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On the Secular Decrease in the Semimajor Axis of Lageos's Orbit

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David Parry Rubincam

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National Aeronautics and
Space Administration

Goddard Space Flight Center
Greenbelt, Maryland 20771

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David Parry Rubincam
Geodynamics Branch
Code 921

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ABSTRACT

The semimajor axis of the Lageos orbit is decreasing secularly at the rate of -1.1 mm day^{-1} due to an unknown force. Nine possible mechanisms are investigated here to discover which one, if any, might be the force. Five of the mechanisms, resonance with the earth's gravitational field, gravitational radiation, the Poynting-Robertson effect, transfer of spin angular momentum to the orbital angular momentum, and drag from near-earth dust are ruled out because they are too small or require unacceptable assumptions to account for the observed rate. Three other mechanisms, the Yarkovsky effect, the Schach effect, and terrestrial radiation pressure could possibly give the proper order-of-magnitude for the decay rate, but the characteristic signatures of these perturbations do not agree with the observed secular decrease. Atmospheric drag from a combination of charged and neutral particles is the most likely cause for the orbital decay. This mechanism explains at least 71 percent of the observed rate of decrease of the semimajor axis. It probably explains all of the decay since (a) the estimate of charged particle drag is conservative and (b) there may be substantial quantities of neutral helium at Lageos's altitude, which helps to solve the "helium problem" accouting for its escape from the earth's atmosphere.

ON THE SECULAR DECREASE IN THE SEMIMAJOR AXIS OF LAGEOS'S ORBIT

1. INTRODUCTION

Lageos, the Laser Geodynamics Satellite, was launched from the Western Test Range into a retrograde orbit on 4 May 1976. It is a completely passive satellite with an outer shell of aluminum. Its spherical surface is studded with laser retroreflectors. Inside it has a cylindrical brass core. Lageos's orbit is very nearly circular, with a semimajor axis of about $2R_E$. The orbital inclination with respect to the earth's equator is about 110 degrees. Other relevant data on Lageos are given in Table 1 and in Smith and Dunn (1980).

Lageos's orbit is suffering an acceleration which is not presently modeled in the orbit determination computer program (Smith and Dunn, 1980). This acceleration is causing the semimajor axis to decrease secularly at the rate of -1.27×10^{-8} ms^{-1} = -1.1 mm day^{-1} (see Fig. 1). In an earlier short paper (Rubincam, 1980) we attributed this decay to atmospheric drag, either from the charged or neutral particles in the earth's atmosphere or a combination of both. The object of the present paper is to discuss this conclusion in more detail, as well as show that other possible mechanisms which might explain the orbital decay appear to be inadequate to do so. These mechanisms are: resonance with the earth's gravitational field, gravitational radiation, the Poynting-Robertson effect, the Yarkovsky effect, the Schach effect, terrestrial radiation, magnetic despin of the satellite, and drag from interplanetary dust. The Schach effect is believed to be new and presented here for the first time.

We will usually be content with order-of-magnitude arguments in our investigation. Also, we use SI units throughout, plus centimeters and millimeters when discussing lengths and days and years when discussing time. These latter non-SI units prove to be very convenient when considering the rate of decay of the semimajor axis. Most of the notation and numerical data are given in Tables 1 and 2. Other symbols are explained as needed in the text.

Table 1

Notation for quantities related to Lageos. The quantities in the first part of the table refer to Lageos's orbit, while the second part refers to the satellite itself. Dash (–) indicates various numerical values are assumed.

Quantity	Symbol	Numerical Value
semimajor axis	a	1.227×10^7 m
eccentricity	e	0.004
true anomaly	f	-
inclination	I	109.9 deg
mean anomaly	M	-
mean motion	n	4.65×10^{-4} s^{-1}
velocity	$\vec{V}$	-
acceleration	$\dot{\vec{V}}$	-
argument of perigee	ω	-
nodal position	Ω	-
cross-sectional area	A	0.2827 m^2
drag coefficient	C_D	-
mass	M_L	411 kg
radius	R_L	0.30 m
temperature	T	-
initial spin rate	ω_L	10.3 rad s^{-1}

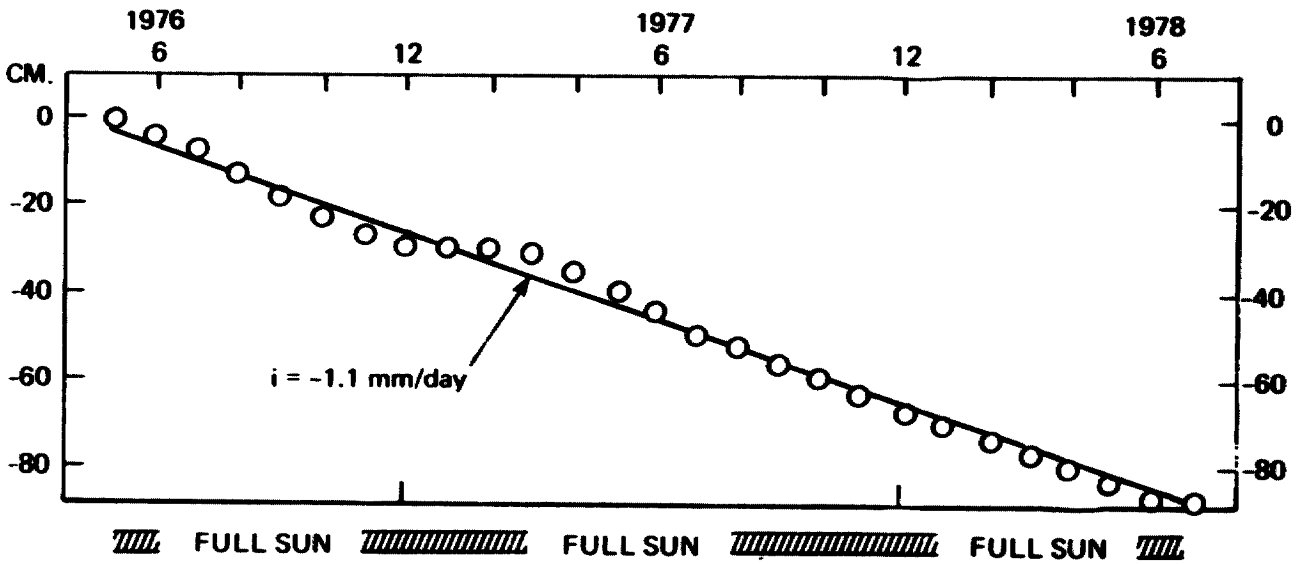

Figure 1. The secular decrease in the semimajor axis of Lageos's orbit. The straight line fitted through the data points has a slope of -1.1 mm day^{-1}. When the orbit is in full sunlight and shadow is shown along the horizontal axis.

Table 2

Notation for quantities related to the earth and sun (first part of the table) and to the universe (the second part). Symbols not shown in Tables 1 and 2 are defined in the text. Dash (–) indicates various numerical values are assumed.

Quantity	Symbol	Numerical Value
gravity field coefficients	$C_{\ell m}$, $S_{\ell m}$	-
solar flux	F_s	1.35×10^3 J m^{-2} s^{-1}
mass of earth	M_E	5.97×10^{24} kg
radial distance from center of earth	r	-
radius of earth	R_E	6.371×10^8 m
rotation angle for earth	θ	-
rotation speed of earth	$\dot{\theta}$	7.29×10^{-5} rad s^{-1}
speed of light	c	3×10^8 m s^{-1}
permittivity of space	ϵ_o	8.85×10^{12} F m^{-1}
gravitation constant	G	6.67×10^{-11} m^3 kg^{-1} s^{-2}
Boltzmann's constant	k	1.38×10^{-23} J deg^{-1}
proton charge	q	1.60×10^{-19} C
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} deg^{-4}
time	t	-

2. RESONANCE

We first enquire into the possibility that the apparent secular decrease in the semimajor axis is due to a long-period resonance with the earth's gravitational field.

Frm Kaula (1966, p. 40) the change in the semimajor axis due to terms of degree ℓ and order m in the spherical harmonic expansion of the field is

$$\Delta a = \frac{2\left(\frac{GM_E}{a}\right)^{1/2}\left(\frac{R_E}{a}\right)^{\ell} F_{\ell mp}(I)\, G_{\ell pq}(e)\,(\ell - 2p + q)\, S_{\ell mpq}}{(\ell - 2p)\,\dot{\omega} + (\ell - 2p + q)\,\dot{M} + m\,(\dot{\Omega} - \dot{\theta})} \tag{1}$$

where (Kaula, 1966, p. 37)

$$S_{\ell mpq} = \begin{bmatrix} C_{\ell m} \\ -S_{\ell m} \end{bmatrix}_{\ell-m \text{ odd}}^{\ell-m \text{ even}} \cos\left[(\ell - 2p)\omega + (\ell - 2p + q)\,M + m\,(\Omega - \theta)\right]$$

$$+ \begin{bmatrix} S_{\ell m} \\ C_{\ell m} \end{bmatrix}_{\ell-m \text{ odd}}^{\ell-m \text{ even}} \sin\left[(\ell - 2p)\omega + (\ell - 2p + q)\,M + m\,(\Omega - \theta)\right]. \tag{2}$$

Here $C_{\ell m}$ and $S_{\ell m}$ are the unnormalized spherical harmonic coefficients and ω, M, Ω, θ are assumed to vary linearly with time. The denominator of (1) is nearly zero when the orbit is close to a resonance.

