

SOLUTION SENSITIVITY AND ACCURACY STUDY OF  
NASTRAN FOR LARGE DYNAMIC PROBLEMS  
INVOLVING STRUCTURAL DAMPING

A. J. KALINOWSKI

NAVAL UNDERWATER SYSTEMS CENTER

SUMMARY

This paper is concerned with both the solution sensitivity and solution accuracy of large dynamic problems involving NASTRAN SOLUTION 8 (i.e., the steady state dynamic response option wherein all response quantities vary as  $e^{i\omega t}$ , where  $\omega$  is the driving frequency and  $t$  is time). Using a submerged steel plate with a viscoelastic layer as the benchmark sample, the solution sensitivity and solution accuracy is checked. The solution sensitivity is examined by running the same finite element model on different computers, different versions of NASTRAN, and different precision levels. The solution accuracy is evaluated for these same runs by comparing the NASTRAN results with the exact solution of the same problem.

SYMBOLS

[B]	Damping Matrix
$c_1$	Dilational Wave Speed in Fluid
{F}	Applied Force Vector
[K]	Stiffness Matrix
$\bar{K}$	Modified Complex Stiffness Matrix
[M]	Mass Matrix
$k$	Wave Number ( $\omega/c_1$ )
$P_i$	Incident Fluid Pressure
$P_0$	Plane Wave Amplitude
$P_B$	Back Side Fluid Pressure
$P_S$	Front Side Fluid Pressure (Scattered Component)
$t$	Time
{U}	Solution Displacement Vector

## SYMBOLS (Cont'd)

x	Spatial Coordinate
$\omega$	Driving Frequency
{ $\Delta$ }	Residual Solution Vector
$\lambda_{,\mu}^r$	Real Elastic Lamé Constants
$\lambda_{,\mu}^i$	Corresponding Viscoelastic Constants
$\rho$	Material Mass Density

## INTRODUCTION

This paper is concerned with the solution accuracy of 1, 2, or 3-dimensional steady state (time harmonic) structural and/or continuum problems whose response quantities all vary in time in proportion to  $e^{i\omega t}$ . The linear equations of motion for such problems usually reduce to an expression of the form

$$[-\omega^2[M] + \underbrace{i\omega[B] + [K]}_{[K]}] \{U\} = \{F\} \quad (1)$$

where [M], [B], and [K] denote the mass, damping and stiffness matrices (MDD, BDD and KDD using usual NASTRAN DMAP notation),  $\omega$  is the driving frequency and {F} are the applied forces. The results presented in this paper focus on continuum type (e.g., figure 1) applications with structural damping, however, once the form of equation (1) has been constructed, the solution becomes a matter of solving large banded symmetrical systems of complex linear algebraic simultaneous equations. Clearly, such equations can also be the end point resulting from many other NASTRAN steady state formulations, either from direct structural formulations or from related fields through analogies. Thus comparisons of solution accuracy, run time, etc. can be viewed and interpreted in a more general vein than simply applying only to problems of the type depicted in figure 1.

The motivation for this comparative study resulted as a consequence of obtaining some unexpected results on some solution 8 (steady state time harmonic rigid format) problems similar to the one shown in figure 2, except for the fact that the initial model had inclusions throughout the rubber thus making analytical solutions to the problem unwieldy.

## PARAMETRIC STUDY MODEL

In order to better understand the accuracy limitations of the results of the initially more complicated inclusion filled model, a simpler homogeneous layered model (figure 2) was constructed and physically corresponds to a totally submerged 2.0" steel plate with a 3.05" viscoelastic rubber layer glued to the steel surface. The input corresponds to an incident pressure wave

