta_o)\cos\psi)\}\Big] \dot{\gamma} \Big\} \\
& + \Big\{ \sum_{n=1}^{N} \sum_{i=1}^{NE} \Big[\int_0^{R-e} m dr\{2\Omega(\phi_{X,j}[\phi_{E,i}(\gamma_o\cos\psi + \sin\psi) + \nu_{1,i}\cos\psi \\
& + \phi_{F,i}\beta_o\cos\psi] + \phi_{Y,j}[- \phi_{E,i}(\cos\psi - \gamma_o\sin\psi) + \nu_{1,i}\sin\psi \\
& + \phi_{F,i}\beta_o\sin\psi] + \phi_{\theta Y,j}[\phi_{E,i}(b_2 + r\beta_o)\sin\psi + b_2(\phi_{F,i}\beta_o \\
& + \phi_{E,i}\gamma_o)\cos\psi] + \phi_{\theta X,j}[- b_2(\phi_{F,i}\beta_o + \phi_{E,i}\gamma_o)\sin\psi + \phi_{E,i}(b_2 + r\beta_o)\cos\psi] \\
& + \phi_{\theta Y,j}\nu_{1,i}b_2\cos\psi - \phi_{\theta X,j}\nu_{1,i}b_2\sin\psi)\} + I_X dr\{\Omega(\phi_{\theta Y,j}[(\sin\theta_o\cos\psi \\
& + (q'_{EO}\sin2\theta_o - \gamma_o\sin\theta_o + \beta_o\cos\theta_o)\sin\psi)\phi'_{E,i} \\
& + (\cos\theta_o\cos\psi + (q'_{EO} - \gamma_o\cos\theta_o + \beta_o\sin\theta_o)\sin\psi)\phi'_{F,i}] \\
& + \phi_{\theta X,j}[(- \sin\theta_o\sin\psi + (q'_{EO}\sin2\theta_o - \gamma_o\sin\theta_o + \beta_o\cos\theta_o)\cos\psi)\phi'_{E,i} \\
& + (- \cos\theta_o\sin\psi + (q'_{EO} - \gamma_o\cos\theta_o + \beta_o\sin\theta_o)\cos\psi)\phi'_{F,i}])\} \; 6
\end{aligned}
$$

$$+ I_Y dr\{\Omega(\phi_{\theta Y,j}\ [\phi'_{E,i}(-\sin\theta_o\cos\psi + (-2q'_{FO}\cos^2\theta_o - q'_{EO}\sin 2\theta_o$$

$$-\beta_o\cos\theta_o + \gamma_o\sin\theta_o)\sin\psi) + \phi'_{F,i}(\cos\theta_o\cos\psi + (-q'_{FO}\sin 2\theta_o + q'_{EO}\cos 2\theta_o$$

$$-\beta_o\sin\theta_o - \gamma_o\cos\theta_o)\sin\psi)] + \phi_{\theta X,j}[\phi'_{E,i}(\sin\theta_o\sin\psi + (-2q'_{FO}\cos^2\theta_o$$

$$-q'_{EO}\sin 2\theta_o - \beta_o\cos\theta_o + \gamma_o\sin\theta_o)\cos\psi) + \phi'_{F,i}(-\cos\theta_o\sin\psi + (q'_{EO}\cos 2\theta_o$$

$$-q'_{FO}\sin 2\theta_o - \beta_o\sin\theta_o - \gamma_o\cos\theta_o)\cos\psi)])\} + I_Z dr\{\Omega(\phi_{\theta Y,j}[(-\sin\theta_o\cos\psi$$

$$+(2q'_{FO}\cos^2\theta_o + \gamma_o\sin\theta_o + \beta_o\cos\theta_o)\sin\psi)\phi'_{E,i} + (-\cos\theta_o\cos\psi$$

$$+(q'_{FO}\sin 2\theta_o + \gamma_o\cos\theta_o + \beta_o\sin\theta_o)\sin\psi)\phi'_{F,i}] + \phi_{\theta X,j}[(\sin\theta_o\sin\psi$$

$$+(2q'_{FO}\cos^2\theta_o + \gamma_o\sin\theta_o + \beta_o\cos\theta_o)\cos\psi)\phi'_{E,i} + (\cos\theta_o\sin\psi + (q'_{FO}\sin 2\theta_o$$

$$-q'_{EO}\cos 2\theta_o + \gamma_o\cos\theta_o + \beta_o\sin\theta_o)\ \cos\psi)\ \phi'_{F,i}])\}\Big]\dot{q}_{T,i}\Big\}$$

$$+ \left\{\sum_{n=1}^{N}\sum_{i=1}^{NA}\left[\int_0^{R-e} m dr\{\Omega^2\ \phi_{Z,j}[\phi_{\theta X,i}(-(a_2 - r\gamma_o)\sin\psi - (e + r + a_2\gamma_o\right.\right.$$

$$-b_2\beta_o)\cos\psi) + \phi_{\theta Y,i}(-(e + r + a_2\gamma_o - b_2\beta_o)\sin\psi$$

$$\left.\left. + (a_2 - r\gamma_o)\cos\psi)]\}\right]\bar{q}_i\right\} + \left\{M_{A,j}\omega^2_{A,j}\bar{q}_j\right\}$$

$$+ \left\{\sum_{n=1}^{N}\left[\int_0^{R-e} m dr\{\Omega^2(\phi_{X,j}[-(b_2\gamma_o + a_2\beta_o)\sin\psi + b_2\cos\psi]\right.\right.$$

$$+\phi_{Y,j}[(b_2\gamma_o + a_2\beta_o)\cos\psi + b_2\sin\psi] + \phi_{\theta Y,j}[(a_2(a_2\gamma_o + r + e - 2b_2\beta_o)$$

$$-b_2^2\gamma_o)\sin\psi + (-a_2(a_2 - r\gamma_o) + b_2(b_2 + r\beta_o))\cos\psi]$$

$$+\phi_{\theta X,j}[(a_2(a_2 - r\gamma_o) - b_2(b_2 + r\beta_o))\sin\psi + (a_2(a_2\gamma_o + r + e - 2b_2\beta_o)$$

$$-b_2^2\gamma_o)\cos\psi])\} + I_X dr\{\Omega^2 q'_{EO}(\phi_{\theta Y,j}\cos\theta_o\sin\psi + \phi_{\theta X,j}\cos\theta_o\cos\psi)$$

$$+\Omega^2[\phi_{\theta Y,j}(\cos 2\theta_o\cos\psi - (\gamma_o\cos 2\theta_o - \beta_o\sin 2\theta_o)\sin\psi) + \phi_{\theta X,j}(-\cos 2\theta_o\sin\psi$$

$$-(\gamma_o\cos 2\theta_o - \beta_o\sin 2\theta_o)\cos\psi)]\} + I_Y dr\{\Omega^2[\phi_{\theta Y,j}(-q'_{EO}\cos\theta_o + q'_{FO}\sin\theta_o)\sin\psi$$

$$+\phi_{\theta X,j}(-q'_{EO}\cos\theta_o + q'_{FO}\sin\theta_o)\cos\psi]\} + I_Z dr\{\Omega^2 q'_{FO}(-\phi_{\theta Y,j}\sin\theta_o\sin\psi\ 7$$

$$
\begin{aligned}
&- \phi_{\theta X,j}\sin\theta_o\cos\psi) + \Omega^2[\ \phi_{\theta Y,j}(- \cos 2\theta_o\cos\psi + (\gamma_o\cos 2\theta_o - \beta_o\sin 2\theta_o)\sin\psi) \\
&+ \phi_{\theta X,j}(\cos 2\theta_o\sin\psi + (\gamma_o\cos 2\theta_o - \beta_o\sin 2\theta_o)\cos\psi)]\}\Big]\ \phi_\theta\theta_T \Big\} \\
&+ \Big\{\sum_{n=1}^{N}\Big[\int_0^{R-e} mdr\{\Omega^2[- \phi_{X,j}(b_2 + r\beta_o)\sin\psi + \phi_{Y,j}(b_2 + r\beta_o)\cos\psi \\
&+ \phi_{\theta Y,j}([(r + e)\ (a_2\gamma_o + r - b_2\beta_o) + r(a_2\gamma_o - b_2\beta_o) - b_2(2r\beta_o + b_2)]\sin\psi \\
&+ [- a_2(a_2\gamma_o + r - b_2\beta_o) - b_2(b_2 + 2r\beta_o) + r^2\gamma_o]\cos\psi) \\
&+ \phi_{\theta X,j}([a_2(a_2\gamma_o + r - b_2\beta_o) + b_2(2r\beta_o + b_2) - r^2\gamma_o]\sin\psi \\
&+ [(r + e)(a_2\gamma_o + r - b_2\beta_o) + r(a_2\gamma_o - b_2\beta_o) - b_2(b_2 + 2r\beta_o)]\cos\psi)]\} \\
&+ I_X dr\{\Omega^2 q'_{EO}(- \phi_{\theta Y,j}\cos\theta_o\cos\psi + \phi_{\theta X,j}\cos\theta_o\sin\psi) + \Omega^2[\phi_{\theta Y,j}(\sin^2\theta_o\sin\psi \\
&+ \gamma_o\cos^2\theta_o\cos\psi) + \phi_{\theta X,j}(\sin^2\theta_o\ \cos\psi + \gamma_o\cos^2\theta_o\sin\psi)]\} \\
&+ I_Y dr\{\Omega^2[\phi_{\theta Y,j}(q'_{EO}\cos\theta_o - q'_{FO}\sin\theta_o)\cos\psi + \phi_{\theta X,j}(- q'_{EO}\cos\theta_o \\
&+ q'_{FO}\sin\theta_o)\sin\psi + \phi_{\theta Y,j}(- \gamma_o\cos\psi - \sin\psi) + \phi_{\theta X,j}(\gamma_o\sin\psi - \cos\psi)]\} \\
&+ I_Z dr\{\Omega^2 q'_{FO}(\phi_{\theta Y,j}\sin\theta_o\cos\psi - \phi_{\theta X,j}\sin\theta_o\sin\psi) + \Omega^2[\ \phi_{\theta Y,j}(\cos^2\theta_o\sin\psi \\
&+ \gamma_o\ \sin^2\theta_o\cos\psi) + \phi_{\theta X,j}(\cos^2\theta_o\cos\psi - \gamma_o\sin^2\theta_o\sin\psi)]\}\Big]\ \beta \Big\} \\
&+ \Big\{\sum_{n=1}^{N}\Big[\int_0^{R-e} mdr\{\ \Omega^2[\ \phi_{X,j}((a_2 - r\gamma_o)\sin\psi + (a_2\gamma_o + r)\cos\psi) \\
&+ \phi_{Y,j}\ ((a_2\gamma_o + r)\sin\psi - (a_2 - r\gamma_o)\cos\psi) + \phi_{\theta Y,j}[(r(a_2\beta_o - b_2\gamma_o) \\
&- b_2(a_2\gamma_o + r) + a_2([\ r + e]\beta_o + b_2) - r^2\beta_o)\sin\psi + (b_2(a_2 - r\gamma_o) \\
&+ b_2(a_2\gamma_o + r) - a_2(a_2\beta_o - r\beta_o) + r^2\beta_o)\cos\psi] + \phi_{\theta X,j}[(-b_2(a_2 - r\gamma_o) \\
&- b_2(a_2\gamma_o + r) + a_2(a_2\beta_o - r\beta_o) - r^2\beta_o)\sin\psi + (r(a_2\beta_o - b_2\gamma_o) - b_2(a_2\gamma_o + r) \\
&+ a_2([r + e]\beta_o + b_2) - r^2\beta_o)\cos\psi]]\}\ \delta
\end{aligned}
$$

$$+ I_X dr\{\Omega^2 q'_{EO}(\phi_{\theta Y,j}\sin\theta_o\cos\psi - \phi_{\theta X,j}\sin\theta_o\sin\psi) + \Omega^2[\phi_{\theta Y,j}(\beta_o\cos^2\theta_o\cos\psi$$

$$- \tfrac{1}{2}\sin 2\theta_o\sin\psi) + \phi_{\theta X,j}(-\beta_o\cos^2\theta_o\sin\psi - \tfrac{1}{2}\sin 2\theta_o\cos\psi)]\}$$

$$+ I_Y dr\{\Omega^2[\phi_{\theta Y,j}(-q'_{EO}\sin\theta_o - q'_{FO}\cos\theta_o)\cos\psi + \phi_{\theta X,j}(q'_{EO}\sin\theta_o$$

$$+ q'_{FO}\cos\theta_o)\sin\psi - \phi_{\theta Y,j}\beta_o\cos\psi + \phi_{\theta X,j}\beta_o\sin\psi]\}$$

$$+ I_Z dr\{\Omega^2 q'_{FO}(\phi_{\theta Y,j}\cos\theta_o\cos\psi - \phi_{\theta X,j}\cos\theta_o\sin\psi) + \Omega^2[\phi_{\theta Y,j}(\tfrac{1}{2}\sin 2\theta_o\sin\psi$$

$$+ \beta_o\sin^2\theta_o\cos\psi) + \phi_{\theta X,j}(\tfrac{1}{2}\sin 2\theta_o\cos\psi - \beta_o\sin^2\theta_o\sin\psi)]\}\Big]\gamma\Big\}$$

$$+ \Big\{\sum_{n=1}^{N}\sum_{i=1}^{NE}\Big[\int_0^{R-e} m dr\{\Omega^2[\phi_{X,j}(-\phi_{E,i}(\gamma_o\sin\psi - \cos\psi) - \nu_{1,i}\sin\psi$$

$$- \phi_{F,i}\beta_o\sin\psi) + \phi_{Y,j}(\phi_{E,i}(\sin\psi + \gamma_o\cos\psi) + \nu_{1,i}\cos\psi + \phi_{F,i}\beta_o\cos\psi)$$

$$+ \phi_{\theta Y,j}((\phi_{F,i}(a_2\gamma_o + r + e - 2b_2\beta_o) - \phi_{E,i}b_2\gamma_o)\sin\psi + (-\phi_{F,i}(a_2 - r\gamma_o)$$

$$+ \phi_{E,i}(b_2 + r\beta_o))\cos\psi) + \phi_{\theta X,j}((\phi_{F,i}(a_2 - r\gamma_o) - \phi_{E,i}(b_2 + r\beta_o))\sin\psi$$

$$+ (\phi_{F,i}(a_2\gamma_o + r + e - 2b_2\beta_o) - \phi_{E,i}b_2\gamma_o)\cos\psi) - \phi_{\theta Y,j}\nu_{1,i}b_2\sin\psi$$

$$- \phi_{\theta X,j}\nu_{1,i}b_2\cos\psi]\} + I_X dr\{\Omega^2[\phi_{\theta Y,j}(-\sin\theta_o\sin\psi - (\gamma_o\sin\theta_o$$

$$- \beta_o\cos\theta_o)\cos\psi)\phi'_{E,i} + \phi_{\theta X,j}(-\sin\theta_o\cos\psi + (\gamma_o\sin\theta_o - \beta_o\cos\theta_o)\sin\psi)\phi'_{E,i}]\}$$

$$+ I_Y dr\{\Omega^2[\phi_{\theta Y,j}(\phi'_{E,i}(\sin\theta_o\sin\psi + (-q'_{EO}\sin 2\theta_o - \beta_o\cos\theta_o - q'_{FO}\cos 2\theta_o$$

$$+ \gamma_o\sin\theta_o)\cos\psi) + \phi'_{F,i}(-\cos\theta_o\sin\psi + (-q'_{FO}\sin 2\theta_o - \beta_o\sin\theta_o$$

$$+ q'_{EO}\cos 2\theta_o - \gamma_o\cos\theta_o)\cos\psi)) + \phi_{\theta X,j}(\phi'_{E,i}(\sin\theta_o\cos\psi + (q'_{EO}\sin 2\theta_o$$

$$+ \beta_o\cos\theta_o + q'_{FO}\cos 2\theta_o - \gamma_o\sin\theta_o)\sin\psi) + \phi'_{F,i}(-\cos\theta_o\cos\psi + (-q'_{EO}\cos 2\theta_o$$

$$+ \beta_o\sin\theta_o + q'_{FO}\sin 2\theta_o + \gamma_o\cos\theta_o)\sin\psi))]\} + I_Z dr\{\Omega^2[\phi_{\theta Y,j}((\cos\theta_o\sin\psi$$

$$+ (-q'_{EO}\cos 2\theta_o + \gamma_o\cos\theta_o + \beta_o\sin\theta_o)\cos\psi)\phi'_{F,i} + \phi'_{E,i}q'_{FO}\cos 2\theta_o\cos\psi)$$

$$+ \phi_{\theta X,j}((\cos\theta_o\cos\psi + (q'_{EO}\cos 2\theta_o - \gamma_o\cos\theta_o - \beta_o\sin\theta_o)\sin\psi)\phi'_{F,i}$$

$$- \phi'_{E,i}q'_{FO}\cos 2\theta_o\sin\psi)]\}\Big]q_{T,i}\Big\} = 0 \qquad (37)$$