We can simplify (1) in the following manner. Since (2) varies sinusoidally with time, it will have its maximum rate of change when we choose the origin of time t such that (2) becomes

$$\begin{aligned} S_{\ell mpq} &\cong C'_{\ell m} \sin\left[(\ell - 2p)\omega + (\ell - 2p + q)M + m\,(\Omega - \theta)\right] \\ &= C'_{\ell m} \sin\left\{\left[(\ell - 2p)\dot{\omega} + (\ell - 2p + q)\dot{M} + m(\dot{\Omega} - \dot{\theta})\right] t\right\} \\ &= C'_{\ell m} \left[(\ell - 2p)\dot{\omega} + (\ell - 2p + q)\dot{M} + m(\dot{\Omega} - \dot{\theta})\right] t \end{aligned}$$

for small t, where $C'_{\ell m}$ is a typical coefficient of degree ℓ and order m. Substitution of the above equation into (1) shows that the dotted expression in the numerator cancels with that in the denominator, leaving

$$\Delta a = \left\{2\left(\frac{GM_E}{a}\right)^{1/2}\left(\frac{R_E}{a}\right)^{\ell} F_{\ell mp}(I) G_{\ell pq}(e)\,(\ell - 2p + q)\, C'_{\ell m}\right\} t$$

which varies linearly with time, so that obviously the expression in curly brackets is

$$\frac{da}{dt} = 2\left(\frac{GM_E}{a}\right)^{1/2}\left(\frac{R_E}{a}\right)^{\ell} \bar{F}_{\ell mp}(I)\, G_{\ell pq}(e)\,(\ell - 2p + q)\,\bar{C}'_{\ell m}, \tag{3}$$

where we have switched to normalized coefficients and inclination functions:

$$\bar{C}'_{\ell m} = \left[\frac{(\ell + m)!}{(2\ell + 1)(2 - \delta_{om})(\ell - m)!}\right]^{1/2} C'_{\ell m}$$

$$\bar{F}_{\ell mp}(I) = \left[\frac{(2\ell + 1)(2 - \delta_{om})(\ell - m)!}{(\ell + m)!}\right]^{1/2} F_{\ell mp}(I).$$

We now simplify (3) in order to estimate its magnitude. We first look at the eccentricity function $G_{\ell pq}(e)$. Since

$$G_{\ell pq}(e) \sim \begin{cases} e^{|q|}, & q \neq 0 \\ 1, & q = 0 \end{cases}$$

the values for q not near zero are negligible due to the small eccentricity of the orbit. Hence we take

$$q \approx 0$$

$$\ell - 2p + q \approx \ell - 2p \tag{4}$$

$$G_{\ell pq}(e) \approx 1. \tag{5}$$

Next, we note that p runs from 0 to ℓ, so that (4) becomes

$$|\ell - 2p| \leq \ell. \tag{6}$$

And since $a \approx 2R_E$, we take

$$\left(\frac{R_E}{a}\right)^{\ell} \approx \frac{1}{2^{\ell}}. \tag{7}$$

Finally, from Kaula's (1966, p. 98) rule-of-thumb we use

$$|\bar{C}'_{\ell m}| \approx \frac{10^{-5}}{\ell^2}. \tag{8}$$

Substituting (4) - (8) in (3) and giving numerical values to GM_E and a yield

$$\frac{da}{dt} \approx \frac{0.114\,\bar{F}_{\ell mp}(I)}{\ell \cdot 2^{\ell}}\,ms^{-1} \approx \frac{10^{7}\,\bar{F}_{\ell mp}(I)}{\ell \cdot 2^{\ell}}\,mm\ day^{-1}.$$

All that remains is to estimate $\bar{F}_{\ell mp}(I)$. Numerical experiments indicate that at most

$$|\bar{F}_{\ell mp}(I)| \approx 1.$$

Using this in the previous equation we have

$$\left|\frac{da}{dt}\right| \approx \frac{10^{7}}{\ell \cdot 2^{\ell}}\,mm\ day^{-1} \tag{9}$$

as an estimate of the rate of change of the semimajor axis with time due to resonance with terms of degree ℓ.

The rate given by (9) falls below the observed rate when $\ell \approx 22$. Terms to degree and order 36 have been numerically investigated to see if a resonance occurs with one of them to account for the observed rate. None of them do. Hence given (9) and the results of the numerical investigation, it is unlikely that the observed rate of decrease of the semimajor axis is due to resonance with a term or even a number of terms above degree 36. We therefore conclude that resonance with the gravity field is probably not the unknown force operating on Lageos.

3. GRAVITATIONAL RADIATION

The drag force easiest to rule out as the unknown force is gravitational radiation. Here the drag is due to radiation reaction; accelerated masses lose energy through gravity waves, just as accelerated charges lose energy through electromagnetic waves. The rate of change in the semimajor axis due to gravitational radiation is given by (Peters, 1964, eq. 5.6):

$$\frac{da}{dt} \cong -\frac{64\,G^2\,M_E^2\,M_L}{5\,c^5\,a^3}$$

where small terms involving the satellite mass and orbital eccentricity have been ignored. Substituting numerical values in the right side of the above equation yields

$$\frac{da}{dt} \cong -1.4 \times 10^{-26} \text{ mm day}^{-1} \tag{10}$$

which fails to explain the observed decay by 26 orders of magnitude. Hence the perturbation is not due to gravitational radiation, which is of no importance for earth satellites. Orbital decay through this mechanism does, however, appear to have been observed in a binary pulsar system, providing an important test of general relativity (Taylor et al., 1979).

4. POYNTING-ROBERTSON EFFECT

The Poynting-Robertson effect also produces a drag force (Robertson, 1937). It is known to be important for small particles in solar orbit (Stacey, 1977, p. 17-20; Lovell, 1954, pp. 402-409). It is accordingly investigated here for Lageos.

The effect is due to the reradiation of light incident upon the satellite. Some of the light falling on Lageos is absorbed by it and is assumed to be reradiated isotropically in its own frame of reference. If the satellite is in motion with respect to an observer in another frame, such as the earth, then the observer sees a Doppler shift in the reradiated light. The light emitted in the direction of motion is shifted towards the blue end of the spectrum while the light emitted opposite to the direction of motion shows a red shift. Since more energy and momentum is carried away from the satellite by the blue-shifted light than the red, the satellite feels a reaction force acting opposite to its direction of motion. It is this force which produces the drag.

The light incident upon Lageos comes from two major sources: earth and sun. We investigate earthlight first. We assume that the earth radiates energy isotropically at the same rate at which it receives energy from the sun. The earth is also taken to be the stationary reference frame.

Soter et al. (1977) gives a simple derivation for the Poynting-Robertson effect. The drag term in the force is

$$M_L \dot{\vec{V}} = -\frac{FA}{c^2}\vec{V} \tag{11}$$

where F is the integrated flux, giving a tangential disturbing function

$$S = -\frac{FA}{M_L c^2} V$$

in the Gaussian form of Lagrange's planetary equations (Blanco and McCuskey, 1961, p. 178):

$$\frac{da}{dt} = \frac{2}{nr\sqrt{1-e^2}} [(er\sin f)R + a(1-e^2)S], \qquad (12)$$

so that

$$\frac{da}{dt} = \frac{2S}{n} = -\frac{2FA}{nM_L c^2} V. \qquad (13)$$

We have assumed the orbit to be circular. Since F varies as r^{-2}, we take

$$F = \frac{F_o R_E^2}{r^2} = \frac{F_o R_E^2}{a^2}, \qquad (14)$$

where F_o is the flux at the earth's surface. Also, for circular orbits

$$V = \frac{\sqrt{GM_E}}{a^{1/2}} \qquad (15)$$

and by definition n is

$$n = \frac{\sqrt{GM_E}}{a^{3/2}}. \qquad (16)$$

Substituting (14) – (16) into (13) gives

$$\frac{da}{dt} = -\frac{2F_o R_E^2 A}{M_L c^2 a}.$$

We need to find F_o in (17), the values of the other quantities being known. This is easily done: the earth intercepts $\pi R_E^2 F_s$ power from the sun and reradiates $4\pi R_E^2 F_o$. Therefore

$$F_o = \frac{F_s}{4}$$

where F_s is taken from Robertson (1937) and shown in Table 2. Plugging numerical values in (17) gives

$$\frac{da}{dt} \cong -1.69 \times 10^{-11} \text{ ms}^{-1}$$

$$= -1 \text{ cm per } 18 \text{ yr} \tag{18}$$

The Poynting-Robertson effect due to earthlight is much too small to account for the observed rate of decrease in a.

Sunlight is also too small to explain the observed orbital decay. Repeating the calculation using the direct solar flux F_s in (13) gives a rate of decay 16 times larger than that due to earthlight (a factor of 4 coming from $F_s = 4F_0$ and another factor of 4 due to the r^{-2} behavior of earthlight given by (14)), or

$$\frac{da}{dt} \cong -2.70 \times 10^{-10} \text{ ms}^{-1}$$

$$= -1 \text{ mm per } 43 \text{ days} \tag{19}$$

The calculation is only approximate since (11) is valid for an inertial frame in which the source of light is at rest. The earth orbiting about the sun is neither an inertial frame, nor is the sun at rest in it. However, these corrections are small (Allan, 1967, pp. 74-75), as are the variations in flux due to the changing satellite-sun distance. The most serious neglect is the effect of the earth's shadow. But the shadow merely turns the drag force off over part of the orbit, slowing the overall decay rate, so that it makes the Poynting-Robertson effect smaller than given in (19). We must conclude that the Poynting-Robertson effect is not the unknown force; it is too small.