$$P_i = P_0 e^{i(kx + \omega t)}, \quad k = \omega/c_1 \quad (2)$$

where  $x$  is the horizontal coordinate along the line of propagation,  $c_1$  is the dilatational wave speed in the fluid, and  $P_0$  is the plane wave amplitude. The exact analytical solution to this problem is known (ref. 1), consequently an accuracy check on the finite element solutions is available. Clearly, the figure 1 model is a spatially one-dimensional problem, consequently the corresponding finite element model need only be one element wide as was done, for example in ref. 1. However, the finite model was made up to eight elements wide for the following reasons: (1) the model simulates the more complex model except for the fact that the inclusions are removed by filling their space are with uniform elements having the same material property as the surrounding rubber material; (2) the problem is artificially made mathematically larger so that more meaningful comparisons of CPU run times could be made; (3) larger problem sizes tend to draw out any potential problems with equation solvers. It is not our intent to discuss or explain the setup of wave propagation problems of the type represented by the figures 1 - 2 example model; the reader is referred to refs. 1 and 2 for supplementary details. In fact, the demonstration problem used here is very similar to the one used in the ref. 1 sample problem except that the plate and viscoelastic thicknesses are different, the damping coefficient in the viscoelastic layer is different and that the steel plate is represented here approximately with CBAR elements rather than with solid elements as in ref. 1. Specifically, the material constants employed are listed below

### DEMONSTRATION PROBLEM PHYSICAL CONSTANTS

MATERIAL	$\lambda^r$ psi	$\mu^r$ psi	$\lambda^i$ psi	$\mu^i$ psi	$\rho$ lb-sec <sup>2</sup> /in <sup>4</sup>
Water	345,600.	0.0	0.0	0.0	.000096
Steel	17,307,000.	11,538,000.	0.0	0.0	.000735
Viscoelastic Material	86,703.	115.9	8670.3	11.59	.0003599

where the meaning of the elastic and viscoelastic constants are defined in detail in ref. 1.

Since the topic of interest here is related to the class of problem treated by ref. 1, it appears appropriate to print an errata to the ref. 1 paper

- in equation (15) of ref. 1, replace  $G_{22} = \lambda^r$  with  $G_{22} = \lambda^r + 2\mu^r$
- in equation (16) of ref. 1, replace  $G_{22} = \epsilon$  with  $G_{22} = \epsilon(1 + 2\mu^i/\lambda^i)$
- in equation (2) of ref. 1, replace  $\omega^2$  with  $-\omega^2$
- in equation (17) of ref. 1, replace  $+n_{d_2}$  with  $+in_{d_2}$  in the  $k_2$  definition

### PARAMETER VARIATIONS

The basic finite element model, figure 2, was exercised for a frequency sweep of 7 different incident frequencies (3.0 kHz, 4.0 kHz, 6.0 kHz, 8.0 kHz, 17.5 kHz, 22.5 kHz, 35.0 kHz). Running the figure 1 model on NASTRAN for the above frequency sweep is designated as a typical run and correspondingly assign it a "run number", which runs from the number 2 through 9. Run number 1 is the exact solution and therefore is the only non NASTRAN designation (it is called a run since even the analytical solution involves a computer evaluation).

Next, the same frequency sweep input data was rerun while varying the following parameters:

- solution precision (S.P. or D.P. on the same computer)
- type of computer (UNIVAC 1108; DEC-VAX; CDC Cyber 175)
- level of NASTRAN (both NASA and MSC versions are considered)
- date (i.e., the same input is resubmitted on the same computer, using the same version of NASTRAN but on different days)

The last parameter (i.e., the date) seems a waste of computer time, however as is shown later, some unexpected results are encountered.

### DMAP INSTRUCTIONS FOR PRINTING SOLUTION ERROR RESIDUAL

It is of interest to know the accuracy of the solution solving capability of the equation solver used by the particular version of NASTRAN employed by the user. Specifically, if the solution  $\{U\}$  is found by NASTRAN, how well does it satisfy the linear simultaneous equations (1)? Consider

substituting the solution {U} into equation (1) and then transposing the applied force vector to the left hand side of equation (1) to obtain

$$[\bar{K}]\{U\} - \{F\} = \{\Delta\} \quad (3)$$

If the equations have been solved exactly, then the residual vector {Δ} will be identically zero. The appearance of large nonzero entries in {Δ} would imply potential inaccuracies in the solution vector {U}. The question of "how large is large?" should be viewed by comparing the size of a particular entry in the {Δ} vector to the size of the applied loads (for this reason, the load vector {F} is also printed). For example, a residual of .2 would be a big residual if the applied forces are on the order of 1.0 lbs.; however, if applied forces are say 100,000 lbs., the .2 residual is acceptable.

