

ON THE APPLICATION OF NONLINEAR LOAD ELEMENTS  
TO THERMAL ANALYSES USING THE NASTRAN THERMAL ANALYZER

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SUMMARY

Using the nonlinear load elements in thermal analysis to simulate an undocumented nonlinear thermal boundary condition is presented. The treatment of the nonlinearity arising from the temperature-dependent convective film coefficients is shown in detail. As an illustration, emphasis is placed on the modeling techniques and their interrelationships with the solution accuracy as affected by a specific integration algorithm of the transient thermal analysis used in the NASTRAN Thermal Analyzer. Briefly shown is the underlying theory on which the maneuvering of terms pertinent to the modeling depends. This demonstration provides some insight into the intricacies of the method that would be general to all applications. A recommendation is also made to modify a nonlinear load element that will enhance the solution capability and broaden the scope of application.

INTRODUCTION

The known solution capabilities of the NASTRAN Thermal Analyzer (NTA) have been well documented (ref. 1 and 2), and a large part of them have been demonstrated in detail (ref. 3). Despite their frequent encounterment in engineering applications, certain nonlinear solution capabilities have not been provided by the NTA in the transient-state thermal analysis (APP HEAT, SOL 9). Those excluded are, for instance, the temperature-dependent thermophysical properties, temperature-dependent convective film coefficients, etc. Their absence from the original NTA's capability list was solely attributed to the fact that the required computer processing time (CPU and I-O times) would be excessive. Computations involving the reassemblages of the thermal conductance matrix, together with the decompositions of the relevant matrices that contain the nonlinear thermophysical properties, would have to be repeated for all steps of the time increment in integration. The program shortcomings, however, may be compensated by an appropriate application of the available nonlinear load elements (NLLEs) in NASTRAN, although some elements were originally developed for structural analyses. The intrinsic modular structure of NASTRAN permits them to be accessible to the NTA. Those NLLEs, obviously, can function as nonlinear thermal loads, and they can be employed to simulate other complex physical phenomena as well. Otherwise, new modules would have to be developed and implemented, even though an excessive computer processing time was tolerable. Such an alternative would be an expensive and time-consuming

proposition. The user would be deprived of immediate solution meanwhile.

Six NLEs are currently available in NASTRAN, namely NOLIN1\*, NOLIN2, NOLIN3, NOLIN4, NOLIN5 and NFTUBE. The last two were developed specifically for thermal analyses: NOLIN5 is capable of simulating temperature sensitive thermal radiation surface properties, such as a louver (ref. 4), and NFTUBE is capable of dealing with thermal energy transfer between the wall and the fluid flowing inside a tube (ref. 5). The remainders are four nonlinear elements of general type capable of accommodating four different functions with a temperature or heat flux being the argument. NOLIN1 allows to specify a table through TABLEDi ( $i = 1,2,3,4$ ), and it is the most versatile NLE in NASTRAN.

The main purpose of this paper is to illustrate how to acquire a non-routine, but essential solution capability by an application of the NLEs through modeling. For a systematic demonstration, NOLIN1 was employed to simulate the temperature-dependent convective film coefficients,  $h(T)$ . The importance of this problem is evident in view of a variety of applications ranging from a domestic heating system, a hot gas flowing through a nozzle, a fluid flow in a nuclear reactor, and a flow in a wind tunnel during the start-up to a thermal ablation including a receding surface. They are all characterized by such a nonlinear boundary condition. More specifically, treatments of the temperature-dependent film coefficients based on one and two reference temperatures are given in detail. The dilemma arising from the presence of the MPC to establish an average temperature of two reference temperatures and the reference with that average temperature by the NLEs was resolved by a trick modeling. A brief review of the theoretical aspect serves not only to provide a basis for the maneuvering in modeling and to provide insight into the intricacies of the method, but to amplify basis of modeling techniques that are equally valid to other applications. A recommendation for the improvement of the NOLIN2 is outlined. It would enhance solution capabilities and broaden the scope of engineering application. Guidelines for the user to apply the NLEs in thermal analysis are provided.

#### THE DEMONSTRATIVE PROBLEM

For the convenience of easy reference and discussion, and later on, to demonstrate the modeling techniques, a relatively simple problem without losing any generality has been designed, and it is described below:

The transient temperature responses in a composite slab (pipe) of an infinite extent are to be determined. Initially, the slab has a uniform temperature of  $293^{\circ}\text{K}$  ( $20^{\circ}\text{C}$ ) throughout its body. The boundary surface of the metallic layer is in contact with a stream of hot gas maintained at  $1422^{\circ}\text{K}$  ( $1149^{\circ}\text{C}$ ) from time  $t = 0$ . The dimensions and thermophysical properties of the slab are tabulated in Table 1. The system depiction, together with its finite element model, is shown in Figure 1.

