SOLUTION OF ENFORCED BOUNDARY MOTION IN DIRECT TRANSIENT AND HARMONIC PROBLEMS

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INTRODUCTION

The current versions of NASTRAN, i.e., NASA, MSC, and MAC support non-zero boundary displacements only in the static analysis. Forcing functions in the dynamic analysis formats allow only forces and pressures to exercise the mathematical model. This limitation can be circumvented by the application of a DMAP alter sequence. For the direct harmonic problem, a simple change to module FRRD can be easily incorporated to effect a more efficient use of the code.

Let the equation of motion be written with the dynamic set of coordinates in partition form with subscript b as the boundary set and subscript c as the complimentary boundary set, i.e.,

$$\begin{bmatrix} m_{cc} & m_{cb} \\ m_{bc} & m_{bb} \end{bmatrix} \begin{Bmatrix} \ddot{X}_c \\ \ddot{X}_b \end{Bmatrix} + \begin{bmatrix} d_{cc} & d_{cb} \\ d_{bc} & d_{bb} \end{bmatrix} \begin{Bmatrix} \dot{X}_c \\ \dot{X}_b \end{Bmatrix} + \begin{bmatrix} k_{cc} & k_{cb} \\ k_{bc} & k_{bb} \end{bmatrix} \begin{Bmatrix} X_c \\ X_b \end{Bmatrix} = \begin{Bmatrix} \bar{P}_c \\ P_b \end{Bmatrix} + \begin{Bmatrix} P_{nl} \\ 0 \end{Bmatrix} \qquad (1)$$

where

m, d, k = mass, damping, and stiffness matrix coefficients

P, P_{nl} = linear and non-linear load vectors

Equation (1) is not solved by the direct transient or frequency formats when p_c, X_b, and therefore $\dot{X}_b$ and $\ddot{X}_b$, are known and P_b, X_c, and therefore $\dot{X}_c$ and $\ddot{X}_c$ are unknown. However, equation (1) can be rewritten in the form needed for solution by the standard NASTRAN modules. The first of these are:

$$[m_{cc}] \; \{\ddot{X}\} + [d_{cc}] \; \{\dot{X}\} + [k_{cc}] \; \{X_c\} = \{P_c\} + \{P_{nl}\} \tag{2}$$

where

$$\{P_c\} = \{\bar{P}_c\} + [m_{cb}] \; \{\ddot{X}_b\} + [d_{cb}] \; \{\dot{X}\} + [k_{cb}] \; \{X_b\}$$

By the use of the partitioning modules, the submatrices in Equations (1) or (2) are easily formed. By letting the boundary displacement vector be input through the FORCE or DLOAD cards, the force vector is actually identified as $\{P_b\} = \{X_b\}$ (or the first or second derivatives).

The formation of the load vector is different for the transient and harmonic cases. These issues will be discussed below. Somewhat independent of the problem is the requirement that the solution vector to be processed by the remaining modules must be of the dimensions of the "d" set. By using once more partitioning vectors and the MERGE module, the solution vector $\{X_c\}$, and in the transient case $\{\dot{X}_c\}$ and $\{\ddot{X}_c\}$, is merged with the boundary vector $\{X_b\}$ to form the dynamic vector $\{X_d\}$. With the "d" set solution vectors formed, the remaining DMAP sequence can be executed without NASTRAN knowing the difference.

In the case of harmonic analysis the non-linear force is zero and equation (2) becomes

$$(w^2 \; [m_{cc}] + iw \; [d_{cc}] + [k_{cc}]) \; \{X_c\} = \{P_c\} \tag{3}$$

where

w = circular frequency, $2 \pi f$.

HARMONIC ANALYSIS

The DMAP alter that was written to partition the matrix equation (1) into the form of equation (2) and then solve the lower order equation (3) is shown in Figure A-1. The following paragraphs discuss the steps involved.