In order to print out the residual vector {Δ} for the 3,000 Hz driving frequency case, the following DMAP instructions were used (note, the {Δ} vector is printed with the heading DELSØL).

- For UNIVAC 1108, SOL 8, LEVEL 17.0 (Runs 3a, 4a)

```
ALTER 159
ADD5 KDD,BDD,MDD,,/KBARX/C,Y,ALPHA=(1.0,0.0)/C,Y,BETA=(0.0,18849.5592)/
      C,Y,GAMA=(-355305758.44,0.0) $
MPYAD KBARX,UDVF,PDF/DELSØL/C,N,0/C,N,1/C,N,-1/ $
MATPRN DELSØL,PDF,,// $
ENDALTER
CEND
```

where frequency dependent input constants BETA and GAMA are to be input by the user and are simply defined as:

$$\begin{aligned} \text{BETA} &= 0.0 + i\omega \\ \text{GAMA} &= -\omega^2 + i 0.0 \end{aligned}$$

where  $\omega$  = the driving frequency in radians/sec

- For UNIVAC 1108, SOL 8, NASA LEVEL 15.5 (Run 2) replace ALTER 159 with ALTER 139
- For VAX, SOL 8, NASA LEVEL 17.5 (Run 5b) same as 1108, NASA LEVEL 17.0
- For CDC CYBER, SOL 8, MSC LEVEL 48B (Run 6) replace ALTER 159 with ALTER 139
- For VAX, SOL 8, MSC LEVEL 52 (Run 7) replace ALTER 159 with ALTER 139
- For VAX, SOL 8, MSC LEVEL 60 (Run 8) replace ALTER 159 with ALTER 139
- For VAX, SOL 26, MSC LEVEL 60 (Runs 9b, 9c, 9d) replace ALTER 159 with ALTER 409  
replace UDFV with UHV (3rd line)  
replace PDF with PD (3rd and 4th line)

When a large residual is encountered, it is desirable to know the node and component number where a large residual appears (i.e., knowing the run number of the questionable residual, what node-component number does this correspond to?). By inserting a DIAG 22 card, the desired correspondence between run number of  $\{\Delta\}$  and the node-component number can be made.

It is important to note that the simple DMAP sequence as presented will apply to only a single frequency; thus, if a frequency sweep is employed, only the  $n^{\text{th}}$  column of the DELSOL vector (i.e.,  $\{\Delta\}$ ) will be correct; the remaining columns of DELSOL should be ignored, where  $n =$  the  $n^{\text{th}}$  value in the frequency sweep appearing on the NASTRAN FREQ card. card.

## DISCUSSION OF RESULTS

The primary variables of interest to us in this study are: (1) the transmitted pressure in the fluid on the back side of the steel plate,  $P_B$ , (e.g., in element number 100352 as shown in figure 2) and (2) the scattered pressure in the fluid on the front side of the plate,  $P_S$  (e.g., in element number 100378 as shown in figure 2). The transmitted pressure is read directly from the NASTRAN printout, whereas the scattered pressure is obtained indirectly from the NASTRAN printout by simply subtracting the incident pressure (equation (2)) from the total pressure printed by NASTRAN. The scattered pressure is of prime importance with regard to establishing the energy absorbing properties of the viscoelastic configuration. As discussed in ref. 2, it has been our experience that for steady state wave propagation problems of the type considered here, at least 10 elements per wave length are needed to adequately compute the pressure response for elements of the type employed in this study. In order to demonstrate this accuracy limitation, the model has been purposely exercised in a driving frequency range that is too high for the mesh to properly produce sufficiently accurate results (i.e., the mesh is too coarse for some of the higher frequencies). For a rubber wave speed of  $C_2 = 15540$ . in/sec, and the coarsest element in the rubber mesh (.1" x .1" elements), the following frequency vs. element/ (wave length) chart is constructed:

freq. (kHz)	elements/wave length (rubber in figure 2)
3.0	51.8
4.0	38.8
6.0	25.9
8.0	19.4
17.5	8.8
22.5	6.9
35.0	4.4

Based on the above chart, it is expected that the accuracy of the finite element solution in relation to the exact solution should start to drift at frequencies of 17.5 kHz and higher.

