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ABSTRACT

Transition probabilities and branching ratios between the upper states $n' k'$ and lower states $n k$ in hydrogen-like atoms in an electric field, and various sums and averages, have been tabulated. n and n' are the principal, and $k = n_1 - n_2$ and $k' = n_1' - n_2'$ are the electric quantum numbers. This is the first time that the transition probabilities and branching ratios are tabulated in terms of these quantum numbers. It is found that the transition probabilities between sublevels of n' and n are almost dependent of k', and are strongly dependent on $|\Delta k| = |k' - k|$, such that the transition probabilities are largest for $\Delta k = 0$, and they exponentially decrease as $|\Delta k|$ increases. Two selection rules are given. As expected, the lifetime of the excited states increases as n' increases, but the lifetime decreases with respect to the increase of $|k'|$.

Transition probabilities are tabulated for the transitions $n' k' \rightarrow n k$, where $2 \leqslant n' \leqslant 11$, $0 \leqslant k' \leqslant k_s$ with k_s the smaller of 5 and $n' -1$, $1 \leqslant n \leqslant 5$, and all possible values of k. Also tabulated are the transitions $n' k' \rightarrow n$, where $2 \leqslant n' \leqslant 10$, all possible values of k', and $1 \leqslant n < n'$. Averages with respect to k', sums with respect to n, and lifetimes of the excited states are also given.

Branching ratios are tabulated for the transitions $n' k' \rightarrow n k$, where $3 \leqslant n' \leqslant 11$, $0 \leqslant k' \leqslant k_s$ with k_s the smaller of 5 and $n' -1$, $2 \leqslant n \leqslant 5$, and all possible values of k. Similarly, the branching

ratios for the transitions $n' k' \rightarrow n$ are tabulated, where $3 \leqslant n' \leqslant 10$, all possible values of k', and $2 \leqslant n \leqslant n'$. In comparing the calculated branching ratios with measurements for $n' = 3$ to $n = 2$ transitions, some unexplained disagreements have been found.

I. INTRODUCTION

Due to an additional degree of energy degeneracy in atomic hydrogen, the linear Stark effect, where energy level splittings are proportional to the applied electric field, exists only in this atom. As a result the splitting of energy levels in atomic hydrogen due to an electric field is considerably larger than other atoms which possess quadratic or higher order Stark effects. In this way, while it is more difficult experimentally to prepare atomic hydrogen in the excited states compared to other atoms, it is easier to observe the effect of a weak electric field.

In the case of nonhydrogen-like atoms, and molecules, in particular heteronuclear molecules, application of an electric field makes the medium birefringent. The light emitted is then elliptically polarized. This property has led to the invention of a number of laser-operated electro-optical devices such as optical limiters, amplitude modulators and tunable filters[1]. In the case of the hydrogen-like atoms on the other hand, due to the property of the linear Stark effect, the light emitted as will be shown is purely linearly polarized.

Transition probabilities and branching ratios between Stark levels are useful in problems dealing with excited atoms in an electric field. In many experimental setups, the excited atoms decay in a stray or external electric field, and a knowledge of the transition probabilities and branching ratios become desirable.

As another example, the electric field of an intense laser beam could be strong enough to cause splitting of atomic energy levels. When the rotational frequency of the transient electron is large compared to the field frequency, the electric field can be considered stationary, and the present results apply. In the case of a hydrogen-like atom, a transient electron acted on by an effective charge Z and having a principal quantum number n has a frequency of $6.58 \times 10^{15} (Z^2/n^3) \mathrm{sec}^{-1}$.

For low n or high Z the frequency may be much higher than the field frequency, and the situation mentioned above may be realized.

Since the present calculation is based on first order perturbation theory, results are valid if the applied electric field is much smaller than the atomic field. On the other hand, from the point of view of observation, the Stark level spacings must be much larger than the fine structure splittings. For the case of a hydrogen-like atom, we must then have

$$\frac{Z^4\alpha^2}{n^3\,(j+½)}\,\mathrm{Ryd} \ll eaF \ll \frac{Z^2}{n^2}\,\mathrm{Ryd} \tag{1}$$

where F is the applied field, e is the absolute value of an electron charge, a and j are the atomic radius and angular momentum, and α is the fine structure constant. Eq. (1) can be written alternatively as

$$1.37 \times 10^5 \,\frac{Z^6}{n^5\,(j+½)}\,\frac{V}{cm} \ll F \ll 2.57 \times 10^9 \frac{Z^4}{n^4}\,\frac{V}{cm}\,. \tag{2}$$

As an example, a laser beam of energy 1 m J, cross-sectional area 10^{-4} cm^2, and pulse duration of 10^{-8} sec has a peak field strength of 4.33×10^5 V/cm, which satisfies Eq. (2) for low n values.

The first calculation on the intensity of Stark level transitions in atomic hydrogen was done by Schroedinger[2] and Epstein[3], and the results have been put to extensive experimental tests[4].

Hiskes and Tarter[5] have tabulated values of transition probabilities for transitions of the type $n'n_1'n_2'm' \rightarrow nn_1\,n_2\,m$, where $n'n_1'n_2'm'$ and $nn_1\,n_2\,m$ are the initial and final states, and $nn_1\,n_2\,m$ are the usual parabolic quantum numbers[6]. In their tables $n' \leqslant 10$, and all possible values of the other quantum numbers have been considered. Similarly, they have tabulated the transition probabilities summed over all final states $nn_1 n_2 m$ for the initial states $n'n_1'n_2'm$ where $1 \leqslant n' \leqslant 25$, and all possible values of $n_1'n_2'm$ have been considered.

A difficulty with the Hiskes and Tarter Tables is that the Stark levels are highly degenerate with respect to energy, and the energy depends only on n and $k = n_1 - n_2$ (Reference 6). As a result many of the transition probabilities tabulated by them belong to a single line. In any application or in any comparison with measurements, the tabulated values should be averaged with respect to $n_1'n_2'm'$ and summed with respect to $n_1 n_2 m$, keeping $k' = n_1' - n_2'$ and $k = n_1 - n_2$ constant. This requires additional work. In this paper, however, the transition probabilities tabulated are given in terms of $n' \, k'$ and $n \, k$, and have the advantage that no additional summing or averaging is necessary. The electric quantum number k plays the role of the angular momentum quantum number ℓ in the presence of an electric field. In addition, in this paper, the branching ratios have also been tabulated.

In Section II, necessary formulas for the transition probabilities and branching ratios are given. Symmetries are discussed and selection rules are given.

Tables for the transition probabilities and branching ratios are given in Section III. As will be discussed later, some disagreements for some branching ratios are found between the present calculation and the measurement of Mark and Wierl[4].

Except for a short table given by Bethe and Salpeter[6] for the branching ratios between Stark levels, the present calculation is the first extensive tabulation of the branching ratios between Stark levels. The transition probability multiplied by the statistical weight of the initial state is called the static intensity J_S by Bethe and Salpeter, while the branching ratios are called the dynamic intensity J_D.

II. FORMULATION

A. Electric Dipole Matrices and Branching Ratios

Due to the spherical symmetry of the atomic Hamiltonian in the absence of an electric field, the x, y, and z components of the electric dipole matrices of the atomic electrons are equal. In a unidirectional electric field, the spherical symmetry is destroyed and is replaced by an axial symmetry. If the field lies along the z-axis, the x and y components of the dipole matrices are in general different from the z-component.

The z-component of the dipole matrices are given in the preceeding paper[7], from now on called I. To evaluate the x and y components, it is more convenient to introduce the components x + iy and x -iy. The matrices due to these components in terms of the hypergeometric functions have been evaluated by Gordon[6]. Below, we give an independent derivation.

In terms of the parabolic coordinates we can write

$$x \pm iy = \sqrt{\xi \eta}\ e^{\pm i \varphi} \tag{3}$$

where ξ, η, and φ are the parabolic coordinates of the running electron[6].

The hydrogenic wave functions in parabolic coordinates are specified by the quantum numbers nn_1n_2m satisfying[6]

$$n = n_1 + n_2 + m + 1 \tag{4}$$

where n is the principal quantum number and m is the absolute value of the magnetic quantum number. We follow Pauli[8] and introduce the electric quantum number k by

$$k = n_1 - n_2 \tag{5}$$

Then we deal with the three independent quantum numbers n k m in terms of which n_1 and n_2 are given by

$$n_1 = \tfrac{1}{2}(n + k - m - 1), \quad n_2 = \tfrac{1}{2}(n - k - m - 1) \tag{6}$$

The energy levels in a linear Stark effect depend only on n and k (Ref. 6).

Evaluation of the matrices in Equation (3) involves a three-dimensional integration with respect to φ, ξ, and η. When the integrations are carried out, we find that the matrix elements of $x \pm iy$ vanish unless $m' = m \pm 1$. These matrices are given by

$$\begin{aligned}
&\langle n\,k\,m \pm 1 \mid x + iy \mid n'\,k'\,m \rangle \\
&= \langle n\,k\,m \pm 1 \mid x - iy \mid n'\,k'\,m \rangle = \frac{1}{2\mu\sqrt{ZZ'}} \left(\frac{2\alpha}{\alpha+\alpha'}\right)^{m+2} \\
&\times \left(\frac{2\alpha'}{\alpha+\alpha'}\right)^{m\pm 1+2} \times \left[\frac{(n_1+m)!\,(n_2+m)!\,(n_1'+m\pm 1)!\,(n_2'+m\pm 1)!}{n_1!\,n_2!\,n_1'!\,n_2'!}\right]^{1/2} \\
&\times \sum_{\nu_1=0}^{n_1} \sum_{\nu_2=0}^{n_2} \sum_{\nu_1'=0}^{n_1'} \sum_{\nu_2'=0}^{n_2'} \left(\frac{-2\alpha}{\alpha+\alpha'}\right)^{\nu_1+\nu_2} \left(\frac{-2\alpha'}{\alpha+\alpha'}\right)^{\nu_1'+\nu_2'} \binom{n_1}{\nu_1}\binom{n_2}{\nu_2}\binom{n_1'}{\nu_1'}\binom{n_2'}{\nu_2'} \\
&\times \frac{\left[\frac{1}{2}(2m\pm 1+1)+\nu_1+\nu_1'\right]!\,\left[\frac{1}{2}(2m\pm 1+1)+\nu_2+\nu_2'\right]!}{(m+\nu_1)!\,(m\pm 1+\nu_1')!\,(m+\nu_2)!\,(m\pm 1+\nu_2')!} \\
&\times (2m+3\pm 1+\nu_1+\nu_2+\nu_1'+\nu_2').
\end{aligned} \tag{7}$$

all notations being defined in I.

Equation (7) is an alternative to an expression in terms of the confluent hypergeometric functions given by Gordon[6].

In order to find the transition probabilities and branching ratios, we have to find the degeneracy of each Stark level. It will be shown below that each level is $n - |k|$ fold degenerate. From (4) and (5) it follows that

$$k = -(n-1) + 2n_1 + m = n - 1 - (2n_2 + m) \tag{8}$$

Keeping in mind that n_1, n_2, and m are positive integers, we have $k_{min} = -(n-1)$ and $k_{max} = n-1$, implying that

$$-(n-1) \leqslant k \leqslant n-1 . \tag{9}$$

We first find the degeneracy for $m = 0$. From Eq. (8) we see that when $m = 0$, $n + |k|$ must be odd. Through Eq. (9) we find that there are n terms for k given by

$$k = -(n-1),\ -(n-3),\ \ldots\ldots,\ (n-3),\ (n-1);\ m = 0. \tag{10}$$

For each k and $m = 0$ there is only one value for n_1 and one value for n_2 specified by Eq. (8). Then the number of states for a given n and k, and $m = 0$ is $\epsilon(n + |k|)$ where we have defined

$$\epsilon(j) = \begin{cases} 1, j \text{ odd} \\ 0, j \text{ even} \end{cases} \tag{11}$$

When $m \neq 0$, the smallest value of m is 1 and through (8) we find the following relationships:

$$\left.\begin{aligned} n_1 &= 0, 1, 2, \ldots, \leqslant \tfrac{1}{2}(n-2+k) \\ n_2 &= -k, -k+1, \ldots, \leqslant \tfrac{1}{2}(n-2-k) \end{aligned}\right\} k < 0, \tag{12}$$

$$\left.\begin{aligned} n_1 &= k, k+1, \ldots, \leqslant \tfrac{1}{2}(n-2+k) \\ n_2 &= 0, 1, 2, \ldots, \leqslant \tfrac{1}{2}(n-2-k) \end{aligned}\right\} k \geqslant 0, \tag{13}$$

Since there is a double degeneracy when $m \neq 0$, we see from Eq. (12) that for $k < 0$, a level with a given n and k is $n + k - \epsilon(n + |k|)$ fold degenerate. Similarly, for $k \geq 0$, we see from Eq. (13) that each level for a given n and k is $n - k - \epsilon(n + |k|)$ fold degenerate. Then for all values of k a level with a given n and k is $n - |k| - \epsilon(n + |k|)$ fold degenerate.

Combining the $m = 0$ and $m \neq 0$ cases, we reach the conclusion that energy levels in a linear Stark effect specified by n and k are $n - |k|$ fold degenerate. As a check the total number of states for a given n is given by

$$\sum_{k=-(n-1)}^{n-1} (n - |k|) = n^2 , \tag{14}$$

as should.