1. FRRD calculates the load vector PDF and exits the module. The parameter ISKP is changed from -1 to a positive number to be transferred to FRRD the second time the module is executed. If the value of ISKP was set to zero, the default value, the module would have been executed normally. A normal

execution would give a solution to equation (1). The FORTRAN listing of module FRRD is shown in Figure A-2. The added code is underlined. Only the subroutines FRRD1A and FRRD1B are executed in this step.

2. The parameter ISKIP is saved for later use.

3. The partition vector DPAR is used to partition the stiffness matrix KDD. The submatrix identification is related to equation (2) by the following:

Figure A-1. DMAP Alter for Harmonic Response

```
ALTER  159.159
FRRD  CASEXX,USETD,DLT,FRL,GMD,GOD,KDD,BDD,MDD,DIT/UDVF,PSF,PDF,PPF/       (1)
         C,N,DISP/C,N,DIRECT/V,N,LUSETD/V,N,MPCF1/V,N,SINGLE/V,N,OMIT/
         V,N,NONCUP/V,N,FRQSET/V,N,ISKIP=-1/  $
SAVE  ISKIP  $                                                              (2)
PARTN  KDD,DPAR,/KD11,KD21,KD12,KD22/  $                                    (3)
PARTN  MDD,DPAR,/MD11,MD21,MD12,MD22/  $                                    (4)
PARTN  PDF,,DPAR/PD11,PD21,PD12,PD22/C,N,1  $                               (5)
MPYAD  KD11,PD21,PD11/P1DF/C,N,0/C,N,-1/  $                                 (6)
FRRD  CASEXX,USETD,DLT,FRL,GMD,GOD,GOD,KD11,,MD11,DIT/UIDVF,PSF,P1DF,       (7)
      PPF/C,N,DISP/C,N,DIRECT/V,N,LUSETD/V,N,MPCF1/V,N,MPCF1/V,N,SINGLE/
      V,N,OMIT/V,N,NONCUP/V,N,FROSET/V,N,ISKIP/  $
MERGE  KD11,KD21,KD12,KD22,DPAR,/KDD/  $                                    (8)
MERGE  MD11,MD21,MD12,MD22,DPAR,/MOD/  $                                    (9)
MERGE  U1DVF,PD21,PD22,,OPAR/UDVF/C,N,1  $                                  (10)

ENDALTER
CEND
```

