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A BOUNDS ON THE RESONANT FREQUENCY OF RECTANGULAR MICROSTRIP ANTENNAS

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SUMMARY

The calculation of currents induced by a transverse electric plane wave normally incident upon an infinite strip embedded in a grounded dielectric slab is used to infer a lower bound on the resonant frequency (or resonant E-plane dimension) for rectangular microstrip antennas. An upper bound is provided by the frequency for which the E-plane dimension is a half-wavelength.

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INTRODUCTION

Microstrip antennas are being used extensively in applications where low-profile, inexpensive, rugged, high efficient antennas are desirable. Due to the thin substrate on which the antenna is constructed, the microstrip antenna is inherently very narrow band (usually less than 5 percent); therefore, an accurate method of determining the resonant frequency is needed in order to adequately design microstrip antennas to meet specified application requirements. The resonant frequency of a microstrip antenna is a function of the dimensions of the metal patch, the dimensions and dielectric properties of the substrate, and the details of the feed. In order to accurately predict the resonant characteristics with consistency, the combined effects of all parameters must be properly modeled. Various approximate and useful models of the microstrip antenna have been developed; however, a complete understanding of the electromagnetic properties of the structure remains questionable.

The purpose of this paper is to show that the analysis of a TE plane wave excited strip in a grounded dielectric can serve as a tool to assist in understanding some of the basic characteristics of rectangular microstrip elements. In particular, a study of the resonant characteristics of the strip provides a lower bound on the natural resonant frequency. An upper bound is provided by the frequency for which the E-plane dimension is $0.5\lambda_{\epsilon}$. The justification for using the TE strip to infer either the resonant frequency of rectangular microstrip antennas of large H-plane dimension or a lower bound on the resonant frequency is based upon the calculation of current distributions for TM excited strips embedded in a grounded dielectric. The current distribution for the TM strip is essentially uniform, except for edge singularities, when the width of the strip is of the order of a wavelength or larger (ref. 1). Use of the TE strip as a model for infering the resonant characteristics of a rectangular patch, therefore, assumes decoupled orthogonal currents on the patch and a uniform distribution of current in the H-plane direction.

SYMBOLS

d	thickness of dielectric
E_y^i	electric field intensity incident on strip
G_y	Green's function
j	$\sqrt{-1}$
J_n	complex amplitude of n^{th} current pulse
J_y	electric current density
k	wave number in dielectric $(2\pi/\lambda_\epsilon)$
k_o	wave number in free space
k_y	Fourier transform variable
m	indicates m^{th} current pulse
n	indicates n^{th} current pulse
N	total number of current pulses
w	width of strip

x,y,z	Cartesian coordinates
y'	variable of integration
y_m	location of center of m^{th} pulse
y_n	location of center of n^{th} pulse
z'	height of strip above ground plane
ϵ	permittivity of dielectric
ϵ_o	permittivity of free space
ϵ_r	dielectric constant (ϵ/ϵ_o)
Δ	half-width of triangular pulse
η_o	intrinsic impedance of free space
λ_ϵ	wavelength in dielectric
μ	permeability of dielectric
μ_o	permeability of free space
μ_r	relative permeability (μ/μ_o)
ω	angular frequency

SYNOPSIS OF THEORY

The geometry for the analytical model is illustrated in Fig. 1. The conducting strip is infinite in the x-direction and parallel to the xy-plane. All electromagnetic quantities are assumed to be invarient in the x-direction. The problem then becomes a 2-dimensional one in the yz-plane.

The problem is analyzed by first deriving a Green's function which satisfies the boundary conditions for an electric-dipole line-source parallel to the x-axis, polarized in the y-direction, and embedded in the grounded dielectric layer. Since the problem is 2-dimensional and only a y-component of electric current is assumed, a scalar Green's function is sufficient to completely characterize the electromagnetic fields. The Green's function for the region inside the dielectric ($0 \leq z \leq d$) is:

$$G_y(y,y',z) = \int_{-\infty}^{\infty} \frac{j\mu}{2k_z'} \left\{ \left[\exp\left(jk_z'(z-z')\right) - \exp\left(-jk_z'|z-z'|\right) \right] \right.$$

$$\left. - 2j \sin(k_z' z) \left[\frac{\epsilon k_z \cos(k_z'(d-z')) + j\epsilon_0 k_z' \sin(k_z'(d-z'))}{\epsilon k_z \cos(k_z' d) + j\epsilon_0 k_z' \sin(k_z' d)} \right] \right\}$$