We now find the transition probabilities and branching ratios. The transition probabilities parallel and perpendicular to the field are different. The transition probability is proportional to the cube of the energy difference between the levels. As a first approximation and as far as the energy difference is concerned, we neglect the energy splitting of the levels due to the electric field. Then in analogy to Eq. (2) of I we have

$$\begin{aligned} A_\pi (n' k', n k) &= \frac{1}{3} \alpha^3 (R_\infty / \hbar a_0^2) \mu Z^4 \left(\frac{1}{n^2} - \frac{1}{n'^2} \right)^3 \\ &\times \frac{1}{n' - |k'|} \sum_{n_1' n_1} [2 - \delta(m,0)] \, | < n n_1 m | z | n' n_1' m > |^2 \end{aligned} \tag{15}$$

where A_π is the probability of a transition parallel to the field, and we have averaged the dipole matrices with respect to n_1' and summed it with respect to n_1. For a given n, k, and n_1, the value of m is given through Eq. (8), and no summation with respect to m is necessary. However, the factor $2 - \delta(m, 0)$ accounts for the fact that the $m = 0$ state is singly degenerate.

Similarly, for transition probabilities perpendicular to the field we get

$$A_\sigma (n' k', n k) = \frac{1}{3}\alpha^3(R_\infty/\hbar a_0^2)\mu Z^4\left(\frac{1}{n^2}-\frac{1}{n'^2}\right)^3$$

$$\times \frac{1}{n' - |k'|} \sum_{n_1' n_1 m' m} \left\{ |\langle nn_1 m \,|\, x \,|\, n' n_1' m' \rangle|^2 + |\langle nn_1 m \,|\, y \,|\, n' n_1' m' \rangle|^2 \right\} \tag{16}$$

This can be written as

$$A_\sigma (n' k', n k) = \frac{1}{3}\alpha^3(R_\infty/\hbar a_0^2)\mu Z^4\left(\frac{1}{n^2}-\frac{1}{n'^2}\right)^3$$

$$\times \frac{1}{n' - |k'|} \sum_{n_1' n_1} \left[2 - \delta(m, 0)\right] \left\{ |\langle nn_1\, m - 1 \,|\, x + iy \,|\, n_1' n_2' m \rangle|^2 \right. \tag{17}$$

$$\left. + \; |\langle nn_1 m + 1 \,|\, x + iy \,|\, n_1' n_2' m \rangle|^2 \right\}$$

The equality of Eqs. (16) and (17) follows from the fact that the matrix elements of x and y are real. The total transition probability is given by

$$A = A_\pi + A_\sigma \tag{18}$$

Since A is proportional to the square of the electric dipole matrices, the value of A when summed and averaged with respect to the internal quantum numbers is the same in spherical and parabolic coordinates. This follows from the fact that the length of a vector is the same in the two coordinates. We therefore have

$$\sum_{\ell'=0}^{n'-1} \sum_{\ell=0}^{n-1} (2\ell' + 1) A(n'\ell', n\ell) = \sum_{k'=-n'+1}^{n'-1} \sum_{k=-n+1}^{n-1} (n - |k'|) A(n'k', nk) \tag{19}$$

Equation (19) shows that A (n′, n) does not change in a weak electric field, where A (n′, n) is the transition probability between the states n′ and n. This has the important consequence that if electrons are distributed according to their statistical weights among the sublevels of the initial state n', they have the same lifetime with and without a weak electric field.

Similarly, when sums and averages are made with respect to the internal quantum numbers, the transition probability should be independent of the direction of the electric field. This means that

$$\sum_{k'=-n'+1}^{n'-1} \sum_{k=-n+1}^{n-1} (n - |k'|) A_\sigma (n'k', nk) = 2 \sum_{k'=-n'+1}^{n'-1} \sum_{k=-n+1}^{n-1} (n - |k'|) A_\pi (n'k', nk) \qquad (20)$$

Eq. (19) and (20) are used as checks on the accuracy of the numerical values of the transition probabilities.

The branching ratios β_T (nk, n'k'), β_T (n, n'k'), β_T (nk, n'), and β_T (n, n') are obtained through relationships similar to Eqs. (3) through (6) of I, except that ℓ and ℓ' are replaced by k and k', and $2\ell' + 1$ is replaced by $n' - |k'|$. Tables for the branching ratios are given in the next section.

B. Parity, Symmetry, and Selection Rules

We first show that all spectral lines emitted in a linear Stark effect are linearly polarized. For a transition $n'k'm' \rightarrow nkm$ through Eq. (8) we obtain

$$n' - n = 2(n_1' - n_1) - \Delta k + \Delta m \qquad (21)$$

where $\Delta k = k' - k$, and $\Delta m = m' - m$. For $n' - n$ even Δk and Δm have the same parity, and for $n' - n$ odd the parity is opposite. This means that for radiation parallel to the field where $\Delta m = 0$, Δk has the same parity as $n' - n$, and for radiation perpendicular to the field where $\Delta m = \pm 1$, Δk has opposite parity to that of $n' - n$. For a given $n' - n$, each transition is either π or σ radiation, and all the emitted radiations are linearly polarized.

Although it is generally believed that there are no selection rules governing transitions between Stark levels, there exists some selection rules as will be given below.

In Eq. (9) of I the interchange of n_1 and n_2 is equivalent to the interchange of ν_1 and ν_2. Similarly, the interchange of n_1' and n_2' is equivalent to the interchange of ν_1' and ν_2'. The operation $\nu_1 \leftrightarrows \nu_2$ and $\nu_1' \leftrightarrows \nu_2'$ in turn changes the sign of the dipole matrix in Eq. (9) of I. Then

$$< n\ -k\ m\ |z|\ n'\ -k'\ m> = -< n\ k\ m\ |z|\ n'k'm> \tag{22}$$

By a similar argument we obtain from (7)

$$\begin{aligned} &< n\ -k\ m \pm 1\ |x \pm iy|\ n'\ -k'\ m > \\ &= < n\ k\ m \pm 1\ |x \pm iy|\ n'k'm > \end{aligned} \tag{23}$$

The two equations (22) and (23), making use of Eqs. (15) and (17), imply that

$$A_\pi (n' - k', n - k) = A_\pi (n'k', nk) \tag{24}$$

$$A_\sigma (n' - k', n - k) = A_\sigma (n'k', nk) \tag{25}$$

Therefore both A_π and A_σ are symmetric with respect to the change in sign of k and k′. Combining the last two equations we obtain

$$A (n' - k', n - k) = A (n'k', nk) \tag{26}$$

Herrick[8] has also derived Eq. (26) using the properties of the Clebsch–Gordan coefficients occurring in the transformation matrix between the spherical and parabolic coordinates.

From (22) we also obtain

$$<n0m|z|n'0m> = 0, \tag{27}$$

from which follows that

$$A_\pi (n'k' = 0, nk = 0) = 0 \tag{28}$$

It may seem at first that k′ = 0 to k = 0 transitions correspond to transitions between two spherically symmetric charge distributions. This, however, is not the case. By expanding the parabolic wave functions in terms of the spherical wave functions, it can be shown that the k′ = 0 to k = 0 transitions correspond to an aggregate of optically forbidden transitions where the conditions $\ell' - \ell = \pm 1$ and $m' - m = 0$ have not been met.

From Eq. (21) we see that when $n' - n$ is even, σ radiation does not exist for $\Delta k = 0$. Due to (28) we therefore have

$$A(n'k' = 0, nk = 0) = 0, \quad n' - n \text{ even.} \tag{29}$$

Eqs. (28) and (29) constitute the two selection rules applicable to transitions among linear Stark levels. Hiskes and Tarter[5] also state that $k' = 0$ to $k = 0$ transition is not allowed, but they don't give any proof for it.

We finally like to enumerate the number of observed lines for transitions between two arbitrary levels n' and n.

For a given n there are $2n - 1$ values for k. Similarly, there are $2n' - 1$ values for k' for a given value of n'. Keeping in mind Eq. (29), the total number of lines observed with both polarities for a given n' and n, which we designate by $N_T(n, n')$, is given by

$$N_T(n', n) = (2n - 1)(2n' - 1) - \epsilon(n' - n - 1), \tag{30}$$

with $\epsilon(j)$ defined by (11).

To find the number of lines due to π-radiation we notice from (10) that for $m = 0$ and a given n there are n values for k. Similarly, there are $n - 1$ values for k for $m = 1$ and a given n; $n - 2$ values for k for $m = 2$, etc. A similar argument applies for the number of terms for k' for given values of m' and n'. In the absence of any degeneracy, the number of lines with π radiations is a sum over the products of the number of states in each group which have the same magnetic quantum numbers. This is given by

$$nn' + (n - 1)(n' - 1) + \dots + 1(n' - n + 1)$$

$$= \sum_{\nu=0}^{n-1} (n - \nu)(n' - \nu) = \frac{1}{6} n(n + 1)(3n' - n + 1) \tag{31}$$

There are, however, degeneracies. It is not difficult to show that m = 0 and m = 2 have n - 2 common k values for a given n. There are n – 3 common values for k for m = 1 and m = 3, n – 4 common values for k for m = 2 and m = 4, etc. Similar argument applies to n′, m′ and k′. From the right hand side of (29) we must then subtract

$$(n - 2)(n' - 2) + (n - 3)(n' - 3) + \dots + 1(n' - n + 1)$$

$$= \sum_{\nu=0}^{n-3} (n - 2 - \nu)(n' - 2 - \nu) = \frac{1}{6} (n - 1)(n - 2)(3n' - n - 3) \tag{32}$$

We also have to take into account the fact that π-radiation does not exist for $\Delta k = 0$. For $n' - n$ odd the $\Delta k = 0$ transition is not automatically allowed for π-radiation, as can be seen from (21), and this has been taken into account in the derivation of (31). For $n' - n$ even the vanishing of $\Delta k = 0$ transition has not been taken into account in (31), but is a consequence of (27). We must therefore subtract 1 from the right-hand side of (31) when $n' - n$ is even. The number of π-radiation lines, $N_\pi(n', n)$, is therefore given by

$$N_\pi(n', n) = \frac{1}{6} n(n + 1)(3n' - n + 1) - \frac{1}{6}(n - 1)(n - 2)(3n' - n - 3)$$

$$- \epsilon(n' - n - 1) \tag{33}$$

$$= nn' + (n - 1)(n' - 1) - \epsilon(n' - n - 1)$$

The number of σ-radiation lines, using (28) and (31), is then given by

$$N_\sigma(n', n) = N_T(n', n) - N_\pi(n', n) = nn' + (n - 1)(n' - 1) - 1 \tag{34}$$

We see that for $n' - n$ even N_π and N_σ are equal, while for $n' - n$ odd, π-radiation has one more line than the σ-radiation. It is also seen that N_π is always even, while N_σ has the parity of $n' - n$.

Due to the symmetry relations (24) and (25) not all values of the transition probabilities are distinct, and there are about half as many distinct transition probability values as there are lines. Let the number of distinct transition probability values for the two kinds of radiations, and their sum, be designated by M_π, M_σ, and M_T. Then detailed calculation shows that

$$M_\pi = M_\sigma = \frac{1}{2} M_T = \frac{1}{2} [nn' + (n-1)(n'-1) - \epsilon(n'-n-1)]. \tag{35}$$

It is also of interest to find the total number of observed lines due to an electron in an initial state n'. Using (30) this number is given by

$$N_T(n') = \sum_{n=0}^{n'-1} N_T(n', n) = \frac{1}{2}[(n'-1)(4n'^2 - 6n' + 1) + \epsilon(n'-1)] \tag{36}$$

This number is considerably less than the number of transitions $\frac{1}{8}(n'-1)(n')(n'+1)(3n'-2)$ given by Hiskes and Tarter[5]. These authors presumably neglect the degeneracy of the lines.

Symmetries similar to (24) through (26) hold for the branching ratios. By replacing ℓ, ℓ', ℓ'', and $2\ell' + 1$ in (3) through (5) of I by k, k′, k″, and $n' - |k'|$, and making use of (26) we find that

$$\begin{cases} \beta_T(n-k, n'-k') = \beta_T(nk, n'k') \\ \beta_T(n-k, n') = \beta_T(nk, n') \\ \beta_T(n, n'-k') = \beta_T(n, n'k') \end{cases} \tag{37}$$

Use have been made of (37) in tabulating the branching ratios.

III. RESULTS AND DISCUSSIONS

Results are given in Tables I through VIII and Figures 1 through 5.

Table I is given as a test and is a comparison of the present calculational results and the calculational results of Schroedinger reconstructed from Table 20a of Bethe and Salpeter[6]. The numbers given in Table I are obtained from Table 20a by multiplying the A values by 2 when $m \neq 0$, summing the A values which have common k and k′, and dividing the results by the statistical weight of the initial state which is $n' - |k'|$. In addition, since A_σ defined here is twice the A_σ defined in Ref. 6, the resultant A_σ is multiplied by a factor of 2. As is seen the present results agree within a few percent with the results of Schroedinger.

In Table I, A values corresponding to $k' = -2$ and -1 are not listed, since these cases can be obtained from the symmetry relations (24) and (25).

Two additional tests for the accuracy of the present calculational results were made. These tests were the satisfaction of the unitarity relation (19) and the isotropy relation (20) for numerous number of cases.

Table II is similar to Table I and gives transition probabilities for transitions $n' = 4$ to $n = 1, 2,$ and 3. Due to their symmetry, A values for $k' = -3, -2,$ and -1 are not given.

Branching ratios for the transitions of $n' = 3$ to $n = 2$ states are given in Table III. For each transition three values are given. "Here" refers to the present calculational results. Ref. 6 refers to the calculated results of Table 20b of Ref. 6, and "Exp" refers to the experimental results reported in the Table 20b. As is seen where the branching ratios are not zero, there are serious disagreements among the three sets of values. Since there are agreements between the A values calculated here and the reference 6, the discrepancies between the branching ratios calculated here and in reference 6 is purely a matter of arithmetical error whose source the author has not been able to find. All possible tests have been made on the present calculation to insure its accuracy. It is more difficult to find the source of the discrepancies between the calculated values and the measurements. The measurements date back to 1929. New measurements of the branching ratios would be helpful in clarifying the disagreements.