Figure A-2. Listing of Module FRRD

LEVEL 2.2.1 (DFC 77)

```
ISN 0002          SUBROUTINE FRRD                                   00000010
            C                                                       00000020
            C     FREQUENCY AND RANDOM RESPONSE MODULE              00000030
            C                                                       00000040
            C     INPUTS CASECC,USETD,ULT,FRL,GMD,GOD,KDD,
                  BCD,MDD,PHIDH,DIT                                 00000050
            C                                                       00000060
            C     OUTPUTS UDV,PS,PD,MP                              00000070
            C                                                       00000080
            C     8 SCRATCHES                                       00000090
            C                                                       00000100
ISN 0003          INTEGER SINGLE,ONIT,CASECC,USETD,DLT,FRL,
                  GMD,GOD,BDD,PHIDH,DIT, 1 SCR1,SCR2,SCR3,
                  SCR5,SCR6,UDV,PS,PD,FP,PDD,OPTION                 00000120
ISN 0004          INTEGER SCP7,SCRB,NAME&2<,MCB&7>                  00000130
```

```
ISN  0006        INTEGER FOL                                          00000140
ISN  0006        COMMON/APP&2<,MODAL&2<,LUSETD,MULTI,SINGLE,
                 OMIT, NONCUP,FRQSET,                                 00000150
              1  ISKIP                                                00000155
            C                                                         00000160
ISN  0007        COMMON/FRRDST/OVF&150<,ICNT,IFRST,ITL&3<IDIT,
                 IFRD,K4DD                                            00000170
ISN  0008        DATA CASECC,USETD,DLT,FRL,GMD,GOD,KDD,HDD,
                 MOD,PHIDH,DIT/                                       00000180
                 1 101,102,103,104,105,106,107,108,109,110,111/       00000190
ISN  0009        DATA UDV,PS,PD,PP,PDD/201,202,203,204,203/           00000200
ISN  0010        DATA SCR1,SCH2,SCR3,SCR4,SCR5,SCR6 /301,302,
                 303,304,305,306/                                     00000210
ISN  0011        DATA SCR7,SSCR8/307.308 /                            00000220
ISN  0012        DATA MODA /4HMODA/                                   00000230
ISN  0013        DATA POL/205/                                        00000240
ISN  0014        DATA NAME /4HFRRD,4H /                               00000250
            C                                                         00000260
            C    BUILD LOADS ON P SET ORDER IS  ALL FREQ.
                 FOR LOAD TOGETHER                                    00000270
            C                                                         00000280
ISN  0015        IF ( ISKIP .GE. 0 ) GO TO 5                          00000281
ISN  0017        NLOAD = ISKIP / 2*16                                 00000282
ISN  0018        NFREQ = ISKIP - NLOAD/*2**16                         00000283
ISN  0019        GO TO 15                                             00000284
ISN  0020      5 CONTINUE                                             00000285
ISN  0021        CALL FRPDIA&DIT,FRL,CASECC,DIT,PF,LUSETD,
                 NFREQ,NLOAD,FRQSET,FOL,                              00000290
                 1 NOTRD<                                             00000300
ISN  0022        1F&MULTI.LT.O.AND.SINGLE,LT.O.AND.OMIT.L.T.O         00000310
                 AND.MODAL
                 1 & 1< .NF. MODA< GO TO 60                           00000320
            C                                                         00000330
            C    REDUCE LOADS TO D OR H SET                           00000340
            C                                                         00000350
ISN  0024        CALL FPRU14$PP.USETD,GMD,MULTI,SINGLE,OMIT,          00000360
                 MODAL&1<,PH1DH,PD,
                 1 PS,SCR5,SCR1,SCR2,SCR3,SCR4<                       00000370
ISN  0025        IF ( ISKIP .LT. 0 ) GO TO 40                         00000375
ISN  0027        15 CONTINUE                                          00000377
ISN  0028        IF ( MULTI .LT. 0 .AND.SINGLE.LT.O .AND.
                 OMIT .LT. O                                          00000378
                 . .AND. MODAL(1) .NE. MODA ) POD = PD                00000379
            C                                                         00000380
            C    SCR5 HAS PH IF MODAL FORMULATION                     00000390
            C                                                         00000400
ISN  0030        IF &MODAL&1< .EQ.MODA< PDD #SCR5                     00000410
            C                                                         00000420
            C    SOLVE PROBLEM FOR EACH FREQUENCY                     00000430
            C                                                         00000440
ISN  0032        IF&NONCUP .LT. 0 .AND. MODAL&1< .EQ. MODA<           00000450
```

```
                   GO TO 50
ISN 0034         10 IF&FREQ .EQ. 1 .OR. NLOAD .EQ 1< SCR6 # UDV
ISN 0036           DO 20 1#1,NFPEQ
ISN 0037           CALL KLOCK&LOCK&ITIME1<
              C
              C    FORM AND DECOMPOSE MATRICES
              C
ISN 0038           CALL FRRD1C&FRL,FROSET,MDD,RDD,KDD.1,SCR1,
                   SCR2,SCR3,SCR4,SCR8,
                   1 SCP7.1GOOD<
              C
              C    ULL IS ON SCR1 -- LLL IS IN SCR2
              C
              C    SOLVE FOR PD LOADS STACK ON SCR6
              C
              C
ISN 0039           CALL FRRD1D&PDD,SCR1,SCR2,SCR3,SCR4,SCR6,
                   NLOAD,1GOOD,NFREQ<
ISN 0040           CALL KLOCK&ITIME2<
ISN 0041           CALL IMTOGO&ITLEF1<
ISN 0042           IF&2*&ITIME2-ITIME1<.GT. ITLEFT .AND. I .NE.
                   NFREQ< GO TO 70
ISN 0044       20  CONTINUE
ISN 0045         1 # NFREQ
ISN 0046       30 CONTINUE
ISN 0047          IF&NFREQ .EQ. 1 .OR. NLOAD .EQ 1< GO TO 40
             C
             C    RESORT SOLUTION VECTORS INTO SAME ORDER AS LOADS
             C
ISN 0049          CALL FRRD1E&SCR6,UDV,NLOAD,I<
ISN 0050          40 ISKIP = NFREQ +NLOAD*2**16
ISN 0051          RETURN
             C
             C    UNCOUPLED MODAL
             C
ISN 0052          50 CALL FRRD1F&MDD,HDD,KDD,FRL,FRQSET,NLOAD,
                  NFREQ,PDD,UDV<
ISN 0053          GO TO 40
ISN 0054          60 PDD # PP
ISN 0055          GO TO 10
             C
             C    INSUFFICIENT TIME TO COMPLETE ANOTHER LOOP
ISN 0056          70 CALL MESAGE&.5.NFREQ-I,NAME<
ISN 0057          MCA&1< # SCR6
ISN 0058          CALL RDTFL&MCA*1<<
ISN 0059          MDONE # MCD&2<
ISN 0060          MCR&1< # PP
ISN 0061          CALL ROTR1&MCH&1<<
ISN 0062          MCR&2< NOONF
ISN 0063          CALL WRT1FL&MCB&1
```

```
ISN  0064        IF&SINGLE .LT. 0< GO TO 80                    00000900
ISN  0066        MCA&1< # PS                                   00000910
ISN  0067        CALL PUTRL&MCA&1<<                            00000920
```