Blade Pitch Equations

$$
\begin{aligned}
&\Bigg\{\sum_{i=1}^{NA}\Bigg[\int_0^{R-e} mdr\{\phi_{X,i}((b_2\gamma_o + a_2\beta_o)\sin\psi - b_2\cos\psi) + \phi_{Y,i}(- (b_2\gamma_o \\
&+ a_2\beta_o)\cos\psi - b_2\sin\psi) + a_2\phi_{Z,i} + \phi_{\theta Y,i}[((a_2^2 + b_2^2)\gamma_o + a_2(r + e))\sin\psi \\
&+ (- a_2(a_2 - r\gamma_o) - b_2(b_2 + r\beta_o))\cos\psi] + \phi_{\theta X,i}[(a_2(a_2 - r\gamma_o) \\
&+ b_2(b_2 + r\beta_o))\sin\psi + ((a_2^2 + b_2^2)\gamma_o + a_2(r + e))\cos\psi]\} \\
&+ I_X dr\{q'_{EO}(\phi_{\theta Y,i}\cos\theta_o\sin\psi + \phi_{\theta X,i}\cos\theta_o\cos\psi)\} + I_Y dr\{\phi_{\theta Y,i}(-q'_{EO}\cos\theta_o \\
&+ q'_{FO}\sin\theta_o)\sin\psi + \phi_{\theta X,i}(- q'_{EO}\cos\theta_o + q'_{FO}\sin\theta_o)\cos\psi + \phi_{\theta Y,i}(\gamma_o\sin\psi \\
&- \cos\psi) + \phi_{\theta X,i}(\gamma_o\cos\psi + \sin\psi)\} + I_Z dr\{q'_{FO}(- \phi_{\theta Y,i}\sin\theta_o\sin\psi \\
&- \phi_{\theta X,i}\sin\theta_o\cos\psi)\}\Bigg]\,\phi_\theta \ddot{q}_i \Bigg\} \\
&+ \Bigg\{\Bigg[\int_0^{R-e} mdr\{(a_2^2 + b_2^2)\} + I_Y dr\{1\}\Bigg]\,\phi_\theta^2\,\ddot{\theta}_T\Bigg\} + \Bigg\{M_1 L_2^2 \phi_{\theta PR}^2 \ddot{\theta}_T\Bigg\} \\
&+ \Bigg\{\Bigg[\int_0^{R-e} mdr\{(a_2^2 + b_2^2)\gamma_o + a_2 r\} + I_X dr\{q'_{EO}\cos\theta_o\} + I_Y dr\{ - q'_{EO}\cos\theta_o \\
&+ q'_{FO}\sin\theta_o + \gamma_o\} \\
&+ I_Z dr\{ - q'_{FO}\sin\theta_o\}\Bigg]\,\phi_\theta \ddot{\beta}\Bigg\} + \Bigg\{ - M_1 L_2 \tan\delta_3 \phi_{\theta PR}\ddot{\beta}\Bigg\} \\
&+ \Bigg\{\Bigg[\int_0^{R-e} mdr\{b_2 r\} + I_X dr\{q'_{EO}\sin\theta_o\} + I_Y dr\{-q'_{EO}\sin\theta_o - q'_{FO}\cos\theta_o\} \\
&+ I_Z dr\{q'_{FO}\cos\theta_o\}\Bigg]\,\phi_\theta\ddot{\gamma}\Bigg\} + \Bigg\{ - M_1 L_2^2 \tan\alpha_1 \phi_{\theta PR}\ddot{\gamma}\Bigg\} \\
&+ \Bigg\{\sum_{i=1}^{NE}\Bigg[\int_0^{R-e} mdr\{a_2\phi_{F,i} + b_2\phi_{E,i}\} + I_X dr\{q'_{EO}\phi'_{F,i}\} + I_Y dr\{ - q'_{FO}\phi'_{E,i}\} \\
&+ I_Z dr\{q'_{FO}\phi'_{E,i}\}\Bigg]\,\phi_\theta\,\ddot{q}_{T,i}\Bigg\} + \Bigg\{\sum_{i=1}^{NE}(-M_1 L_2(\phi_{FPR,i} \\
&+ [L_2 \tan\alpha_1/(R-e)]\phi_{ET,i}))\phi_{\theta PR}\,\ddot{q}_{T,i}\Bigg\} \\
&+ \Bigg\{\sum_{i=1}^{NA}\Bigg[\int_0^{R-e} mdr\{2\Omega[\phi_{\theta Y,i}(a_2(a_2 - r\gamma_o)\sin\psi + a_2(a_2\gamma_o + r + e - b_2\beta_o)\cos\psi) \\
&+ \phi_{\theta X,i}(- a_2(a_2\gamma_o + r + e - b_2\beta_o)\sin\psi + a_2(a_2 - r\gamma_o)\cos\psi)]\}\quad 10
\end{aligned}
$$

$$
\begin{aligned}
&+ I_X dr\{2\Omega q'_{EO}[\phi_{\theta Y,i}\cos\theta_o\cos\psi - \phi_{\theta X,i}\cos\theta_o\sin\psi] + \Omega[\phi_{\theta Y,i}(-\cos 2\theta_o\sin\psi \\
&- (\gamma_o\cos 2\theta_o - \beta_o\sin 2\theta_o)\cos\psi) + \phi_{\theta X,i}(-\cos 2\theta_o\cos\psi + (\gamma_o\cos 2\theta_o \\
&- \beta_o\sin 2\theta_o)\sin\psi)]\} + I_Y dr\{\Omega[\phi_{\theta Y,i}(-q'_{EO}\cos\theta_o + q'_{FO}\sin\theta_o)2\cos\psi \\
&+ \phi_{\theta X,i}(q'_{EO}\cos\theta_o - q'_{FO}\sin\theta_o)2\sin\psi + \phi_{\theta Y,i}(\gamma_o\cos\psi + \sin\psi) + \phi_{\theta X,i}(-\gamma_o\sin\psi \\
&+ \cos\psi)]\} + I_Z dr\{2\Omega q'_{FO}[-\phi_{\theta Y,i}\sin\theta_o\cos\psi + \phi_{\theta X,i}\sin\theta_o\sin\psi] \\
&+ \Omega[\phi_{\theta Y,i}(\cos 2\theta_o\sin\psi + (\gamma_o\cos 2\theta_o - \beta_o\sin 2\theta_o)\cos\psi) + \phi_{\theta X,i}(\cos 2\theta_o\cos\psi \\
&- (\gamma_o\cos 2\theta_o - \beta_o\sin 2\theta_o)\sin\psi)]\}\Big]\phi_\theta\dot{\bar{q}}_i \Big\} + \Big\{(2\zeta_\theta I_T\Omega + C_1L_2^2\phi_{\theta PR}^2)\dot{\theta}_T\Big\} \\
&+ \Big\{\Big[\int_0^{R-e} mdr\{2\Omega[-b_2(b_2 + r\beta_o)]\} + I_X dr\{-\Omega\cos 2\theta_o\} + I_Y dr\{-\Omega\} \\
&+ I_Z dr\{\Omega\cos 2\theta_o\}\Big]\phi_\theta\dot{\beta}\Big\} + \Big\{-C_1L_2^2\tan\delta_3\phi_{\theta PR}\dot{\beta}\Big\} + \Big\{\Big[\int_0^{R-e} mdr\{2\Omega[a_2(b_2 + r\beta_o)]\} \\
&+ I_X dr\{-\Omega\sin 2\theta_o\} + I_Z dr\{\Omega\sin 2\theta_o\}\Big]\phi_\theta\dot{\gamma}\Big\} + \Big\{-C_1L_2^2\tan\alpha_1\phi_{\theta PR}\dot{\gamma}\Big\} \\
&+ \Big\{\sum_{i=1}^{NE}\Big[\int_0^{R-e} mdr\{2\Omega(-\phi_{F,i}b_2\beta_o + \phi_{E,i}a_2\beta_o) - 2\Omega v_{1,i}b_2\} + I_X dr\{\Omega[-\phi'_{E,i}\sin\theta_o \\
&- \phi'_{F,i}\cos\theta_o]\} + I_Y dr\{\Omega[\phi'_{E,i}\sin\theta_o - \phi'_{F,i}\cos\theta_o]\} + I_Z dr\{\Omega[\phi'_{E,i}\sin\theta_o \\
&+ \phi'_{F,i}\cos\theta_o]\}\Big]\phi_\theta\dot{q}_{T,i}\Big\} \quad + \Big\{\sum_{i=1}^{NE}(-C_1L_2(\phi_{FPR,i} \\
&+ [L_2\tan\alpha_1/(R-e)]\phi_{ET,i}))\phi_{\theta PR}\dot{q}_{T,i}\Big\} + \Big\{\Big[\int_0^{R-e} mdr\{\Omega^2[e(a_2\gamma_o - b_2\beta_o) \\
&- b_2(b_2 + r\beta_o) + a_2^2]\} + I_X dr\{-\Omega^2\cos 2\theta_o\} + I_Z dr\{\Omega^2\cos 2\theta_o\}\Big]\phi_\theta^2\theta_T \Big\} \\
&+ \Big\{(\int_0^{R-e} K\phi_\theta{}^2)\theta_T\Big\} + \Big\{K_1L_2^2\phi^2{}_{\theta PR}\theta_T\Big\} \\
&+ \Big\{\Big[\int_0^{R-e} mdr\{\Omega^2[a_2(a_2\gamma_o + r + e) - b_2(b_2\gamma_o - 2a_2\beta_o)]\} + I_X dr\{\Omega^2[q'_{EO}\cos\theta_o \\
&- \gamma_o\cos 2\theta_o]\} + I_Y dr\{\Omega^2(-q'_{EO}\cos\theta_o + q'_{FO}\sin\theta_o)\} + I_Z dr\{\Omega^2[-q'_{FO}\sin\theta_o \\
&+ \gamma_o\cos 2\theta_o]\}\Big]\phi_\theta\beta\Big\} + \Big\{-K_1L_2^2\tan\delta_3\phi_{\theta PR}\beta\Big\} + \Big\{\Big[\int_0^{R-e} mdr\{\Omega^2[-(a_2^2 + b_2^2)\beta_o \\
&+ eb_2]\} + I_X dr\{-\Omega^2\beta_o\cos 2\theta_o\} + I_Z dr\{\Omega^2\beta_o\cos 2\theta_o\}\Big]\phi_\theta\gamma\Big\} \quad 11
\end{aligned}
$$

$$+\left\{-K_1L_2^2\tan\alpha_1\phi_{\theta PR}\gamma\right\} +\left\{\sum_{i=1}^{NE}\left[\int_0^{R-e} m\,dr\{\Omega^2[\phi_{F,i}(a_2+e\gamma_o)-\phi_{E,i}(b_2\right.\right.$$

$$+(r+e)\beta_o)]\}+I_X dr\{-\Omega^2\beta_o\phi'_{E,i}\cos\theta_o\}+I_Y dr\{\Omega^2\beta_o(\phi'_{E,i}\cos\theta_o+\phi'_{F,i}\sin\theta_o)\}$$

$$\left.\left.+I_Z dr\{\Omega^2[-\phi'_{E,i}q'_{FO}\cos2\theta_o+(q'_{EO}\cos2\theta_o-\beta_o\sin\theta_o)\phi'_{F,i}]\}\right]\phi_\theta\, q_{T,i}\right\}$$

$$+\left\{\sum_{i=1}^{NE}(-K_1L_2(\phi_{FPR,i}+[L_2\tan\alpha_1/(R-e)]\phi_{ET,i}))\phi_{\theta PR}q_{T,i}\right\}=0 \qquad (38)$$