A total of 14 computer runs were made (designated as runs number 2, 3a, 3b, . . . . 9d) and are tabulated in Table 1 (for the transmitted back side pressure  $P_B$ ) and in Table 2 (for the scattered front side pressure,  $P_S$ ). The pressure results are normalized by the magnitude of the incident pressure wave,  $P_0$ . In addition to the pressure result, Table 1 has two additional pieces of information, namely the magnitude of the largest complex residual,  $|\Delta|$ , appearing in the residual vector  $\{\Delta\}$  as computed by equation (3), (the  $|\Delta|$  value is obtained by scanning the DMAP printed DELSØL printout and seeking out the largest absolute value of all the rows of the  $\{\Delta\}$  vector corresponding to the 3,000 Hz frequency case. Note that the residual for only the 3,000 Hz case is reported. Also listed is the total CPU time required to execute the full frequency sweep solution for the run in question.

#### Word Length (Precision) Sensitivity

The earlier Level 15.5 version of NASA NASTRAN for a UNIVAC 1108 computer, does not efficiently solve complex systems of equations of the type given by equation (1), (i.e., steady state time harmonic rigid format 8) when the double precision option is used. Experience on large problems (e.g., the size of the one in ref. 2, pg 435) has demonstrated that in the solving of the equation (1), NASTRAN has spent literally hours in the decomposition operation. In order to obtain reasonable run times, a PARAM DECOMOPT4 card is added to the bulk data in order to force the decomposition to work in single precision. In comparing the NASTRAN Run 2 (Table 1) (1108-S.P.) results to the exact solution over the frequency range

(3. - 8. kHz) where the mesh is sufficiently fine,\* it is noted that at 6.0 kHz, a 154.0% error in the transmitted pressure is experienced. The corresponding error on the scattered pressure (Run 2, Table 2) is not as severe, namely 16.5%. It is noted that the 154% error is the situation that motivated this entire comparative study. The percent errors at 3. kHz, 4.0 kHz and 8 kHz are also much larger than should be expected for the mesh size employed. The largest residual,  $|\Delta|$ , in the residual vector  $\{\Delta\}$ , is .272 at 3.0 kHz; this is in comparison to a load vector component of the size (.20) (the largest residual was not at a loaded node however). This is a further indication that the solution resulting from decomposition in single precision is not accurate enough. Upon running the same problem on a Level 17.0 version of NASA-NASTRAN on an 1108 computer (single precision must be invoked with DMAP) in single precision, Run 3a still results in a similar bad solution with a similar worse residual  $|\Delta|$ . The fact that Level 17.0 uses a different decomposition algorithm did not improve the bad results. However, again running the same problem on Level 17.0 on the UNIVAC 1108 in double precision (the default situation) resulted in excellent results in comparison to the exact solution. For example, at the 6.0 kHz, the percent error reduced from 154.0% down to 0.4% error. Similarly good results were obtained in the entire (3.0 - 8.0 kHz range) for both the transmitted and scattered pressure. The double precision gave a very small worse residual at 3,000 Hz,  $|\Delta| = 2.099 \times 10^{-8}$ , which suggests that equation (1) has been solved accurately. In comparing CPU times between Run 3a and Run 4a in Table 1, it appears there is little penalty in CPU time between single and double precision runs for Level 17.0. Consequently, there is really no incentive (from a time saving point of view) to make Sol. 8 type runs in single precision, as there was for Level 15.5 NASTRAN.

Again running the same problem on Level 48B, MSC version of NASTRAN on a CDC CYBER 175 computer (Run 6), very good results were obtained in relation to the exact solution over the (3.0 - 8.0 kHz range); further, excellent consistency with the UNIVAC 1108 double precision runs is demonstrated at all frequencies by comparing Run 4a with Run 6. The worst residual,  $|\Delta|$ , on the CDC computer is not as good as the 1108 double precision run, but this is expected since the CDC single precision word length is slightly smaller than the 1108 double precision word length; however, it should be noted that differences in the decomposition algorithms could also account for differences in the worse residual, even if the word lengths were the same.