5. YARKOVSKY EFFECT

We consider now the Yarkovsky effect (Öpik, 1951, pp. 194-197; Lovell, 1954, pp. 410-411). This is a differential thermal effect caused by light falling on a rotating satellite with appreciable thermal inertia.

Radiation from a light source warms the surface of the satellite. The hottest part of the satellite would be the sub-source point if the satellite were not rotating. However, suppose light is falling on a rotating satellite as shown in Fig. 2. Here areas on the satellite's surface are carried around from the shadowed side into the light and are warmed by it. There is a delay in heating up due to the thermal inertia of the satellite, so that the hottest part is the "afternoon" side of the satellite and not the sub-source point. This is similar to temperature variations here on earth, were afternoon is the hottest time of day, instead of noon.

The asymmetric heating causes a net force to act on the satellite, since photons from the hotter areas of the surface carry away more momentum than the colder areas. This force, which makes a definite angle with the satellite-source line, perturbs the orbit.

Light for the Yarkovsky effect comes from the earth and sun, just as for the Poynting-Robertson effect. We again investigate earthlight first.

To obtain a qualitative idea of how the Yarkovsky effect from earthlight can affect the orbit we refer to Fig. 3. Here the geometry of the orbit and Lageos spin axis is such that there is a component of force acting consistently opposite to the satellite velocity vector. This component will of course cause the orbit to decay. If the satellite is spinning in the sense opposite to that shown in Fig. 3, then the force will increase the size of the orbit. Thus the effect on the orbit depends crucially on the orientation of the spin axis in space. The presumption at this stage of the investigation must be that the spin axis is oriented largely as shown in Fig. 3, in order to explain the observed decay shown in Fig. 1.

We turn now to estimating the magnitude of the acceleration due to the Yarkovsky effect. We will assume that Lageos is composed of two hemispheres, one at temperature T and the other at temperature T + ΔT. If each hemisphere radiates like a blackbody over a characteristic area of πR_L^2, then the acceleration is

$$|\dot{\vec{V}}| = \frac{\sigma\pi R_L^2\,[(T + \Delta T)^4 - T^4]}{M_L c} \cong \frac{4\pi R_L^2 \sigma T^3 \Delta T}{M_L c} \qquad (20)$$

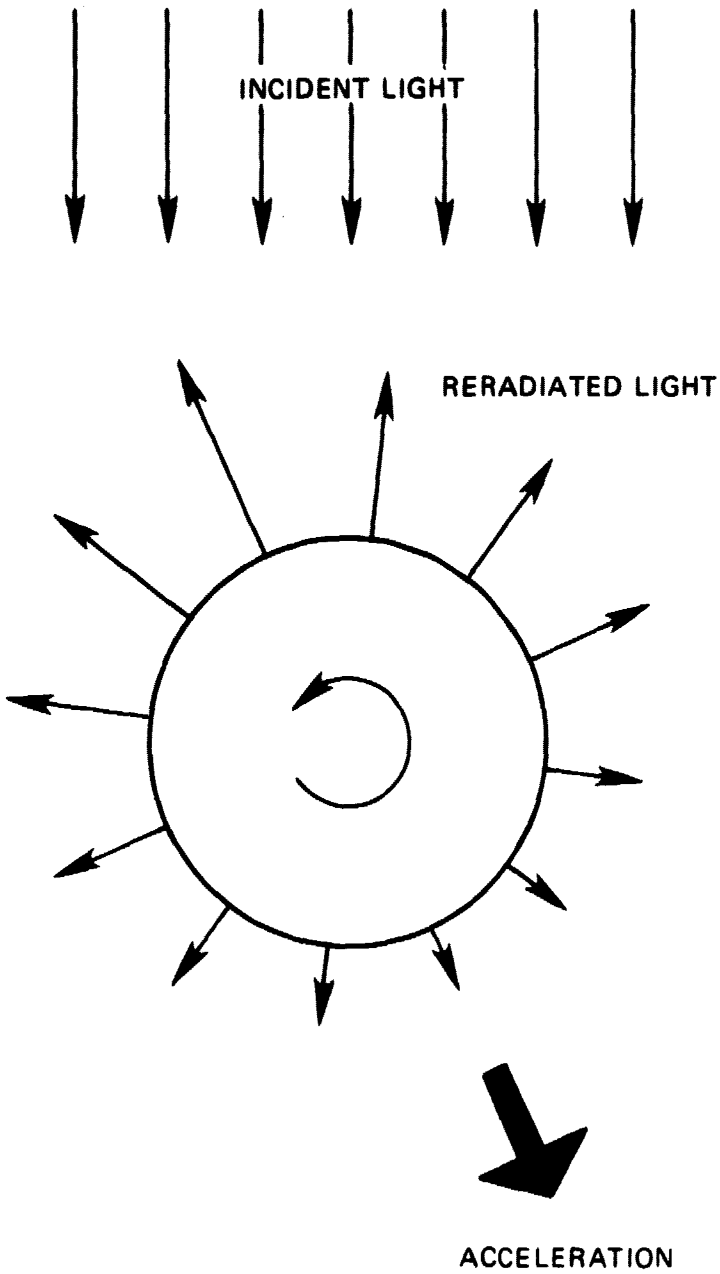

Figure 2. Schematic diagram illustrating the Yarkovsky effect on a satellite. Light impinges on the satellite from the top, as illustrated by the parallel vertical arrows. Some of the light is absorbed by the satellite and re-emitted. The re-emitted light is denoted by the arrows pointing outwards from the satellite. The radiation reaction forces from this light give a net acceleration $\dot{\vec{V}}$ to the satellite, shown as the thick arrow. The acceleration is not in the direction of the incident light, due to the rotation of the satellite.

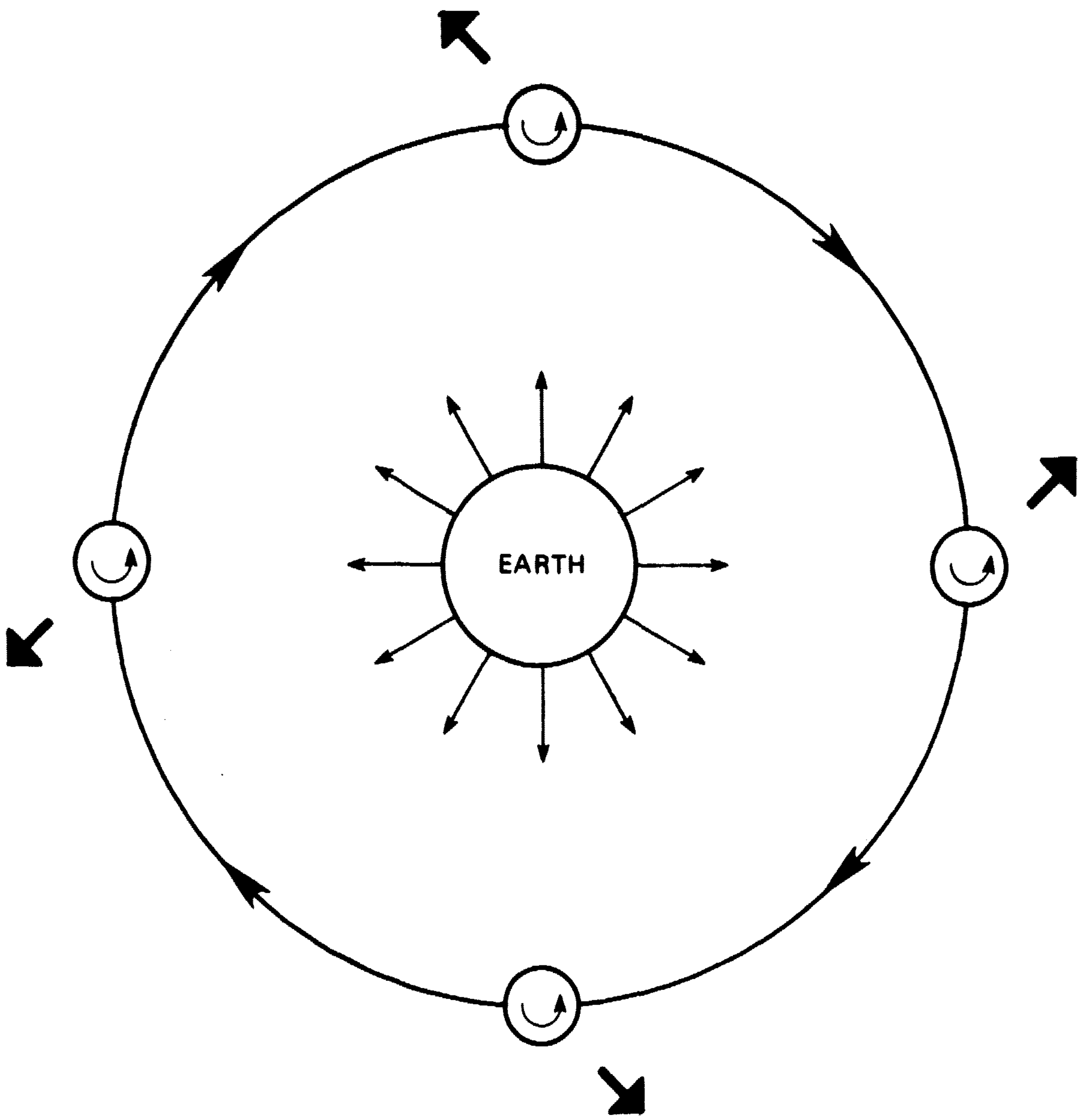

Figure 3. Schematic illustration showing that the Yarkovsky effect acceleration $\dot{\vec{V}}$ is cumulative around the orbit. $\dot{\vec{V}}$ is again represented by the thick arrows.