$$\cdot \exp\left(jk_y(y-y')\right) \; dk_y \qquad (1)$$

where,

$$k_z = \begin{cases} \sqrt{k_0^2 - k_y^2} & k_y \leq k_0 \\ -j\sqrt{k_y^2 - k_0^2} & k_y \geq k_0 \end{cases}$$

$$k_z' = \begin{cases} \sqrt{k^2 - k_y^2} & k_y \leq k \\ -j\sqrt{k_y^2 - k^2} & k_y \geq k \end{cases}$$

By weighting the Green's function with the electric current density and integrating across the strip, the radiated electromagnetic fields can be calculated at any point inside the dielectric layer. Since the current distribution on the strip is not known, it must first be determined. It is in this calculation of the current density on the strip which allows one to infer a lower bound on the natural resonant frequency of rectangular microstrip elements. The determination of the strip current is accomplished by imposing the restraint of zero tangential electric field on the surface of the perfectly conducting strip to arrive at the following integro-differential equation for the unknown current density:

$$E_y^i(y,z') = \frac{j\omega}{k^2}\left[k^2 + \frac{\partial^2}{\partial y^2}\right]\int_{-w/2}^{w/2} J_y(y')\ G_y(y,y',z')\ dy' \qquad (2)$$

where the Green's function is evaluated at the strip (z=z'). The electric field incident on the strip is a known quantity since plane wave excitation is assumed.

The integro-differential equation is solved for the unknown current density by employing the method of moments (ref. 2) using piecewise linear expansion of the current and triangular pulse testing to arrive at a set of N simultaneous equations.

$$E_y^i(y_m,z') = C\sum_{n=1}^{N} J_n\ Z_{mn} \quad ; \quad m = 1,2,3,\cdots,N \qquad (3)$$

where,

$$C = (jk_o\eta_o z'\Delta)/(\pi\epsilon_r)$$

and where the generalized impedance elements, Z_{mn}, are given by

$$Z_{mn} = \int_0^\infty \left\{ \cos(k_y(y_m-y_n)) \left[\frac{\sin(k_y \Delta/2)}{(k_y \Delta/2)} \right]^2 \right.$$

$$\cdot \left[\mu_r \epsilon_r - \left(\frac{2}{k_o \Delta}\right)^2 \sin^2 (k_y \Delta/2) \right] \left[\sin(k_z' z') \,/\, (k_z' z') \right]$$

$$\left. \left[\frac{\epsilon k_z \cos(k_z'(d-z')) + j\epsilon_o k_z' \sin(k_z'(d-z'))}{\epsilon k_z \cos(k_z' d) + j\epsilon_o k_z' \sin(k_z' d)} \right] \right\} dk_y \qquad (4)$$

The generalized impedance elements, Z_{mn}, represent the mutual impedance between the m^{th} and n^{th} current pulses. Equation 4 must be evaluated by numerical integration and by properly including the residue at the surface wave poles.

For a plane wave with an electric field of unit amplitude and normal incidence on the strip, the incident electric field at the strip location (z=z') is

$$E_y^i(y_m,z') = \left[\frac{2j \sin(kz') \exp(jk_o d)}{\sqrt{\epsilon_r} \cos(kd) + j \sin(kd)} \right] \qquad (5)$$

By setting equation 5 equal to equation 3, the complex amplitudes of the current pulses can be calculated by solving the resulting set of simultaneous equations using matrix inversion.

RESULTS

All calculations presented here are for a lossless dielectric with a dielectric constant of 2.5, which corresponds closely to the properties of teflon-fiberglass. All current densities are normalized to the electric field that would exist at the strip location with the strip removed (eq. 5).

The distribution of current density across a strip located at the surface (z'=d) of a $0.02\lambda_\epsilon$ grounded dielectric is plotted in Fig. 2 for strip widths of 1/4, 1/2, and 3/4 wavelength. One readily notices that the current on the strip is excited more strongly when the strip width is near a half-wavelength. If one plots the real and imaginary parts of the strip current at the center of the strip as a function of the strip width, as is done in Fig. 3, the plot has the characteristic resonant behavior of a dipole antenna.