The Level 52, MSC version of NASTRAN on a DEC-VAX computer (Run 7) gave comparable results to the Level 17.0 NASA NASTRAN double precision 1108, and to Level 48B, MSC NASTRAN on a CDC-CYBER 175. The word length in double precision on the VAX is slightly more than the single precision CDC and slightly less than the double precision 1108. It is noted the Run 7 gave the smallest worse residual.

\* It should be emphasized that in comparing any NASTRAN result to the exact solution, for the purpose of measuring the formulation quality, it should be done only in the frequency range of 3.0 - 8.0 kHz where the mesh satisfies the 10 element/wave length criterion.



## Computer Type/Level of NASTRAN Sensitivity

It is inconvenient to run the same exact version of NASTRAN on different computers due to leasing restrictions, consequently this combination was not done. Thus running NASTRAN on different computers always involved running a different NASTRAN version as well. In scanning the results of Table 1 and Table 2, all the runs performed with decomposition precision using word lengths between 60 - 72 bits (i.e., Runs 3a, 4a, 6, 7) gave both accurate results in comparison to the exact solution (3.0 - 8.0 kHz range) and consistent results from machine-to-machine and version-to-version. We have purposely not commented on the accuracy of Runs 5, 8 and 9 involving the DEC-VAX computer due to some reservations we have regarding the operating conditions of the particular VAX on which these runs were made, and will be discussed next.

## Solution Repeatability

During the process of preparing this collection of comparative runs, an unexplained phenomenon (which is still unexplained as of this writing) occurred, namely the fact that Level 60 MSC-NASTRAN run on a DEC-VAX computer gave different results to the same problem upon rerunning the same data. The Run 9 series of runs were made on rigid format 26 which is comparable (there are differences in the decomposition routine) to NASA-NASTRAN rigid format 8. For example the input producing Run 9a was resubmitted over again (producing Run 9b) so that the residual vector  $\{\Delta\}$  could be printed (employing the DMAP instructions given earlier). In comparing the solutions, the results were slightly different (e.g., 5.2% at 3.0 kHz). The same input data was again rerun on successive days producing Runs 9c and 9d. Run 9c is the closest to the more stable results made on the 1108 and CDC computers.

Using the same DEC-VAX computer facility, the base case input was resubmitted again employing the Level 17.5 NASA/GODDARD NASTRAN and thus producing Runs 5a and 5b. Again the nonrepeatability of the solution on the VAX was experienced, this time with an entirely different version of NASTRAN.

An MSC Level 60 DEC-VAX computer run was made similar to Run 9a through Run 9d, except that the older rigid format 8 instead of the newer MSC rigid format 26 was employed. The results were poor in comparison to the exact solution, further, the worse residual of  $|\Delta| = .2127$  was unacceptably high. No reruns of the same input were made on this version and level of NASTRAN.

In order to demonstrate that repeatable results are possible (a notion we usually assume is true on most modern computers), the base case data was rerun on the UNIVAC 1108 computer; single precision Runs 3a and 3b were totally repeatable as well as double precision Runs 4a and 4b.

The facts that (1) the VAX computer resulted in nonrepeatable results employing two separate versions of NASTRAN and (2) the Level 60 MSC NASTRAN for

the VAX computer, Sol. 8 (Run 8) gave poor results, strongly suggests a problem with the particular VAX computer on which the runs were made. The following list provides some possible reasons for nonrepeatability:

- computer central processing drops a bit in the main memory or operating register
- main memory itself drops a bit between storing and retrieving data
- floating point accelerator drops a bit during calculations
- disk subsystem (drive or interface drops a bit)
- computer temperature rises due to air conditioning not keeping up with thermal load during the summer months when the runs were made; this could result in dropping a bit by one of the four above mentioned possibilities.

Finally it is noted that as of this writing, non NASTRAN users of the same VAX computer that made the runs reported here, did not report any repeatability problems with computer program results totally unrelated to NASTRAN.