assuming $\Delta T << T$. The temperature T of a blackbody at distance a from the earth is (Blanco and McCuskey, 1961, p. 53):

$$T = T_E \left(\frac{R_E}{2a}\right)^{1/2} \cong \frac{T_E}{2}$$

where $T_E \cong 280$ K is the blackbody temperature of the earth. Substituting numerical values in (20) gives

$$|\dot{\vec{V}}| = 1.43 \times 10^{-12}\ \Delta T\ \mathrm{ms}^{-2}. \tag{21}$$

Taking

$$S \cong -|\dot{\vec{V}}|$$

we find

$$\frac{da}{dt} = -0.51\ \Delta T\ \mathrm{mm\ day}^{-1}. \tag{22}$$

A temperature difference of 2 K in our model will explain the entire perturbation. Since this does not seem unreasonable, it would appear a detailed treatment of the problem involving the albedo of the satellite, its thermal properties, etc. is needed before we can make a judgment as to the importance of the Yarkovsky effect. However, we will leave our result (22) in abeyance for the moment and show later that such a detailed treatment is not needed. We turn our attention now to the Yarkovsky effect due to sunlight.

We can compute the approximate magnitude of the acceleration from (20), this time taking $T = T_E$, since Lageos is at about the same distance from the sun as it is the earth. This yields in analogy to (21)

$$|\dot{\vec{V}}| = 1.14 \times 10^{-11}\ \Delta T\ \mathrm{ms}^{-2} \tag{23}$$

To compute the effect of this acceleration on the orbit, let $\dot{\vec{V}}$ have the components $\dot{\vec{V}} = (f_x, f_y, f_z)$ in the coordinate system shown in Fig. 4. The orthogonal accelerations R, S, and W are then given by

$$\begin{bmatrix} R \\ \\ S \\ \\ W \end{bmatrix} = \begin{bmatrix} \cos(\omega+f)\cos\Omega & \cos(\omega+f)\sin\Omega & \sin(\omega+f)\sin I \\ -\cos I\sin\Omega\sin(\omega+f) & +\cos I\cos\Omega\sin(\omega+f) & \\ -\sin(\omega+f)\cos\Omega & -\sin(\omega+f)\sin\Omega & \cos(\omega+f)\sin I \\ -\cos I\sin\Omega\cos(\omega+f) & +\cos I\cos\Omega\cos(\omega+f) & \\ \sin I\sin\Omega & -\sin I\cos\Omega & \cos I \end{bmatrix} \begin{bmatrix} f_x \\ \\ f_y \\ \\ f_z \end{bmatrix}$$

where the rotation matrix comes from Goldstein (1950, p. 109). Substituting the expressions R and S into (12), noting that

$$r = \frac{a(1-e^2)}{1+e\cos f}$$

and integrating the true anomaly f from 0 to 2π yields the average rate of change for the semimajor axis

$$\frac{da}{dt} = \frac{2ea^{3/2}}{\sqrt{GM_E}} [(-\sin\omega\cos\Omega - \cos I\sin\Omega\cos\omega)f_x$$

$$+ (-\sin\omega\sin\Omega + \cos I\cos\Omega\cos\omega)f_y$$

$$+ (\cos\omega\sin I)f_z] \qquad (24)$$

This procedure assumes that the magnitude and direction of $\dot{\vec{V}}$ does not change appreciably over one revolution (certainly a good assumption), and that the orbit is oriented so that the satellite does not enter the earth's shadow (to be discussed shortly).

From (23) and (24) we find that the rate of change of semimajor axis with time is roughly

$$\left|\frac{da}{dt}\right| \cong 1.96 \times 10^{-10}\ \Delta T\ \mathrm{ms^{-1}} = 0.017\ \Delta T\ \mathrm{mm\ day^{-1}}. \qquad (25)$$

An unreasonably large temperature difference of 10 K is still much too small to contribute appreciably to the rate of change of the semimajor axis. Thus the Yarkovsky effect when the orbit is in full sunlight is unimportant.

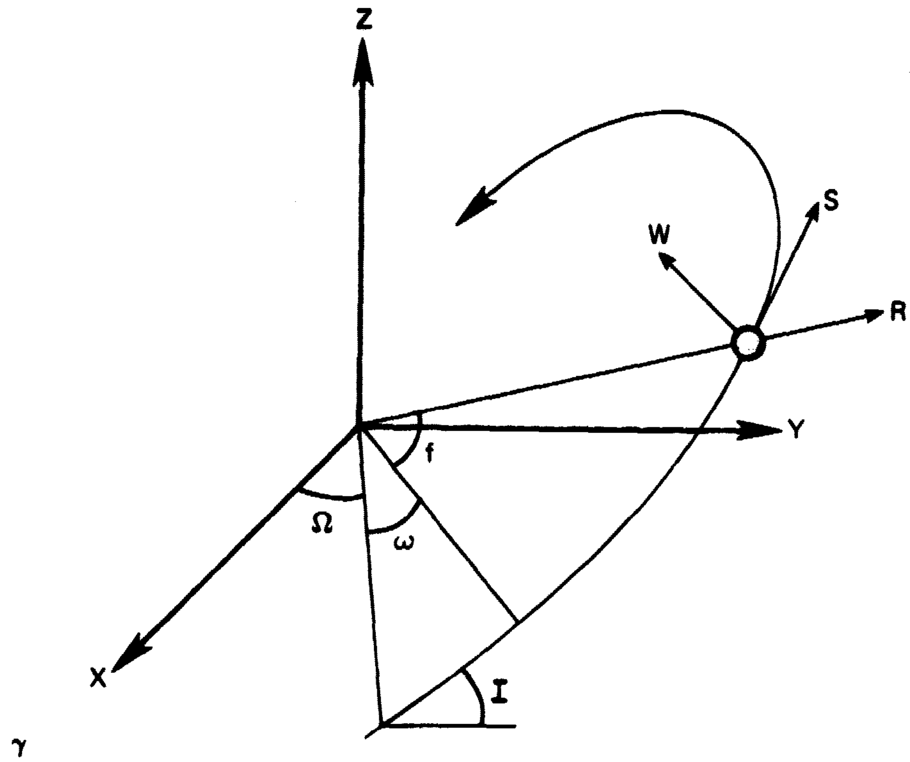

Figure 4. The orbital geometry used in computing the Yarkovsky effect from sunlight.

This result does not hold if part of the orbit falls in the earth's shadow. In this case $\dot{\vec{V}}$ is zero in the shadow, which is equivalent to integrating f over less than 2π when averaging the perturbation over one revolution. In fact, as much as one-sixth of the orbit call fall in the earth's shadow. In these circumstances large terms in the expression for da/dt do not average out to zero and we have approximately

$$\left|\frac{da}{dt}\right| \cong \frac{2a^{3/2}}{\sqrt{GM_E}} \left|\dot{\vec{V}}\right| \cong 4.91 \times 10^{-8}\ \Delta T\ \mathrm{ms}^{-1}$$

$$= 4.24\ \Delta T\ \mathrm{mm\ day}^{-1} \tag{26}$$

or a factor of e^{-1} larger than given by (25) when the orbit is in full sunlight. This is even larger than (22), assuming the same temperature difference. We cannot rule out the Yarkovsky effect from sunlight using order-of-magnitude arguments only when the orbit is shadowed, just as we could not rule out the Yarkovsky effect from earthlight.

We will, however, attempt to rule out the Yarkovsky effect from both sources on the basis of the orbital geometry and the known behavior of the semimajor axis shown in Fig. 1.

Apogee kick stage separation occurred at +4.5 deg geocentric latitude and +20.8 deg east longitude. At this time the Lageos spin axis had elevation -11.0 deg and azimuth +158.7 deg. The nodal position was +28.7 deg. The use of these data plus the known orbital inclination show that the spin axis was approximately in the plane of the orbit, as might be expected. The spin axis must have slowly moved out of the orbital plane as the node progressed along the equator, assuming the position of the spin axis stayed fixed in space.

The Yarkovsky effect from sunlight or earthlight will have no long-term effect on the semimajor axis when the spin axis lies in the orbital plane, as may be seen from considerations like those shown in Fig. 3. Hence, in the beginning part of the curve shown in Fig. 1, when the satellite is in full sunlight (so that the contribution from the sun is negligible) and the spin axis is nearly in the orbital plane (so that the contribution from the earth is negligible), the curve should have near zero slope, assuming the unknown force is due to the Yarkovsky effect. The slope should become steeper as the spin axis moves out of the orbital plane and the contribution from the earth becomes appreciable.

This does not occur, as is obvious from Fig. 1. If anything the slope is steeper at this part of the curve than at later times and then flattens out a little. This is just the opposite from what we would expect from the Yarkovsky effect. We conclude that the unknown force is not due to the Yarkovsky effect. Moreover, the temperature differences across Lageos must be small, since the curve deviates but little from a straight line, indicating the Yarkovsky effect from both the earth and sun are operating at a low level. Still, analysis of the irregularities in the curve may yield information about the Yarkovsky effect.