Blade Rigid-Body Flapping Equations

$$
\begin{aligned}
&\Big\{\sum_{i=1}^{NA}\Big[\int_0^{R-e} m\,dr\{\phi_{X,i}(b_2 + r\beta_o)\sin\psi - \phi_{Y,i}(b_2 + r\beta_o)\cos\psi + \phi_{Z,i}(r + a_2\gamma_o \\
&- b_2\beta_o) + \phi_{\theta Y,i}[(e(a_2\gamma_o - b_2\beta_o) + r(r + e + 2a_2\gamma_o) + b_2^2)\sin\psi + (- a_2(a_2\gamma_o \\
&- b_2\beta_o) - r(a_2 - r\gamma_o))\cos\psi] + \phi_{\theta X,i}[(a_2(a_2\gamma_o - b_2\beta_o) + r(a_2 - r\gamma_o))\sin\psi \\
&+ (e(a_2\gamma_o - b_2\beta_o) + r(r + e + 2a_2\gamma_o) + b_2^2)\cos\psi]\} + I_X dr\{q'_{EO}(\\
&- \phi_{\theta Y,i}\cos\theta_o\cos\psi + \phi_{\theta X,i}\cos\theta_o\sin\psi) + \phi_{\theta Y,i}(\cos^2\theta_o\sin\psi + (\gamma_o\cos^2\theta_o \\
&- \tfrac{1}{2}\beta_o\sin 2\theta_o)\cos\psi) + \phi_{\theta X,i}(\cos^2\theta_o\cos\psi - (\gamma_o\cos^2\theta_o - \tfrac{1}{2}\beta_o\sin 2\theta_o)\sin\psi)\} \\
&+ I_Y dr\{\phi_{\theta Y,i}(q'_{EO}\cos\theta_o - q'_{FO}\sin\theta_o - \gamma_o)\cos\psi \\
&+ \phi_{\theta X,i}(- q'_{EO}\cos\theta_o + q'_{FO}\sin\theta_o + \gamma_o)\sin\psi\} + I_Z dr\{q'_{FO}(\phi_{\theta Y,i}\sin\theta_o\cos\psi \\
&- \phi_{\theta X,i}\sin\theta_o\sin\psi) + \phi_{\theta Y,i}(\sin^2\theta_o\sin\psi + (\gamma_o\sin^2\theta_o + \tfrac{1}{2}\beta_o\sin 2\theta_o)\cos\psi) \\
&+ \phi_{\theta X,i}(\sin^2\theta_o\cos\psi - (\gamma_o\sin^2\theta_o + \tfrac{1}{2}\beta_o\sin 2\theta_o)\sin\psi)\}\Big]\ddot{q}_i\Big\} \\
&+ \Big\{\Big[\int_0^{R-e} m\,dr\{((a_2^2 + b_2^2)\gamma_o + a_2 r)\} + I_X dr\{q'_{EO}\cos\theta_o\} + I_Y dr\{(- q'_{EO}\cos\theta_o \\
&+ q'_{FO}\sin\theta_o + \gamma_o)\} + I_Z dr\{- q'_{FO}\sin\theta_o\}\Big]\phi_\theta\ddot{\theta}_T\Big\} + \Big\{- M_1 L_2^2\tan\delta_3\phi_{\theta PR}\ddot{\theta}_T\Big\} \\
&+ \Big\{\Big[\int_0^{R-e} m\,dr\{r^2 + b_2^2 + 2a_2 r\gamma_o\} + I_X dr\{\cos^2\theta_o\} + I_Z dr\{\sin^2\theta_o\}\Big]\ddot{\beta}\Big\} \\
&+ \Big\{M_1 L_2^2\tan^2\delta_3\ddot{\beta}\Big\} + \Big\{\Big[\int_0^{R-e} m\,dr\{- b_2(a_2 - r\gamma_o)\} + I_X dr\{\tfrac{1}{2}\sin 2\theta_o\} \\
&+ I_Z dr\{- \tfrac{1}{2}\sin 2\theta_o\}\Big]\ddot{\gamma}\Big\} + \Big\{M_1 L_2^2\tan\delta_3\tan\alpha_1\ddot{\gamma}\Big\} + \Big\{\sum_{i=1}^{NE}\Big[\int_0^{R-e} m\,dr\{\phi_{F,i}(a_2\gamma_o \\
&+ r) + \phi_{E,i}b_2\gamma_o + \nu_{1,i}b_2\} + I_X dr\{\phi'_{F,i}\cos\theta_o\} + I_Z dr\{- \phi'_{E,i}\sin\theta_o\}\Big]\ddot{q}_{T,i}\Big\} \\
&+ \Big\{\sum_{i=1}^{NE}(M_1 L_2\tan\delta_3(\phi_{FPR,i} + [L_2\tan\alpha_1/(R-e)]\phi_{ET,i}))\ddot{q}_{T,i}\Big\} \\
&+ \Big\{\sum_{i=1}^{NA}\Big[\int_0^{R-e} m\,dr\{2\Omega[\phi_{\theta Y,i}((r(r + e + 2a_2\gamma_o) + e(a_2\gamma_o - b_2\beta_o) + b_2^2)\cos\psi \\
&+ (a_2(a_2\gamma_o - b_2\beta_o) - r^2\gamma_o)\sin\psi) + \phi_{\theta X,i}((- r(r + e + 2a_2\gamma_o) - e(a_2\gamma_o \qquad 13
\end{aligned}
$$

$$- b_2\beta_o) - b_2^2)\sin\psi + (a_2(a_2\gamma_o - b_2\beta_o) - r^2\gamma_o)\cos\psi)]\}$$

$$+ I_X dr\{2\Omega q'_{EO}(\phi_{\theta Y,i}\cos\theta_o\sin\psi + \phi_{\theta X,i}\cos\theta_o\cos\psi)$$

$$+ \Omega[\phi_{\theta Y,i}(\cos\psi - 2(\gamma_o\cos^2\theta_o - \tfrac{1}{2}\beta_o\sin 2\theta_o)\sin\psi) + \phi_{\theta X,i}(- \sin\psi - 2(\gamma_o\cos^2\theta_o$$

$$- \tfrac{1}{2}\beta_o\sin 2\theta_o)\cos\psi)]\} + I_Y dr\{\Omega[\phi_{\theta Y,i}(- q'_{EO}\cos\theta_o + q'_{FO}\sin\theta_o)2\sin\psi$$

$$+ \phi_{\theta X,i}(- q'_{EO}\cos\theta_o + q'_{FO}\sin\theta_o)2\cos\psi$$

$$+ \phi_{\theta Y,i}(2\gamma_o\sin\psi - \cos\psi) + \phi_{\theta X,i}(2\gamma_o\cos\psi + \sin\psi)]\} + I_Z dr\{2\Omega q'_{FO}[$$

$$- \phi_{\theta Y,i}\sin\theta_o\sin\psi - \phi_{\theta X,i}\sin\theta_o\cos\psi] + \Omega[\phi_{\theta Y,i}(\cos\psi - (2\gamma_o\sin^2\theta_o$$

$$+ \beta_o\sin 2\theta_o)\sin\psi) + \phi_{\theta X,i}(- \sin\psi - (2\gamma_o\sin^2\theta_o + \beta_o\sin 2\theta_o)\cos\psi)]\}\Big]\dot{\bar{q}}_i \Big\}$$

$$+ \Big\{ \Big[\int_0^{R-e} m dr\{2\Omega b_2(b_2 + r\beta_o)\} + I_X dr\{\Omega\cos 2\theta_o\} + I_Y dr\{\Omega\} + I_Z dr\{- \Omega\cos 2\theta_o\}\Big]\phi_\theta\dot{\theta}_T \Big\}$$

$$+ \Big\{- C_1 L_2^2\tan\delta_3\phi_{\theta PR}\,\dot{\theta}_T\Big\} + \Big\{C_1 L_2^2\tan^2\delta_3\,\dot{\beta}\Big\} + \Big\{\Big[\int_0^{R-e} m dr\{2\Omega(b_2(a_2\gamma_o + r)$$

$$+ r^2\beta_o)\} + I_X dr\{\Omega(2q'_{EO}\sin\theta_o + \beta_o - \gamma_o\sin 2\theta_o)\} + I_Y dr\{\Omega(- 2q'_{EO}\sin\theta_o$$

$$- 2q'_{FO}\cos\theta_o - \beta_o)\} + I_Z dr\{\Omega(2q'_{FO}\cos\theta_o + \gamma_o\sin 2\theta_o + \beta_o)\}\Big]\dot{\gamma}\Big\}$$

$$+ \Big\{C_1 L_2^2\tan\delta_3\tan\alpha_1\,\dot{\gamma}\Big\} + \Big\{\sum_{i=1}^{NE}\Big[\int_0^{R-e} m dr\{2\Omega[\phi_{E,i}(b_2 + r\beta_o)]\}$$

$$+ I_X dr\{\Omega[\phi'_{E,i}(q'_{EO}\sin 2\theta_o + \beta_o\cos\theta_o - \gamma_o\sin\theta_o) + \phi'_{F,i}(q'_{EO} + \beta_o\sin\theta_o$$

$$- \gamma_o\cos\theta_o)]\} + I_Y dr\{\Omega[\phi'_{E,i}(- q'_{EO}\sin 2\theta_o - 2q'_{FO}\cos^2\theta_o + \gamma_o\sin\theta_o - \beta_o\cos\theta_o)$$

$$+ \phi'_{F,i}(q'_{EO}\cos 2\theta_o - q'_{FO}\sin 2\theta_o - \gamma_o\cos\theta_o - \beta_o\sin\theta_o)]\}$$

$$+ I_Z dr\{\Omega[\phi'_{E,i}(2q'_{FO}\cos^2\theta_o + \gamma_o\sin\theta_o + \beta_o\cos\theta_o) + \phi'_{F,i}(q'_{FO}\sin 2\theta_o + \gamma_o\cos\theta_o$$

$$+ \beta_o\sin\theta_o)]\}\Big]\dot{q}_{T,i}\Big\} + \Big\{\sum_{i=1}^{NE}(C_1 L_2\tan\delta_3(\phi_{FPR,i} + [L_2\tan\alpha_1/(R-e)]\phi_{ET,i}))\dot{q}_{T,i}\Big\}$$

$$+ \Big\{\sum_{i=1}^{NA}\Big[\int_0^{R-e} m dr\{\Omega^2[\phi_{\theta Y,i}(- b_2(b_2 + 2r\beta_o))\cos\psi + \phi_{\theta X,i}(b_2(b_2 + 2r\beta_o))\sin\psi]\}\Big]\bar{q}_i\Big\}$$

$$+ \Big\{\Big[\int_0^{R-e} m dr\{\Omega^2[a_2(a_2\gamma_o + r + e) - b_2(b_2\gamma_o - 2a_2\beta_o)]\} + I_X dr\{\Omega^2[q'_{EO}\cos\theta_o$$

$$- \gamma_o\cos 2\theta_o + \beta_o\sin 2\theta_o]\} + I_Y dr\{\Omega^2[- q'_{EO}\cos\theta_o - q'_{FO}\sin\theta_o]\} + I_Z dr\{\Omega^2[q'_{FO}\sin\theta_o$$

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$$+ \gamma_o \cos 2\theta_o - \beta_o \sin 2\theta_o]\}\Big]\phi_\theta \theta_T \Big\} + \Big\{ - K_1 L_2^2 \tan\delta_3 \phi_{\theta PR} \theta_T \Big\}$$

$$+ \Big\{ \Big[\int_0^{R-e} mdr\{\Omega^2[e(a_2\gamma_o - b_2\beta_o) + r(r + e) - 4r\beta_o b_2 + 2a_2 r\gamma_o - b_2^2]\}$$

$$+ I_X dr\{\Omega^2 \sin^2\theta_o\} + I_Y dr\{- \Omega^2\} + I_Z dr\{\Omega^2 \cos^2\theta_o\}\Big]\beta \Big\} + \Big\{ K_1 L_2^2 \tan^2\delta_3 \beta \Big\}$$

$$+ \Big\{ K_\beta \beta \Big\} + \Big\{ \Big[\int_0^{R-e} mdr\{\Omega^2[b_2(a_2 - r\gamma_o) + a_2\beta_o(2r + e)]\} + I_X dr\{$$

$$- \tfrac{1}{2}\Omega^2 \sin 2\theta_o\} + I_Z dr\{\tfrac{1}{2}\Omega^2 \sin 2\theta_o\}\Big]\gamma \Big\} + \Big\{ K_1 L_2^2 \tan\delta_3 \tan\alpha_1 \gamma \Big\}$$

$$+ \Big\{ \sum_{i=1}^{NE} \Big[\int_0^{R-e} mdr\{\Omega^2[- \phi_{E,i} b_2 \gamma_o + \phi_{F,i}(a_2\gamma_o + r + e + 2b_2\beta_o) - \nu_{1,i} b_2]\}$$

$$+ I_X dr\{- \Omega^2 \phi'_{E,i} \sin\theta_o\} + I_Y dr\{\Omega^2[\phi'_{E,i} \sin\theta_o + \phi'_{F,i} \cos\theta_o]\}$$

$$+ I_Z dr\{- \Omega^2[\phi'_{F,i} \cos\theta_o]\}\Big] q_{T,i} \Big\} + \Big\{ \sum_{i=1}^{NE} (K_1 L_2 \tan\delta_3 (\phi_{FPR,i}$$

$$+ [L_2 \tan\alpha_1/(R-e)]\phi_{ET,i}))q_{T,i} \Big\} = 0 \qquad (39)$$

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Blade Rigid-Body Lagging Equations 16

$$
\begin{aligned}
&\Big\{\sum_{i=1}^{NA}\Big[\int_0^{R-e} m\,dr\{\phi_{X,i}(-(a_2 - r\gamma_0)\sin\psi - (a_2\gamma_0 + r)\cos\psi) + \phi_{Y,i}(-(a_2\gamma_0 + r)\sin\psi \\
&+ (a_2 - r\gamma_0)\cos\psi) + \phi_{Z,i}a_2\beta_0 + \phi_{\theta Y,i}(([e\beta_0 - b_2](a_2 - r\gamma_0))\sin\psi \\
&+ (-b_2(a_2\gamma_0 + r) - (a_2^2 + r^2)\beta_0)\cos\psi) + \phi_{\theta X,i}((b_2(a_2\gamma_0 + r) + (a_2^2 \\
&+ r^2)\beta_0)\sin\psi + ([e\beta_0 - b_2](a_2 - r\gamma_0))\cos\psi)\} + I_X dr\{q'_{EO}(-\phi_{\theta Y,i}\sin\theta_0\cos\psi \\
&+ \phi_{\theta X,i}\sin\theta_0\sin\psi) + \phi_{\theta Y,i}(\tfrac{1}{2}\sin 2\theta_0\sin\psi + (\tfrac{1}{2}\gamma_0\sin 2\theta_0 - \beta_0\sin^2\theta_0)\cos\psi) \\
&+ \phi_{\theta X,i}(\tfrac{1}{2}\sin 2\theta_0\cos\psi - (\tfrac{1}{2}\gamma_0\sin 2\theta_0 - \beta_0\sin^2\theta_0)\sin\psi)\} + I_Y dr\{\phi_{\theta Y,i}(q'_{EO}\sin\theta_0 \\
&+ q'_{FO}\cos\theta_0)\cos\psi + \phi_{\theta X,i}(-q'_{EO}\sin\theta_0 - q'_{FO}\cos\theta_0)\sin\psi\} + I_Z dr\{q'_{FO}(\\
&- \phi_{\theta Y,i}\cos\theta_0\cos\psi + \phi_{\theta X,i}\cos\theta_0\sin\psi) + \phi_{\theta Y,i}(-\tfrac{1}{2}\sin 2\theta_0\sin\psi - (\tfrac{1}{2}\gamma_0\sin 2\theta_0 \\
&+ \beta_0\cos^2\theta_0)\cos\psi) + \phi_{\theta X,i}(-\tfrac{1}{2}\sin 2\theta_0\cos\psi + (\tfrac{1}{2}\gamma_0\sin 2\theta_0 + \beta_0\cos^2\theta_0)\sin\psi)\}\Big]\ddot{q}_i\Big\} \\
&+ \Big\{\Big[\int_0^{R-e} m\,dr\{b_2 r\} + I_X dr\{q'_{EO}\sin\theta_0\} + I_Y dr\{-q'_{EO}\sin\theta_0 - q'_{FO}\cos\theta_0\} \\
&+ I_Z dr\{q'_{FO}\cos\theta_0\}\Big]\phi_\theta\ddot{\theta}_T\Big\} + \Big\{-M_1L_2^2\tan\alpha_1\phi_{\theta PR}\ddot{\theta}_T\Big\} + \Big\{\Big[\int_0^{R-e} m\,dr\{-b_2(a_2 \\
&- r\gamma_0)\} + I_X dr\{\tfrac{1}{2}\sin 2\theta_0\} + I_Z dr\{-\tfrac{1}{2}\sin 2\theta_0\}\Big]\ddot{\beta}\Big\} + \Big\{M_1L_2^2\tan\delta_3\tan\alpha_1\ddot{\beta}\Big\} \\
&+ \Big\{\Big[\int_0^{R-e} m\,dr\{a_2^2 + r^2\} + I_X dr\{\sin^2\theta_0\} + I_Z dr\{\cos^2\theta_0\}\Big]\ddot{\gamma}\Big\} \\
&+ \Big\{M_1L_2^2\tan^2\alpha_1\ddot{\gamma}\Big\} + \Big\{\sum_{i=1}^{NE}\Big[\int_0^{R-e} m\,dr\{\phi_{E,i}r - a_2\nu_{1,i}\} + I_X dr\{\phi'_{F,i}\sin\theta_0\} \\
&+ I_Z dr\{\phi'_{E,i}\cos\theta_0\}\Big]\ddot{q}_{T,i}\Big\} + \Big\{\sum_{i=1}^{NE}(M_1L_2\tan\alpha_1(\phi_{FPR,i} \\
&+ [L_2\tan\alpha_1/(R-e)]\phi_{ET,i}))\ddot{q}_{T,i}\Big\} + \Big\{\sum_{i=1}^{NA}\Big[\int_0^{R-e} m\,dr\{2\Omega[\phi_{\theta Y,i}(a_2^2\beta_0\sin\psi \\
&+ (r + e)a_2\beta_0\cos\psi) + \phi_{\theta X,i}(-(r + e)a_2\beta_0\sin\psi + a_2^2\beta_0\cos\psi)]\} \\
&+ I_X dr\{\Omega[-\phi_{\theta Y,i}\beta_0\cos 2\theta_0\sin\psi - \phi_{\theta X,i}\beta_0\cos 2\theta_0\cos\psi]\} + I_Y dr\{\Omega\beta_0(\phi_{\theta Y,i}\sin\psi \\
&+ \phi_{\theta X,i}\cos\psi)\} + I_Z dr\{\Omega\beta_0(\phi_{\theta Y,i}\cos 2\theta_0\sin\psi + \phi_{\theta X,i}\cos 2\theta_0\cos\psi)\}\Big]\dot{q}_i\Big\} \\
&+ \Big\{\Big[\int_0^{R-e} m\,dr\{-2\Omega a_2(b_2 + r\beta_0)\} + I_X dr\{\Omega\sin 2\theta_0\} + I_Z dr\{-\Omega\sin 2\theta_0\}\Big]\phi_\theta\dot{\theta}_T\Big\}
\end{aligned}
$$