It should be noted that the branching ratios B_T is called the dynamical intensity J_D in reference 6, while the transition probability A multiplied by the statistical weight of the initial state, $n' - |k'|$, is called the static intensity J_S in this reference.

In Figure 1, the transition probabilities for transitions between sublevels of $n' = 10$ and $n = 9$ are plotted versus $\Delta k = k' - k$. Three interesting features of this figures are the following: (1) For a given n and n′ the transition probability is a function of Δk only, and is almost independent of the values of the initial state electric quantum numbers k′, (2) the probability, with a few exceptions, decreases exponentially as $|\Delta k|$ increases. As Δk changes by 8 units, the A values change by some 16 orders of magnitudes. It is largest for $\Delta k = 0$, then for $\Delta k = \pm 1$, etc. This conclusion is in agreement with the statement by Bethe and Salpeter[6] that the outermost components of the Stark spectra are the weakest, (3) For a fixed value of $|\Delta k|$, the $k' - k$ transition probability is larger than the $k - k'$, i.e., the transition probability is larger, when the change in k′ is in the same direction as the change in n′.

In Figure 2 the transition probabilities for the initial states $n' = 10$, $k' = 0 - 9$ are plotted versus the final states $n = 1 - 9$. It is seen that while for $n = 9$ the transition probabilities increase as k′ decreases, the situation reverses itself for lower values of n. Figure 3 is similar to Figure 2, except that spherical coordinates have been used and is given for comparison.

The lifetimes of the excited states in an electric field for $2 \leqslant n' \leqslant 10$, and $-(n-1) \leqslant k' \leqslant n-1$ are plotted versus $|k'|$ in Figure 4. Due to the symmetry relation (26) the lifetime is independent of the sign of k'. It should be noted that the lifetime has two distinct behavior depending on whether k' is even or odd. In each case, the lifetime increases as n' increases, as expected. However, it decreases with the increase of $|k'|$.

Hiskes and Tarter[5] have similarly calculated the lifetime of the excited states in an electric field. However, their figures are obscured by the fact that they plot the lifetime versus the three quantum numbers n, m, and n_1, and they have not taken the degeneracy of the states into account. They show that the lifetime is independent of the sign of k' for $m = 0$, while this independence holds for all values of m.

We now describe Tables IV and V dealing with the transition probabilities. In Table IV transition probabilities A for the initial states $n' k'$, $2 \leqslant n' \leqslant 10$, $0 \leqslant k' \leqslant n' - 1$, and the final states n, where $1 \leqslant n < n'$, and various sums and averages are given. It is worth noting that, as expected, lifetimes of the excited states n' are the same as the lifetimes of these states without an electric field (I and Ref. 6).

Tables V give transition probabilities A for the transitions $n' k' \rightarrow n k$, where the ranges of $n' k'$ and $n k$ are specified in the tables. Due to the limitation of space, values of k' larger than 5 and less than $n' -1$ are not given. However, to allow interpolation, values of $k' - n' -1$ are given. Similarly, transition probabilities for negative values of k and k' are not given. These can be calculated from the positive values of k and k' using the symmetry relation (26). It should be noted that each transition probability in this table corresponds to a linearly polarized π or σ radiation. This can be seen from (21), which states that for $n' - n + k' - k$ even (odd), the radiation is parallel (perpendicular) to the field. The properties of the transition probabilities given in these tables are stated in the discussion of Figure 1.

In Figure 5, the branching ratios for the initial states $n' = 10$ and $k' = 0 - 9$ are plotted versus the final states $n = 1 - 9$. It is seen that the branching ratios for the ground state as the final state is equal to unity which is due to the lack of the metastable states. From there on it decreases monotonically as n increases.

In Table VI branching ratios β_T for the initial states $n'k'$, $3 \leq n' \leq 10$, $0 \leq k' \leq n' - 1$, and the final states n, where $1 < n < n'$, are given. Averages with respect to k' are also given. Since the branching ratio between any initial state and the ground state is unity, they are not listed in the table. Similarly, due to the symmetry relations (37), the branching ratios for negative values of k' are not given.

In Tables VII branching ratios β_T for the transitions $n'k' \rightarrow nk$, where the ranges of $n'k'$ and nk are specified in the tables, are given. Due to the symmetry relationships (37) the branching ratios for the negative values of k' are not given.

A study of this table shows that for given n', n, and k, the branching ratios have different behavior depending whether k' is even or odd. For k' either even or odd, the branching ratios are smooth functions of k', monotonically increasing with k', monotonically decreasing, or having a maximum with respect to k'.

The transition probabilities between the principal quantum numbers n and n' do not change in an electric field. However, the branching ratios do change. Tables VIII gives ratios of branching ratios with and without an electric field. As is seen, this ratio for transitions to the ground state is larger than 1, and for transitions to other states is less than 1. This means that on the average the electrons decay faster to the ground state with an electric field. The ratios go through a minimum as n increases.

In tables that follow transition probabilities and branching ratios are tabulated for atomic hydrogen. They can be applied to the hydrogen-like atoms, such as the alkali atoms, muonium, positronium, etc., by realizing that the electric dipole matrices as given by Eq. (7) scale as $(\mu Z)^{-1}$, where μ is the reduced mass and Z is the effective charge of the hydrogen-like atom. Since the energy levels scale as μZ^2, through Eq. (1) of I we see that the transition probability scale as μZ^4. To find the transition probability of a hydrogen-like atom, we then have to multiply the tabulated values for the transition probability by μZ^4.

On the other hand, since the branching ratios are ratios of transition probabilities, they are independent of μ and Z, and the tabulated values are valid for all hydrogen-like atoms.

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TABLE I. Transition probabilities A_π and A_σ for $n' = 3$ to $n = 1$ and 2 transitions. The A values in parenthesis are reconstructed from Table 20a of Bethe and Salpeter[6] as described in the text. Probabilities for the negative values of k′ are obtained through the symmetry relations (24) and (25).

		A_π (sec^{-1})			A_σ (sec^{-1})		
n	k \ k′	0	1	2	0	1	2
1	0	0.00	0.00	8.37+7 (8.2+7)	0.00	8.37+7 (8.2+7)	0.00
2	-1	3.41+6 (3.37+6)	0.00	1.40+4 (1.39+4)	0.00	2.25+5 (2.22+5)	0.00
	0	0.00	1.62+7 (1.60+7)	0.00	5.14+7 (5.08+7)	0.00	5.05+5 (5.00+5)
	+1	3.41+6 (3.37+6)	0.00	2.36+7 (2.33+7)	0.00	2.72+7 (2.69+7)	0.00

TABLE II. Transition probabilities A_π and A_σ for $n' = 4$ to $n = 1, 2,$ and 3 transitions.

		A_π (sec^{-1})				A_σ (sec^{-1})			
n	k \ k'	0	1	2	3	0	1	2	3
1	0	0.0	1.14+6	0.0	3.07+7	1.36+7	0.0	2.05+7	0.0
2	-1	0.0	7.25+5	0.0	2.69+4	9.67+5	0.0	1.61+5	0.0
	0	0.0	0.0	5.16+6	0.0	0.0	8.17+6	0.0	4.30+5
	+1	0.0	8.06+4	0.0	9.70+6	9.67+5	0.0	7.90+6	0.0
3	-2	0.0	4.91+2	0.0	1.44-1	1.15+4	0.0	7.80	0.0
	-1	8.99+5	0.0	2.00+3	0.0	0.0	4.54+4	0.0	1.85
	0	0.0	2.54+6	0.0	2.63+3	9.90+6	0.0	8.06+4	0.0
	+1	8.99+5	0.0	3.69+6	0.0	0.0	6.89+6	0.0	9.32+4
	+2	0.0	3.45+5	0.0	4.35+6	1.15+4	0.0	3.53+6	0.0

TABLE III. Branching ratios $\beta_T(\pi)$ and $\beta_T(\sigma)$ for transitions $n' = 3$ to $n' = 2$. The branching ratios for the negative values of k' are obtained through the symmetry relations (37).

		$\beta_T(\pi)$			$\beta_T(\sigma)$		
	$k \backslash k'$	0	1	2	0	1	2
Here	-1	50		0		0	
Ref. 6		89		0		0	
Exp*		79		0		0	
Here	0		100		100		0
Ref. 6			100		100		0
Exp			100		100		0
Here	+1			22		24	
Ref. 6				86		17	
Exp				92		38	

*Exp: Mark and Wierl, Z. Physics 53, 526; 55, 126; 57, 494 (1929)

TABLE IV. Transition probabilities A in sec^{-1} for the initial states n′k′, where $2 \leqslant n' \leqslant 10$, $0 \leqslant k' \leqslant n' - 1$, and the final states n, where $1 \leqslant n < n'$. Averages with respect to k′, and sums with respect to n are also given. Lifetimes of each excited state n′k′ and n′ in sec are also given. The probability for $n' - k' \rightarrow n$ transition is the same as that for $n'k' \rightarrow n$ transition.

n′k′ \ n	1	2	3	4	5	6	Total	Lifetime
20	6.27+8						6.27+8	1.60−9
21	3.13+8						3.13+8	3.19−9
Mean	4.70+8						4.70+8	2.13−9
30	0.00	5.82+7					5.82+7	1.72−8
31	8.37+7	4.36+7					1.27+8	7.86−9
32	8.37+7	2.41+7					1.08+8	9.28−9
Mean	5.57+7	4.41+7					9.98+7	1.00−8
40	1.36+7	1.93+6	1.17+7				2.73+7	3.66−8
41	1.14+6	8.97+6	9.81+6				1.99+7	5.02−8
42	2.05+7	1.32+7	7.30+6				4.10+7	2.44−8
43	3.07+7	1.02+7	4.45+6				4.53+7	3.21−8
Mean	1.28+7	8.42+6	8.99+6				3.02+7	3.31−8
50	0.00	2.21+6	8.12+5	3.49+6			6.51+6	1.54−7
51	5.16+6	1.08+6	1.98+6	3.07+6			1.13+7	8.86−8
52	1.15+6	2.27+6	2.97+6	2.54+6			8.93+6	1.12−7
53	6.88+6	5.03+6	3.18+6	1.92+6			1.70+7	5.88−8
54	1.38+7	4.93+6	2.32+6	1.26+6			2.23+7	4.49−8
Mean	4.13+6	2.53+6	2.20+6	2.70+6			1.16+7	8.65−8
60	1.69+6	2.45+5	8.12+5	3.51+5	1.32+6		4.20+6	2.38−7
61	5.64+4	8.91+5	1.98+6	6.39+5	1.19+6		3.22+6	3.10−7
62	2.26+6	6.94+5	2.97+6	9.14+5	1.04+6		5.49+6	1.82−7
63	8.46+5	7.97+5	3.18+6	1.07+6	8.62+5		4.67+6	2.14−7
64	2.82+6	2.25+6	2.32+6	1.02+6	6.65+5		8.28+6	1.21−7
65	7.05+6	2.68+6	1.31+6	7.44+5	4.59+5		1.22+7	8.17−8
Mean	1.65+6	9.74+5	7.79+5	7.72+5	1.03+6		5.21+6	1.92−7
70	0.00	3.50+5	1.32+5	2.17+5	1.67+5	5.83+5	1.45+6	6.90−7
71	8.83+5	1.53+5	2.57+5	1.91+5	2.68+5	5.39+5	2.28+6	4.38−7
72	8.83+4	3.91+5	2.69+5	2.22+5	3.50+5	4.84+5	1.81+6	5.54−7
73	1.10+6	4.40+5	2.61+5	3.39+5	4.19+5	4.21+5	2.98+6	3.35−7
74	5.89+5	3.56+5	4.73+5	4.93+5	4.39+5	3.51+5	2.70+6	3.70−7
75	1.33+6	1.12+6	8.14+5	5.69+5	3.98+5	2.76+5	4.51+6	2.22−7
76	3.98+6	1.58+6	7.97+5	4.62+5	2.95+5	1.99+5	7.30+6	1.37−7
Mean	7.57+5	4.39+5	3.36+5	3.04+5	3.25+5	4.56+5	2.62+6	3.82−7

TABLE IV. (continued)

n'k' \ n	1	2	3	4	5	6	7	8	Total	Lifetime
80	3.93+5	5.73+4	1.12+5	6.66+4	9.52+4	8.67+4	2.90+5		1.10+6	9.08-7
81	7.02+3	1.90+5	8.18+4	9.96+4	9.11+4	1.21+5	2.71+5		8.61+5	1.16-6
82	4.92+5	1.12+5	1.21+5	1.13+5	1.01+5	1.57+5	2.48+5		1.34+6	7.44-7
83	8.85+4	1.89+5	1.62+5	1.15+5	1.35+5	1.88+5	2.22+5		1.10+6	9.09-7
84	5.90+5	2.81+5	1.49+5	1.54+5	1.88+5	2.05+5	1.93+5		1.76+6	5.68-7
85	4.10+5	1.91+5	2.31+5	2.50+5	2.35+5	2.03+5	1.62+5		1.68+6	5.95-7
86	6.88+5	6.14+5	4.65+5	3.38+5	2.45+5	1.79+5	1.30+5		2.66+6	3.76-7
87	2.41+6	9.88+5	8.13+5	3.02+5	1.95+5	1.35+5	9.69+4		4.64+6	2.16-7
Mean	3.87+5	2.22+5	1.65+5	1.43+5	1.39+5	1.56+5	2.27+5		1.44+6	6.95-7
90	0.0	9.38+4	3.54+5	4.65+4	3.53+4	4.74+4	4.83+4	1.57+5	4.63+5	2.16-6
91	2.41+5	3.90+4	6.32+4	4.16+4	4.63+4	4.73+4	6.32+4	1.48+5	6.90+5	1.45-6
92	1.38+4	1.06+5	5.68+4	5.11+4	5.31+4	5.17+4	7.91+4	1.37+5	5.49+5	1.82-6
93	2.89+5	8.37+4	6.44+4	6.64+4	5.64+4	6.38+4	9.33+4	1.26+5	8.43+5	1.19-6
94	7.72+4	1.00+5	9.84+4	7.08+4	6.67+4	8.34+4	1.03+5	1.12+5	7.12+5	1.40-6
95	3.38+5	1.84+5	9.60+4	8.23+4	9.43+4	1.05+5	1.07+5	9.82+4	1.10+6	9.06-7
96	2.89+5	1.16+5	1.25+5	1.37+5	1.33+5	1.20+5	1.03+5	8.31+4	1.10+6	9.05-7
97	3.86+5	3.58+5	2.82+5	2.11+5	1.57+5	1.18+5	8.96+4	6.76+4	1.67+6	5.99-7
98	1.54+6	6.51+5	3.45+5	2.07+5	1.35+5	9.36+4	6.87+4	5.18+4	3.10+6	3.23-7
Mean	2.14+5	1.22+5	8.91+4	7.46+4	6.91+4	7.07+4	8.24+4	1.23+5	8.45+5	1.18-6