K_{dd} = KD11
K_{cb} = KD12
K_{bc} = KD21
K_{bb} = KD22

4. The partition of the mass matrix, MDD, is similar to the stiffness matrix.

5. Because the load vector is calculated for all frequencies and loading conditions at once, PDF is a load matrix, a load vector in each column. The partition vector DPAR is used again to separate the enforced displacements from the forces. The relationship to equation (2) is

P_c = PD11
P_b = PD21

6. The module MPYAD performs the matrix multiplication and additions required by equation (2). Here

P_c = P1DF

7. Module FRRD is executed again, but this time the parameter ISKIP is positive. A jump to statement 15, underlined in Figure A-2, causes only the subroutines FRRD1C, FRRD1E and FRRD1F to be executed. The solution to equation (3) is obtained in this step. The code uses the following names related to equation (3).

M_{cc} = MD11
K_{cc} = KD11
P_c = P1DF
X_c = U1DF

8. The stiffness matrices are merged to form the system stiffness matrix. This is the inverse of operation 3.

9. Similar to the stiffness matrix, this operation is the inverse of operation 4.

10. Merges the solution vector X_c of equation (6-7) with X_b to form the system solution vector X_d.

The three merges, operations 8, 9, and 10, are made necessary because NASTRAN uses the displacement approach to the problem solution. In order to calculate stress and forces in the members, the solution vector must contain all grid points.

TRANSIENT ANALYSIS

The DMAP Alter required for the Rigid Format 9, Direct Transient Response, is shown in Figure A-3. The discussions below relates to the circled numbers in the DMAP listing.