## CONCLUSIONS

The paper is concerned with both the accuracy of equation solvers and with the accuracy of the problem formulation for large dynamic steady state problems (e.g., rigid format 8). Based on a series of computer runs on different versions of NASTRAN on different computers, the following set of conclusions are drawn:

- NASTRAN solution decomposition algorithms employing less than a 60 bit word could lead to serious errors in the results (e.g., employing 36 bit single precision words on an 1108 computer gave up to 154% error in the solution with both Level 15.5 and Level 17.0 versions of NASA NASTRAN.
- Correlation between results run on the 1108 double precision Level 17.0 NASA NASTRAN; CDC-CYBER 175 single precision Level 48B MSC-NASTRAN; and DEC-VAX double precision Level 52 MSC-NASTRAN were excellent.
- Unexplained unrepeatability of results were experienced on the DEC-VAX computer for both MSC-Level 60 NASTRAN and NASA-GODDARD Level 17.5 NASTRAN; Level 17.0 of NASA NASTRAN had no repeatability problems on an 1108 computer.
- A minimum of 10 elements per wave length should be used to model traveling wave propagation problems of the type treated in this paper; coarser meshes lead to increasingly bad results when comparing NASTRAN results to the exact continuum solution to the same problem.

As a final comment regarding repeatability, it is noted that it is not being suggested that this problem is one to be found in all VAX computers employing NASTRAN. The spirit of the NASTRAN Colloquium is to share USER's experiences, thus it was felt that our problem should be brought to the attention of the NASTRAN USER's community in the event that similar problems are encountered by others.

#### REFERENCES

1. Kalinowski, A. J., "Modeling Structural Damping for Solids Having Distinct Shear and Dilatational Loss Factors", Seventh NASTRAN USER's COLLOQUIUM, Oct. 1978 (NASA CP-2062).
2. Kalinowski, A. J., "Fluid/Structure Interaction", Shock and Vibration Computer Programs - Review and Summaries, edited by W. Pilkey and B. Pilkey, The Shock and Vibration Information Center, 1975, U.S. Dept. of Defense.