TABLE IV. (continued)

n'k' \ n	1	2	3	4	5	6	7	8	9	Total	Lifetime
100	1.28+5	1.86+4	3.33+4	1.98+4	2.27+4	1.98+4	2.58+4	2.85+4	9.07+4	3.87+5	2.58-6
101	1.42+3	5.89+4	2.41+4	2.74+4	2.23+4	2.42+4	2.64+4	3.57+4	8.62+4	3.07+5	3.26-6
102	1.53+5	3.01+4	3.66+4	2.83+4	2.55+4	2.76+4	2.86+4	4.33+4	8.10+4	4.54+5	2.20-6
103	1.64+4	6.12+4	4.02+4	3.01+4	3.14+4	2.99+4	3.37+4	5.05+4	7.50+4	3.68+5	2.72-6
104	1.79+5	6.23+4	3.87+4	4.01+4	3.54+4	3.38+4	4.18+4	5.62+4	6.84+4	5.55+5	1.80-6
105	6.38+4	5.75+4	6.14+4	4.67+4	3.91+4	4.29+4	5.17+4	5.95+4	6.13+4	4.84+5	2.07-6
106	2.04+5	1.23+5	6.61+4	4.99+4	5.24+4	5.80+4	6.07+4	5.96+4	5.38+4	7.28+5	1.37-6
107	2.08+5	7.70+4	7.32+4	7.99+4	7.97+4	7.39+4	6.53+4	5.60+4	4.59+4	7.59+5	1.32-6
108	2.30+5	2.20+5	1.79+5	1.38+5	1.05+5	8.03+4	6.21+4	4.87+4	3.78+4	1.10+6	8.09-7
109	1.03+6	4.46+5	2.41+5	1.47+5	9.63+4	6.72+4	4.94+4	3.79+4	2.96+4	2.15+6	4.65-7
Mean	1.26+5	7.13+4	5.16+4	4.24+4	3.80+4	3.69+4	3.91+4	4.68+4	7.16+4	5.24+5	1.91-6

TABLE V. Transition probabilities A in sec^{-1} for $n'k' \rightarrow nk$ transitions, where $2 \leq n' \leq 11$, $-(n'-1) \leq k' \leq n'-1$, $1 \leq n \leq 5$, and $0 \leq k \leq n-1$. Due to the space limitation values of k' that fall between $-(n'-1)$ and -5, and 5 and $n-1$ are not listed. The probabilities for the negative values of k are obtained using symmetry relation (26).

n' \ k'	-n'+1	-5	-4	-3	-2	-1	0	1	2	3	4	5	n'-1	Mean
							A. n = 1, k = 0							
2							6.27+8	3.13+8						4.70+8
3							0.00	8.37+7	8.37+7					5.58+7
4							1.36+7	1.14+6	2.05+7	3.07+7				1.28+7
5							0.00	5.16+6	1.15+6	6.88+6	1.38+7			4.13+6
6							1.69+6	5.64+4	2.26+6	8.46+5	2.82+6	7.05+6		1.65+6
7							0.00	8.83+5	8.83+4	1.10+6	5.89+5	1.33+6	3.98+6	7.57+5
8							3.93+5	7.02+3	4.92+5	8.85+4	5.90+5	4.10+5	2.41+6	3.87+5
9							0.00	2.41+5	1.38+4	2.89+5	7.72+4	3.38+5	1.54+6	2.14+5
10							1.28+5	1.42+3	1.53+5	1.64+4	1.79+5	6.38+4	1.03+6	1.26+5
11							0.00	8.62+4	3.19+3	1.01+5	1.64+4	1.15+5	7.19+5	7.84+4
							B. n = 2, k = 0							
3							5.14+7	1.62+7	5.05+5					2.44+7
4							0.00	8.17+6	5.16+6	4.30+5				4.41+6
5							1.99+6	1.68+5	2.06+6	2.02+6	2.84+5			1.29+6
6							0.00	8.09+5	1.84+5	6.74+5	9.19+5	1.84+5		4.90+5
7							3.17+5	1.24+4	3.57+5	1.40+5	2.63+5	4.66+5	1.21+5	2.19+5
8							0.00	1.72+5	2.04+4	1.71+5	9.78+4	1.17+5	8.25+4	1.10+5
9							8.50+4	1.92+3	9.64+4	2.08+4	8.81+4	6.73+4	5.77+4	6.02+4
10							0.00	5.35+4	3.88+3	5.58+4	1.81+4	4.82+4	4.14+4	3.52+4
11							3.03+4	4.45+2	3.41+4	4.68+3	3.35+4	1.49+4	3.04+4	2.17+4
							C. n = 2, k = 1							
3					1.40+4	2.25+5	3.41+6	2.72+7	2.36+7					9.85+6
4				2.69+4	1.61+5	7.25+5	9.67+5	8.06+4	7.90+6	9.70+6				2.01+6
5			2.56+4	9.09+4	1.95+5	1.02+5	1.10+5	8.09+5	1.78+4	2.92+6	4.62+6			6.20+5
6		2.04+4	5.11+4	6.13+4	1.02+4	6.54+4	1.23+5	1.63+4	5.00+5	6.13+4	1.28+6	2.47+6		2.42+5
7	1.55+4	2.97+4	2.15+4	3.57+2	3.35+4	3.05+4	1.67+4	1.10+5	5.16+2	3.00+5	7.14+4	6.29+5	1.44+6	1.10+5
8	1.16+4	8.07+3	1.91+2	1.70+4	7.80+3	1.28+4	2.87+4	4.60+3	8.42+4	9.20+2	1.84+5	6.54+4	8.94+5	5.57+4
9	8.73+3	7.02+2	8.77+3	1.88+3	8.45+3	1.03+4	4.38+3	2.68+4	8.13+2	6.11+4	3.18+3	1.16+5	5.84+5	3.07+4
10	6.63+3	4.66+3	3.62+2	5.34+3	3.81+3	3.73+3	9.32+3	1.66+3	2.24+4	1.48+1	4.39+4	4.66+3	3.98+5	1.80+4
11	5.09+3	3.01+1	3.35+3	1.40+3	2.81+3	4.15+3	1.54+3	8.89+3	5.03+2	1.80+4	1.31+2	3.16+4	2.80+5	1.12+4

TABLE V. (continued)

n' \ k'	$-n'+1$	-5	-4	-3	-2	-1	0	1	2	3	4	5	$n'-1$	Mean
							D. n = 3, k = 0							
4							9.90+6	2.54+6	8.06+4	2.63+3				3.45+6
5							0.00	1.67+6	8.54+5	7.99+4	7.27+3			7.54+5
6							5.19+5	4.52+4	4.11+5	3.33+5	5.72+4	8.92+3		2.53+5
7							0.00	2.19+5	5.10+4	1.26+5	1.46+5	3.84+4	8.58+3	1.06+5
8							9.75+4	4.33+3	9.51+4	3.87+4	4.53+4	6.99+4	7.51+3	5.11+4
9							0.00	5.42+4	7.25+3	4.36+4	2.64+4	1.88+4	6.30+3	2.72+4
10							2.90+4	7.84+2	3.00+4	7.37+3	2.11+4	1.75+4	5.20+3	1.56+4
11							0.00	1.85+4	1.61+3	1.68+4	6.33+3	1.08+4	4.27+3	9.50+3
							E. n = 3, k = 1							
4				1.85+1	2.00+3	4.54+4	8.99+5	6.89+6	3.69+6	9.32+4				1.99+6
5			1.25+2	4.49+3	3.68+4	2.15+5	3.75+5	1.35+4	2.05+6	1.62+6	1.05+5			4.96+5
6		2.33+2	4.66+3	2.18+4	5.96+4	4.52+4	3.76+4	3.30+5	1.49+4	7.39+5	7.87+5	8.41+4		1.77+5
7	2.89+2	3.90+3	1.24+4	1.83+4	5.29+3	2.54+4	6.10+4	4.06+3	2.06+5	3.43+4	3.08+5	4.19+5	6.28+4	7.63+4
8	2.99+2	7.08+3	5.97+3	4.78+2	1.35+4	1.62+4	6.56+3	5.65+4	3.98+1	1.21+5	3.65+4	1.43+5	4.63+4	3.76+4
9	2.84+2	1.99+3	2.27+2	6.81+3	4.30+3	5.67+3	1.64+4	1.42+3	4.32+4	1.70+3	7.17+4	3.18+4	3.43+4	2.03+4
10	2.57+2	3.73+2	3.41+3	1.08+3	3.92+3	6.15+3	1.89+3	1.56+4	8.14+1	3.09+4	3.29+3	4.34+4	2.58+4	1.17+4
11	2.27+2	1.72+3	2.42+2	2.49+3	2.30+3	1.79+3	5.86+3	5.85+2	1.31+4	7.86+1	2.17+4	4.01+3	1.97+4	7.19+3
							F. n = 3, k = 2							
4				1.44-1	7.80	4.91+2	1.15+4	3.45+5	3.53+6	4.35+6				7.81+5
5			2.49	4.49+1	8.51+2	6.60+3	3.12+4	7.12+4	2.46+4	1.47+6	2.21+6			2.28+5
6		7.28	7.28+1	7.01+2	2.85+3	5.42+3	1.75+3	1.67+4	9.79+4	1.19+2	6.85+5	1.22+6		8.65+4
7	1.18+1	7.95+1	4.75+2	1.20+3	9.72+2	5.17	4.83+3	8.70+3	5.31+3	8.17+4	6.07+3	3.53+5	7.25+5	3.88+4
8	1.46+1	3.02+2	5.09+2	1.51+2	1.64+2	1.70+3	7.51+2	3.09+3	1.20+4	1.08+3	6.04+4	1.06+4	4.59+5	1.95+4
9	1.58+1	2.21+2	1.26+1	1.89+2	5.76+2	3.94+1	1.28+3	1.93+3	1.52+3	1.21+4	5.04+1	4.33+4	3.05+5	1.07+4
10	1.57+1	3.11-1	1.46+2	1.89+2	4.23	6.04+2	2.96+2	9.07+2	2.65+3	6.19+2	1.07+4	8.18+1	2.10+5	6.24+3
11	1.50+1	9.82+1	5.85+1	2.57+1	2.74+2	4.00+1	4.51+2	5.99+2	5.49+2	2.89+3	1.94+2	9.00+3	1.50+5	3.85+3

TABLE V. (continued)

n' \ k'	-n'+1	-5	-4	-3	-2	-1	0	1	2	3	4	5	n'-1	Mean
G. n = 4, k = 0														
5							2.88+6	6.56+5	2.07+4	6.75+2	5.13			7.91+5
6							0.00	5.01+5	2.25+5	2.19+4	2.00+3	4.50+1		1.93+5
7							1.83+5	1.63+4	1.18+5	8.57+4	1.60+4	2.55+3	1.02+2	7.05+4
8							0.00	7.88+4	1.87+4	3.36+4	3.56+4	1.97+4	1.47+2	3.16+4
9							3.88+4	1.88+3	3.35+4	1.40+4	1.16+4	1.59+4	1.73+2	1.61+4
10							0.00	2.19+4	3.18+3	1.46+4	9.25+3	4.25+3	1.81+2	8.98+3
11							1.25+4	3.84+2	1.19+4	3.21+3	6.61+3	5.87+3	1.78+2	5.35+3
H. n = 4, k = 1														
5			4.84-2	5.06	4.96+2	1.31+4	2.99+5	2.24+6	9.32+5	2.50+4	6.07+2			5.34+5
6		1.15	3.87+1	1.21+3	1.13+4	7.64+4	1.56+5	3.06+3	6.59+5	4.13+5	3.04+4	2.08+3		1.48+5
7	4.24	7.81+1	1.30+3	6.81+3	2.14+4	2.01+4	1.47+4	1.41+5	8.29+3	2.27+5	1.97+5	2.54+4	3.00+3	5.72+4
8	8.22	1.09+3	3.81+3	6.35+3	2.48+3	1.08+4	2.98+4	1.20+3	8.74+4	1.64+4	8.86+4	1.01+5	3.27+3	2.65+4
9	1.18+1	2.11+3	1.93+3	2.88+2	5.84+3	8.18+3	2.83+3	2.81+4	1.76+2	4.99+4	1.66+4	3.83+4	3.16+3	1.37+4
10	1.43+1	5.69+2	1.53+2	2.91+3	2.19+3	2.66+3	8.88+3	4.89+2	2.14+4	1.35+3	2.83+4	1.38+4	2.88+3	7.76+3
11	1.58+1	1.98+2	1.40+3	5.50+2	1.89+3	3.39+3	8.68+2	8.57+3	1.41	1.50+4	2.20+3	1.62+4	2.55+3	4.66+3
I. n = 4, k = 2														
5			1.52-4	3.90-2	2.80	2.01+2	5.36+3	1.67+5	1.53+6	1.12+6	2.37+4			3.02+5
6		9.70-3	7.76-1	1.81+1	3.88+2	3.46+3	1.96+4	5.11+4	8.21+3	6.24+5	5.89+5	3.15+4		9.74+4
7	5.81-2	2.48	3.12+1	3.37+2	1.58+5	3.68+3	1.94+3	9.97+3	6.91+4	1.25+3	2.79+5	3.27+5	2.85+4	4.03+4
8	1.52-1	3.52+1	2.33+2	6.73+2	6.83+2	2.71+1	3.50+3	8.11+3	2.55+3	5.65+4	7.05+3	1.36+5	2.34+4	1.93+4
9	2.66-1	1.48+2	2.81+1	1.06+2	8.75+1	1.32+3	9.04+2	2.11+3	1.09+4	2.32+2	4.05+4	9.98+3	1.87+4	1.03+4
10	3.76-1	1.17+2	8.06	1.18+2	4.58+2	8.48+1	1.01+3	2.11+3	8.96+2	1.07+4	5.18+1	2.79+4	1.47+4	5.88+3
11	4.64-1	4.06-1	9.45+1	1.49+2	4.83	5.09+2	3.86+2	6.82+2	2.80+3	2.62+2	9.24+3	4.26+2	1.17+4	3.57+3