We should note that we have assumed two different temperatures for Lageos: one due to sunlight and the other to earthlight. This inconsistency makes no essential change in our argument, since we were interested only in obtaining a rough estimate of the possible importance of the Yarkovsky effect.

6. SCHACH EFFECT

Related to the Yarkovsky effect is the Schach effect, which was discovered by Milton Schach of the NASA Goddard Space Flight Center in the course of this investigation. Its operation depends upon the cooling off and heating up of the satellite as it moves in and out of the earth's shadow. We refer to Fig. 5 for its explanation.

The satellite is warmed up by sunlight when it is outside of the shadow. When it enters the shadow it will take some time to cool off, due to its thermal inertia. Likewise, it will take some time to warm up when the satellite exits the shadow into sunlight. Qualitatively we can think of the satellite cooling off and heating up instantaneously as it moves through a <u>rotated</u> shadow, as indicated in Fig. 5.

This can affect the orbit in two ways. One is a delay in turning off and on the Poynting-Robertson effect due to sunlight. This causes no essential change in the long-term operation of the Poynting-Robertson effect or our arguments based upon it. The other way it can affect the orbit is when the satellite is a poor heat conductor and rotating slowly (or not at all) or with its spin axis pointing towards the sun. In this case the hemisphere facing the sun becomes hot, so that the

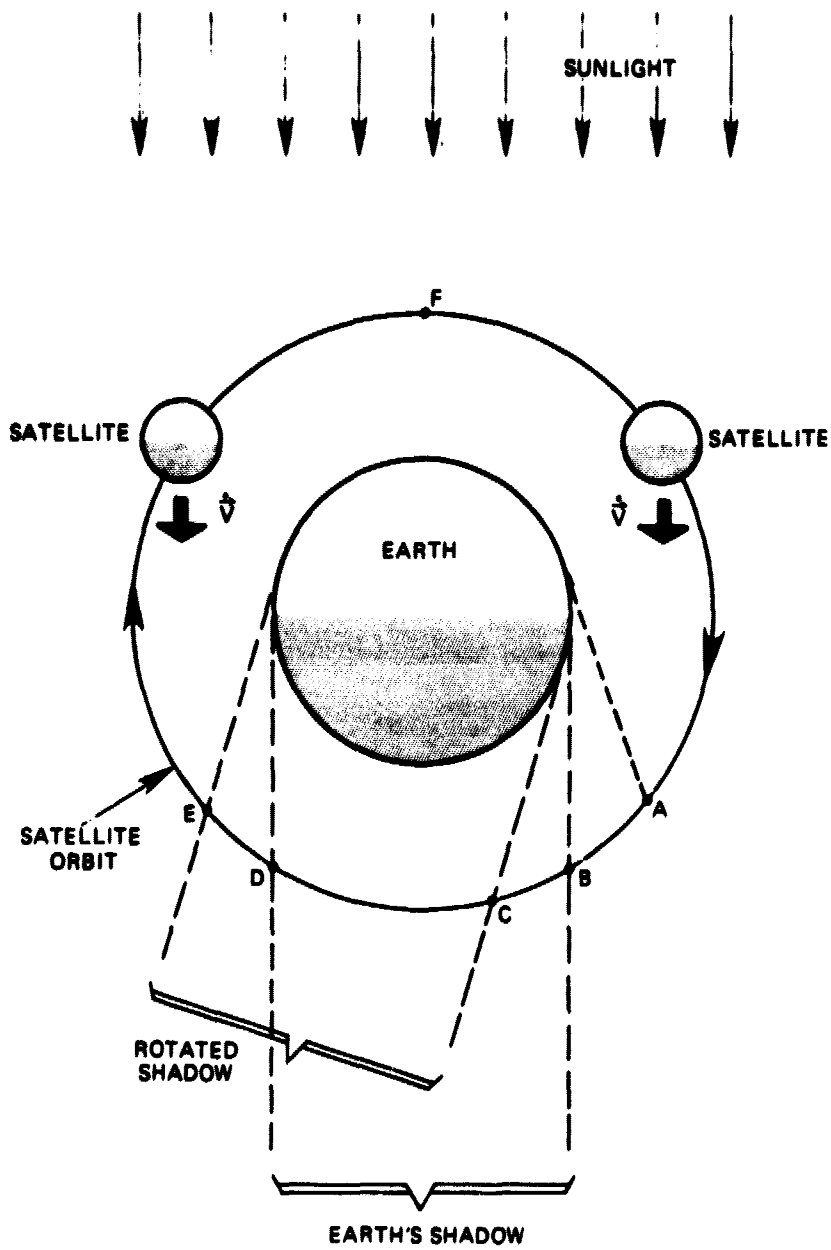

Figure 5. Schematic illustration of the Schach effect. A satellite is shown at two points along its orbit. Note that the acceleration $\vec{V}$ is in the direction of the sunlight (so that the Yarkovsky rotation is ignored) when the satellite is not in shadow. If the satellite cools off and warms up instantly when moving through the shadow, then the acceleration is zero in the shadow and the acceleration over arc FED tends to cancel with that over arc FAB by symmetry. However, if the satellite has some thermal inertia, then it takes some time to cool off and heat up when moving through the shadow. Qualitatively we can think of a thermally inertialess satellite passing through a rotated shadow inside of which $\vec{V}$ is zero. In this case the acceleration over arc FE cancels that over arc FA, but no cancellation occurs for arc ABC. This leads to a net acceleration when averaging over one revolution which tends to increase the semimajor axis.

acceleration due to the reradiated sunlight points in the direction opposite to the sun (ignoring the Yarkovsky effect). This acceleration does not cancel over all parts of the orbit, giving a net impulse to the satellite when it is averaged over one revolution, as explained in the figure caption.

The Schach effect cannot be the unknown force causing the orbit to decay. For one thing, the force operates whether the orbit intersects the shadow or not, while the Schach effect operates only when Lageos moves through the shadow. For another the force acts to increase the semimajor axis, as should be clear from Fig. 5. Further, the Schach effect must operate at a low level, like the Yarkovsky effect, since the curve shows only small deviations from a straight line. Again the analysis of the deviations may be of some interest in trying to detect the effect.

While the Schach effect is not important for Lageos, it may have some interesting consequences for the orbits of the particles comprising the rings of Jupiter, Saturn, Uranus, and possible microtektite rings of the earth (O'Keefe, 1980). But we will not pursue this here, being off the main topic.

7. TERRESTRIAL RADIATION

Yet another radiation force arises fromthe pressure of terrestrial radiation on the satellite. This topic has been treated by many authors, including Wyatt (1963), Sehnal (1970), Prior (1970), and Smith (1970). Recent work has been done by Saslaw (1978, 1979) and Lautman (1977a, 1977b). See the references of these papers for other relevant work.

Terrestrial radiation falling on the satellite comes from three sources (Smith, 1970). One is the diffuse reflection of sunlight from land, water, clouds, etc. Another is the specular reflection of sunlight from still water, ice, or snow. These two together constitute the "albedo problem," although most attention is given to diffuse reflection. Last is the thermal infrared radiation coming from the absorption of sunlight and its reradiation by the earth.

Analytical treatment of diffuse reflection is difficult because of the complicated way it varies in both space and time. The simple early models of diffuse reflection are probably unrealistic. Hence, to obtain a general indication of how diffuse radiation affects a satellite, we will rely on the work of Lautman (1977a, 1977b), who gives the most detailed treatment of the problem to date.

Lautman (1977b) assumes the albedo a_ℓ varies according to the equation $a_\ell = a_o + a_2 \sin^2\phi$, where ϕ is latitude and a_o, a_2 are constants. It is also assumed that "the perturbing acceleration is given by the nonterminator expressions during half of the orbit that the satellite is closest to the sun and that it is zero during the other half," with the more complicated terminator expressions to be given in a future paper (Lautman, 1977b, p. 3). He finds (see his Fig. 5) in a sample calculation for the Pageos balloon satellite that the semimajor axis a decreases by about 160 m over 24 days and then increases by about 280 m over the next 36 days and was still increasing when the 60 day integration time was reached. The following parameters were assumed in this calculation: $a = 1.06 \times 10^7$ m, $e = 0.0628$, $I = 86.9$ deg, $\Omega = 329.8$ deg, $\omega = 215.8$ deg, T_o = MJD 39384.0, $a_o = 0.219$, $a_2 = 0.410$, and the area-to-mass ratio was 11.8 $m^2\ kg^{-1}$.

Both Pageos and Lageos are in polar orbits; their semimajor axes are comparable, and the eccentricities of their orbits are small. Hence it is reasonable to assume that the qualitative behavior of Lageos's orbit will be similar to Pageos's. This qualitative behavior is inconsistent with what is observed on Lageos. Pageos's semimajor axis decreases and then increases over a short time period of 60 days. Lageos's semimajor axis shows a steady decrease over the long time period of 3 years. Since diffuse radiation from the earth is capable of increasing the semimajor axis over short time intervals but Lageos's semimajor axis does not do so, we conclude that diffuse radiation is a most unlikely candidate for being the unknown force acting on Lageos.