Fig. 4 shows the strip resonant width (width for zero imaginary current) as a function of the thickness of the dielectric. The width of the strip for resonance decreases in a monotonic fashion as the dielectric thickness increases. This behavior is also generally characteristic of the resonant size of microstrip antennas (refs. 3,4). The measured resonant E-plane dimensions for several rectangular microstrip antennas (refs. 4-7) with substrate dielectric constants between 2.48 and 2.62 are included in Fig. 4 for comparison. Although the actual resonant frequency will also depend upon the H-plane dimension of the patch, the data of Fig. 4 indicate that the variation of the resonant frequency will be bracketed on the upper end by $W=0.5\lambda_\epsilon$ and on the lower end by the resonant width of the TE strip. As noted by the measurements in reference 5, there is about a 3% variation in the resonant frequency versus the H-plane dimension of the patch for a dielectric thickness of $0.013\lambda_\epsilon$; however, if the thickness is increased, as is sometimes done in order to increase bandwidth, a corresponding increase in the sensitivity of the resonant frequency may be expected. For example, with a thickness of $0.05\lambda_\epsilon$, the TE strip results predict a possible downward shift in the resonant size of as much as 12% from the half-wavelength value. The measured data in Fig. 4 tend to verify this trend for thicker dielectrics. Of additional interest is that the TE strip calculations tend to predict the resonant frequency quite accurately for rectangular microstrip antennas whose H-plane dimension is one wavelength or greater, as shown by the solid symbols in Fig. 4.

Fig. 5 shows the effect of placing a protective teflon-fiberglass cover over the conducting patch. Again the TE strip calculations predict an additional downward shift in resonant frequency due to the dielectric cover, which is verified by measurements with a rectangular patch whose H-plane dimension exceeds one wavelength.

CONCLUSION

It is demonstrated that a plane-wave excited infinite strip on, or embedded in, a grounded dielectric sheet can be used as an analytical aid to determine the maximum expected shift in the natural resonant frequency for rectangular microstrip antennas with or without a dielectric cover. It is also demonstrated that the TE strip model consistently predicts quite accurately the resonant frequency for rectangular microstrip antennas with a large H-plane dimension (greater than one wavelength).

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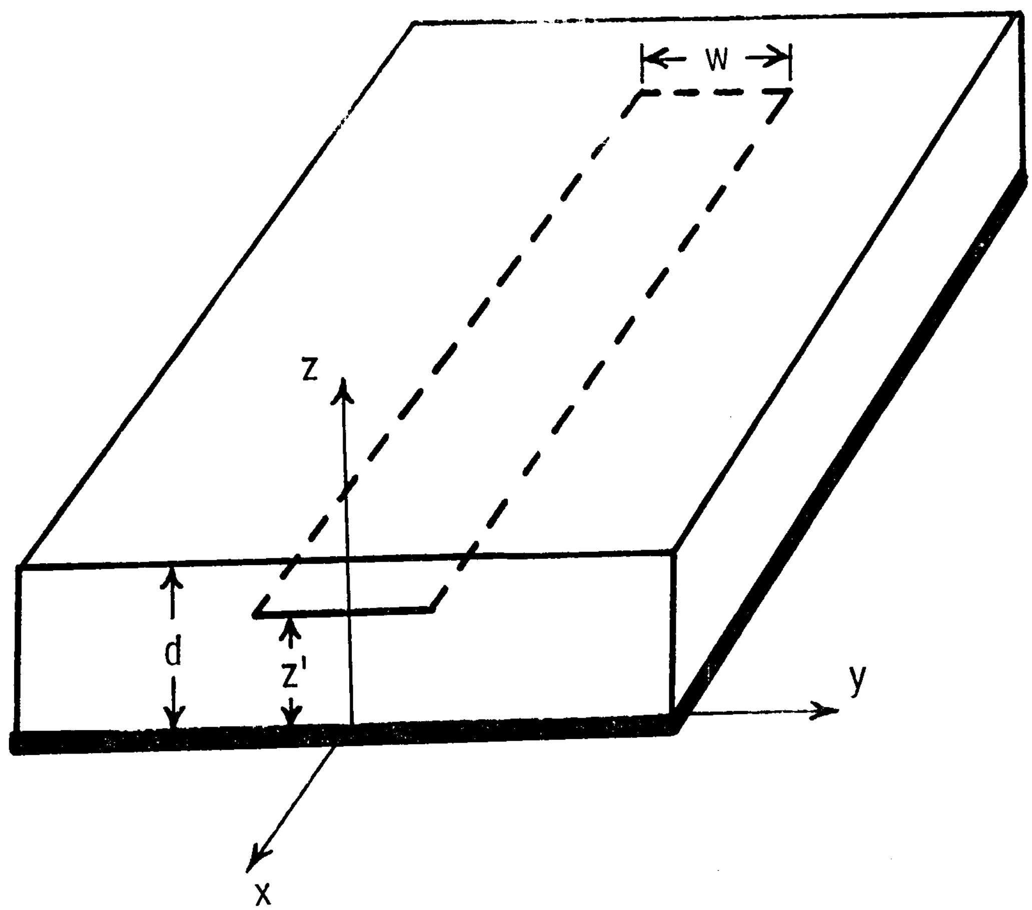