TABLE V. (contined)

n' \ k'	-n'+1	-5	-4	-3	-2	-1	0	1	2	3	4	5	n'-1	Mean
	J. n = 4, k = 3													
5			5.07-7	6.49-5	1.03-2	6.90-1	4.32+1	1.85+3	6.15+4	7.74+5	1.23+6			1.19+5
6		8.98-5	3.59-3	1.68-1	3.52	6.25+1	5.50+2	4.00+3	9.68+3	7.53+3	3.97+5	7.10+5		4.41+4
7	8.90-4	1.90-2	4.51-1	4.94	4.19+1	1.78+2	3.22+2	6.60+1	3.29+3	1.82+4	8.10+1	2.15+5	4.30+5	1.94+4
8	3.16-3	6.58-1	4.59	2.26+1	5.31+1	2.89+1	2.13+1	6.75+2	9.68+2	1.56+3	1.85+4	8.25+2	2.76+5	9.66+3
9	6.85-3	3.58	1.12+1	1.50+1	5.85-1	3.16+1	1.26+2	2.87+1	6.21+2	1.86+3	5.36+2	1.59+4	1.85+5	5.25+3
10	1.13-2	5.35	3.86	7.31-1	2.02+1	2.66+1	1.28	1.85+2	1.77+2	4.22+2	2.32+3	1.14+2	1.29+5	3.06+3
11	1.57-2	7.99-1	1.72	1.02+1	4.33	6.79	5.10+1	1.00+1	1.81+2	3.46+2	2.40+2	2.40+3	9.25+4	1.88+3
	K. n = 5, k = 0													
6							1.07+6	2.26+5	7.03+3	2.26+2	1.95	1.41-2		2.43+5
7							0.00	1.89+5	7.80+4	7.66+3	6.92+2	1.83+1	4.10-1	6.35+4
8							7.82+4	7.04+3	4.30+4	2.89+4	5.61+3	8.90+2	1.78	2.47+4
9							0.00	3.41+4	8.13+3	1.13+4	1.14+4	3.70+3	3.93	1.17+4
10							1.81+4	9.35+2	1.42+4	5.99+3	3.39+3	4.72+3	6.28	6.20+3
11							0.00	1.03+4	1.59+3	5.88+3	3.83+3	1.23+3	8.36	3.58+3
	L. n = 5, k = 1													
6		5.91-5	1.80-2	1.76	1.67+2	4.78+3	1.20+5	8.84+5	3.13+5	8.58+3	2.31+2	1.49		1.79+5
7	4.79-3	4.58-1	1.44+1	4.25+2	4.30+3	3.19+4	7.22+4	8.87+2	2.55+5	1.37+5	1.08+4	8.18+2	1.55+1	5.27+4
8	3.46-2	3.01+1	4.64+2	2.61+3	8.94+3	9.68+3	6.59+3	6.65+4	4.46+3	8.35+4	6.34+4	9.07+3	4.03+1	2.16+4
9	1.05-1	3.89+2	1.43+3	2.57+3	1.22+3	5.11+3	1.55+4	4.17+2	4.07+4	8.14+3	3.04+4	3.11+4	6.47+1	1.05+4
10	2.09-1	2.61+2	7.28+2	1.56+2	2.81+3	4.35+3	1.36+3	1.48+4	1.88+2	2.25+4	7.87+3	1.22+4	8.28+1	5.67+3
11	3.27-1	1.91+2	9.19+1	1.38+3	1.16+3	1.37+3	4.98+3	1.92+2	1.12+4	9.13+2	1.22+4	6.28+3	9.36+1	3.32+3

TABLE V. (continued)

k' / n'	-n' + 1	-5	-4	-3	-2	-1	0	1	2	3	4	5	n, -1	Mean
	M. n = 5, k = 2													
6		2.64-7	6.34-5	1.39-2	1.10	8.54+1	2.47+3	7.90+4	6.77+5	3.79+5	8.92+3	1.76+2		1.19+5
7	6.15-5	4.52-3	3.01-1	7.64	1.76+2	1.72+3	1.10+4	3.09+4	2.80+3	2.68+5	1.98+5	1.24+4	6.94+2	4.04+4
8	7.55-4	1.01	1.37+1	1.58+2	8.12+2	2.17+3	1.51+3	5.35+3	4.11+4	1.42+3	1.14+5	1.07+5	1.12+3	1.76+4
9	3.19-3	1.57+1	1.09+2	3.46+2	4.09+2	4.15+1	2.17+3	5.84+3	1.11+3	3.27+4	5.08+3	5.22+4	1.33+3	8.82+3
10	7.97-3	6.83+1	1.42+2	6.24+1	4.60+1	8.55+2	7.62+2	1.24+3	7.70+3	2.40+1	2.26+4	6.49+3	1.39+3	4.87+3
11	1.48-2	5.64+1	4.46	6.64+1	3.02+2	8.96+1	6.72+2	1.70+3	4.54+2	7.39+3	1.64+2	1.49+4	1.35+3	2.88+3
	N. n = 5, k = 3													
6		4.41-10	2.20-7	3.58-5	5.97-3	4.43-1	3.07+1	1.31+3	4.30+4	4.58+5	4.25+5	7.69+3		6.70+4
7	2.95-7	4.43-5	2.20-3	1.08-1	2.49	4.92+1	4.78+2	3.76+3	1.01+4	3.84+3	2.27+5	2.49+5	1.14+4	2.62+4
8	6.16-6	1.25-2	3.07-1	3.69	3.49+1	1.67+2	3.64+2	1.55+2	2.86+3	1.80+4	6.35+1	1.17+5	1.12+4	1.22+4
9	3.62-5	4.64-1	3.52	1.93+1	5.20+1	3.77+1	1.20+1	7.31+2	1.37+3	1.14+3	1.78+4	1.67+3	9.87+3	6.33+3
10	1.14-4	2.77	9.62	1.50+1	1.32	2.82+1	1.56+2	7.21+1	6.28+2	2.46+3	2.58+2	1.47+4	8.32+3	3.57+3
11	2.49-4	4.51	3.84	4.71-1	2.03+1	3.57+1	7.85-1	2.21+2	3.10+2	3.79+2	2.94+3	8.46	6.89+3	2.15+3
	O. n = 5, k = 4													
6		7.65-13	1.91-10	6.10-8	8.74-6	1.25-3	9.21-2	8.62	4.13+2	1.54+4	2.31+5	4.52+5		2.67+4
7	1.51-9	1.13-7	1.04-5	4.48-4	1.82-2	4.02-1	7.33	9.62+1	7.46+2	1.93+3	2.62+3	1.36+5	2.83+5	1.17+4
8	5.41-8	9.73-5	2.15-3	4.27-2	4.75-1	3.95	1.76+1	4.02+1	2.00	8.13+2	4.48+3	1.15+2	1.83+5	5.75+3
9	4.46-7	4.51-3	5.40-2	3.66-1	1.67	3.73	1.20	7.87	1.22+2	1.48+2	4.95+2	5.27+3	1.23+5	3.10+3
10	1.78-6	5.15-2	2.34-1	6.28-1	6.53-1	5.31-2	4.87	1.57+1	6.68-1	1.45+2	3.73+2	2.27+2	8.65+4	1.79+3
11	4.64-6	1.35-1	2.15-1	7.15-2	2.85-1	2.11	1.54	1.56	3.21+1	2.01+1	1.23+2	5.47+2	6.25+4	1.09+3

TABLE VI. Branching ratios β_T for the initial states n'k', where $3 \leqslant n' \leqslant 10$, $0 \leqslant k' \leqslant n' - 1$, and the final states n, where $1 < n < n'$. Averages with respect to k' are also given. Branching ratio for $n' - k' \rightarrow n$ transition is the same as for $n'k' \rightarrow n$ transition.

n'k' \ n	2	3	4	5	6	7
30	1.00					
31	3.42-1					
32	2.24-1					
Mean	5.35-1					
40	4.56-1	4.29-1				
41	7.01-1	4.92-1				
42	3.75-1	1.78-1				
43	2.46-1	9.81-2				
Mean	5.01-1	3.49-1				
50	6.48-1	3.60-1	5.36-1			
51	4.24-1	3.01-1	2.72-1			
52	5.71-1	4.16-1	2.85-1			
53	3.90-1	2.04-1	1.13-1			
54	2.60-1	1.10-1	5.65-2			
Mean	4.85-1	3.10-1	2.85-1			
60	4.34-1	2.90-1	2.36-1	3.13-1		
61	6.03-1	3.38-1	3.18-1	3.71-1		
62	4.14-1	2.55-1	2.19-1	1.90-1		
63	4.99-1	3.59-1	2.64-1	1.85-1		
64	3.98-1	2.18-1	1.31-1	8.03-2		
65	2.69-1	1.18-1	6.29-2	3.75-2		
Mean	4.74-1	2.90-1	2.38-1	2.39-1		
70	5.83-1	3.12-1	2.83-1	2.46-1	4.02-1	
71	4.24-1	2.68-1	2.06-1	1.95-1	2.36-1	
72	5.60-1	3.29-1	2.53-1	2.58-1	2.68-1	
73	4.10-1	2.36-1	1.88-1	1.66-1	1.41-1	
74	4.55-1	3.18-1	2.40-1	1.80-1	1.30-1	
75	4.04-1	2.29-1	1.42-1	9.25-2	6.12-2	
76	2.76-1	1.24-1	6.74-2	4.14-2	2.72-2	
Mean	4.68-1	2.79-1	2.17-1	1.94-1	2.15-1	

Table VI. (continued)

n'k' \ n	2	3	4	5	6	7	8	9
80	4.29-1	2.66-1	1.97-1	1.76-1	1.76-1	2.63-1		
81	5.67-1	3.05-1	2.51-1	2.13-1	2.24-1	3.14-1		
82	4.19-1	2.53-1	1.91-1	1.60-1	1.64-1	1.85-1		
83	5.24-1	3.19-1	2.27-1	2.04-1	2.07-1	2.02-1		
84	4.09-1	2.28-1	1.70-1	1.48-1	1.31-1	1.10-1		
85	4.25-1	2.89-1	2.19-1	1.70-1	1.30-1	9.67-2		
86	4.08-1	2.36-1	1.50-1	1.01-1	7.00-2	4.89-2		
87	2.82-1	1.29-1	7.11-2	4.41-2	2.97-2	2.09-2		
Mean	4.64-1	2.72-1	2.04-1	1.73-1	1.68-1	1.94-1		
90	5.57-1	2.95-1	2.39-1	1.91-1	1.90-1	1.97-1	3.38-1	
91	4.24-1	2.57-1	1.88-1	1.56-1	1.46-1	1.55-1	2.14-1	
92	5.47-1	3.02-1	2.29-1	1.94-1	1.77-1	1.98-1	2.51-1	
93	4.16-1	2.42-1	1.81-1	1.45-1	1.33-1	1.39-1	1.49-1	
94	4.96-1	3.08-1	2.14-1	1.77-1	1.69-1	1.68-1	1.58-1	
95	4.09-1	2.25-1	1.60-1	1.35-1	1.20-1	1.06-1	8.89-2	
96	4.04-1	2.66-1	2.02-1	1.59-1	1.26-1	9.86-2	7.52-2	
97	4.10-1	2.42-1	1.57-1	1.07-1	7.61-2	5.54-2	4.05-2	
98	2.87-1	1.33-1	7.44-2	4.65-2	3.14-2	2.25-2	1.67-2	
Mean	4.61-1	2.67-1	1.97-1	1.61-1	1.47-1	1.49-1	1.82-1	
100	4.28-1	2.57-1	1.84-1	1.51-1	1.34-1	1.32-1	1.47-1	2.34-1
101	5.49-1	2.92-1	2.27-1	1.81-1	1.65-1	1.62-1	1.83-1	2.81-1
102	4.22-1	2.50-1	1.81-1	1.44-1	1.30-1	1.24-1	1.37-1	1.78-1
103	5.27-1	3.00-1	2.15-1	1.79-1	1.57-1	1.53-1	1.73-1	2.04-1
104	4.14-1	2.35-1	1.73-1	1.37-1	1.18-1	1.14-1	1.20-1	1.23-1
105	4.73-1	2.98-1	2.06-1	1.62-1	1.46-1	1.42-1	1.38-1	1.27-1
106	4.09-1	2.24-1	1.54-1	1.26-1	1.11-1	9.96-2	8.79-2	7.39-2
107	3.89-1	2.49-1	1.89-1	1.50-1	1.20-1	9.67-2	7.75-2	6.04-2
108	4.12-1	2.46-1	1.62-1	1.13-1	8.11-2	6.00-2	4.54-2	3.44-2
109	2.91-1	1.37-1	7.73-2	4.86-2	3.30-2	2.37-2	1.79-2	1.38-2
Mean	4.58-1	2.64-1	1.92-1	1.53-1	1.35-1	1.28-1	1.36-1	1.70-1