1. The Stiffness matrix is partitioned in accordance with Equation (2) where

$$KD11 = K_{cc}$$
$$KD12 = K_{cb}$$
$$KD21 = K_{bc}$$
$$KD22 = K_{bb}$$

2. The Mass matrix is partitioned similar to the Stiffness matrix

$$\left[MDD \right] = \left[\begin{array}{c|c} MD11 & MD12 \\ \hline MD21 & MD22 \end{array} \right]$$

Figure A-3. DMAP Alter to Rigid Format 9

```
ALTER   163
PARTN   KDD,OPAR,/KD11,KD21,KD12,KD22/  $                                  (1)
PARTN   MDD,OPAR,/MDLL,MD21,MD12,MD22/  $                                  (2)
PARTN   PD,OPAR/PD11,PA21,PD12,PD22/C,N,1  $                               (3)
MPYAD   PA21,MV1,/PBT21/C,N,O/C,N,1/C,N,O/C,N,2  $                    (4) (5)
ADD     PBT21,PA21/PB21/C,Y,ALPHA=(0.550E-2.0)/C,Y,BETA=(0.550E-2.0)$      (6)
MPYAD   PB21,MAIT,/PV21/C,N,O/C,N,1/C,N.O/C,N,2  $                         (7)
MPYAD   PV21,MV1,/PCT21/C,N,O/C,N,1/C,N,O/C,N,2  $                         (8)
ADD     PCT21,PV21/PC21/C,Y,ALPHA=(0.550E-2.0.)/C,Y,BETA=(0.550E-2.0)$     (9)
MPYAD   PC21,MAIT,/PU21/C,N,O/C,N,1/C,N,O/C,N,2  $                         (10)
MPYAD   KD12,PU21,PD11/P1D/C,N,O/C,N,1 /$                                  (11)
ALTER   165,165
TRD     CASEXX,TRL,NLFT,DIT,KD11,MD11,PID/UIDVT,P1LD/C,N,DIRECT/
        V,N,NOUE/V,N,NONCUP/V,N,NCOL $                                     (12)
ALTER   166
MERGE   KD11,KD21,KD12,KD22,OPAR/KDD/  $                                   (13)
MERGE   MD11,MD21,MD12,MD22,OPAR,/MDD/  $                                  (14)
MERGE   PD11,PILD,PD12,PD22,,OPAR/PNLD/C,N,1  $                            (15)
PARTN   PA21,PVA,/A21,,PDA12,/C,N,1  $                                     (16)
PARTN   PV21,PVA,/V21,,PDA12,/C,N,1  $                                     (17)
PARTN   PU21,PVA,/U21,,PDA12,/C,N,1  $                                     (18)
MERGE   A21,,V21,,PVVA,/PVA21/C,N,1  $                                     (19)
MERGE   PVA21,,U21,,PVUVA,/PUVA21/C,N,1  $                                 (20)
MERGE   U1DVT,PUVA21,,,,DPAR/UDVT/C,N,1  $                                 (21)
ENDALTER
```

3. The load vector, PD, which is output from module TRLG, is partitioned according to Equation (2), where

$$PD = \{P(t_1)\}, \{P(t_2)\}, \ldots$$

$$PD11 = \{\bar{P}_c(t_1)\}, \{\bar{P}_c(t_2)\}, \ldots$$

$$PA21 = \{P_b(t_1)\}, \{P_b(t_2)\}, \ldots$$

Note that PD is a matrix formed by columns of load vectors, one column for each time step. The matrices PD22 and PD12 are not generated, i.e.

$$PD = \begin{bmatrix} PD11 \\ \hline PA21 \end{bmatrix}$$

4. Direct input matrices, MV1 and MA1T, are used subsequently to calculate the velocity and displacement matrices from the acceleration matrix. The forms of MV1 AND MA1T are

$$MV1 = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & 1 & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & \cdot & & & \end{bmatrix}}_{N+2} \updownarrow M$$

$$MA1T = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 & \cdot & \cdot & \cdot \\ 0 & 1 & 1 & 1 & \cdot & \cdot & \cdot \\ 0 & 0 & 1 & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & \cdot & & & \end{bmatrix}}_{N+2} \updownarrow M$$

The dimensions of both matrices are M X N + 2 where M is the number of coordinates in the b-set and N is the number of time steps.