$$+ \left\{ - C_1 L_2^2 \tan\alpha_1 \phi_\theta \dot{\theta}_T \right\}$$

$$+ \left\{ \left[\int_0^{R-e} mdr\{- 2\Omega(r^2\beta_o + b_2(a_2\gamma_o + r))\} + I_X dr\{\Omega(- 2q'_{EO}\sin\theta_o + \gamma_o \sin 2\theta_o - \beta_o)\} + I_Y dr\{\Omega\beta_o\} + I_Z dr\{\Omega(- 2q'_{FO}\cos\theta_o - \gamma_o\sin 2\theta_o - \beta_o)\} \right] \dot{\beta} \right\}$$

$$+ \left\{ C_1 L_2^2 \tan\delta_3 \tan\alpha_1 \dot{\beta} \right\} + \left\{ (2\zeta_\gamma I_\gamma \omega_\gamma + C_1 L_2^2 \tan^2\alpha_1)\dot{\gamma} \right\}$$

$$\left\{ \sum_{i=1}^{NE} \left[\int_0^{R-e} mdr\{- 2\Omega(a_2\phi_{E,i} + r\beta_o\phi_{F,i}) - 2\Omega r \nu_{1,i}\} + I_X dr\{\Omega[\phi'_{E,i}(2q'_{EO}\sin^2\theta_o + \beta_o\sin\theta_o) - \phi'_{F,i}\beta_o\cos\theta_o]\} + I_Y dr\{\Omega[\phi'_{E,i}(- 2q'_{EO}\sin^2\theta_o - q'_{FO}\sin 2\theta_o - \beta_o\sin\theta_o) + \phi'_{F,i}(q'_{EO}\sin 2\theta_o + 2q'_{FO}\cos^2\theta_o + \beta_o\cos\theta_o)]\} + I_Z dr\{\Omega[\phi'_{E,i}(q'_{FO}\sin 2\theta_o + \beta_o\sin\theta_o) + \phi'_{F,i}(- 2q'_{FO}\cos^2\theta_o - \beta_o\cos\theta_o)]\} \right] \dot{q}_{T,i} \right\} + \left\{ \sum_{i=1}^{NE} (C_1 L_2 \tan\alpha_1(\phi_{FPR,i} + [L_2\tan\alpha_1/(R-e)]\phi_{ET,i}))\dot{q}_{T,i} \right\}$$

$$+ \left\{ \sum_{i=1}^{NA} \left[\int_0^{R-e} mdr\{\Omega^2[\phi_{\theta Y,i}((- b_2(a_2\gamma_o + r) - r^2\beta_o)\sin\psi + (b_2(a_2 - r\gamma_o) + a_2 r\beta_o)\cos\psi) + \phi_{\theta X,i}((- b_2(a_2 - r\gamma_o) - a_2 r\beta_o)\sin\psi + (- b_2(a_2\gamma_o + r) - r^2\beta_o)\cos\psi)]\} \right] \bar{q}_1 \right\} + \left\{ \left[\int_0^{R-e} mdr\{\Omega^2[- (b_2^2 + a_2^2)\beta_o + eb_2]\} + I_X dr\{ - \Omega^2\beta_o\cos 2\theta_o\} + I_Z dr\{\Omega^2\beta_o\cos 2\theta_o\} \right] \phi_\theta\theta_T \right\} + \left\{ - K_1 L_2^2 \tan\alpha_1 \phi_{\theta PR}\theta_T \right\}$$

$$+ \left\{ \left[\int_0^{R-e} mdr\{\Omega^2[b_2(a_2 - r\gamma_o) + a_2\beta_o(2r + e)]\} + I_X dr\{- \tfrac{1}{2}\Omega^2\sin 2\theta_o\} + I_Z dr\{\tfrac{1}{2}\Omega^2\sin 2\theta_o\} \right] \beta \right\} + \left\{ K_1 L_2^2 \tan\delta_3 \tan\alpha_1 \beta \right\} + \left\{ \left[\int_0^{R-e} mdr\{\Omega^2[e(r - a_2\gamma_o) - b_2 r\beta_o]\} + I_Z dr\{- \Omega^2\beta_o\sin^2\theta_o\} \right]\gamma \right\} + \left\{ K_1 L_2^2 \tan^2\alpha_1 \gamma \right\} + \left\{ K_\gamma \gamma \right\}$$

$$+ \left\{ \sum_{i=1}^{NE} \int_0^{R-e} mdr\{\Omega^2[- \phi_{E,i}(b_2\beta_o - e) - a_2\beta_o\phi_{F,i}]\} q_{T,i} \right\} + \left\{ \sum_{i=1}^{NE} (K_1 L_2 \tan\alpha_1(\phi_{FPR,i} + [L_2\tan\alpha_1/(R-e)]\phi_{ET,i})) q_{T,i} \right\} = 0 \qquad (40)$$

Blade Bending Equations

$$\Big\{\sum_{i=1}^{NA}\Big[\int_0^{R-e} m\,dr\{\phi_{X,i}[\phi_{E,j}(\gamma_o\sin\psi - \cos\psi) + \nu_{1,j}\sin\psi + \phi_{F,j}\beta_o\sin\psi]$$

$$+ \phi_{Y,i}[\phi_{E,j}(-\gamma_o\cos\psi - \sin\psi) - \nu_{1,j}\cos\psi - \phi_{F,j}\beta_o\cos\psi] + \phi_{Z,i}\phi_{F,j}$$

$$+ \phi_{\theta Y,i}[(\phi_{E,j}b_2\gamma_o + \phi_{F,j}(a_2\gamma_o + r + e))\sin\psi + (-\phi_{F,j}(a_2 - r\gamma_o)$$

$$- \phi_{E,j}(b_2 + r\beta_o))\cos\psi] + \phi_{\theta X,i}[(\phi_{F,j}(a_2 - r\gamma_o) + \phi_{E,j}(b_2 + r\beta_o))\sin\psi$$

$$+ (\phi_{E,j}b_2\gamma_o + \phi_{F,j}(a_2\gamma_o + r + e))\cos\psi] + \phi_{\theta Y,i}b_2\nu_{1,j}\sin\psi + \phi_{\theta X,i}b_2\nu_{1,j}\cos\psi\}$$

$$+ I_X dr\{\phi'_{F,j}[\phi_{\theta Y,i}(\cos\theta_o\sin\psi + (\gamma_o\cos\theta_o - \beta_o\sin\theta_o - q'_{EO})\cos\psi)$$

$$+ \phi_{\theta X,i}(\cos\theta_o\cos\psi - (\gamma_o\cos\theta_o - \beta_o\sin\theta_o - q'_{EO})\sin\psi)]\}$$

$$+ I_Y dr\{\phi'_{E,j}[\phi_{\theta Y,i}q'_{FO}\cos\psi - \phi_{\theta X,i}q'_{FO}\sin\psi]\} + I_Z dr\{\phi'_{E,j}[\phi_{\theta Y,i}(-\sin\theta_o\sin\psi$$

$$- (\gamma_o\sin\theta_o + \beta_o\cos\theta_o + q'_{FO})\cos\psi) + \phi_{\theta X,i}(-\sin\theta_o\cos\psi + (\gamma_o\sin\theta_o + \beta_o\cos\theta_o$$

$$+ q'_{FO})\sin\psi)]\}\Big]\ \ddot{q}_i\Big\} + \Big\{\Big[\int_0^{R-e} m\,dr\{b_2\phi_{E,j} + a_2\phi_{F,j}\} + I_X dr\{q'_{EO}\phi'_{F,j}\}$$

$$+ I_Y dr\{-q'_{FO}\phi'_{E,j}\} + I_Z dr\{q'_{FO}\phi'_{E,j}\}\Big]\ \phi_\theta\ddot{\theta}_T\Big\} + \Big\{-M_1L_2\phi_{\theta PR}(\phi_{FPR,j}$$

$$+ [L_2\tan\alpha_1/(R-e)]\phi_{ET,j})\ \ddot{\theta}_T\Big\} + \Big\{\Big[\int_0^{R-e} m\,dr\{\phi_{F,j}(a_2\gamma_o + r) + \phi_{E,j}b_2\gamma_o$$

$$+ b_2\nu_{1,j}\} + I_X dr\{\phi'_{F,j}\cos\theta_o\} + I_Z dr\{-\phi_{E,j}\sin\theta_o\}\Big]\ \ddot{\beta}\Big\} + \Big\{M_1L_2\tan\delta_3(\phi_{FPR,j}$$

$$+ [L_2\tan\alpha_1/(R-e)]\phi_{ET,j})\ \ddot{\beta}\Big\} + \Big\{\Big[\int_0^{R-e} m\,dr\{\phi_{E,j}r - a_2\nu_{1,j}\} + I_X dr\{\phi'_{F,j}\sin\theta_o\}$$

$$+ I_Z dr\{\phi'_{E,j}\cos\theta_o\}\Big]\ \ddot{\gamma}\Big\} + \Big\{M_1L_2\tan\alpha_1(\phi_{FPR,j} + [L_2\tan\alpha_1/(R-e)]\phi_{ET,j})\ \ddot{\gamma}\Big\}$$

$$+\Big\{\sum_{i=1}^{NE}\Big[\int_0^{R-e} m\,dr\{\phi^2_{E,j} + \phi^2_{F,j}\} + I_X dr\{\phi'_{F,j}\phi'_{F,i}\} + I_Z dr\{\phi'_{E,j}\phi'_{E,i}\}\Big]\ \ddot{q}_{T,i}\Big\}$$

$$+ \Big\{(\phi_{FPR,j} + [L_2\tan\alpha_1/(R-e)]\phi_{ET,j})\ \sum_{i=1}^{NE}(M_1(\phi_{FPR,i}$$

$$+ [L_2\tan\alpha_1/(R-e)]\phi_{ET,i}))\ \ddot{q}_{T,i}\Big\} + \Big\{\sum_{i=1}^{NA}\Big[\int_0^{R-e} m\,dr\{2\Omega[\phi_{\theta Y,i}(\phi_{F,j}(a_2 - r\gamma_o)\sin\psi$$

$$+ \phi_{F,j}(a_2\gamma_o + r + e - b_2\beta_o)\cos\psi) + \phi_{\theta X,i}(-\phi_{F,j}(a_2\gamma_o + r + e - b_2\beta_o)\sin\psi$$

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$$+ \phi_{F,j}(a_2 - r\gamma_o)\cos\psi)]\} + I_X dr\{\Omega[\phi'_{F,j}(\phi_{\theta Y,i}(\cos\theta_o\cos\psi - (\gamma_o\cos\theta_o - q'_{EO}$$

$$- \beta_o \sin\theta_o)\sin\psi) + \phi_{\theta X,i}(- \cos\theta_o \sin\psi - (\gamma_o \cos\theta_o - q'_{EO} - \beta_o \sin\theta_o)\cos\psi))$$

$$+ \phi'_{E,j}(\phi_{\theta Y,i}(- \sin\theta_o \cos\psi + (\gamma_o \sin\theta_o - \beta_o \cos\theta_o - q'_{EO}\sin 2\theta_o)\sin\psi)$$

$$+ \phi_{\theta X,i}(\sin\theta_o \sin\psi + (\gamma_o \sin\theta_o - \beta_o \cos\theta_o - q'_{EO}\sin 2\theta_o)\cos\psi))]\}$$

$$+ I_Y dr\{\Omega[\phi'_{E,j}(\phi_{\theta Y,i}(\sin\theta_o \cos\psi + (q'_{EO}\sin 2\theta_o - 2q'_{FO}\sin^2\theta_o$$

$$+ \beta_o \cos\theta_o - \gamma_o \sin\theta_o)\sin\psi) + \phi_{\theta X,i}(- \sin\theta_o \sin\psi + (q'_{EO}\sin 2\theta_o - 2q'_{FO}\sin^2\theta_o$$

$$+ \beta_o \cos\theta_o - \gamma_o \sin\theta_o)\cos\psi)) + \phi'_{F,j}(\phi_{\theta Y,i}(- \cos\theta_o \cos\psi + (- q'_{EO}\cos 2\theta_o$$

$$+ q'_{FO}\sin 2\theta_o + \beta_o \sin\theta_o + \gamma_o \cos\theta_o)\sin\psi) + \phi_{\theta X,i}(\cos\theta_o \sin\psi + (- q'_{EO}\cos 2\theta_o$$

$$+ q'_{FO}\sin 2\theta_o + \beta_o \sin\theta_o + \gamma_o \cos\theta_o)\cos\psi))]\} + I_Z dr\{\Omega[\phi'_{F,j}(\phi_{\theta Y,i}(\cos\theta_o \cos\psi$$

$$-(q'_{FO}\sin 2\theta_o - q'_{EO}\cos 2\theta_o + \gamma_o \cos\theta_o + \beta_o \sin\theta_o)\sin\psi) + \phi_{\theta X,i}(- \cos\theta_o \sin\psi$$