Figure 1: Strip embedded in a dielectric slab on an infinite ground plane.

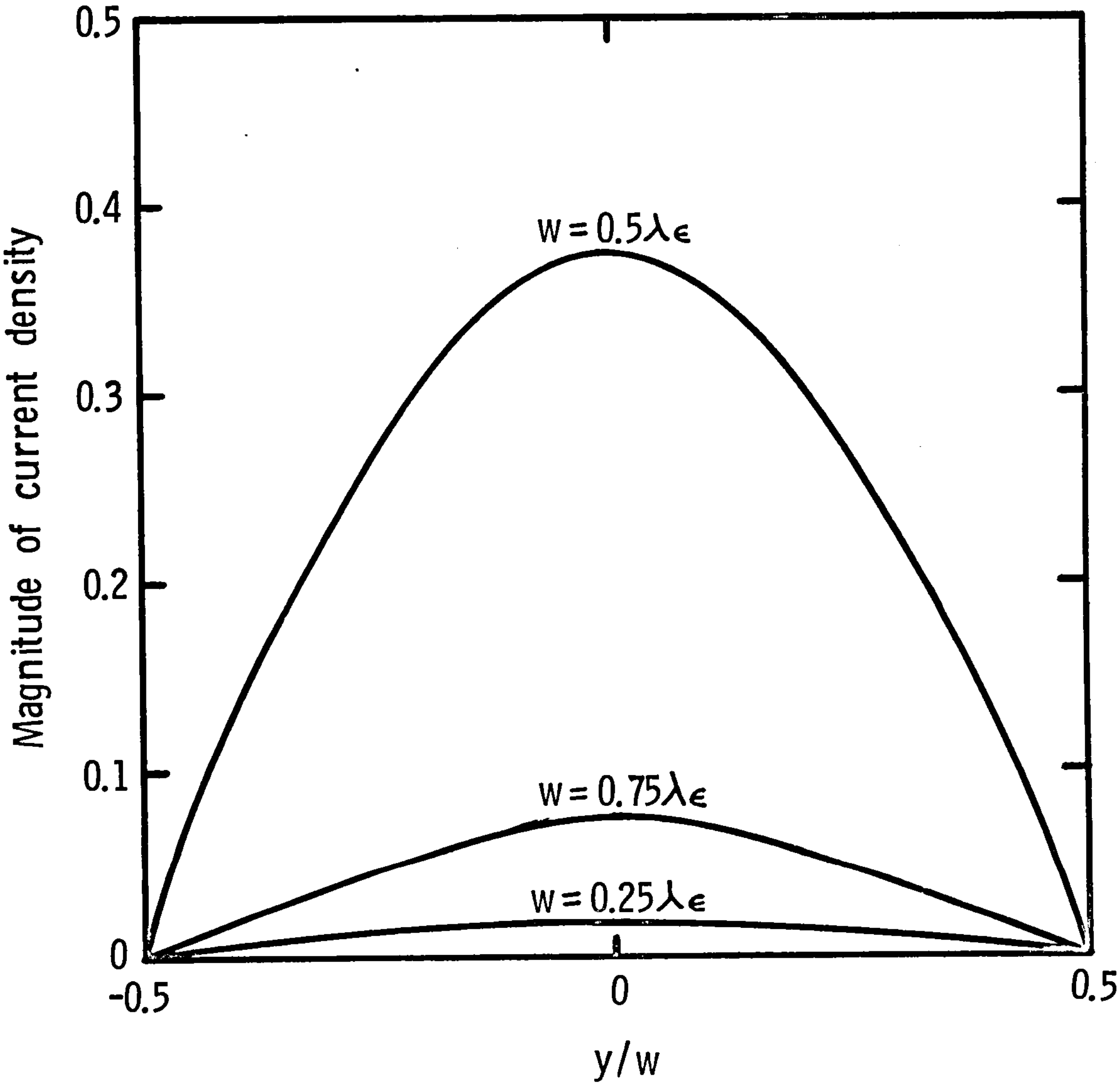

Figure 2: Magnitude of current density across a TE plane wave excited strip at surface of a grounded dielectric