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\*The names of actual NTA cards are capitalized.

## THE THEORETICAL CONSIDERATION

The success of attempting a solution utilizing any NLEs to simulate a problem with nonlinear thermophysical properties relies upon the following factors:

- (1) The nonlinear terms, arising from the temperature-dependent thermophysical properties, which are embraced in the square matrices of the thermal conductance and/or the thermal capacitance, must be extractable and separable from their normal positions. The resulting products of the temperature-dependent coefficients ( $kA/l$  and  $hA$  or  $C$ ) and their respectively associated variables ( $T$  or  $\dot{T}$ ) must be transposed and merged into the nonlinear thermal load vector.
- (2) An NLE of an appropriate form suited to accommodate the transposed quantities must be selected. Any restraints associated with the use of the NLEs must be strictly observed.
- (3) A full acquaintance with the characteristics of each NLE and the integration algorithm for the transient heat transfer analysis (APP HEAT, SOL 9) is essential in controlling and accomplishing desired solution accuracy, stability and efficiency at wish.

To serve as the theoretical basis for such a maneuvering, a brief review of pertinent mathematical operations is in order.

A transient thermal analysis in the NTA is based on the general heat equation in the matrix form as follows (ref. 1):

$$[C] \{ \dot{T} \} + [K] \{ T \} = \{ Q^l \} + \{ Q^n \} \quad (1)$$

where

- $T$  = the vector of temperatures at grid points
- $\dot{T}$  = the rate change of temperature vector
- $K$  = the thermal conductance matrix of constant elements
- $C$  = the thermal capacitance matrix of constant elements
- $Q^l$  = the vector of applied linear thermal loads
- $Q^n$  = the vector of applied nonlinear thermal loads

The convective coupling terms are normally incorporated with the thermal conductive couplings in the thermal conductance matrix  $[K]$  regardless of their dependency. When the convective film coefficient is temperature-dependent, the convective coupling terms embracing  $h(T)$  have to be separated from their original  $[K_c]$ , and thus  $[K_c] \{ T \}$ . The extracted terms being a new vector  $\{ H(T) \}$  are transposed to the other side of the equal sign in equation (1), and they become or merge with the vector of nonlinear thermal loads,  $\{ Q^n \}$ . The rearranged new equation conforms exactly to equation (1), meaning that it is then within the solution capability of the NTA. Mathematically, it can be expressed by

$$[K_0(k, h(T))]\{T\} = [K_1(k)]\{T\} + \{H(T)\} \quad (2)$$

The integration algorithm for equation (1) is based on the modified Newmark Beta Method (ref. 6). It is a forward and backward differencing method with a parameter  $\beta$  that enables the user to select a value in the range of  $0 < \beta < 1$ . When equation (1) is transformed into such a difference expression, it reads (ref. 1):

$$\begin{aligned} & [K]\{\beta T_{n+1} + (1-\beta)T_n\} + \frac{1}{\Delta t}[C]\{T_{n+1} - T_n\} \\ & = \{\beta Q_{n+1}^l + (1-\beta)Q_n^l\} + (1+\beta)\{Q_n^n\} - \beta\{Q_{n-1}^n\} \end{aligned} \quad (3)$$

the subscript  $n$  refers to the  $n$ th time step. The accuracy and stability of solution are closely related to  $\beta$ . Equation (3) can be rearranged to give

$$\begin{aligned} & \left[ \frac{1}{\Delta t}[C] + \beta[K] \right] \{T_{n+1}\} = \left[ \frac{1}{\Delta t}[C] - (1-\beta)[K] \right] \{T_n\} \\ & + \{\beta Q_{n+1}^l + (1-\beta)Q_n^l\} + (1+\beta)\{Q_n^n\} - \beta\{Q_{n-1}^n\} \end{aligned} \quad (4)$$

The matrix  $[1/\Delta t C + \beta K]$  is decomposed into its triangular factors and solved at each time step via the forward and backward substitution process. If  $[K]$  contains terms of  $k(T)$  or  $h(T)$ , a search from the tables and replacement of new values based on  $\{T_n\}$  have to be carried out for all terms residing inside those two square matrices that are associated with  $\{T_{n+1}\}$  and  $\{T_n\}$ . Consequently, the matrix decomposition has to be performed for each time step. This requires a very excessive computer processing time. However,  $[K]$  needs be evaluated only once if its constituent terms are all constant. This situation is equally valid in  $[C]$ , although  $C(T)$  is not considered in this study.