$$- (q'_{FO}\sin 2\theta_o - q'_{EO}\cos 2\theta_o + \gamma \cos\theta_o + \beta_o \sin\theta_o)\cos\psi)) + \phi'_{E,j}((- \sin\theta_o \cos\psi$$

$$+ (2q'_{FO}\sin^2\theta_o + \gamma_o \sin\theta_o + \beta_o \cos\theta_o)\sin\psi)\phi_{\theta Y,i} + \phi_{\theta X,i}(\sin\theta_o \sin\psi$$

$$+ (2q'_{FO}\sin^2\theta_o + \gamma_o \sin\theta_o + \beta_o \cos\theta_o)\cos\psi))]\}\Big] \hat{q}_i \Big\} + \Big\{ \Big[\int_0^{R-e} mdr\{2\Omega(\phi_{F,j} b_2 \beta_o$$

$$- \phi_{E,j} a_2 \beta_o) + 2\Omega v_{1,j} b_2\} + I_X dr\{\Omega(\phi'_{F,j}\cos\theta_o + \phi'_{E,j}\sin\theta_o)\}$$

$$+ I_Y dr\{\Omega(- \phi'_{E,j}\sin\theta_o + \phi'_{F,j}\cos\theta_o)\} + I_Z dr\{\Omega($$

$$- \phi'_{E,j}\sin\theta_o - \phi'_{F,j}\cos\theta_o)\}\Big] \phi_\theta \dot{\theta}_T \Big\} + \Big\{- C_1 L_2 \phi_{\theta PR}(\phi_{FPR,j}$$

$$+ [L_2 \tan\alpha_1/(R-e)]\phi_{ET,j}) \dot{\theta}_T \Big\} + \Big\{ \Big[\int_0^{R-e} mdr\{- 2\Omega\phi_{E,j}(b_2 + r\beta_o)\}$$

$$+ I_X dr\{\Omega[\phi'_{F,j}(\gamma_o \cos\theta_o - \beta_o \sin\theta_o - q'_{EO}) + \phi'_{E,j}(\gamma_o \sin\theta_o - \beta_o \cos\theta_o$$

$$- q'_{EO}\sin 2\theta_o)]\} + I_Y dr\{\Omega[\phi'_{E,j}(q'_{EO}\sin 2\theta_o + 2q'_{FO}\cos^2\theta_o - \gamma_o \sin\theta_o$$

$$+ \beta_o \cos\theta_o) + \phi'_{F,j}(- q'_{EO}\cos 2\theta_o + q'_{FO}\sin 2\theta_o + \gamma_o \cos\theta_o + \beta_o \sin\theta_o)]\}$$

$$+ I_Z dr\{\Omega[\phi'_{E,j}(- 2q'_{FO}\cos^2\theta_o - \gamma_o \sin\theta_o - \beta_o \cos\theta_o) + \phi'_{F,j}(- q'_{FO}\sin 2\theta_o$$ 19

$$+ q'_{EO}\cos 2\theta_o - \gamma_o\cos\theta_o - \beta_o\sin\theta_o)]\}\Big]\,\dot{\beta}\Big\} + \Big\{ C_1 L_2 \tan\delta_3(\phi_{FPR,j}$$

$$+ [L_2\tan\alpha_1/(R-e)]\phi_{ET,j})\,\dot{\beta}\Big\} + \Big\{\Big[\int_0^{R-e} m\,dr\{2\Omega(a_2\phi_{E,j} + r\beta_o\phi_{F,j} + \nu_{1,j}r)\}$$

$$+ I_X dr\{\Omega[\phi'_{F,j}\beta_o\cos\theta_o - \phi'_{E,j}(\beta_o\sin\theta_o + 2q'_{EO}\sin^2\theta_o)]\}$$

$$+ I_Y dr\{\Omega[\phi'_{E,j}(2q'_{EO}\sin^2\theta_o + q'_{FO}\sin 2\theta_o + \beta_o\sin\theta_o) + \phi'_{F,j}(- q'_{EO}\sin 2\theta_o$$

$$- 2q'_{FO}\cos^2\theta_o - \beta_o\cos\theta_o)]\} + I_Z dr\{\Omega[\phi'_{E,j}(- q'_{FO}\sin 2\theta_o - \beta_o\sin\theta_o)$$

$$+ \phi'_{F,j}(2q'_{FO}\cos^2\theta_o + q'_{EO}\sin 2\theta_o + \beta_o\cos\theta_o)]\}\Big]\dot{\gamma}\Big\} + \Big\{ C_1 L_2\tan\alpha_1(\phi_{FPR,j}$$

$$+ [L_2\tan\alpha_1/(R-e)]\phi_{ET,j})\dot{\gamma}\Big\}\ \Big\{\sum_{i=1}^{NE}\Big[\int_0^{R-e} m\,dr\{2\Omega[\nu_{1,j}\phi_{E,i} - \nu_{1,i}\phi_{E,j}$$

$$+ \beta_o(\phi_{F,j}\phi_{E,i} - \phi_{F,i}\phi_{E,j})]\} + I_X dr\{\Omega(\beta_o$$

$$+ q'_{EO}\sin\theta_o)(\phi'_{F,j}\phi'_{E,i} - \phi'_{E,j}\phi'_{F,i})\}$$

$$+ I_Y dr\{\Omega(q'_{EO}\sin\theta_o + 2q'_{FO}\cos\theta_o + \beta_o)(\phi'_{E,j}\phi'_{F,i} - \phi'_{F,j}\phi'_{E,i})\}$$

$$+ I_Z dr\{\Omega(q'_{EO}\sin\theta_o - 2q'_{FO}\cos\theta_o - \beta_o)(- \phi'_{F,j}\phi'_{E,i} + \phi'_{E,j}\phi'_{F,i})\}\Big]\dot{q}_{T,i}\Big\}$$

$$+ \Big\{ C_1(\phi_{FPR,j} + [L_2\tan\alpha_1/(R-e)]\phi_{ET,j})\sum_{i=1}^{NE}(\phi_{FPR,i}$$

$$+ [L_2\tan\alpha_1/(R-e)]\phi_{ET,i})\dot{q}_{T,i}\Big\} + \Big\{ 2\zeta_{q,j}M_{q,j}\omega_{q,j}\,\dot{q}_{T,j}\Big\}$$

$$+ \Big\{\Big[\int_0^{R-e} m\,dr\{\Omega^2[\phi_{F,j}(a_2 + e\gamma_o) - \phi_{E,j}(\beta_o(r + e) + b_2)]\}$$

$$+ I_X dr\{- \Omega^2\phi'_{E,j}(q'_{EO}\sin 2\theta_o + \beta_o\cos\theta_o)\} + I_Y dr\{\Omega^2[\phi'_{E,j}(q'_{EO}\sin 2\theta_o$$

$$+ q'_{FO}\cos 2\theta_o + \beta_o\cos\theta_o) + \phi'_{F,j}(- q'_{EO}\cos 2\theta_o + q'_{FO}\sin 2\theta_o + \beta_o\sin\theta_o)]\}$$

$$+ I_Z dr\{\Omega^2[- \phi'_{E,j}q'_{FO}\cos 2\theta_o + \phi'_{F,j}(- q'_{FO}\sin 2\theta_o + q'_{EO}\cos 2\theta_o$$

$$- \beta_o\sin\theta_o)]\}\Big]\phi_\theta\theta_T\Big\} + \Big\{ - K_1 L_2\phi_{\theta PR}(\phi_{FPR,j} + [L_2\tan\alpha_1/(R-e)]\phi_{ET,j})\theta_T\Big\}$$

$$+ \Big\{\Big[\int_0^{R-e} m\,dr\{\Omega^2[- \phi_{E,j}b_2\gamma_o + \phi_{F,j}(a_2\gamma_o + r + e + 2b_2\beta_o) - \nu_{1,j}b_2]\}$$

$$+ I_X dr\{- \Omega^2\phi'_{E,j}\sin\theta_o\} + I_Y dr\{\Omega^2(\phi'_{E,j}\sin\theta_o - \phi'_{F,j}\cos\theta_o)\} \quad 20$$

$$
\begin{aligned}
&+ I_Z dr\{\Omega^2 \phi'_{F,j} \cos\theta_o\}\Big]\beta \Big\} + \Big\{ K_1 L_2 \tan\delta_3 (\phi_{FPR,j} + [L_2 \tan\alpha_1/(R-E)]\phi_{ET,j})\ \beta \Big\} \\
&+ \Big\{ \Big[\int_0^{R-e} m\,dr\{\Omega^2[-\ \phi_{E,j}(b_2\beta_o - e) - \phi_{F,j} a_2 \beta_o]\}\Big]\gamma \Big\} + \Big\{ K_1 L_2 \tan\alpha_1 (\phi_{FPR,j} \\
&+ [L_2 \tan\alpha_1/(R-e)]\phi_{ET,j})\gamma \Big\} + \Big\{ \sum_{i=1}^{NE} \Big[\int_0^{R-e} m\,dr\{-\ \Omega^2 b_2 \beta_o \nu_{2,i,j}\} \\
&+ I_X dr\{\Omega^2 \sin^2\theta_o \phi'_{E,j} \phi'_{E,i}\} + I_Y dr\{\Omega^2[\phi'_{E,j}(-\ \phi'_{E,i} \sin^2\theta_o + \tfrac{1}{2}\phi'_{F,i} \sin 2\theta_o) \\
&+ \phi'_{F,j}(\tfrac{1}{2}\phi'_{E,i} \sin 2\theta_o - \phi'_{F,i} \cos^2\theta_o)]\} + I_Z dr\{\Omega^2[-\ \tfrac{1}{2}\phi'_{E,j} \phi'_{F,i} \sin 2\theta_o \\
&+ \phi'_{F,j}(-\ \tfrac{1}{2}\phi'_{E,i} \sin 2\theta_o + \phi'_{F,i} \cos^2\theta_o)]\}\Big] q_{T,i} \Big\} + \Big\{ K_1 (\phi_{FPR,j} \\
&+ [L_2 \tan\alpha_1/(R-e)]\phi_{ET,j}) \sum_{i=1}^{NE} (\phi_{FPR,i} + [L_2 \tan\alpha_1/(R-e)]\phi_{ET,i}) q_{T,i} \Big\} \\
&+ \Big\{ \omega^2_{q,j} M_{q,j} q_{T,j} \Big\} = 0
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
dT &= \tfrac{1}{2}\rho c U[C_L U_T - C_D U_P] \\
dH &= \tfrac{1}{2}\rho c U[C_L U_P + C_D U_T] \\
dM &= \tfrac{1}{2}\rho c^2 U^2 C_M \\
t_1 &= \frac{\partial(dT)}{\partial U_T} = \tfrac{1}{2}\rho c\,{}^{1}/_{U}[C_L(U^2 + U_T^2) - C_D U_T U_P + C_{L,\alpha} U_T U_P \\
&\quad - C_{D,\alpha} U_P^2] + \tfrac{1}{2}\rho c\,{}^{1}/_{V}[C_{L,M} U_T^2 - C_{D,M} U_T U_P] \\
t_2 &= \frac{\partial(dT)}{\partial U_P} = \tfrac{1}{2}\rho c\,{}^{1}/_{U}[C_L U_T U_P - C_D(U^2 + U_P^2) - C_{L,\alpha} U_T^2 \\
&\quad + C_{D,\alpha} U_T U_P] + \tfrac{1}{2}\rho c\,{}^{1}/_{V}[C_{L,M} U_T U_P - C_{D,M} U_P^2] \\
t_3 &= \frac{\partial(dT)}{\partial\alpha} = \tfrac{1}{2}\rho c U[C_{L,\alpha} U_T - C_{D,\alpha} U_P] \\
h_1 &= \frac{\partial(dH)}{\partial U_T} = \tfrac{1}{2}\rho c\,{}^{1}/_{U}[C_L U_T U_P + C_D(U^2 + U_T^2) + C_{L,\alpha} U_P^2 \\
&\quad + C_{D,\alpha} U_T U_P] + \tfrac{1}{2}\rho c\,{}^{1}/_{V}[C_{L,M} U_T U_P + C_{D,M} U_T^2] \\
h_2 &= \frac{\partial(dH)}{\partial U_P} = \tfrac{1}{2}\rho c\,{}^{1}/_{U}[C_L(U^2 + U_P^2) + C_D U_T U_P - C_{L,\alpha} U_T U_P \\
&\quad - C_{D,\alpha} U_T^2] + \tfrac{1}{2}\rho c\,{}^{1}/_{V}[C_{L,M} U_P^2 + C_{D,M} U_T U_P] \\
h_3 &= \frac{\partial(dH)}{\partial\alpha} = \tfrac{1}{2}\rho c U[C_{L,\alpha} U_P + C_{D,\alpha} U_T] \\
m_1 &= \frac{\partial(dM)}{\partial U_T} = \tfrac{1}{2}\rho c^2[C_M 2U_T + C_{M,\alpha} U_P] \\
&\quad + \tfrac{1}{2}\rho c^2\,{}^{1}/_{V} C_{M,M} U_T U \\
m_2 &= \frac{\partial(dM)}{\partial U_P} = \tfrac{1}{2}\rho c^2[C_M 2U_P - C_{M,\alpha} U_T] \\
&\quad + \tfrac{1}{2}\rho c^2\,{}^{1}/_{V} C_{M,M} U_P U \\
m_3 &= \frac{\partial(dM)}{\partial\alpha} = \tfrac{1}{2}\rho c^2 C_{M,\alpha} U^2
\end{aligned}
$$

$$
\begin{aligned}
\delta U_T &= \frac{\partial U_T}{\partial q} + \frac{\partial U_T}{\partial \dot{q}} \\
&= \sum_{i=1}^{NA} \{\phi_{\theta Y,i}(U_P\cos\psi - U_P[\gamma_o - q'_{EO}]\sin\psi) \\
&\quad - \phi_{\theta X,i}(U_P[\gamma_o - q'_{EO}]\cos\psi + U_P\sin\psi)\}\bar{q}_i \\
&\quad - \{\Omega(a\beta_o + bq'_{EO})\}\phi_\theta\theta_T \\
&\quad - \{\Omega(b + r\beta_o) + U_P(\gamma_o - q'_{EO})\}\beta \\
&\quad - \{\Omega e(\gamma_o - q'_{EO}) + U_P\beta_o\}\gamma \\
&\quad + \sum_{i=1}^{NE} \{\Omega[\phi'_{E,i}(- a - e\gamma_o + (r + e)q'_{EO}) - \phi_{E,i}q'_{EO} - \phi_{F,i}\beta_o \\
&\quad + r_i] - U_P[\phi'_{E,i}(\beta_o + q'_{FO}) + \phi'_{F,i}(\gamma_o - q'_{EO})]\}q_{T,i} \\
&\quad + \sum_{i=1}^{NA} \{\phi_{\theta Y,i}[b(\gamma_o - q'_{EO})\sin\psi - (b + r\beta_o)\cos\psi] + \phi_{\theta X,i}[b(\gamma_o - q'_{EO})\cos\psi \\
&\quad + (b + r\beta_o)\sin\psi] + \phi_{X,i}[(\gamma_o - q'_{EO})\sin\psi - \cos\psi] + \phi_{Y,i}[-(\gamma_o \\
&\quad - q'_{EO})\cos\psi - \sin\psi]\}\dot{\bar{q}}_i \\
&\quad + \{b\}\phi_\theta\dot{\theta}_T \\
&\quad + \{b(\gamma_o - q'_{EO})\}\dot{\beta} \\
&\quad + \{r + aq'_{EO}\}\dot{\gamma} \\
&\quad + \sum_{i=1}^{NE} \{\dot{r}_i q'_{EO} + \phi_{E,i}\}\dot{q}_{T,i}
\end{aligned}
$$

$$
\begin{aligned}
\delta U_P &= \frac{\partial U_P}{\partial q} + \frac{\partial U_P}{\partial \dot{q}} \\
&= \sum_{i=1}^{NA} \{\phi_{\theta Y,i}(- U_P(\beta_o + q'_{FO})\sin\psi) - \phi_{\theta X,i}(U_P(\beta_o + q'_{FO})\cos\psi)\}\bar{q}_i \\
&\quad + \{\Omega b(\beta_o + q'_{FO})\phi_\theta\}\theta_T \\
&\quad - \{\Omega(a - r\gamma_o) + U_P(\beta_o + q'_{FO})\}\beta
\end{aligned}
$$