It is of some interest to estimate the magnitude of the perturbation due to diffuse reflection for Lageos. If Lageos were in Pageos's orbit, then the change in a would be very roughly 1.6 cm over the time involved. This is due to the very much smaller area-to-mass ratio for Lageos. If the orbit had Lageos's eccentricity of 0.004, then the change in a would be about 16 times smaller, or about 1 mm over the time involved, since the long-period terms in Lautman's (1977b) expression for the change in a (his equation 34) are proportional to the first or greater power of e. Thus it would appear that the magnitude of the diffuse reflection effect is small compared to that of the unknown force is small. This is consistent with the small magnitude of the fluctuations about a straight line shown in Fig. 1.

Wyatt (1963) discussed the specular component of albedo radiation pressure. Again the mathematical difficulties in producing a realistic model are great, but a rough estimate of its magnitude can be made. Wyatt's (1963) work suggests that the specular component is probably only a few percent the size of the diffuse component. The latter is already judged to be small. Further, we would expect its operation to be erratic, since the necessary conditions of calm water or snow or ice fields and cloudless skies above them are erratic. This is inconsistent with the regularity of behavior shown in Fig. 1. We dismiss the possibility of specular reflection being the unknown perturbation for these reasons.

The effect of infrared radiation pressure on Lageos is probably also too small. We would expect the intensity of the radiation to be roughly that of diffuse reflection, but more symmetrically disposed over the earth. Since the effect of diffuse radiation is judged to be small, and since symmetry tends to produce no secular effects as found by Wyatt (1963), we will not consider it further.

8. MAGNETIC DESPIN OF LAGEOS

Lageos's spin rate is being slowed down by the earth's magnetic field via eddy currents (Zonov, 1961; Smythe, 1950, pp. 390-420). If we assume that the lost spin angular momentum is gained by the orbital angular momentum in analogy to tidal friction, then this will affect the semimajor axis. We will compute the magnitude of this effect.

Assume a circular orbit: the orbital angular momentum L is then (Blanco and McCuskey, 1961, p. 133):

$$L = \sqrt{GM_E}\, M_L\, a^{1/2}$$

so that the change in the semimajor axis Δa is then

$$\Delta a = \frac{2a^{1/2}\, \Delta L}{\sqrt{GM_E}\, M_L} \tag{27}$$

due to a change in angular momentum ΔL, assuming the spin axis is perpendicular to the orbital plane. We take ΔL to be the entire spin angular momentum of the satellite

$$|\Delta L| = C\omega_L$$

where we take the moment of inertia C to be that of a homogeneous sphere:

$$C = 0.4\, M_L R_L^2.$$

Using the two equations above in (27) and numerical values from Tables 1 and 2 yield

$$|\Delta a| \approx 1.28 \times 10^{-4}\ \mathrm{m} = 0.128\ \mathrm{mm}.$$

Since this is the entire effect of stopping the spin and not just a rate, there is no possibility of magnetic despin being the unknown force acting on Lageos's orbit.

9. DRAG FROM INTERPLANETARY DUST

Collisions with dust particles will cause a drag force to be exerted on Lageos. We will use the standard drag equation (Blanco and McCuskey, 1961, p. 204)

$$\frac{da}{dt} = -\frac{C_D A \rho V^2}{M_L n} \tag{28}$$

to estimate the magnitude of the drag force, where in this equation ρ is the density of near earth dust in kg m^{-3} and a circular orbit has been assumed. Using (15) and (16) in (28) and numerical values from Tables 1 and 2 give

$$\frac{da}{dt} = -\frac{C_D A \rho \sqrt{GM_E a}}{M_L} = -4.81 \times 10^7\ C_D \rho\ \mathrm{ms^{-1}} \tag{29}$$

for Lageos. We now need C_D and ρ.

For C_D we take the standard value 2.2 (Cook, 1965, p. 929). From Hughes (1975) we discover that the upper limit on the interplanetary dust density ρ_I at 1 AU from the sun is

$$\rho_I = 1.5 \times 10^{-19}\ \mathrm{kg\ m^{-3}}.$$

The absence of dust belts around the earth indicates that a generous estimate in the concentration of dust near the earth is (Shapiro, et al., 1966):

$$\rho \approx 10\,\rho_I = 1.5 \times 10^{-18}\ \mathrm{kg\ m^{-3}}$$

Using these values for C_D and ρ in (29) yield

$$\frac{da}{dt} \cong -1.59 \times 10^{-10}\ \mathrm{ms^{-1}} = -0.0137\ \mathrm{mm\ day^{-1}} \tag{30}$$

for the rate of decrease in a due to dust drag. This is two orders-of-magnitude too small to explain the observed rate. Since our assumptions were rather generous in making this estimate, we conclude there is no possibility of attributing the orbital decay to this mechanism.

10. ATMOSPHERIC DRAG

We finally consider atmospheric drag from neutral and charged particles as being the unknown force acting on Lageos. We look at neutral particle drag first. We can rewrite (29) as

$$mC_D N(a) = \frac{-M_L}{A\sqrt{GM_E a}} \frac{da}{dt} = 1.58 \times 10^{11}\ \mathrm{m^{-3}} \tag{31}$$

where the right side of (30) uses the observed decay rate of $-1.27 \times 10^{-8}\ \mathrm{ms^{-1}}$, and where $\rho = N(a)m$, with N(a) being the number density in particles $\mathrm{m^{-3}}$ at Lageos's altitude and m being the particle mass. This equation must be satisfied if we are to ascribe the observed orbital decay as being solely due to some constituent of the neutral atmosphere.

We confine the discussion to hydrogen and helium only. The atomic hydrogen comes from the photodissociation of water vapor, while helium comes from the decay of uranium and thorium inside the earth. Other constituents of the earth's atmosphere, such as nitrogen, are too heavy to be found in appreciable amounts at Lageos's altitude (Jacchia, 1977, Table 10).

We consider hydrogen first. We take m in (31) as the mass of the hydrogen atom. Cook (1965, Table 2) finds that C_D tends to increase slowly with height, ranging from 3.6 at 1000 km altitude to 3.8 at 3000 km altitude for an exospheric temperature of 1000 K. We will assume $C_D = 3.8$ for hydrogen at Lageos's altitude for all temperatures. In this case (31) reduces to

$$N(a) = 4.15 \times 10^{10}\ \mathrm{m^{-3}} \tag{32}$$

which is the number density necessary to explain the observed orbital decay due solely to neutral hydrogen drag. We now need to estimate N(a) for hydrogen and compare it to (32).

We estimate N(a) using the theory of planetary coronae (Chamberlain 1963, 1968; see also Brinkmann, 1971 and Chamberlain and Smith, 1971). For a spherically symmetric, nonrotating atmosphere the number density at radial distance r from the center of the earth is given by Chamberlain (1963, p. 906):

$$N(r) = N(r_c)\, e^{-(\lambda_c - \lambda)} \zeta(\lambda) \tag{33}$$

where

$$\lambda = \lambda(r) = \frac{GM_E m}{kT_c r}. \tag{34}$$

Here r_c and T_c are the radial distance to, and temperature at, the critical level, respectively, while $\lambda_c = \lambda(r_c)$. The critical level is the altitude at which a neutral particle escaping towards infinity has something like a 1/e chance of colliding with another molecule, where e is the base of the natural logarithm (Blanco and McCuskey, 1961, p. 58). We take the critical level to be 600 km altitude, so that $r_c = 6371 + 600 = 6971$ km. The $\zeta(\lambda)$ appearing in (33) is a partition function which in turn is the sum of three other functions: $\zeta(\lambda) = \zeta_{bal} + \zeta_{sat} + \zeta_{esc}$. Numerical values for the ζ_{bal}, ζ_{sat} and ζ_{esc} for various values of λ may be found in Chamberlain's (1963) Tables 1 and 2. The values for $N(r_c)$ at temperature T_c come from Jacchia's (1977) Table 10.

Table 3 shows the number density for atomic hydrogen for various temperatures T_c for Lageos's altitude using (33) and (34). It also compares the number densities at 2500 km altitude using (33) and (34) with those of Jacchia's (1977) model. The agreement between the two is quite good.

Table 3 shows that N(a) reaches a maximum of about $0.54 \times 10^{10}\ m^{-3}$ near $T_c = 1000$ K at Lageos's altitude. Thus the number density found from the theory of planetary coronae falls short by at least a factor of 8 from the number required by (32) to explain the orbital decay as being due entirely to neutral hydrogen, regardless of exospheric temperature. Hence unless the number density at the critical level or the drag coefficient are greatly increased somehow, it appears that neutral hydrogen drag can explain no more than about 12 percent of the observed decrease in a.

We turn now to drag from neutral helium. Only ^4He need be considered, since the abundance ^3He is too small to exert appreciable drag on Lageos (Mac Donald, 1963). We find

Table 3

Number densities for hydrogen at various exospheric temperatures. The first column shows the temperature. The second shows the number density at 2500 km altitude computed according to equations (33) and (34), and the third column the number density at 2500 km according to Jacchia's (1977) Table 10. Note that the agreement between the two is quite good. The last column shows the number density at Lageos's altitude.

T_c (K)	N(10^{10} m^{-3}		
	2500 km (computed)	2500 km (Jacchia)	5856 km (computed, Lageos)
600	4.28	4.30	0.35
800	3.26	3.27	0.50
1000	2.44	2.45	0.54
1200	1.88	1.87	0.54
1400	1.48	1.47	0.50
1600	1.19	1.17	0.46
1800	0.97	0.96	0.42
2000	0.81	0.79	0.38
2200	0.69	0.67	0.35
2400	0.59	0.57	0.31
2600	0.51	0.49	0.29

$$C_D N(a) = 3.95 \times 10^{10}\ m^{-3}$$

from (31) where the mass m appearing in that equation is now 4 times that of hydrogen.