TABLE VII. Branching ratios β_T for $n'k' \rightarrow nk$ transitions where $3 \leqslant n' \leqslant 11$, $-(n'-1) \leqslant k' \leqslant n'-1$, $2 \leqslant n \leqslant 5$, $0 \leqslant k \leqslant n-1$. Due to the space limitation, values of k′ that fall between $-(n'-1)$ and -5, and 5 and $n-1$ are not listed. The branching ratios for the negative values of k are obtained using symmetry relations (37).

n′ \ k′	-n′+1	-5	-4	-3	-2	-1	0	1	2	3	4	5	n′-1	Mean
A. n = 2, k = 0														
3							8.83-1	1.27-1	4.69-3					3.52-1
4							3.28-1	5.67-1	1.39-1	1.03-2				3.31-1
5							5.18-1	2.82-1	4.29-1	1.46-1	1.48-2			3.21-1
6							3.03-1	4.70-1	2.55-1	3.48-1	1.51-1	1.87-2		3.14-1
7							4.52-1	2.89-1	4.23-1	2.37-1	2.95-1	1.54-1	2.20-2	3.10-1
8							2.98-1	4.35-1	2.75-1	3.83-1	2.24-1	2.58-1	2.49-2	3.07-1
9							4.26-1	2.91-1	4.12-1	2.63-1	3.50-1	2.15-1	2.75-2	3.05-1
10							2.97-1	4.17-1	2.83-1	3.89-1	2.52-1	3.22-1	2.98-2	3.03-1
11							4.11-1	2.92-1	4.04-1	2.74-1	3.67-1	2.43-1	3.19-2	
B. n = 2, k = 1														
3					1.30-4	1.77-3	5.86-2	2.14-1	2.19-1					9.17-2
4				6.13-4	4.23-3	4.50-2	6.39-2	8.91-2	2.31-1	2.36-1				8.53-2
5			1.23-3	6.23-3	3.43-2	[illegible]	6.49-2	1.07-1	1.08-1	2.37-1	2.44-1			8.20-2
6		1.85-3	7.77-3	2.70-2	2.33-2	[illegible]	6.52-2	8.06-2	1.36-1	1.24-1	2.40-1	2.49-1		8.02-2
7	2.41-3	8.95-3	2.19-2	1.79-2	4.34-2	4.56-2	6.56-2	8.97-2	9.32-2	1.56-1	1.37-1	2.41-1	2.52-1	7.92-2
8	2.92-3	1.84-2	1.52-2	3.69-2	3.44-2	5.56-2	6.55-2	7.70-2	1.10-1	1.04-1	1.70-1	1.49-1	2.55-1	7.85-2
9	3.37-3	1.37-2	3.19-2	2.75-2	4.82-2	5.08-2	6.58-2	8.28-2	8.67-2	1.26-1	1.14-1	1.81-1	2.56-1	7.80-2
10	3.76-3	2.79-2	2.31-2	4.24-2	4.09-2	5.76-2	6.57-2	7.49-2	9.80-2	9.54-2	1.39-1	1.23-1	2.58-1	7.77-2
11	4.11-3	2.01-2	3.77-2	3.41-2	5.11-2	5.38-2	6.59-2	7.91-2	8.29-2	1.11-1	1.03-1	1.50-1	2.59-1	
C. n = 3, k = 0														
4							3.63-1	1.27-1	1.97-3	5.81-5				1.39-1
5							1.72-1	1.95-1	1.10-1	5.03-3	3.35-4			1.24-1
6							1.88-1	1.42-1	1.30-1	9.44-2	7.99-3	7.77-4		1.16-1
7							1.42-1	1.61-1	1.25-1	9.66-2	8.17-2	1.06-2	1.30-3	1.12-1
8							1.60-1	1.32-1	1.37-1	1.13-1	7.66-2	7.17-2	1.85-3	1.09-1
9							1.32-1	1.49-1	1.24-1	1.17-1	1.03-1	6.37-2	2.41-3	1.08-1
10							1.49-1	1.28-1	1.37-1	1.17-1	1.02-1	9.47-2	2.94-3	1.06-1
11							1.28-1	1.44-1	1.23-1	1.24-1	1.10-1	8.95-2	3.46-3	

TABLE VII (continued)

n' \ k'	-n'+1	-5	-4	-3	-2	-1	0	1	2	3	4	5	n'-1	Mean
	D. n = 3, k = 1													
4				4.08-7	4.87-5	2.28-3	3.30-2	3.46-1	9.00-2	2.06-3				8.49-2
5			5.74-6	2.72-4	4.47-3	2.18-2	8.82-2	7.29-2	2.81-1	1.01-1	4.93-3			7.55-2
6		2.02-5	6.03-4	5.56-3	1.51-2	3.98-2	4.76-2	1.43-1	8.55-2	2.34-1	1.09-1	7.53-3		7.07-2
7	4.33-5	9.67-4	5.99-3	1.10-2	2.21-2	3.27-2	7.93-2	6.41-2	1.66-1	9.24-2	2.00-1	1.13-1	9.83-3	6.82-2
8	7.29-5	6.05-3	8.36-3	1.45-2	2.42-2	5.01-2	4.90-2	1.10-1	7.51-2	1.73-1	9.72-2	1.75-1	1.19-2	6.65-2
9	1.07-4	6.64-3	1.10-2	1.88-2	3.36-2	3.75-2	7.50-2	6.07-2	1.30-1	8.29-2	1.73-1	1.01-1	1.37-2	6.54-2
10	1.43-4	9.10-3	1.50-2	2.40-2	2.98-2	5.43-2	4.95-2	9.63-2	6.98-2	1.42-1	8.87-2	1.69-1	1.53-2	6.46-2
11	1.81-4	1.23-2	1.82-2	2.42-2	4.03-2	4.02-2	7.26-2	5.88-2	1.13-1	7.71-2	1.50-1	9.32-2	1.68-2	
	E. n = 3, k = 2													
4				3.19-9	1.90-7	2.46-5	4.21-4	1.73-2	8.61-2	9.60-2				2.01-2
5			1.13-7	2.71-6	9.99-5	6.35-4	5.84-3	1.11-2	2.00-2	9.65-2	1.05-1			1.74-2
6		6.10-7	9.35-6	1.69-4	6.38-4	2.85-3	3.31-3	1.07-2	2.41-2	2.47-2	1.01-1	1.09-1		1.59-2
7	1.68-6	1.95-5	2.14-4	5.69-4	1.54-3	1.62-3	6.08-3	8.65-3	1.45-2	3.56-2	3.03-2	1.04-1	1.13-1	1.51-2
8	3.33-6	2.36-4	4.87-4	9.39-4	1.11-3	3.82-3	4.13-3	9.31-3	1.52-2	1.78-2	4.52-2	3.60-2	1.15-1	1.46-2
9	5.48-6	4.12-4	6.48-4	8.93-4	2.50-3	2.39-3	6.18-3	7.69-3	1.22-2	2.20-2	2.12-2	5.31-2	1.17-1	1.43-2
10	7.98-6	5.00-4	7.67-4	1.71-3	1.63-3	4.34-3	4.49-3	8.62-3	1.20-2	1.48-2	2.86-2	2.45-2	1.18-1	1.40-2
11	1.07-5	6.74-4	1.22-3	1.25-3	3.12-3	2.90-3	6.22-3	7.17-3	1.09-2	1.66-2	1.74-2	3.46-2	1.20-1	
	F. n = 4, k = 0													
5							4.43-1	5.81-2	2.31-3	3.97-5	2.30-7			1.08-1
6							1.16-1	2.02-1	4.51-2	5.01-3	2.48-4	3.75-6		8.65-2
7							1.86-1	8.61-2	1.16-1	3.57-2	6.85-3	6.02-4	1.47-5	7.79-2
8							8.99-2	1.49-1	6.81-2	7.45-2	2.90-2	8.01-3	3.43-5	7.27-2
9							1.44-1	8.02-2	1.17-1	5.60-2	5.21-2	2.40-2	6.20-5	6.98-2
10							8.21-2	1.30-1	7.10-2	9.17-2	4.72-2	3.90-2	9.64-5	6.76-2
11							1.28-1	7.74-2	1.13-1	6.29-2	7.31-2	4.05-2	1.36-4	

TABLE VII (continued)

n′ \ k′	-n′+1	-5	-4	-3	-2	-1	0	1	2	3	4	5	n′-1	Mean
							G. n = 4, k = 1							
5			2.18-9	2.98-7	5.55-5	1.16-3	4.59-2	1.98-1	1.04-1	1.47-3	2.73-5			5.37-2
6		9.50-8	4.79-6	2.68-4	2.19-3	2.76-2	5.45-2	6.10-2	1.44-1	9.75-2	3.86-3	1.74-4		4.60-2
7	5.98-7	1.84-5	5.24-4	2.66-3	1.69-2	2.26-2	4.20-2	8.27-2	6.67-2	1.10-1	8.82-2	6.28-3	4.31-4	4.24-2
8	1.86-6	7.49-4	2.77-3	1.08-2	1.17-2	3.00-2	4.71-2	5.04-2	9.10-2	7.04-2	8.80-2	7.96-2	7.62-4	4.04-2
9	4.08-6	2.71-3	7.25-3	7.25-3	2.22-2	2.85-2	3.95-2	6.38-2	5.60-2	9.03-2	7.22-2	7.27-2	1.13-3	3.91-2
10	7.29-6	5.11-3	5.29-3	1.66-2	1.82-2	2.12-2	4.41-2	4.61-2	7.34-2	6.03-2	8.60-2	7.26-2	1.53-3	3.83-2
11	1.14-5	4.31-3	1.26-2	1.25-2	2.50-2	3.10-2	3.84-2	5.62-2	5.12-2	7.80-2	6.35-2	8.04-2	1.93-3	
							H. n = 4, k = 2							
5			6.84-12	2.29-9	3.14-7	1.78-5	8.24-4	1.48-2	1.71-1	6.61-2	1.07-3			2.84-2
6		8.02-10	9.49-8	3.98-6	7.33-5	1.16-3	5.39-3	2.42-2	2.40-2	1.54-1	7.47-2	2.66-3		2.42-2
7	8.19-9	5.68-7	1.24-5	1.24-4	1.07-3	2.47-3	6.13-3	1.18-2	4.93-2	3.11-2	1.36-1	7.99-2	4.17-3	2.20-2
8	3.43-8	2.55-5	1.56-4	8.69-4	1.26-3	2.57-3	6.30-3	1.76-2	1.77-2	6.65-2	3.71-2	1.21-1	5.56-3	2.07-2
9	9.19-8	1.70-4	6.74-4	7.21-4	1.58-3	3.63-3	7.46-3	1.07-2	3.06-2	2.29-2	7.71-2	4.25-2	6.83-3	1.99-2
10	1.90-7	5.16-4	4.76-4	1.20-3	2.21-3	3.85-3	6.73-3	1.49-2	1.51-2	4.25-2	2.76-2	8.30-2	7.99-3	1.93-2
11	3.34-7	3.60-4	9.87-4	1.42-3	2.35-3	4.37-3	7.95-3	1.01-2	2.35-2	1.91-2	5.24-2	3.18-2	9.05-3	
							I. n = 4, k = 3							
5			2.28-14	3.82-12	1.16-9	6.11-8	6.63-6	1.64-4	6.89-3	4.55-2	5.54-2			6.71-3
6		7.38-12	4.39-10	3.67-8	6.60-7	2.03-5	1.42-4	1.47-3	2.98-3	6.83-3	5.18-2	6.01-2		5.68-3
7	1.23-10	4.34-9	1.75-7	1.78-6	2.65-5	9.77-5	4.03-4	5.20-4	2.89-3	7.77-3	8.20-3	5.51-2	6.27-2	5.10-3
8	6.96-10	4.24-7	2.97-6	2.61-5	6.17-5	1.50-4	2.18-4	1.18-3	1.72-3	4.03-3	1.28-2	1.03-2	6.47-2	4.74-3
9	2.28-9	3.93-6	2.28-5	3.84-5	7.23-5	1.37-4	5.06-4	5.69-4	2.06-3	3.64-3	5.09-3	1.75-2	6.64-2	4.50-3
10	5.46-9	1.88-5	2.46-5	4.65-5	9.70-5	2.47-4	2.80-4	1.05-3	1.28-3	2.92-3	6.01-3	6.17-3	6.77-2	4.33-3
11	1.07-8	1.66-5	3.62-5	7.03-5	1.33-4	1.75-4	5.51-4	5.71-4	1.68-3	2.35-3	3.75-3	8.60-3	6.89-2	