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$$+ \{\Omega(r\beta_o - eq'_{FO})\}\gamma$$

$$+ \sum_{i=1}^{NE} \{\Omega[\phi'_{F,i}(-a + r\gamma_o) + \phi_{E,i}(\beta_o + q'_{FO})] - U_P\phi'_{F,i}(\beta_o + q'_{FO})\}q_{T,i}$$

$$+ \sum_{i=1}^{NA} \{(-(a - r\gamma_o)\cos\psi + (r + e + bq'_{FO} + a\gamma_o)\sin\psi)\phi_{\theta Y,i}$$

$$+ ((a - r\gamma_o)\sin\psi + (r + e + bq'_{FO} + a\gamma_o)\cos\psi)\phi_{\theta X,i}$$

$$+ \phi_{X,i}(\beta_o + q'_{FO})\sin\psi - \phi_{Y,i}(\beta_o + q'_{FO})\cos\psi$$

$$+ \phi_{Z,i}\}\dot{\bar{q}}_i$$

$$+ \{a\}\phi_\theta\dot{\theta}_T$$

$$+ \{a\gamma_o + r + bq'_{FO}\}\dot{\beta}$$

$$- \{aq'_{FO}\}\dot{\gamma}$$

$$+ \sum_{i=1}^{NE} \{-\dot{r}_i q'_{FO} + \phi_{F,i}\}\dot{q}_{T,i}$$

$$\delta\alpha = \frac{\partial\alpha}{\partial q} + \frac{\partial\alpha}{\partial\dot{q}} = \{-1\}\phi_\theta\theta_T$$

$$Q_{\bar{q}_j} = \left\{\int_0^{R-e} \left[\sum_{n=1}^{N} \{ \sum_{i=1}^{NA} \{\phi_{\theta X,i}[\phi_{\theta Y,j}((-b_1\beta_o + a_1q'_{EO} + r + e)dH\right.\right.$$

$$+ a_1(\beta_o + q'_{FO})dT)] + \phi_{X,j}[\phi_{\theta Y,i}dT] - \phi_{Y,j}[\phi_{\theta X,i}dT]\}\bar{q}_i$$

$$+ \{\phi_{\theta X,j}[(-a_1\sin\psi - a_1(\gamma_o - q'_{EO})\cos\psi)dH - (b_1\sin\psi - (a_1q'_{FO}$$

$$- b_1\dot{\gamma}_o)\cos\psi)dT] + \phi_{\theta Y,j}[(a_1\cos\psi - a_1(\gamma_o - q'_{EO})\sin\psi)dH + (b_1\cos\psi$$

$$+ (a_1q'_{FO} - b_1\gamma_o)\sin\psi)dT]\}\phi_\theta\theta_T$$

$$+ \{\phi_{\theta X,j}[((b_1\beta_o - r - a_1q'_{EO})\sin\psi + e(\gamma_o - q'_{EO})\cos\psi)dH$$

$$- (a_1(\beta_o + q'_{FO})\sin\psi + e(\beta_o + q'_{FO})\cos\psi)dT] + \phi_{\theta Y,j}[((- b_1\beta_o + r$$

$$+ a_1 q'_{EO})\cos\psi + e(\gamma_o - q'_{EO})\sin\psi)dH + (a_1(\beta_o + q'_{FO})\cos\psi - e(\beta_o$$

$$+ q'_{FO})\sin\psi)dT] + \phi_{X,j}\sin\psi dT - \phi_{Y,j}\cos\psi dT + \phi_{Z,j}((\gamma_o - q'_{EO})dH$$

$$- (\beta_o + q'_{FO})dT)\}\beta$$

$$+ \{\phi_{\theta X,j}[(b_1(\gamma_o - q'_{EO})\sin\psi + (e\beta_o - b_1)\cos\psi)dH - ((b_1 q'_{FO} + a_1\gamma_o$$

$$+ r)\sin\psi - (a_1 - r\gamma_o)\cos\psi)dT] - \phi_{\theta Y,j}[(b_1(\gamma_o - q'_{EO})\cos\psi - (e\beta_o$$

$$- b_1)\sin\psi)dH + ((- b_1 q'_{FO} - a_1\gamma_o - r)\cos\psi - (a_1 - r\gamma_o)\sin\psi)dT]$$

$$- \phi_{X,j}[((\gamma_o - q'_{EO})\cos\psi + \sin\psi)dH - q'_{FO}\cos\psi dT] - \phi_{Y,j}[((\gamma_o - q'_{EO})\sin\psi$$

$$- \cos\psi)dH - q'_{FO}\sin\psi dT] + \phi_{Z,j}\beta_o dH\}\gamma$$

$$+ \sum_{i=1}^{NE} \{\phi_{\theta X,j}[((- \phi_{F,i} + b_1(\gamma_o - q'_{EO})\phi'_{E,i} + a_1\beta_o\phi'_{E,i})\sin\psi$$

$$- (\phi_{F,i}(\gamma_o - q'_{EO}) + b_1\phi'_{E,i} - e\beta_o\phi'_{E,i})\cos\psi)dH - ((\phi_{E,i} + b_1(\gamma_o$$

$$- q'_{EO})\phi'_{F,i} + a_1(\beta_o + q'_{FO})\phi'_{F,i} + b_1 q'_{FO}\phi'_{E,i})\sin\psi + (\phi_{E,i}\gamma_o$$

$$- r_i - q'_{FO}\phi_{F,i} + (r + e)q'_{FO}\phi'_{F,i})\cos\psi)dT] - \phi_{\theta Y,j}[((- \phi_{F,i}$$

$$+ b_1(\gamma_o - q'_{EO})\phi'_{E,i} + a_1\beta_o\phi'_{E,i})\cos\psi + (\phi_{F,i}(\gamma_o - q'_{EO}) + b_1\phi'_{E,i}$$

$$- e\beta_o\phi'_{E,i})\sin\psi)dH + ((- \phi'_{E,i} - b_1(\gamma_o - q'_{EO})\phi'_{F,i} - a_1(\beta_o + q'_{FO})\phi'_{F,i}$$

$$- b_1 q'_{FO}\phi'_{E,i})\cos\psi + (\phi_{E,i}\gamma_o - r_i - q'_{FO}\phi_{F,i} + (r + e)q'_{FO}\phi'_{F,i})\sin\psi)dT]$$

$$- \phi_{X,j}[(\phi'_{E,i}\sin\psi + \phi'_{E,i}(\gamma_o - q'_{EO})\cos\psi)dH + (- \phi'_{F,i}\sin\psi - (\phi'_{F,i}(\gamma_o$$

$$- q'_{EO}) + q'_{FO}\phi'_{E,i})\cos\psi)dT] - \phi_{Y,j}[(- \phi'_{E,i}\cos\psi + \phi'_{E,i}(\gamma_o$$

$$- q'_{EO})\sin\psi)dH + (\phi'_{F,i}\cos\psi - (\phi'_{F,i}(\gamma_o - q'_{EO}) + q'_{FO}\phi'_{E,i})\sin\psi)dT]$$

$$- \phi_{Z,j}[- \beta_o\phi'_{E,i}dH + \phi'_{F,i}(\beta_o + q'_{FO})dT]\}q_{T,i} + [(\phi_{\theta X,j}(a_1\gamma_o + r + e$$

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$$+ b_1 q'_{FO}) - \phi_{\theta Y,j}(a_1 - r\gamma_o)$$

$$- \phi_{Y,j}(\beta_o + q'_{FO}))\cos\psi + (\phi_{\theta X,j}(a_1 - r\gamma_o) + \phi_{\theta Y,j}(a_1\gamma_o + r + e$$

$$+ b_1 q'_{FO}) + \phi_{X,j}(\beta_o + q'_{FO}))\sin\psi + \phi_{Z,j}][t_1\delta U_T + t_2\delta U_P + t_3\delta\alpha]$$

$$+ [(- \phi_{\theta X,j} b_1(\gamma_o - q'_{EO}) + \phi_{\theta Y,j}(b_1 + r\beta_o) + \phi_{Y,j}(\gamma_o - q'_{EO})$$

$$+ \phi_{X,j})\cos\psi - (\phi_{\theta X,j}(b_1 + r\beta_o) + \phi_{\theta Y,j} b_1(\gamma_o - q'_{EO}) + \phi_{X,j}(\gamma_o - q'_{EO})$$

$$- \phi_{Y,j})\sin\psi][h_1\delta U_T + h_2\delta U_P + h_3\delta\alpha]\}\Big]dr\Big\} \qquad (42)$$

$$Q_{\theta T} = \Big\{\int_0^{R-e}\Big[(- a_1 dH - b_1 dT)\phi_\theta^2\theta_T$$

$$+ \sum_{i=1}^{NE}((- \phi_{F,i} - \phi'_{E,i}q'_{EO}b_1)dH - (\phi_{E,i} + \phi'_{E,i}q'_{FO}b_1$$

$$- \phi'_{F,i}(q'_{EO}b_1 - q'_{FO}a_1))dT)\phi_\theta q_{T,i}$$

$$+ a_1(t_1\delta U_T + t_2\delta U_P + t_3\delta\alpha)\phi_\theta$$

$$- b_1(h_1\delta U_T + h_2\delta U_P + h_3\delta\alpha)\phi_\theta$$

$$- (m_1\delta U_T + m_2\delta U_P + m_3\delta\alpha)\phi_\theta\Big]dr\Big\} \qquad (43)$$

$$Q_\beta = \Big\{\int_0^{R-e}\Big[(- a_1(\gamma_o - q'_{EO})dH - (b_1\gamma_o - a_1 q'_{FO})dT)\phi_\theta\theta_T$$

$$- (b_1 dH - (a_1 - r\gamma_o)dT)\gamma$$

$$+ \sum_{i=1}^{NE}((- \phi_{F,i}(\gamma_o - q'_{EO}) - \phi'_{E,i}b_1)dH - (\phi_{E,i}\gamma_o - \phi_{F,i}q'_{FO}$$

$$- r_i - \phi'_{F,i}(b_1 - rq'_{FO}))dT)q_{T,i}$$

$$+ (a_1\gamma_o + r + b_1 q'_{FO})(t_1\delta U_T + t_2\delta U_P + t_3\delta\alpha)$$

$$- (b_1(\gamma_o - q'_{EO}))(h_1\delta U_T + h_2\delta U_P + h_3\delta\alpha)\Big]dr\Big\} \qquad (44)$$

$$
\begin{aligned}
Q_{\gamma} = \Bigg\{ \int_0^{R-e} \Big[& (b_1 q'_{EO} dH + b_1 q'_{FO} dT) \phi_\theta \theta_T \\
& + \sum_{i=1}^{NE} ((\phi_{E,i} q'_{EO} - r_i + \phi'_{E,i}(a_1 - r q'_{EO}))dH \\
& + (\phi_{E,i} q'_{FO} - r\phi'_{E,i} q'_{FO} - \phi'_{F,i}(a_1 - r q'_{EO}))dT) q_{T,i} \\
& - (a_1 q'_{FO})(t_1 \delta U_T + t_2 \delta U_P + t_3 \delta\alpha) \\
& - (a_1 q'_{EO} + r)(h_1 \delta U_T + h_2 \delta U_P + h_3 \delta\alpha) \Big] dr \Bigg\}
\end{aligned} \tag{45}
$$

$$
\begin{aligned}
Q_{q_{T,j}} = \Bigg\{ \int_0^{R-e} \Big[& (- \phi_{F,j} dH - \phi_{E,j} dT) \phi_\theta \theta_T \\
& + \sum_{i=1}^{NE} ((- q'_{EO} r_{j,i} - \phi'_{E,i}(q'_{EO} \phi_{E,j} - r_j))dH \\
& - (q'_{FO} \phi_{E,j} \phi'_{E,i} + \phi'_{F,i}(- q'_{EO} \phi_{E,j} + r_j + q'_{FO} \phi_{F,j}) \\
& + q'_{FO} r_{j,i})dT) q_{T,i} \\
& + \phi_{F,j}(t_1 \delta U_T + t_2 \delta U_P + t_3 \delta\alpha) \\
& - \phi_{E,j}(h_1 \delta U_T + h_2 \delta U_P + h_3 \delta\alpha) \Big] dr \Bigg\}
\end{aligned} \tag{46}
$$

LIST OF SYMBOLS

a	$= q_{EO} + (^c/_4 - EA)\cos\theta_o$
a_1	$= q_{EO} - AC\cos\theta_o$
a_2	$= q_{EO} - CG\cos\theta_o$
a_h	$= EA/b_h$
AC	Distance from blade elastic axis to aerodynamic center--positive toward leading edge.
L_2	Distance from blade elastic axis to pushrod: positive toward leading edge.
L_3	Radial location of blade pitch horn.
L_1	Length of one arm of tail rotor blade pitch spider beam.
L_s	Length of tail rotor pitch beam arm.
m	Blade elemental mass
M_G	Airframe mode generalized mass.
M_q	Generalized mass of blade bending modes.
M_A	Generalized mass of fixed system modes.
M_1	Mass at pushrod.
M	Mach number.
m_1	Partial derivative of pitching moment with respect to local blade tangential velocity.
m_2	Partial derivative of pitching moment with respect to local blade vertical velocity.
m_3	Partial derivative of pitching moment with respect to local blade angle of attack.
n	Blade number.
N	Number of blades.
NE	Number of blade bending modes.