Cook (1965, Table 2) finds the values for C_D for helium to be somewhat lower than for hydrogen, ranging from 2.9 at 2000 km altitude to 3.0 at 3000 km altitude for a temperature of 1500 K. He also finds that $C_D = 3.1$ for the same two altitudes for a temperature of 2000 K. Since C_D is rather insensitive to both altitude and temperature, we will set $C_D = 3.0$ without any great error. The above equation now becomes

$$N(a) = 1.32 \times 10^{10}\ m^{-3}. \tag{35}$$

This is the number density necessary to explain the secular decrease in a as being due solely to neutral helium drag.

The exospheric temperature needed to produce the number density (35) is 2200 K. The variation in exospheric temperature since Lageos's launch is shown in Figure 6. The curve is based on the solar flux and geomagnetic activity in the following manner (Jacchia, 1977, equations 20, 31a, and 31b):

$$T_C = 5.48\ \bar{F}^{0.8} + 101.8\ F^{0.4} + \Delta_G T_c$$

where

$$\Delta_G T_c = \int_0^{\pi} A_p \sin^4 \phi_I\, d\phi_I$$

and

$$A_p = 57.5\ K_p\ [1 + 0.027 \exp (0.4\ K_p)].$$

Here F is the solar flux in 10^4 Jansky (10^{-22} W m^{-2} Hz^{-1}), $\bar{F}$ is the average of F over six rotations, ϕ_I is the invariant magnetic latitude, and K_p is the geomagnetic activity index. Obviously the temperature at no time approaches the 2200 K necessary to explain the orbital decay as helium drag. In fact, assuming a typical temperature of 1000 K from Fig. 6 in (33) and (34) gives $N(a) = 10^7\ m^{-3}$. The number density thus derived from the actual exospheric temperature falls about 3 orders-of-magnitude short of the necessary number (35). Neutral helium appears to be a poor candidate for explaining the secular decrease in a.

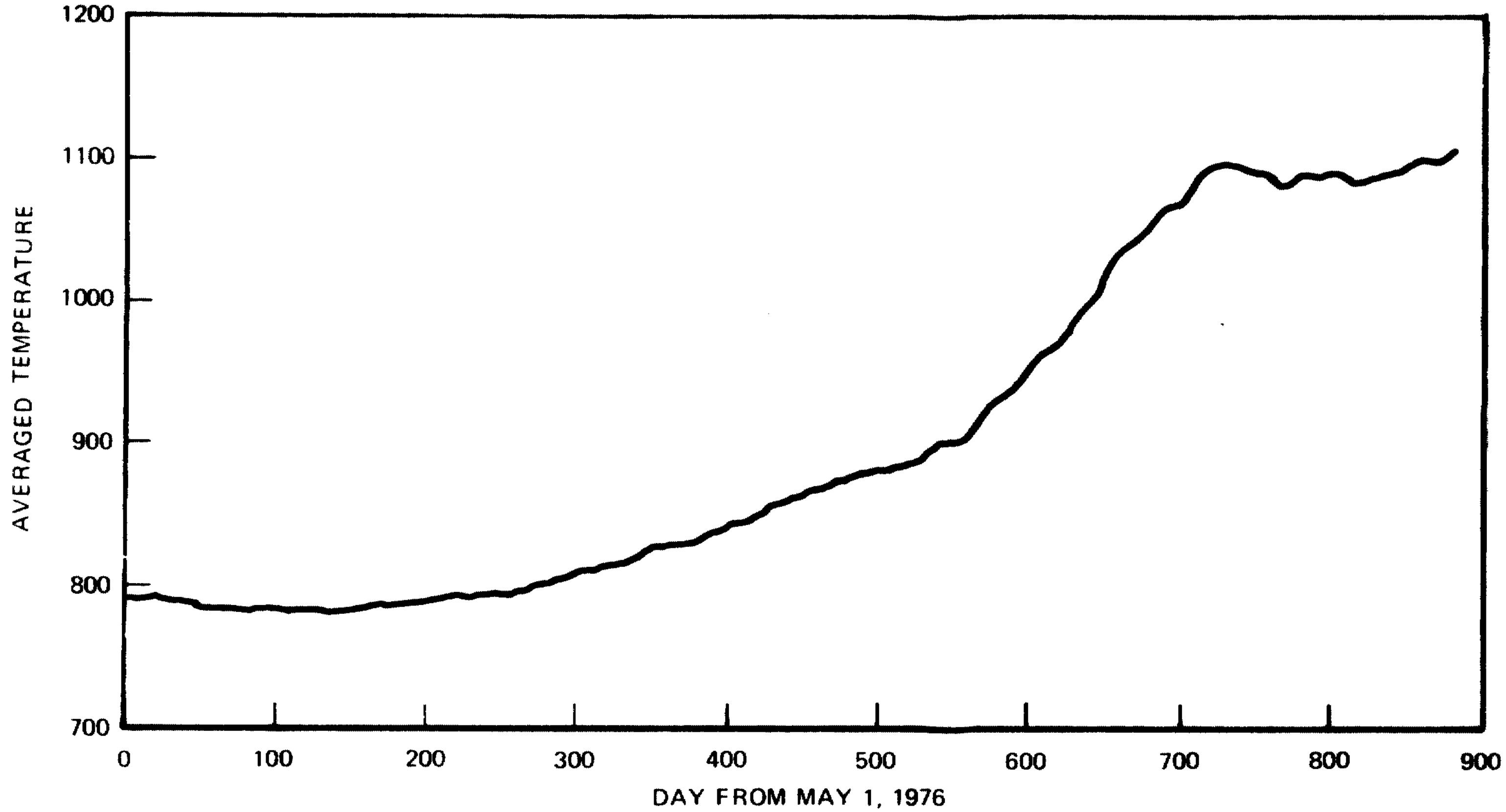

Figure 6. The exospheric temperatures as a function of time since Lageos's launch, based on the solar flux and geomagnetic activity.

In spite of this, however, we do not rule it out, since helium drag helps solve the "helium problem."

The helium problem is reviewed by MacDonald (1963), Kockarts (1973), Hunten (1973), Hartle and Mayr (1976), and Chamberlain (1978). It consists of accounting for the outgassing from the earth's interior, escape, and abundance of both ^3He and ^4He in the earth's atmosphere. The influx of ^4He from radioactive decay greatly exceeds the rate at which it is being lost through known mechanisms. A loss mechanism must exist, since otherwise its present abundance would have been built up in only 10^6 years. Further, if one assumes the loss mechanism for ^4He is thermal escape (as considered here), with the temperature of the exosphere being elevated in some unknown manner, then the loss of ^3He cannot be accounted for by thermal escape (Chamberlain, 1978, p. 278). As Chamberlain (1978, p. 278) says, "The problem will not go away and is unsolved."

The exospheric temperature required for the thermal escape flux of ^4He to be equal to its influx into the atmosphere from the earth's interior was found by MacDonald (1963) to be 2200 K. This is the same temperature required to explain the drag on Lageos as being due entirely to helium. Hence helium is an attractive candidate for being the drag on Lageos, since it takes care of the drag force and the ^4He part of the helium problem in one stroke. However, if this mechanism is at work then the temperature of the exosphere would have to be elevated by an as yet unrecognized heat source, as pointed out by MacDonald (1963). Moreover, it would have to be a steady source largely uncorrelated with the solar flux and geomagnetic activity, as demanded by the lack of correlation between Figs. 1 and 6. Finally, it does not help the ^3He part of the helium problem.

We wish to point out that our assumption of a spherically symmetric atmosphere is oversimplified. The helium distribution shows a distinct bulge over the winter hemisphere (Kockarts, 1973) which has recently been directly measured (Mauersberger et al., 1976).

The thermal mechanism is not the only one which can increase the helium abundance in the upper atmosphere. Winds can greatly enhance helium escape, as shown by Hartle and Mayr (1976). Also, helium may become ionized and escape over the poles on the open magnetic field lines

(Chamberlain, 1978, p. 278). There is the further possibility that charge exchange between He or H and hot He^+ in the plasmasphere produces energetic He atoms (Chamberlain, 1978, p. 278). It is of some interest to note that the trend of both theory and measurement has been towards a higher abundance of helium in the upper atmosphere (Nicolet, 1961; Young et al., 1977; Hartle and Mayr, 1976). And, as pointed out before, an escape mechanism must exist; a high helium density at Lageos:s altitude may allow a steady-state between influx and loss, removing the need for assuming that we live in a special period in which the helium abundance is building up (Kockarts, 1973). Helium may well play a significant role in the drag on Lageos.

We turn our attention now to charged particle drag. A satellite can become charged from collisions with charged particles and from the photoejection of electrons from the satellite's surface. The satellite charge will interact electromagnetically with the charged particles in its vicinity and cause the satellite to lose momentum; whence the drag (Chopra, 1961).

Let us first estimate the number density of the charged particles. The dominant ions at Lageos's altitude are H^+, He^+, and O^+ with number densities of about 2×10^9, 2×10^8, and 2×10^8 m^{-3}, respectively. (Chappell et al., 1970, p. 51). In Rubincam (1980) we assumed that H^+ was the only ion present with a number density of 10^8 m^{-3}. Thus the data of Chappell et al. (1970) show that the charged particle density at Lageos's altitude is much greater than assumed in the previous paper.