slab ($z' = d = 0.02\lambda_\epsilon, \epsilon_r = 2.5$).

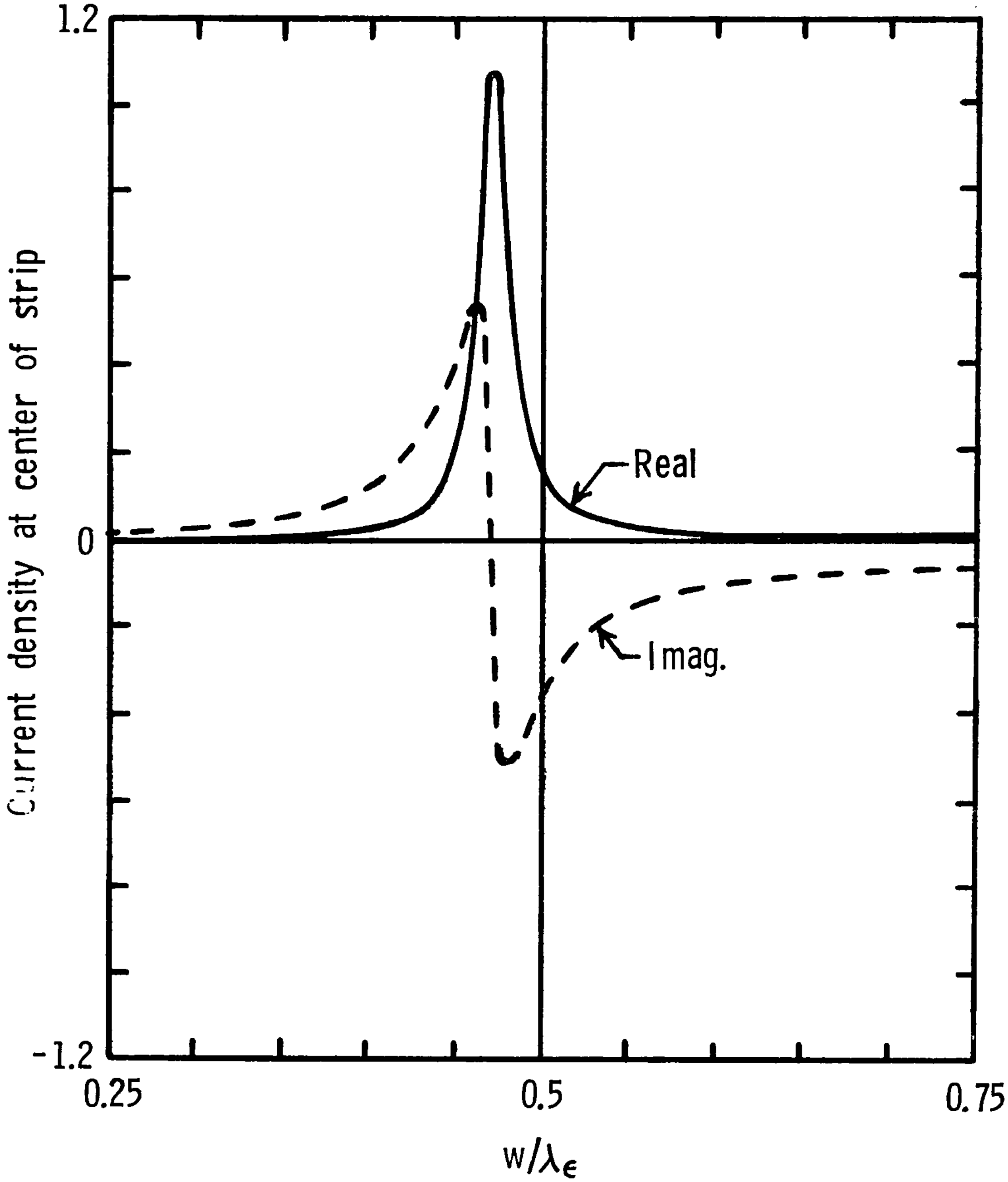

Figure 3: Complex current density at center of strip versus strip width ($z' = d = 0.02\lambda_\epsilon, \epsilon_r = 2.5$).