NA	Number of fixed system modes.
h_3	Partial derivative of drag with respect to local blade angle of attack.
I	Blade flatwise, edgewise, torsional mass moment of inertia matrix.
I_θ	Blade torsional mass moment of inertia.
I_{YY}	Blade edgewise second moment of area.
I_X	Elemental blade flatwise mass moment of inertia.
I_Y	Elemental blade torsional mass moment of inertia.
I_Z	Elemental blade chordwise mass moment of inertia.
I_T	Total blade torsional mass moment of inertia.
I_γ	Blade mass moment of inertia about lag hinge.
J	Local blade polar second moment of area.
K	Blade torsional stiffness.
K_G	Airframe mode generalized stiffness.
K_γ	Blade lag hinge spring rate.
K_β	Blade flapping hinge spring rate.
K_1	Pitch beam stiffness or stiffness at main or tail rotor blade pushrod.
K_{MA}	Stiffness of tail rotor pitch actuator for pure moment applied at pitch beam end.
$C_{M,M}$	Partial derivative of pitching moment coefficient with respect to Mach number.
CG	Distance from blade elastic axis to center of gravity--positive toward leading edge.
dL	Elemental lift.
dD	Elemental drag.
dM	Elemental pitching moment.

dT	Elemental thrust.
dH	Elemental inplane force.
e	Blade offset.
EA	Distance from blade semi-chord to elastic axis-positive toward trailing edge.
G	Blade torsional modulus of elasticity.
h_1	Partial derivative of drag with respect to local blade tangential velocity.
h_2	Partial derivative of drag with respect to local blade vertical velocity.
$\bar{q}_j$	Fixed system mode generalized coordinate.
$Q_{T,j}$	Blade bending mode generalized coordinate--bending up and leading positive.
q_{FO}	Steady blade flatwise deflection--up positive.
Q_{EO}	Steady blade inplane deflection--lag positive.
$\bar{q}_{\theta X}$	Hub pitch coordinate.
$\bar{q}_{\theta Y}$	Hub roll coordinate.
$\bar{q}_X$	Hub lateral coordinate.
$\bar{q}_Y$	Hub longitudinal coordinate.
$\bar{q}_Z$	Hub vertical coordinate.
q_o	Collective mode coordinate.
q_D	Reactionless mode coordinate.
q_s	Sine cyclic coordinate.
q_c	Cosine cyclic coordinate.

q'_{F01}	
q'_{F0s}	Blade flatwise bending slope steady, and sine and cosine coefficient components.
q_{F0c}	
$\bar{Q}$	Generalized forces.
$Q_{\bar{q},j}$	Generalized force on j'th fixed system mode.
$Q_{\theta T}$	Generalized force on blade pitch.
Q_{β}	Generalized force on blade flapping.
Q_{γ}	Generalized force on blade lagging.
$Q_{q_{T},j}$	Generalized force on j'th blade bending mode.
Q_{j}	Generalized aerodynamic force.
r	Radius of local blade element from offset.
r_1	Radial location of inner snubber of crossbeam rotor.
r_2	Radial location of outer snubber of crossbeam rotor.
R	Rotor radius.
R_S	Radius to servo connections on main rotor swash plate.
R_B	Radius to pushrod connections on main rotor swash plate.
t_1	Partial derivative of thrust with respect to local blade tangential velocity.
t_2	Partial derivative of thrust with respect to local blade vertical velocity.
t_3	Partial derivative of thrust with respect to local blade angle of attack.
U	Total local blade inflow velocity.
U_P	Local blade vertical velocity.
U_T	Local blade tangential velocity.

v	Speed of sound.
V	Potential energy.
V_A	Rotor axial velocity.
V_F	Forward-flight speed.
X_1	Displacement at blade pushrod.
α	Local blade angle of attack.
α_1	Pitch-lag coupling--lead, pitch-up positive.
β	Blade rigid-body flapping generalized coordinate--up positive.
β_o	Steady blade coning--up positive.
γ	Blade rigid-body lag generalized coordinate--lead positive.
γ_o	Steady blade lag--lead positive.
δ_3	Pitch-flap coupling--flap up, pitch down positive.
ζ_q	Fraction of critical structural damping of blade bending modes--based on modal frequency.
ζ_θ	Fraction of critical structural damping of blade pitch mode--based on rotor speed.
ζ_γ	Fraction of critical rigid-body lag damping--based on uncoupled lag frequency.
ζ_A	Fraction of critical structural damping of fixed system modes--based on modal frequency.
θ	Blade pitch generalized coordinate--leading edge down positive.
θ_o	Steady blade pitch angle--leading edge down positive.
θ_T	Blade pitch normal coordinate.
θ_P	Geometric blade pitch angle--leading edge up positive.
ν_1	$= (\int_0^r(-\phi'_{E,i}q'_{EO} + \phi'_{F,i}q'_{FO})dr) = -r_i$
ν_2	$= (\int_0^r(\phi'_{E,i}\phi'_{E,j} + \phi'_{F,i}\phi'_{F,j})dr) = \dot{r}_i = r_{i,j}$
ρ	Air mass density.

ϕ	Local blade inflow angle.
$\phi_{X,Y,Z}$	Fixed system translational mode shapes at hub.
$\phi_{\theta X,\theta Y}$	Fixed system rotational mode shapes at hub.
ϕ_F	Blade flatwise bending mode shape.
ϕ_E	Blade inplane bending mode shape.
ϕ_θ	Blade torsional mode shape.
$\phi_{\theta PR}$	Blade torsional mode shape at pushrod radial station.
ϕFPR	Blade flatwise bending mode shape at pushrod radial station.
ϕ_{ET}	Blade inplane bending mode shape at tip radius.
ψ	Blade azimuthal angle.
ω_{T1}	Blade asymmetric torsional frequency.
ω_{T2}	Blade symmetric torsional frequency.
ω_P	Hub pitch frequency.
ω_Y	Hub yaw frequency.
ω_{EN}	Blade edgewise natural frequency.
ω_q	Frequency of blade bending modes.
ω_γ	Uncoupled rigid-body lag frequency.
ω_A	Frequency of fixed system modes.
ω	Flutter frequency.
Ω	Rotor speed.

Subscripts

i	Refers to blade element or mode number.
j	Refers to mode or force number.

s

Refers to mode number.

t

n Refers to blade number.

PR Refers to pushrod.

X

Y Refers to hub lateral, longitudinal, and vertical directions.

Z

θ_X Refers to hub pitch and roll directions.

θ_Y

Superscripts

c Menas coupled.

u Means uncoupled.

Differential Notation

' Differentiation with respect to radius.

. Differentiation with respect to time.

.. Second differential with respect to time.

D.4 Fuselage Dynamic Representation

The fuselage math model is presented by equation (47), shown in Figure D.4-1. It consists of up to sixteen rigid and/or elastic mode shapes. For this modal representation we need the generalized masses (M_{gi}), frequencies (ω_{ni}) and damping (ζ_i) for these mode shapes. We also need the modal components at any point on the aircraft where forces and moments are applied.

Obtaining the generalized coordinates, q_i, one can then evaluate the response at any point on the aircraft using equation (48), see Figure D.4-1.

$$m_{gi}\,\ddot{q}_i + 2\zeta_i m_{gi}\,\omega_i\,\dot{q}_i + m_{gi}\,\omega_i^2\,q_i = \phi_{Hi}^T \cdot F_H + \sum_{k=1}^{N} \phi_{ki}^T \cdot F_k \tag{47}$$

WHERE:

ϕ_{Hi}^T = TRANSPOSE OF MODE SHAPES AT HUB

F_H = FORCES AND MOMENTS AT HUB

ϕ_{ki}^T = TRANSPOSE OF MODE SHAPES AT K^{TH} AIRCRAFT POINT

F_k = FORCES AND MOMENTS AT K^{TH} AIRCRAFT POINT

N = TOTAL NUMBER OF AIRCRAFT POINTS WHERE EXCITATIONS APPLIED

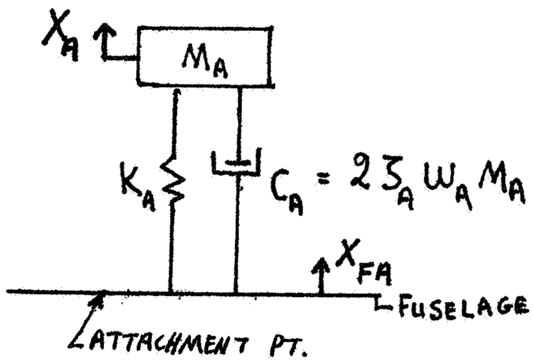
$$X_k = \phi_{ki} \cdot q_i \tag{48}$$

X_k = RESPONSE VECTOR AT K^{TH} POINT ON AIRCRAFT.

FIGURE D.4-1: FUSELAGE MATH MODEL

D.5 Fixed System Absorber

The math model of the fixed system absorber is shown in Figure D.5-1. It is a one degree of freedom spring-mass system. The kinetic and potential energies of the system are given by equations (49) and (50) respectively, shown in Figure D.5-1. Substituting into Lagrange's equation results in the equation of motion for the fixed absorber shown in Figure D.5-1 by equation (51).



$$K.E. = \frac{1}{2} M_A \left(\dot{X}_A + \dot{X}_{FA}\right)^2 \qquad (49)$$

$$P.E. = \frac{1}{2} K_A X_A^2 \qquad (50)$$

WHERE :

$$\omega_A^2 = \frac{K_A}{M_A}$$

THUS :

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = 0 \qquad (51)$$

D.5-1: FIXED SYSTEM ABSORBER

WHERE:

$$\{X\} = \left\{\begin{array}{c} X_{FA} \\ \hdashline X_A \end{array}\right\}$$

$$[M] = \left[\begin{array}{c:c} M_A & M_A \\ \hdashline M_A & M_A \end{array}\right] \quad ; \quad [C] = \left[\begin{array}{c:c} 0 & 0 \\ \hdashline 0 & 2\zeta_A \omega_A M_A \end{array}\right]$$

AND

$$[K] = \left[\begin{array}{c:c} 0 & 0 \\ \hdashline 0 & M_A \omega_A^2 \end{array}\right]$$

D.5-1: FIXED SYSTEM ABSORBER (CONCLUDED)

D.6 Assembly of Coupled Equations

To assemble the coupled equations of motion we need to know the modal components, of the same mode shapes used in Section D.4, at any aircraft point where the rotor, fixed and rotating absorbers are attached.

Here we will demonstrate the procedure used to couple the fixed system with that of the rotor. This same technique is used to couple the bifilars and fixed absorbers with the fixed system.

Equation (52) in Figure D.6-1 shows the rotor equation of motion in general form. We partitioned the mass, stiffness and damping matrices into submatrices associated with the hub (attachment point) and rotor degrees of freedom, see equation (53) Figure D.6-1.

The hub degrees of freedom are related to the fuselage (generalized) degrees of freedom by equation (54) shown in Figure D.6-1. Thus equation (53) can be transformed into equation (55) whose state vector consists of the fuselage and rotor degrees of freedom, as shown in Figure D.6-2.

With the hub degrees of freedom replaced by the generalized degrees of freedom, q's, we can now combine the rotor and fuselage equations of motion, as shown by equation (56) in Figure D.6-2.

$$[M]\cdot\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\cdot\{X\} = \{F\} \qquad (52)$$

$$\begin{bmatrix} M_{HH} & M_{HR} \\ M_{RH} & M_{RR} \end{bmatrix}\begin{Bmatrix} \ddot{X}_H \\ \ddot{X}_R \end{Bmatrix} + \begin{bmatrix} C_{HH} & C_{HR} \\ C_{RH} & C_{RR} \end{bmatrix}\begin{Bmatrix} \dot{X}_H \\ \dot{X}_R \end{Bmatrix} + \begin{bmatrix} K_{HH} & K_{HR} \\ K_{RH} & K_{RR} \end{bmatrix}\begin{Bmatrix} X_H \\ X_R \end{Bmatrix} = \begin{Bmatrix} F_R \\ 0 \end{Bmatrix} \qquad (53)$$

WHERE:

$\{X_H\}$ = HUB DEGREES OF FREEDOM, (6×1)

$\{X_R\}$ = ROTOR DEGREES OF FREEDOM, $(n_R \times 1)$

$$\{X_H\} = [\phi_H]\cdot\{q\} \qquad (54)$$

WHERE:

$\{q\}$ = FUSELAGE DEGREES OF FREEDOM $(n_F \times 1)$

$[\phi_H]$ = HUB MODE SHAPES $(6 \times n_F)$

Figure D.6-1: Uncoupled Equations of Motion

$$
\left[\begin{array}{c|c}
[\phi_H]^T\cdot[M_{HH}]\cdot[\phi_H] & [\phi_H]^T\cdot[M_{HR}] \\
\hline
[M_{RH}]\cdot[\phi_H] & [M_{RR}]
\end{array}\right]
\left\{\begin{array}{c}
\ddot{q} \\
\hline
\ddot{X}_R
\end{array}\right\} +
$$

$$
\left[\begin{array}{c|c}
[\phi_H]^T\cdot[C_{HH}]\cdot[\phi_H] & [\phi_H]^T\cdot[C_{HR}] \\
\hline
[C_{RH}]\cdot[\phi_H] & [C_{RR}]
\end{array}\right]
\left\{\begin{array}{c}
\dot{q} \\
\hline
\dot{X}_R
\end{array}\right\} +
$$

$$
\left[\begin{array}{c|c}
[\phi_H]^T\cdot[K_{HH}]\cdot[\phi_H] & [\phi_H]^T\cdot[K_{HR}] \\
\hline
[K_{RH}]\cdot[\phi_H] & [K_{RR}]
\end{array}\right]
\left\{\begin{array}{c}
q \\
\hline
X_R
\end{array}\right\} =
\left\{\begin{array}{c}
[\phi_H]^T\cdot\{F_R\} \\
\hline
0
\end{array}\right\} \qquad (55)
$$

$$
\left[\begin{array}{c|c}
[m_{g}] + [\phi_H]^T\cdot[M_{HH}]\cdot[\phi_H] & [\phi_H]^T\cdot[M_{HR}] \\
\hline
[M_{RH}]\cdot[\phi_H] & [M_{RR}]
\end{array}\right]
\left\{\begin{array}{c}
\ddot{q} \\
\hline
\ddot{X}_R
\end{array}\right\} +
$$

Figure D.6-2: Procedure to Form Coupled Equations (Continued)

$$
\left[\begin{array}{c|c}
\left[2\zeta m_g \omega\right] + [\phi_H]^T \cdot [C_{HH}] \cdot [\phi_H] & [\phi_H]^T \cdot [C_{HR}] \\
\hline
[C_{RH}] \cdot [\phi_H] & [C_{RR}]
\end{array}\right]
\left\{\begin{array}{c} \dot{q} \\ \hline \dot{X}_R \end{array}\right\} +
$$

$$
\left[\begin{array}{c|c}
\left[m_g \omega^2\right] + [\phi_H]^T \cdot [K_{HH}] \cdot [\phi_H] & [\phi_H]^T \cdot [K_{HR}] \\
\hline
[K_{RH}] \cdot [\phi_H] & [K_{RR}]
\end{array}\right]
\left\{\begin{array}{c} q \\ \hline X_R \end{array}\right\} =
\left\{\begin{array}{c} [\phi_H]^T \cdot \{F_R\} + [\phi_{AP}]^T \cdot \{F_{AP}\} \\ \hline 0 \end{array}\right\}
\tag{56}
$$

WHERE:

$[\phi_{AP}]$ = MODE SHAPES AT ANY AIRCRAFT POINT $(6 \times n_F)$

$\{F_{AP}\}$ = FORCE APPLIED AT ANY AIRCRAFT POINT (6×1)

Figure D.6-2: Procedure to Form Coupled Equations (Concluded)

D.7 Forced Response Solution

The final math model is represented by equation (57) of Figure D.7-1, where the state vector, X , consists of the fuselage, rotor, fixed and rotating absorbers degrees of freedom.