FIGURE 1 - TEST PROBLEM

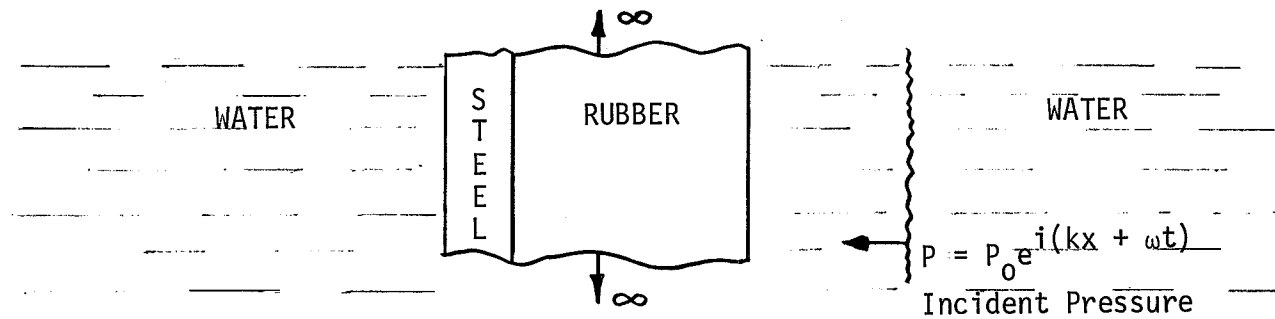
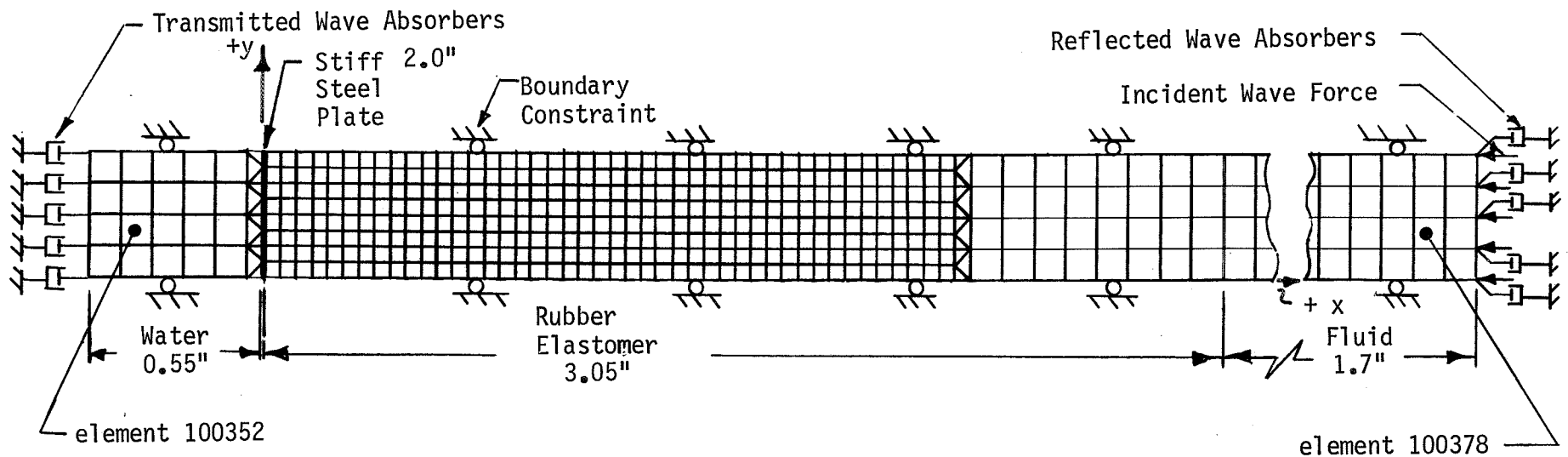


FIGURE 2 - FINITE ELEMENT MODEL



- 856 Elements (CQDMEM, (TRMEM, CBAR)
- 588 Grid Points

S.P. (Single Precision)  
D.P. (Double Precision)

TABLE 1 - NASTRAN COMPARATIVE SOLUTIONS  
(BACK SIDE PRESSURE, element 100352)

SOLUTION SOURCE	BACK SIDE PRESSURE, $P_B/P_0$ (amplitude)							Largest Residual $ \Delta $ at 3. kHz	TOTAL CPU (min)	RUN NUMBER
	35. kHz	22.5 kHz	17.5 kHz	8. kHz	6. kHz	4. kHz	3. kHz			
EXACT ANALYTICAL (Ref. 1)	.00809	.01776	.02796	.09550	.13893	.22863	.31324	not apply	not apply	1
Level 15.5 NASA-NASTRAN, UNIVAC 1108 Computer, 36 Bit S.P. Decomposition (Sol.8)	.00284	.01343	.02573	.16978	.35292	.34048	.27799	.272	21.12	2
Level 17.0 NASA-NASTRAN, UNIVAC 1108 Computer, 36 Bit S.P. Decomposition (Sol.8)	.00282	.01318	.02454	.13781	.30044	.38303	.33906	.247 not avail.	19.15	3a
	.00282	.01318	.02454	.13781	.30044	.38303	.33906		19.07	3b
Level 17.0 NASA-NASTRAN, UNIVAC 1108 Computer, 72 Bit D.P. Decomposition (Sol.8)	.00277	.01268	.02307	.09247	.13638	.22677	.31191	2.099x10 <sup>-8</sup> not avail.	20.01	4a
	.00277	.01268	.02307	.09247	.13638	.22677	.31191		19.29	4b
Level 17.5 NASA/GODDARD NASTRAN, DEC-VAX Computer, 64 Bit D.P. Decomposition (Sol.8)	.00285	.01289	.02278	.08913	.13305	.21937	.30407	not avail. .07426	52.18	5a
	.00279	.01139	.02248	.08145	.12512	.22887	.27257		59.45	5b
Level 48B MSC-NASTRAN CDC CYBER 175 Computer, 60 Bit S.P. Decomposition (Sol.8)	.00277	.01268	.02307	.09247	.13638	.22678	.31192	1.24x10 <sup>-5</sup>	3.03	6
Level 52 MSC-NASTRAN, DEC-VAX Computer, 64 Bit D.P. Decomposition (Sol.8)	.00277	.01268	.02307	.09247	.13638	.22677	.31191	7.422x10 <sup>-9</sup>	22.33	7
Level 60 MSC-NASTRAN, DEC-VAX Computer, 64 Bit D.P. Decomposition (Sol.8)	.00259	.01268	.02304	.09266	.13659	.22602	.45052	.2127	54.28	8
Level 60 MSC-NASTRAN, DEC-VAX Computer 64 Bit D.P. Decomposition (Sol.26) (4 runs on same data)	.00272	.01263	.02261	.08125	.13343	.22745	.27247	not avail. 5.65x10 <sup>-5</sup> 20.6 x10 <sup>-5</sup> 9.82x10 <sup>-5</sup>	not avail.	9a
	.00277	.01265	.02314	.09278	.13627	.21877	.28690		51.56	9b
	.00276	.01268	.02325	.09360	.13633	.22709	.31192		51.81	9c
	.00277	.01268	.02159	.09246	.13912	.22467	.31191		23.44	9d