TABLE VII (continued)

n′ \ k′	-n′+1	-5	-4	-3	-2	-1	0	1	2	3	4	5	n′-1	Mean
J. n = 5, k = 0														
6							2.55-1	7.01-2	1.28-3	4.84-5	2.36-7	1.15-9		6.23-2
7							8.83-2	1.06-1	4.88-2	2.73-3	2.64-4	4.14-6	5.68-8	4.89-2
8							9.98-2	6.51-2	5.52-2	3.43-2	3.66-3	5.70-4	3.95-7	4.29-2
9							6.33-2	7.70-2	5.18-2	3.27-2	2.47-2	4.18-3	1.33-6	3.96-2
10							7.59-2	5.62-2	5.72-2	4.22-2	2.15-2	1.83-2	3.13-6	3.76-2
11							5.57-2	6.70-2	4.98-2	4.26-2	3.46-2	1.54-2	5.92-6	
K. n = 5, k = 1														
6		4.83-12	2.17-9	3.78-7	3.04-5	1.48-3	2.85-2	2.74-1	5.70-2	1.84-3	2.79-5	1.21-7		4.95-2
7	6.63-10	1.03-7	5.46-6	1.47-4	2.55-3	1.57-2	6.96-2	5.09-2	1.71-1	4.94-2	4.20-3	1.85-4	2.16-6	4.01-2
8	7.66-9	1.90-5	2.84-4	2.78-3	8.93-3	2.63-2	3.20-2	1.01-1	5.19-2	1.14-1	4.21-2	6.03-3	9.02-6	3.57-2
9	3.55-8	4.01-4	2.62-3	5.35-3	1.21-2	2.02-2	5.51-2	4.04-2	1.03-1	4.98-2	7.99-2	3.60-2	2.22-5	3.34-2
10	1.04-7	2.32-3	3.39-3	6.78-3	1.36-2	3.15-2	3.02-2	7.33-2	4.41-2	9.46-2	4.70-2	5.90-2	4.20-5	3.18-2
11	2.30-7	2.28-3	4.57-3	9.39-3	1.87-2	2.20-2	4.90-2	3.64-2	8.10-2	4.55-2	8.30-2	4.39-2	6.77-5	
L. n = 5, k = 2														
6		2.16-14	7.66-12	2.98-9	2.01-7	2.65-5	5.86-4	2.45-2	1.23-1	8.13-2	1.08-3	1.43-5		2.41-2
7	8.48-12	1.02-9	1.13-7	2.63-6	1.01-4	8.06-4	8.78-3	2.01-2	2.69-2	1.03-1	7.88-2	2.86-3	9.66-5	1.99-2
8	1.66-10	6.27-7	8.31-6	1.59-4	7.20-4	3.80-3	5.32-3	1.51-2	3.88-2	3.22-2	8.51-2	7.36-2	2.50-4	1.79-2
9	1.07-9	1.59-5	1.85-4	5.64-4	1.76-3	2.09-3	8.52-3	1.48-2	1.97-2	4.98-2	3.77-2	7.15-2	4.58-4	1.68-2
10	3.87-9	1.86-4	4.20-4	9.06-4	1.19-3	4.97-3	6.43-3	1.29-2	2.51-2	2.37-2	5.51-2	4.23-2	7.02-4	1.60-2
11	1.01-8	3.08-4	5.37-4	8.53-4	2.94-3	3.19-3	8.45-3	1.27-2	1.66-2	3.37-2	2.77-2	5.68-2	9.68-4	

TABLE VII (continued)

k′ / n′	-n′+1	-5	-4	-3	-2	-1	0	1	2	3	4	5	n′-1	Mean
M. n = 5, k = 3														
6		3.60-17	2.66-14	7.67-12	1.09-9	1.37-7	7.30-6	4.07-4	7.83-3	9.82-2	5.13-2	6.29-4		1.20-2
7	4.07-14	9.89-12	8.26-10	3.66-8	1.42-6	2.24-5	3.55-4	1.89-3	8.45-3	1.02-2	9.37-2	5.75-2	1.60-3	1.01-2
8	1.35-12	7.67-9	1.82-7	3.62-6	2.89-5	2.39-4	5.47-4	1.35-3	4.08-3	2.03-2	1.34-2	8.59-2	2.55-3	9.03-3
9	1.20-11	4.53-7	5.67-6	2.81-5	1.44-4	1.98-4	4.76-4	1.67-3	4.81-3	6.27-3	3.04-2	1.69-2	3.44-3	8.38-3
10	5.51-11	7.02-6	2.40-5	8.37-5	9.10-5	2.88-4	7.13-4	1.45-3	3.11-3	9.95-3	8.42-3	3.81-2	4.27-3	7.94-3
11	1.71-10	1.92-5	4.94-5	5.57-5	2.02-4	3.35-4	6.16-4	1.56-3	3.49-3	4.67-3	1.57-2	1.06-2	5.04-3	
N. n = 5, k = 4														
6		6.25-20	2.31-17	1.31-14	1.59-12	3.87-10	2.19-8	2.67-6	7.52-5	3.31-3	2.79-2	3.69-2		2.86-3
7	2.07-16	2.52-14	3.89-12	1.52-10	1.03-8	1.81-7	5.27-6	4.52-5	4.80-4	1.06-3	2.99-3	3.20-2	3.97-2	2.44-3
8	1.18-14	5.92-11	1.27-9	4.09-8	3.82-7	5.23-6	2.03-5	8.61-5	1.17-4	1.02-3	3.14-3	3.42-3	4.13-2	2.17-3
9	1.46-13	4.36-9	8.37-8	5.03-7	3.96-6	9.16-6	2.14-5	4.43-5	3.06-4	4.46-4	1.47-3	5.58-3	4.25-2	2.00-3
10	8.42-13	1.24-7	5.31-7	2.68-6	4.19-6	9.00-6	2.48-5	9.40-5	1.09-4	5.97-4	1.09-3	1.87-3	4.36-2	1.88-3
11	3.09-12	4.97-7	1.73-6	2.10-6	5.79-6	1.44-5	3.30-5	4.84-5	2.33-4	2.73-4	9.08-4	2.00-3	4.45-2	

TABLE VIII. Ratios of the branching ratios with and without an electric field, $\beta_T^F(n, n')/\beta_T(n, n')$, where $\beta_T^F(n, n')$ is the branching ratio with an electric field.

n' \ n	1	2	3	4	5	6	7	8	9
2	1.333								
3	1.041	0.758							
4	1.036	0.668	0.635						
5	1.029	0.606	0.490	0.576					
6	1.024	0.564	0.417	0.417	0.515				
7	1.018	0.539	0.375	0.344	0.364	0.483			
8	1.015	0.468	0.348	0.300	0.295	0.332	0.449		
9	1.013	0.508	0.328	0.274	0.254	0.265	0.307	0.431	
10	1.011	0.497	0.315	0.255	0.227	0.226	0.241	0.289	0.411

FIGURE CAPTIONS

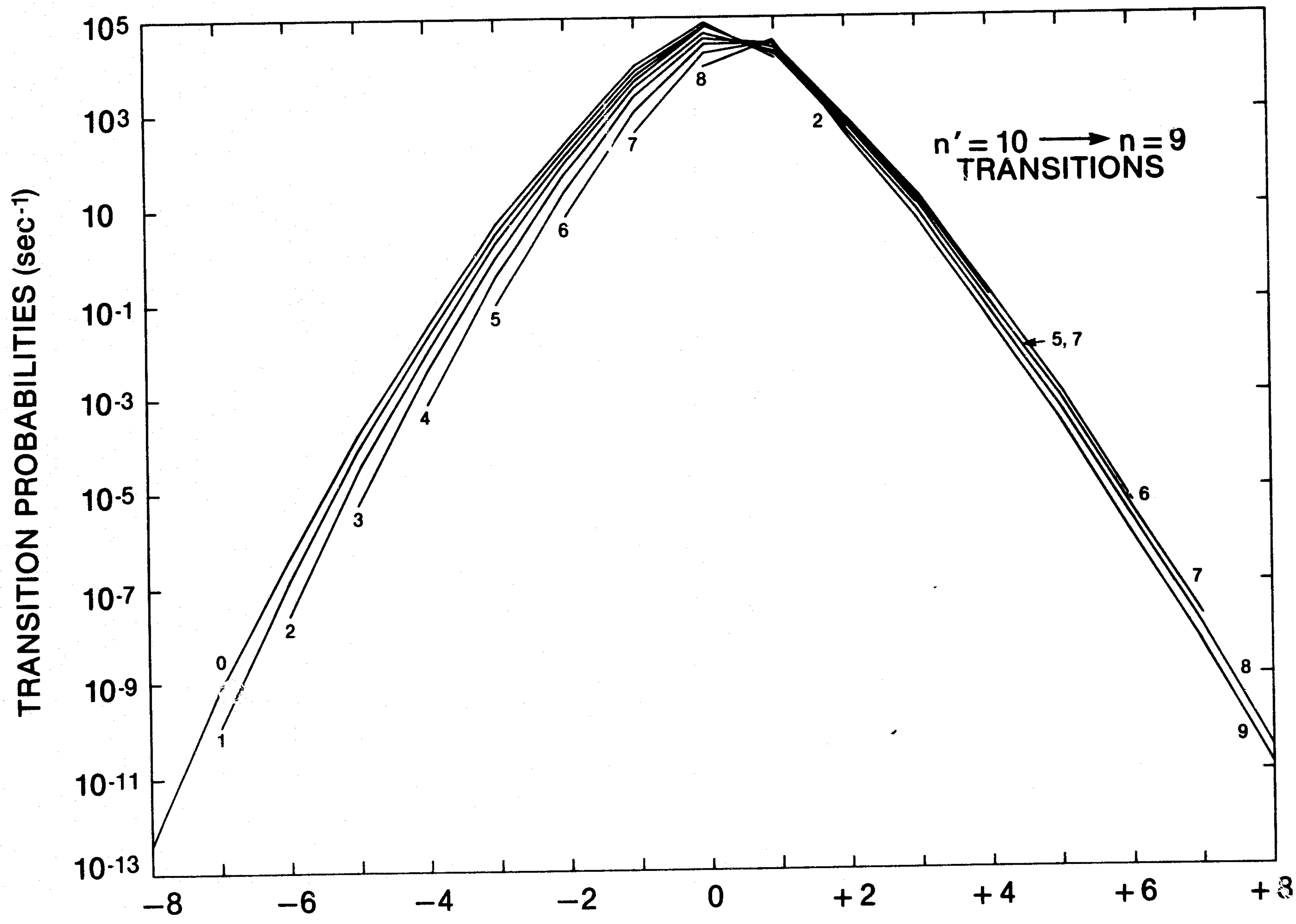
Figure 1. Transition probabilities for transitions between sublevels of $n' = 10$ and $n = 9$ versus $\Delta k = k' - k$. Numbers on the curves are the values of k'.

Figure 2. Transition probabilities for the initial states $n' = 10$ and $k' = 0 - 9$, and the final states $n = 1 - 9$. Numbers on the curves are the values of k'.

Figure 3. Transition probabilities in spherical coordinates for the initial states $n' = 10$ and $\ell' = 0 - 8$, and the final states $n = 1 - 9$. Numbers on the curves are the values of ℓ'.

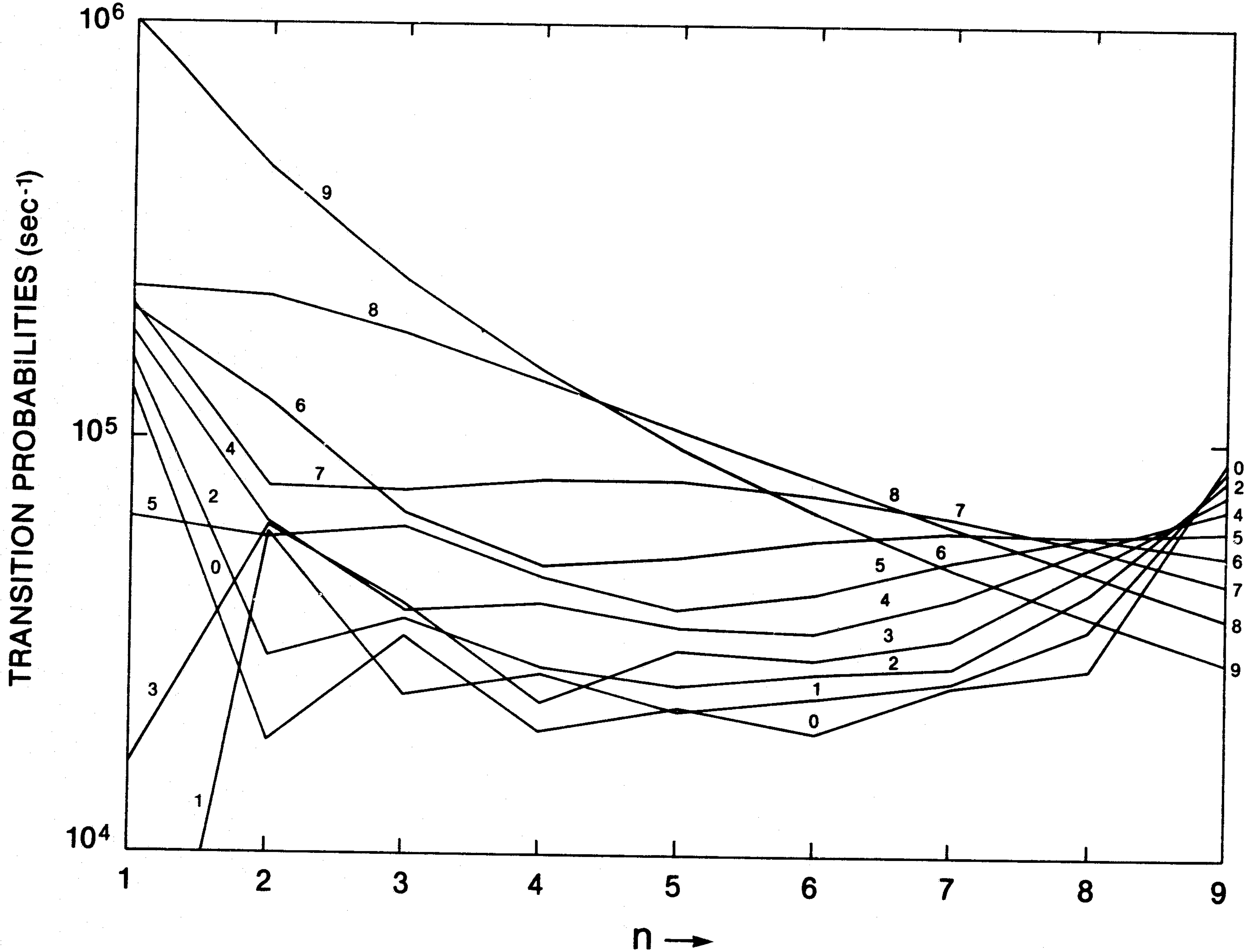
Figure 4. Lifetimes of the excited states $n' = 2 - 10$ and $-(n-1) \leqslant k' \leqslant n - 1$. The state $n', -k'$ has the same lifetime as the state n', k'.