A conservative estimate of the drag due to H^+, He^+, and O^+ can be made by assuming that these ions behave like neutral atoms. In other words, we take the drag coefficient C_D to be 3.8 for H^+ and 3.0 for He^+, just as we did in computing the neutral particle drag for these constituents. We take C_D to be equal to the standard value of 2.2 for O^+ (Cook, 1965, p. 943).

The contribution of each type of ion to the decrease in the semimajor axis using the above values for number densities and drag coefficients can be computed from (29). The results are summarized in Table 4. H^+ can account for about 4.4 percent of the observed decay rate, while He^+ and O^+ account for 1.4 and 4.1 percent, respectively. Altogether the ions account for about 10 percent of the observed decay rate, assuming the ions drag like neutral atoms. The neutral atmosphere accounts for another 10 percent (Table 3) as explained previously, leaving aside the problematical contribution

Table 4
Amount of observed drag explained by charged particle drag, assuming the ions behave like neutral particles. The drag due to neutral hydrogen is also given.

Constituent	Number Density N(a) (m^{-3})	Drag Coefficient C_D	Percent of Observed Drag
H^+	2.0×10^9	3.8	4.4
He^+	2.0×10^8	3.0	1.4
O^+	2.0×10^8	2.2	4.1
			Ion subtotal: 9.9
Neutral H	4.5×10^9	3.8	Total: 10.0
			19.9

of neutral helium. Thus we can conservatively attribute at least 20 percent of the observed rate of decrease in the semimajor axis of Lageos to atmospheric drag from the combined effects of charged and neutral particles.

Charged particle drag is in general greater than neutral particle drag for equal particle densities (Chopra, 1961). Unfortunately, the various theories developed to assess the importance of charged particle drag on satellites do not agree on how much greater. For example, Chopra (1961) in his Table VI compares values for C_D found from the Jastrow-Pearse, Kraus-Watson, and Chopra-Singer theories of Coulomb drag. The values range from a low of about 2 in the Kraus-Watson theory, to about 6 in the Jastrow-Pearse theory, to a high of about 8×10^3 in the Chopra-Singer theory for a high altitude (20,000 km) satellite charged at -21 v. In Rubincam (1980) we used the Chopra-Singer result to show that the decay of Lageos's orbit falls within the limit for drag predicted by this theory.

However, the experiments by Knechtel and Pitts (1964) using mercury ions to measure charged particle drag in the laboratory indicates that the Jastrow-Pearse theory is more nearly correct. So we will use their results to estimate the importance of charged particle drag on Lageos.

Knechtel and Pitts (1964) find that multiplying the charged particle drag for an uncharged satellite by

$$s \cong 1 - 1.33 \frac{q\phi_o}{E} \tag{36}$$

gives the total (neutral plus charged particle) drag on the satellite for $R_L/\lambda_D = 7.5$, where q is the proton charge, E is the kinetic energy of the ions, ϕ_o is the satellite potential, and λ_D is the Debye length (Reitz and Milford, 1967, p. 272):

$$\lambda_D = \left(\frac{\epsilon_o kT}{2N(a)q^2}\right)^{1/2} \tag{37}$$

where T is the temperature of the ions. Equation (36) is derived from Fig. 5 of Knechtel and Pitts (1964).

Since Lageos orbits inside the plasmasphere where the thermal regime is one of cold ions (Russell, 1972), we will take T = 2000 K and E = 3kT/2. The number densities N(a) for H^+, He^+, and O^+

have already been given. Using these data in (37) give $R_L/\lambda_D = 6.15$ for H^+, and 1.94 for He^+ and O^+. These values are lower than the $R_L/\lambda_D = 7.5$ required by (36), but Knechtel and Pitts (1964) did not run the experiment for these low values. However, since the factor s increases as R_L/λ_D decreases (Knechtel and Pitts, 1964, Fig. 5), we can use (36) to estimate a lower bound on the contribution of charged particles to the drag. In this case (36) becomes

$$s = 1 - 5.14\,\phi_o \tag{38}$$

It remains to find the satellite potential ϕ_o to use in (38). Satellites at the geosynchronous altitude can become charged up to hundreds or even thousands of volts (e.g. Garrett et al., (1980). At lower altitudes (~ 1000 km) satellite potentials appear to be on the order of a volt (Samir et al., 1979, p. 102). We will assume the lower figure is more nearly correct and take $\phi_o = -1$ v. (The voltage is negative since electrons collide with a satellite much more frequently than with positive ions, because of their higher speeds at any given temperature.)

Using this voltage in (38) yields s = 6.14. We will approximate the ion drag for an uncharged satellite by the neutral particle assumption (Table 4)). We have already found that the ions account for about 10 percent of the observed decrease in the semimajor axis if they behave like neutral particles. Thus multiplication of this figure by s gives 61 percent as the amount of the observed decay which can be explained as being due to charged particle drag, using the data of Knechtel and Pitts (1964). Neutral hydrogen drag accounts for another 10 percent, so that 71 percent of the observed drag can be reasonably attributed to the combined effects of charged and neutral particle atmospheric drag. A similar conclusion was reached by Barlier (1980).

We conclude our investigation of atmospheric drag by saying that if we can reasonably attribute 71 percent of the observed decay to a combination of neutral and charged particle drag, then it is likely that this mechanism explains the entire effect. This is because our estimate of the magnitude of charged particle drag is a probable lower bound and because the 71 percent figure does not include a possible neutral helium contribution.

11. DISCUSSION

We have examined nine mechanisms to discover which one, if any, might be the force causing the semimajor axis to decrease at the rate of -1.1 mm day^{-1}. Gravitational radiation and transfer of spin angular momentum to orbital angular momentum are incontrovertibly too small to be the unknown force. The Poynting-Robertson effect from sun light fails by a factor of 40 to explain the observed rate; the effect from earthlight is even smaller. While a factor of 40 is close enough to give one pause, there are no parameters to adjust to try and nudge the result up to the required value. We rule these three mechanisms out.

Resonance with the gravitational field and drag from interplanetary dust appear to be at least two orders-of-magnitude too small. However, these mechanisms contain "adjustable parameters" (the magnitudes of the gravity field coefficients and the density of the dust, for example) which might be increased to give the desired value. Also, we have used a simple linear theory in treating the resonance and ignored other complicating perturbations. Still, these two mechanisms appear to require unreasonable assumptions to attribute the force to either one of them. Szebehely (1980), however, suggests the gravitational field is in fact responsible for the secular decrease; but our results on atmospheric drag strongly contraindicate this.

Of the radiation effects the Schach effect may be ruled out. It operates only when part of the orbit is in shadow, while the secular decrease continues whether the orbit is in full sunlight or shadow. Further, it tends to increase the semimajor axis rather than decrease it. Terrestrial radiation pressure or the Yarkovsky effect do not seem to be the cause of the orbital decay. Terrestrial radiation pressure can increase and decrease the semimajor axis over a short period of time, while the Yarkovsky effect should be absent in the early part of Fig. 1 when the orbit is in full sunlight. But their characteristic signature on the orbit is absent from Fig. 1. However, the treatment of terrestrial radiation pressure and the Yarkovsky effect are the least satisfactory of all of those given here, the former because of its complexity and the latter because of its possibly large magnitude. More research is needed into their effect on satellite orbits.

Atmospheric drag seems to be the most likely cause for the secular decrease in the semimajor axis of Lageos's orbit. Neutral hydrogen drag accounts for about 10 percent of the observed decrease. Charged particle drag from H^+, He^+, and O^+ accounts for at least another 61 percent. This makes a grand total of 71 percent of the observed decay which can be attributed to atmospheric drag. This is the largest effect found so far with the proper signature on the semimajor axis. It is not hard to imagine that if this mechanism explains at least 71 percent, then it in fact accounts for 100 percent of the observed decrease. The argument is that our estimate of charged particle drag was somewhat conservative, so that in fact it contributes more than 61 percent of the total drag; and that our estimate of the neutral particle drag entirely omitted helium. A neutral helium component helps explain the helium problem.

In conclusion, it appears that atmospheric drag from a combination of charged and neutral particles adequately explains the secular decrease of 1.1 mm day^{-1} in the semimajor axis of Lageos's orbit.

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16. Abstract The semimajor axis of the Lageos orbit is decreasing secularly at the rate of -1.1 mm day^{-1} due to an unknown force. Nine possible mechanisms are investigated here to discover which one, if any, might be the force. Five of the mechanisms, resonance with the earth's gravitational field, gravitational radiation, the Poynting-Robertson effect, transfer of spin angular momentum to the orbital angular momentum, and drag from near-earth dust are ruled out because they are too small or require unacceptable assumptions to account for the observed rate. Three other mechanisms, the Yarkovsky effect, the Schach effect, and terrestrial radiation pressure could possibly give the proper order-of-magnitude for the decay rate, but the characteristic signatures of these perturbations do not agree with the observed secular decrease. Atmospheric drag from a combination of charged and neutral particles is the most likely cause for the orbital decay. This mechanism explains at least 71 percent of the observed rate of decrease of the semimajor axis. It probably explains all of the decay since (a) the estimate of charged particle drag is conservative and (b) there may be substantial quantities of neutral helium at Lageos's altitude ...

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