S.P. (Single Precision)  
D.P. (Double Precision)

TABLE 2 - NASTRAN COMPARATIVE SOLUTIONS  
(FRONT SIDE PRESSURE, element 100378)

SOLUTION SOURCE	FRONT SIDE SCATTERED PRESSURE, $P_S/P_0$ (amplitude)							RUN NUMBER
	35. kHz	22.5 kHz	17.5 kHz	8. kHz	6. kHz	4. kHz	3. kHz	
EXACT ANALYTICAL (Ref. 1)	.04253	.08580	.13310	.34059	.44266	.59075	.59907	1
Level 15.5 NASA-NASTRAN, UNIVAC 1008 Computer, 36 Bit S.P. Decomposition (Sol.8)	.15093	.05609	.11772	.32372	.36949	.59695	.62678	2
Level 17.0 NASA-NASTRAN, UNIVAC 1108 Computer, 36 Bit S.P. Decomposition (Sol.8)	.15093	.05603	.11759	.32875	.39233	.58218	.60932	3a
	.15093	.05603	.11759	.32875	.39233	.58218	.60932	3b
Level 17.0 NASA-NASTRAN, UNIVAC 1108 Computer, 72 Bit D.P. Decomposition (Sol.8)	.15093	.05592	.11744	.33419	.44176	.59134	.59943	4a
	.15093	.05592	.11744	.33419	.44176	.59134	.59943	4b
Level 17.5 NASA/GODDARD NASTRAN, DEC-VAX Computer, 64 Bit D.P. Decomposition (Sol.8)	.14389	.06016	.12378	.29474	.40715	.58416	.59813	5a
	.14385	.06717	.11805	.24716	.32194	.50502	.56985	5b
Level 48B MSC-NASTRAN CDC CYBER 175 Computer, 60 Bit S.P. Decomposition (Sol.8)	.15093	.05591	.11744	.33419	.44176	.59134	.59942	6
Level 52 MSC-NASTRAN, DEC-VAX Computer, 64 Bit D.P. Decomposition (Sol.8)	.15093	.05592	.11744	.33419	.44176	.59134	.59942	7
Level 60 MSC-NASTRAN, DEC-VAX Computer, 64 Bit D.P. Decomposition (Sol.8)	.16173	.05591	.11745	.33203	.43973	.59726	.59972	8
Level 60 MSC-NASTRAN, DEC-VAX Computer, 64 Bit D.P. Decomposition (Sol.26) (4 runs on same data)	.15304	.05422	.11353	.28611	.44209	.55800	.50161	9a
	.15100	.05838	.11587	.33329	.44178	.59291	.60261	9b
	.15095	.05592	.11822	.33563	.44131	.59211	.59944	9c
	.15093	.05592	.12031	.33385	.45400	.59269	.59942	9d