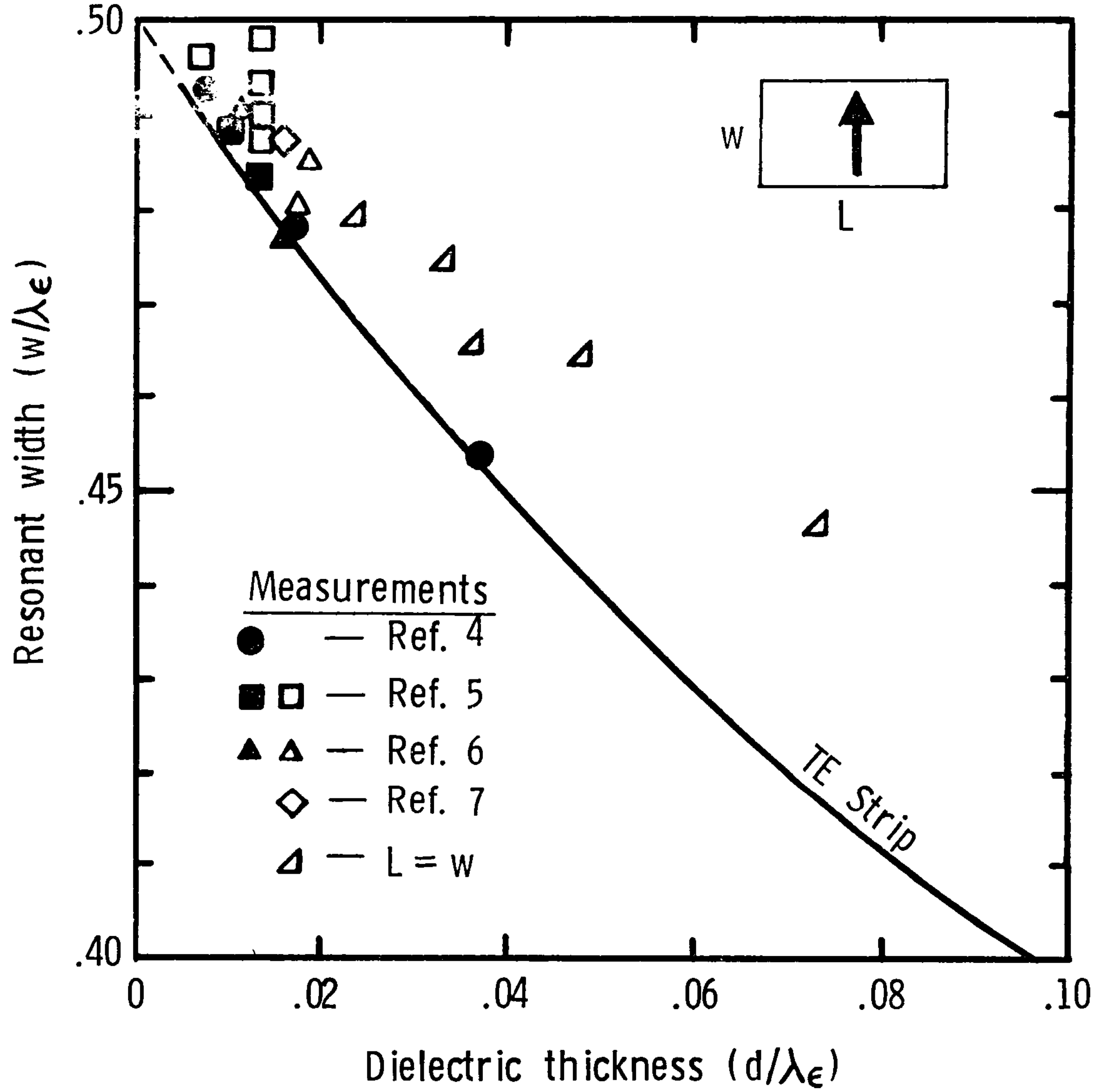

Figure 4: Comparison between calculated resonant width of strip and measured resonant width of microstrip antenna ($z' = d, \epsilon_r = 2.5$).