Figure 5. Branching ratios for the initial states $n' = 10$ and $k' = 0 - 9$, and the final states $n = 1 - 9$. Numbers on the curves are the values of k'.



TRANSITION PROBABILITIES (sec-1)
10-13
10-11
10-9
10-7
10-5
10-3
10-1
10
103
105
−8
−6
−4
−2
0
+2
+4
+6
+8
Δk
n′ = 10 → n = 9
TRANSITIONS
0
1
2
3
4
5
6
7
8
2
5, 7
6
7
8
9

FIGURE 1



TRANSITION PROBABILITIES (sec-1)
10^6
10^5
10^4
1 2 3 4 5 6 7 8 9
n →
9
8
7
6
5
4
3
2
1
0

FIGURE 2

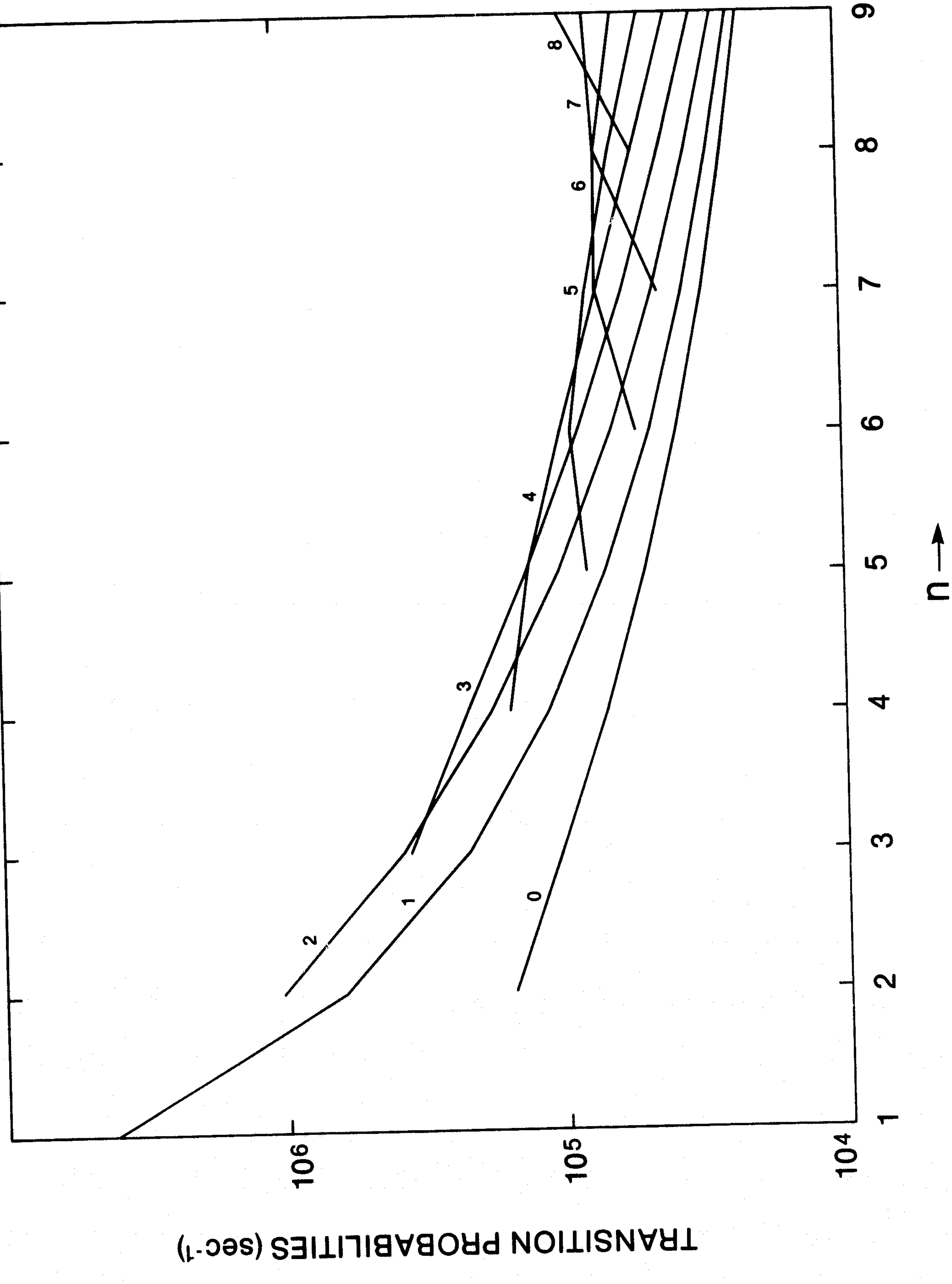
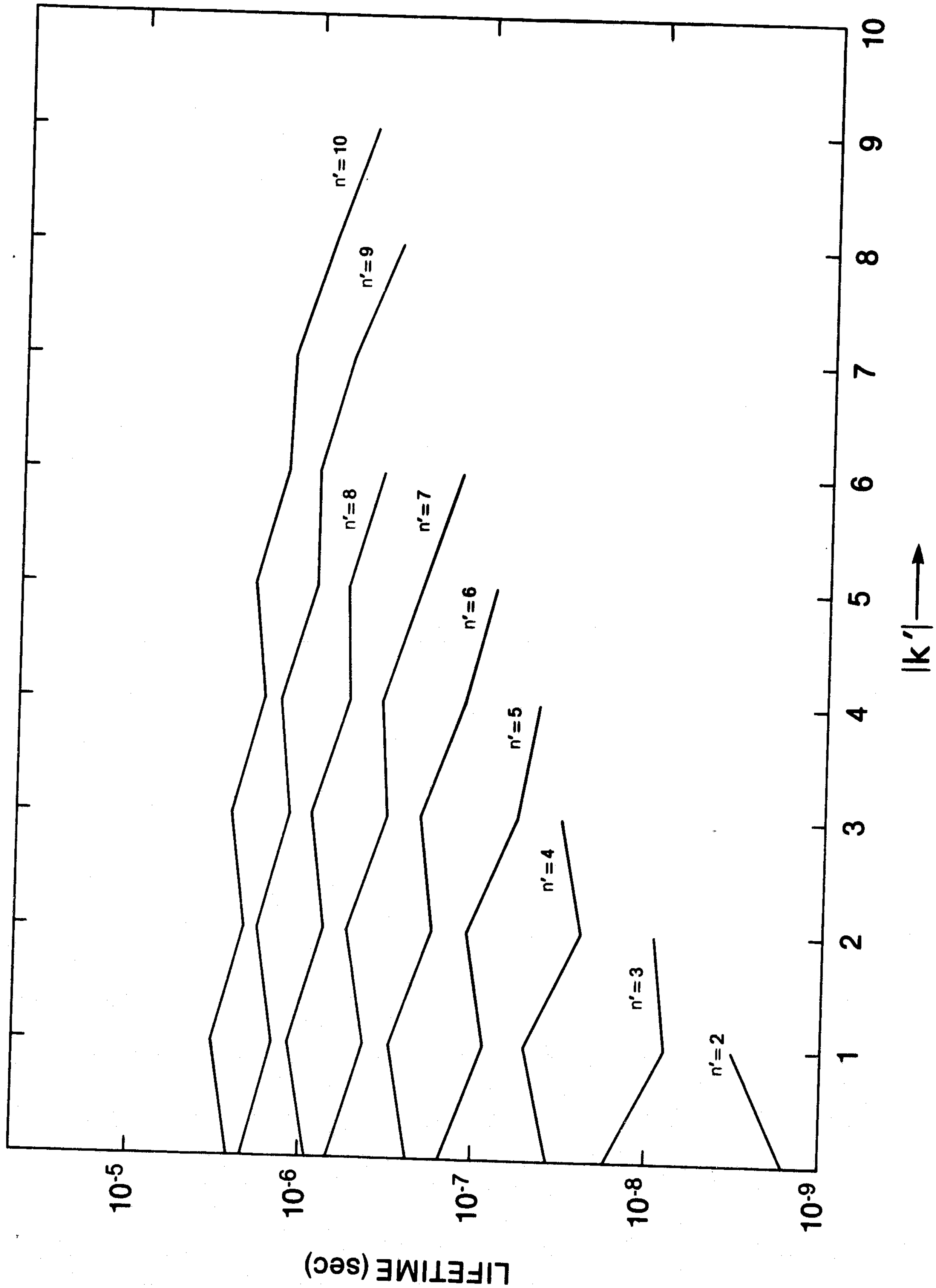

TRANSITION PROBABILITIES (sec-1)
106
105
104
1
2
3
4
5
6
7
8
9
n
0
1
2
3
4
5
6
7
8

FIGURE 3



LIFETIME (sec)
10-5
10-6
10-7
10-8
10-9
|k′|
1
2
3
4
5
6
7
8
9
10
n′=2
n′=3
n′=4
n′=5
n′=6
n′=7
n′=8
n′=9
n′=10

FIGURE 4

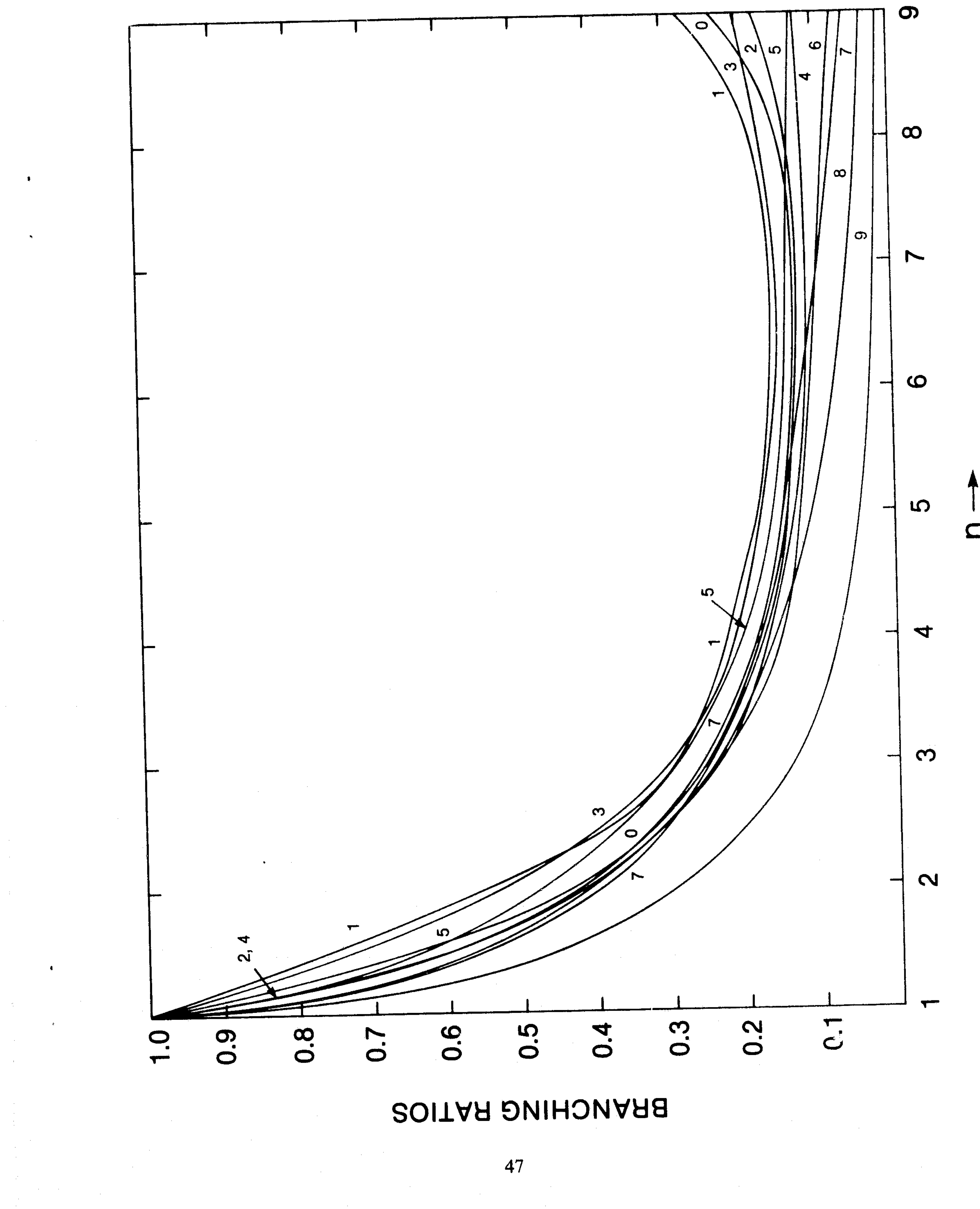


FIGURE 5