5. Produces the matrix product

$$[PBT21] = [PA21]*[MV1]$$

$$= [\{P_b\ (t_1)\}, \{P_b\ (t_2)\}, \ldots] \begin{bmatrix} 0 & 1 & 0 & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & 1 & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & \cdot & & & \end{bmatrix}$$

$$= [0, \{P_b\ (t_1)\}, \{P_b\ (t_2)\}, \ldots]$$

It is seen that this operation moves the columns of the acceleration vectors from time t_i to t_i+1.

6. Produces the matrix sum

$$[PB21] = \alpha[PBT21] + \beta[PAZ1]$$

The coefficients α and β are set equal to one-half of the integration time step, Δt:

$$[PB21] = \frac{\Delta t}{2}[\{P_1 + P_2\}, \{P_2 + P_3\}, \ldots]$$

$$=[\{\Delta V_1\}, \{\Delta V_2\}, \ldots]$$

where $\{P_i\} = \{P_c(t_i)\}$; i = 1 to N + 2

The matrix PB21 represents the change in velocity, ΔV_i, between time steps, t_i and t_{i+1}. The calculation is based on the trapesoidal rule for numerical integration.

7. The final step in producing the matrix of velocity vectors, PV21 from the matrix of acceleration vectors, PA21, this module produces the matrix product

$$[PV21] = [PB21]\ [MA1T]$$

$$= [\{\Delta V_1\}, \{\Delta V_2\}, \ldots] \begin{bmatrix} 1 & 1 & 1 & 1 & . & . \\ 0 & 1 & 1 & 1 & . & . \\ 0 & 0 & 1 & 1 & . & . \\ 0 & 0 & 0 & 1 & . & . \\ . & . & . & . & & \\ . & . & . & . & & \end{bmatrix}$$

$$= [\{\Delta V_1\}, \{\Delta V_1 + \Delta V_2\}, \{\Delta_1 + \Delta_2 + \Delta V_3\}, \ldots]$$

8., 9. A repeat of operations e, f, g. The matrix of displacement
and 10. PU21, is calculated from the matrix of velocity vectors, PV21.

11. The load vector is calculated in accordance with Equation (2).

KD12 = K_{cb}

PU21 = $\{X_b\}_1, \{X_b\}_2, \ldots$

PD11 = $\{\bar{P}_b\}_1, \{\bar{P}_b\}_2, \ldots$

P1D = $\{P_c\}_1, \{P_c\}_2, \ldots$

12. The module TRD calculates the solution to Equation (2).

KD11 = [Kcc]

MD11 = [Mcc]

P1D = $P_1, P_2, \ldots$

$$U1DVT = \begin{bmatrix} X & X & \\ \dot{X} & \dot{X} & \\ \ddot{X}_1, & \ddot{X}_2, & \cdots \end{bmatrix}$$

$$[P1LD] = \{P_{\eta\ell}\} .$$

The solution vector, U1DVT, is a matrix of displacements, velocity and acceleration vectors for each grid point; a column for each time step.

13. The system stiffness matrix is formed

$$\begin{bmatrix} KD11 & KD12 \\ \hline KD21 & KD22 \end{bmatrix} = [KDD]$$

14. The system mass matrix is formed similar to the operation (13.)

15. The system load vector is formed

$$\begin{bmatrix} PD11 \\ \hline P1LD \end{bmatrix} = [PNLD]$$

16., 17. and 18. Partition the acceleration, PA21, velocity, PV21, and displacement, PU21, matrices to the correct size to be merged with U1DVT.

19., 20. and 21. These operations merge the solution matrix, UD1VT, with the excitation matrix, PUVA21, to form the final system solution matrix, UDVT.

$$\begin{bmatrix} UD1VT \\ \hline PUVA21 \end{bmatrix} = [UDVT]$$

From this point on, the solution is calculated according to the Standard Rigid Format 9 procedure.