Relating the terms of nonlinear thermal load to the specific demonstrative problem, it is obtained from equations (1) and (2) that

$$\{Q^n\} = -\{H(T)\} = \begin{Bmatrix} H_1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ H_{200} \end{Bmatrix} \quad (5)$$

where

$$\begin{cases} H_1 = h(T_r)AT_1 \\ H_{200} = -h(T_r)AT_{200} \end{cases} \quad (6)$$

and  $h(T_r)$  denotes the dependency of the convective film coefficient in a reference temperature  $T_r$  which can be the wall temperature or an average temperature of the free stream and the wall temperature as will be discussed and treated in following sections. In the NTA, the NOLIN1 card defines an NLLE of the form

$$Q_i^n(t) = S_i F(T_j(t)) \quad (7)$$

where  $Q_i^n(t)$  is the load being applied to the GRID  $i$  at time  $t$ ,  $S_i$  is a scale factor.  $F(T_j(t))$  denotes a tabulated function in  $T_j$  to accommodate  $H_1(T_j)$  and  $H_2(T_j)$  as defined in equation (6). The equivalent nonlinear thermal load functions are entered through a TABLEDi ( $i = 1, 2, 3, 4$ ) card.  $T(t)$  is any permissible time-varying temperature on which the functions depend. Other NLLEs, NOLIN2, NOLIN3 and NOLIN4, can be selected as long as their specific forms are suitable for respective applications.

#### THE CONVECTIVE FILM COEFFICIENT $h(T_r)$

The convective film coefficient  $h$  governs the magnitude of heat transfer between the hot gas and the boundary surface of structure.  $h$ , in general, can be expressed as  $h(T_r)$  where the reference temperature  $T_r$  may refer to only one temperature, such as the wall temperature, or more than one temperature through an intermediary, such as the average value of the free stream temperature and the surface temperature of the wall. A typical example is a fully developed pipe flow. The expression for  $h$  is (ref. 7)

$$h = 0.023 \frac{k_\infty}{D_h} (\text{Re})^{0.8} (\text{Pr})^{0.4} \quad (8)$$

For large temperature differences, a mean temperature is used in evaluating all physical properties except the flow velocity. This procedure would involve very tedious interrelations. To simplify computations, it is desirable to separate the temperature-dependent from the independent factors. An alternative form is, therefore, obtained by an application of the equation of state and the continuity equation, which leads to

$$h = 0.023 C_{p\infty} \mu_\infty^{0.2} \text{Pr}_\infty^{-0.6} G_\infty^{0.8} D_h^{-0.2} \left(\frac{T_w}{T_\infty}\right)^{0.8} \quad (9)$$

As the free stream temperature  $T_\infty$  is known, all quantities in the preceding equation except  $T_w$  are constants.  $h$  is, therefore, a function of  $T_w$ . For the problem of concern, it is  $h = h(T_1)$ . The values of this function obtained by using equation (9) are listed in Table 2. Similar expression, but referencing with an average value of the free stream temperature and the surface temperature is also widely used in engineering practices. For this demonstrative problem, it is  $h = h(T_r)$  and  $T_r = \frac{1}{2}(T_1 + T_{200})$ . The values of  $h$  vs  $T_r$  are given in Table 3.

## THE MODELING TECHNIQUES AND DISCUSSION

A one-dimensional finite element model is sufficient to represent the demonstrative problem, and this model is shown in Figure 1. A listing of the input data deck is reproduced in Figure 2. With comments appropriately inserted, functions regarding various segments of the deck are self-explanatory. The space bounded by two rows of asterisks is reserved for adding a packet of cards which simulate the effect of the convective film coefficients. Three cases of  $h$  are considered: (1)  $h$  of a constant value, (2)  $h = h(T_1)$ , and (3)  $h = h(T_r)$  with  $T_r = \frac{1}{2}(T_1 + T_{200})$ .

### h of a constant Value

To assure correctness of modeling and to evaluate the obtainable accuracy of solution by the nonlinear load approach, the case of a constant film coefficient with  $h = 0.467 \text{ W/cm}^2\text{-K}$  was tested first. Because, this case is well within the documented capability of the NTA. It can be accomplished using a train of cards CHBDY, PHBDY and MAT4 by the conventional modeling method. The temperature results so obtained serve as a base to compare with that obtained by the nonlinear load approach. The two packets of input data cards, whose images are shown in Figures 3(a) and 3(b), were used in respective computer runs. The equivalent nonlinear thermal loads for the pair of NLEs through NOLIN1 405 with TABLED1 4051 and TABLED1 4052 are given in Table 4. The effect of selecting the time-step size  $\Delta t$  for integration on solution accuracy was also studied.