A solution of the form shown by equation (58) is assumed which after substitution in equation (57) results in equation (59), see Figure D.7-1. Expressing the state vector, X_C and X_S in terms of the sine and cosine components of the fuselage and remaining degrees of freedom, see equation (60) Figure D.7-1, and substituting it into equation (59) gives equation (61) from which equation (62) immediately follows after grouping like degrees of freedom. The form of equation (62) is preferred over that of equation (61) since the size of the inverse matrices involved in the solution is greatly reduced. From equations (62) the solutions for q and γ are expressed by equations (63) and (64) respectively, see Figure D.7-1.

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{F\} \tag{57}$$

$$\{X\} = \{X_s\}\sin\omega_f t + \{X_c\}\cos\omega_f t \tag{58}$$

$$\begin{aligned} \left[[K] - \omega_f^2[M]\right]\{X_c\} + \omega_f[C]\{X_s\} &= \{F_c\} \\ -\omega_f[C]\{X_c\} + \left[[K] - \omega_f^2[M]\right]\{X_s\} &= \{F_s\} \end{aligned} \tag{59}$$

WHERE:

$$\{X_c\} = \left\{\begin{array}{c} q_c \\ \hdashline \gamma_c \end{array}\right\} \quad \text{AND} \quad \{X_s\} = \left\{\begin{array}{c} q_s \\ \hdashline \gamma_s \end{array}\right\} \tag{60}$$

$\{q_c\}$ AND $\{q_s\}$ ARE THE COSINE AND SINE COMPONENTS OF THE FUSELAGE DEGREES OF FREEDOM

$\{\gamma_c\}$ AND $\{\gamma_s\}$ ARE THE COSINE AND SINE COMPONENTS OF THE ROTOR, FIXED ABSORBERS AND ROTATING ABSORBERS DEGREES OF FREEDOM

Figure D.7-1: Forced Response Solution (Continued)

$$\left[\begin{array}{c|c} [K]-\omega_f^2[M] & \omega_f[C] \\ \hline -\omega_f[C] & [K]-\omega_f^2[M] \end{array}\right]\left\{\begin{array}{c} q_c \\ \gamma_c \\ \hline q_s \\ \gamma_s \end{array}\right\} = \left\{\begin{array}{c} F_c \\ 0 \\ \hline F_s \\ 0 \end{array}\right\} \tag{61}$$

$$\left[\begin{array}{c|c} [E] & [F] \\ \hline [G] & [H] \end{array}\right]\left\{\begin{array}{c} q_c \\ q_s \\ \hline \gamma_c \\ \gamma_s \end{array}\right\} = \left\{\begin{array}{c} F_c \\ F_s \\ \hline 0 \\ 0 \end{array}\right\} \tag{62}$$

$$\left[[E]-[F][H]^{-1}[G]\right]\left\{\begin{array}{c} q_c \\ q_s \end{array}\right\} = \left\{\begin{array}{c} F_c \\ F_s \end{array}\right\} \tag{63}$$

$$\left\{\begin{array}{c} \gamma_c \\ \gamma_s \end{array}\right\} = [H]^{-1}[G]\left\{\begin{array}{c} q_c \\ q_s \end{array}\right\} \tag{64}$$

Figure D.7-1: Forced Response Solution (Concluded)

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D.8 Time History Solution

We have shown previously the technique used to couple the equations of motion of the fixed system, rotor, fixed absorber, and the linear bifilar transfered to fixed system coordinates. The final equation we arrived at is of the form shown by equation (65), Figure D.8-1. We can rewrite equation (65) in the form shown by equation (66), Figure D.8-2, where the right-hand-side of the equation is replaced by a forcing vector. The non-linear inplane bifilar equations of motion, shown in Figure D.1-3, can be rewritten in the compact form shown in equation (67), Figure D.8-2. Using the coupling technique described in section D.6 we can also couple equations (66) and (67). The resultant coupled mass matrix and force vector are shown in Figure D.8-2 by equations (68) and (69) respectively. The state vector is expanded and consists of the fuselage, rotor, fixed absorber, linear inplane and/or vertical bifilar in the fixed system coordinate, and non-linear inplane bifilar degrees of freedom. The final equations of motion of the system can be rewritten as shown by equation (70) Figure D.8-2. The reason for partitioning the matrix as shown in equation (70) is that the submatrix D has been reduced to the identity matrix and the only inversion required for solution is that of a matrix whose dimension is much smaller than that of the total mass matrix as shown in equation (9). This saves considerable computer time and in addition it is more stable.

Solving equations (71) and (72), shown in Figure D.8-3, we get the acceleration vector. The velocity and displacement vector are updated by equations (73). This procedure is repeated until the state variable, X , have converged within the specified constraints.

$$
\begin{bmatrix}
[M_q] & [M_{qR}] & [M_{qFA}] & [M_{qLB}] \\
[M_{Rq}] & [M_R] & 0 & 0 \\
[M_{FAq}] & 0 & [M_{FA}] & 0 \\
[M_{LBq}] & 0 & 0 & [M_{LB}]
\end{bmatrix}
\begin{Bmatrix} \ddot{q} \\ \ddot{X}_R \\ \ddot{X}_{FA} \\ \ddot{X}_{LB} \end{Bmatrix}
=
\begin{Bmatrix} F_q \\ 0 \\ 0 \\ 0 \end{Bmatrix}
-
\begin{bmatrix}
[C_q] & [C_{qR}] & [C_{qFA}] & [C_{qLB}] \\
[C_{Rq}] & [C_R] & 0 & 0 \\
[C_{FAq}] & 0 & [C_{FA}] & 0 \\
[C_{LBq}] & 0 & 0 & [C_{LB}]
\end{bmatrix}
\begin{Bmatrix} \dot{q} \\ \dot{X}_R \\ \dot{X}_{FA} \\ \dot{X}_{LB} \end{Bmatrix}
-
\begin{bmatrix}
[K_q] & [K_{qR}] & [K_{qFA}] & [K_{qLB}] \\
[K_{Rq}] & [K_R] & 0 & 0 \\
[K_{FAq}] & 0 & [K_{FA}] & 0 \\
[K_{LBq}] & 0 & 0 & [K_{LB}]
\end{bmatrix}
\begin{Bmatrix} q \\ X_R \\ X_{FA} \\ X_{LB} \end{Bmatrix}
\qquad (65)
$$

FIGURE D.8-1: COUPLED EQUATIONS OF MOTION

$$\begin{bmatrix} [M_q] & [M_{qR}] & [M_{qFA}] & [M_{qLB}] \\ [M_{Rq}] & [M_R] & 0 & 0 \\ [M_{FAq}] & 0 & [M_{FA}] & 0 \\ [M_{LBq}] & 0 & 0 & [M_{LB}] \end{bmatrix} \begin{Bmatrix} \ddot{q} \\ \ddot{X}_R \\ \ddot{X}_{FA} \\ \ddot{X}_{LB} \end{Bmatrix} = \begin{Bmatrix} F_q \\ F_R \\ F_{FA} \\ F_{LB} \end{Bmatrix} \tag{66}$$

$$\begin{bmatrix} [M_H(\psi)] & [M_{HB}(\psi)] \\ [M_{BH}(\psi)] & [M_B(\psi)] \end{bmatrix} \begin{Bmatrix} \ddot{X}_H \\ \ddot{\gamma} \end{Bmatrix} = \begin{Bmatrix} F_H(\psi) \\ F_\gamma(\psi) \end{Bmatrix} \tag{67}$$

$$\begin{bmatrix} [M_q]+[\Phi_H]^T\cdot[M_H]\cdot[\Phi_H] & [M_{qR}] & [M_{qFA}] & M_{qLB} & [\Phi_H]^T\cdot[M_{HB}] \\ [M_{Rq}] & [M_R] & 0 & 0 & 0 \\ [M_{FAq}] & 0 & [M_{FA}] & 0 & 0 \\ [M_{LBq}] & 0 & 0 & [M_{LB}] & 0 \\ [M_{BH}]\cdot[\Phi_H] & 0 & 0 & 0 & [M_B] \end{bmatrix} = [M] \tag{68}$$

FIGURE D.8-2: COUPLED EQUATIONS OF MOTION (CONTINUED)

$$\left\{\begin{array}{c} \{F_Q\} + [\phi_H]^T \cdot \{F_H\} \\ \hline F_R \\ \hline F_{FA} \\ \hline F_{LB} \\ \hline [\phi_H]^T \cdot \{F_\gamma\} \end{array}\right\} = \{F\} \tag{69}$$

$$\left[\begin{array}{c|c} [A] & [B] \\ \hline [C] & [D] \end{array}\right] \left\{\begin{array}{c} \ddot{X}_1 \\ \hline \ddot{X}_2 \end{array}\right\} = \left\{\begin{array}{c} F_1 \\ \hline F_2 \end{array}\right\} \tag{70}$$

WHERE:

$$\{X_1\} = \left\{\begin{array}{c} q \\ \hline X_R \end{array}\right\} \quad \text{AND} \quad \{\dot{X}_2\} = \left\{\begin{array}{c} X_{FA} \\ \hline X_{LB} \\ \hline \gamma \end{array}\right\}$$

X_R = ROTOR DEGREES OF FREEDOM

X_{FA} = FIXED ABSORBER DEGREES OF FREEDOM

X_{LB} AND γ = LINEAR (TRANF. TO FIX SYSTEM) AND N.L. BIFILAR DEGREES OF FREEDOM.

FIGURE D.8-2: COUPLED EQUATIONS OF MOTION (CONCLUDED)

$$\{\ddot{X}_1\} = \left[[A]-[B][C]\right]^{-1} \cdot \left\{\{F_1\} - [B]\cdot\{F_2\}\right\} \quad (71)$$

$$\{\ddot{X}_2\} = \{F_2\} - [C]\cdot\{\ddot{X}_1\} \quad (72)$$

SINCE:

$$[D] \equiv [I] \quad (\text{IDENTITY MATRIX})$$

$$\{\dot{X}_1\}_{NEW} = \{\dot{X}_1\}_{OLD} + (\Delta T)\{\ddot{X}_1\}_{NEW}$$

$$\{X_1\}_{NEW} = \{X_1\}_{OLD} + (\Delta T)\{\dot{X}_1\}_{NEW} \quad (73)$$

$$\{\dot{X}_2\}_{NEW} = \{\dot{X}_2\}_{OLD} + (\Delta T)\{\ddot{X}_2\}_{NEW}$$

$$\{X_2\}_{NEW} = \{X_2\}_{OLD} + (\Delta T)\{\dot{X}_2\}_{NEW}$$

FIGURE D.8-3: SOLUTION OF COUPLED EQUATIONS OF MOTION

D.9 List of Symbols

M_A	Fixed absorber mass
M_{G_i}, m_{gi}	Generalized masses
M_T	Total bifilar mass
m	Individual bifilar mass
N	Total number of bifilars
n	Bifilar tuning
q_i	Generalized coordinates
R	Distance from center of bifilar tracking hole to center of rotation
r	Equivalent pendulum arm
X_A	Vertical motion of fixed system absorber
X_{FA}	Vertical motion of attachment point of fixed absorber and fuselage
X	Rotating coordinate system
X_I	Inertia coordinate system
w_A	Fixed absorber tuning frequency
w_i	Generalized frequencies
w_β	Vertical bifilar tuning frequency
w_γ	Inplane bifilar tuning frequency
β	Vertical bifilar degree of freedom
γ	Inplane bifilar degree of freedom
ζ_A	Fixed absorber damping
ζ_β	Vertical bifilar damping
ζ_γ	Inplane bifilar damping

ζ_i	Generalized damping
ϕ	Mode shapes
ψ	Bifilar arm angle of rotation
Ω	Rotor speed
θ_x	Hub roll
θ_y	Hub pitch
θ_z	Hub yaw
ζ_x	Hub longitudinal modal damping
ζ_y	Hub lateral modal damping
ζ_z	Hub vertical modal damping
$\zeta_{\theta x}$	Hub roll modal damping
$\zeta_{\theta y}$	Hub pitch modal damping
$\zeta_{\theta z}$	Hub yaw modal damping
w_x	Hub longitudinal modal frequency
w_y	Hub lateral modal frequency
w_z	Hub vertical modal frequency
$w_{\theta x}$	Hub roll modal frequency
$w_{\theta y}$	Hub pitch modal frequency
$w_{\theta z}$	Hub yaw modal frequency

Subscripts

H, h	Hub
HR	Hub-Rotor
i	Fixed system mode
k	k^{th} bifilar
R	Rotor

Differential Notation

$\cdot$	Differentiation with respect to time
$\cdot\cdot$	Second differential with respect to time

Matrix Definitions

C	Damping matrix
C_{HH}	Hub damping sub-matrix
C_{HR}	Hub/rotor damping coupled sub-matrix
C_{RH}	Rotor/hub damping coupled sub-matrix
C_{RR}	Rotor damping sub-matrix
C_q	Generalized fuselage damping sub-matrix
C_{qR}	Generalized fuselage/rotor damping coupled sub-matrix
C_{Rq}	Rotor/generalized fuselage damping coupled sub-matrix
C_{qFA}	Generalized fuselage/fixed absorber damping coupled sub-matrix
C_{FAq}	Fixed absorber/generalized fuselage damping coupled sub-matrix
C_{qLB}	Generalized fuselage/linear bifilar damping coupled sub-matrix
C_{LBq}	Linear bifilar/generalized fuselage damping coupled sub-matrix
F	Force vector
F_{AP}	Force vector at any point on the aircraft
F_C	Cosine component of generalized force vector
F_S	Sine component of generalized force vector
F_H	Force vector at hub
F_R	Force vector at hub due to rotor
K	Stiffness matrix
K_{HH}	Hub stiffness sub-matrix
K_{HR}	Hub/rotor stiffness coupled sub-matrix

K_{RH}	Rotor/hub stiffness coupled sub-matrix
K_{RR}	Rotor stiffness sub-matrix
K_q	Generalized fuselage stiffness sub-matrix
K_{qR}	Generalized fuselage/rotor stiffness coupled sub-matrix
K_{Rq}	Rotor/generalized fuselage stiffness coupled sub-matrix
K_{qFA}	Generalized fuselage/fixed absorber stiffness coupled sub-matrix
K_{FAq}	Fixed absorber/generalized fuselage stiffness coupled sub-matrix
K_{qLB}	Generalized fuselage/linear bifilar stiffness coupled sub-matrix
K_{LBq}	Linear bifilar/generalized fuselage stiffness coupled sub-matrix
M	Mass matrix
M_{HH}	Hub mass matrix
M_{HR}	Hub/rotor mass coupled sub-matrix
M_{RH}	Rotor/hub mass coupled sub-matrix
M_{RR}	Rotor mass sub-matrix
M_q	Generalized fuselage mass sub-matrix
M_{qR}	Generalized fuselage/rotor mass coupled sub-matrix
M_{Rq}	Rotor/generalized fuselage mass coupled sub-matrix
M_{qFA}	Generalized fuselage/fixed absorber mass coupled sub-matrix
M_{FAq}	Fixed absorber/generalized fuselage mass coupled sub-matrix
M_{qLB}	Generalized fuselage/linear bifilar mass coupled sub-matrix
M_{LBq}	Linear bifilar/generalized fuselage mass coupled sub-matrix