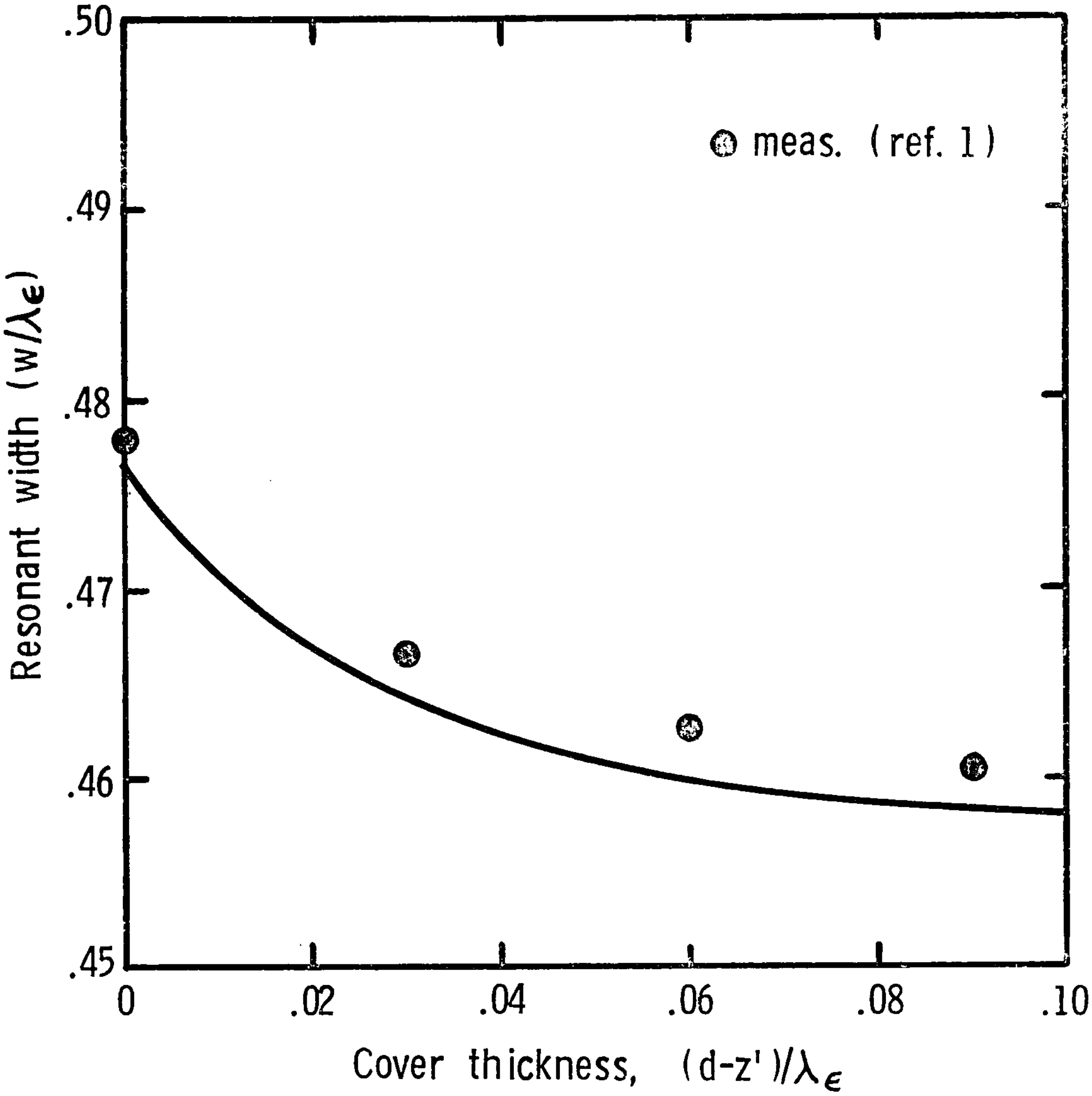

Figure 5: Comparison between calculated resonant width of strip and measured resonant width of microstrip antenna with cover ($z'=0.0175\lambda_\epsilon, \epsilon_r=2.5$).

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16. Abstract

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