Temperature results summarized in Table 5 with the headings of Cases 1 and 3 are solutions yielded by these two approaches. In both cases, the same time-step size (the  $\Delta t$ -set (a) in Figure 4) was used. The largest difference between the two temperature profiles occurs at  $t = 0.25 \text{ sec.}$ , suggesting that a refinement of the time-step size is warranted. To maximize the efficiency of the computer processing time, refinements of  $\Delta t$  were made only in the beginning segments of time. Three sets of time-step size, which are presented in the NTA TSTEP card format and identified as the  $\Delta t$ -sets (a), (b) and (c), are shown in Figure 4. A total of five cases were computed, two (Cases 1 and 2) by the conventional modeling method and the other three (Cases 3, 10 and 11) by the nonlinear load approach. Temperature results are tabulated in Table 5 for comparison. It is evident that both modeling methods can yield comparable temperature solutions with a desired accuracy, provided an appropriate  $\Delta t$ -set is used (e.g., to compare Case 2 with Case 11).

This test gave confidence in the nonlinear load approach and verified the adequacy of the two sets of time-step size, namely the  $\Delta t$ -sets (b) and (c). They were, therefore, selected in all following computations. Inevitably, efficiencies of those runs using the  $\Delta t$ -sets (b) and (c) were penalized by a certain amount. They are reflected by the CPU and I-O times, which are also summarized in Table 5.

$$h = h(T_1)$$

The modeling procedure similar to that used in the preceding case of a constant  $h$  is equally applicable to this case of a convective film coefficient  $h$ , which is dependent on the surface temperature of the wall. The preparation of the equivalent nonlinear thermal loads, however, is much more involved. Computations of those equivalent loads were based on the expressions given in equation (6). The computational steps and results are detailed in Table 3. The relevant packet of input data cards is shown in Figure 3(c).

Two computer runs were executed with the specified  $\Delta t$ -sets (b) and (c). Table 6 displays their temperature results under the headings of Cases 12 and 13. Results are in excellent agreement, meaning that either  $\Delta t$ -set is satisfactory to render accurate solutions. The  $\Delta t$ -set (b) was then selected in studying the effect of the parameter  $\beta$  in the integration algorithm on temperature solutions.

The integration algorithm available in the NTA for its transient thermal analysis is a modified Newmark Beta Method. It uses a fixed time-step size, but permits different time-step sizes to be changed at discrete times upon the user's specification. To avoid excessive matrix decompositions, the number of changes should be kept low. This algorithm is known to be an efficient one for large systems of linear equations, but it is uncertain for problems with strong nonlinearities. As accuracy and stability of a solution are closely related to the parameter  $\beta$ , a numerical investigation with different values of  $\beta$  was conducted. The values of  $\beta$ , including 0, 0.25, 1 and 0.55 (the default value), were executed. Except the case of using  $\beta = 0.5$  (the central differences), whose temperature results are virtually identical to that of Case 13 (using  $\beta = 0.55$ ) and, therefore, eliminated from the tabulation in Table 6. Case 16 (using  $\beta = 1.0$ , the backward differences), yields comparable temperature results as that of Case 13. However, the solution instabilities caused both cases with  $\beta = 0.25$  and 0 (Euler integration) to terminate in their mid-courses. They are respectively labeled as Cases 17 and 18 and also included in Table 6.

$$h = h(T_r) \text{ with } T_r = \frac{1}{2}(T_1 + T_{200})$$

When the average temperatures  $T_r$  of the free stream temperature  $T_{200}$  and the surface temperatures of wall  $T_1$  are referenced by the temperature-dependent convective film coefficients  $h = h(T_r)$ , the MPC (Multipoint constraint) card is suited to provide such a relation. When  $T_{100} = T_r$  is assigned, the equivalent nonlinear thermal loads are functions of  $T_{100}$ , meaning that  $T_{100}(t)$  will be referenced by the nonlinear thermal load tables, if the modeling technique similar to that used in the two preceding cases is followed. Since  $T_{100}$  is constrained by an MPC, this modeling has violated one cardinal rule that must be strictly observed by a user in applying the nonlinear loads. The grid points to which the nonlinear loads are applied and the variables (either temperature or heat flux) on which they depend must not be variables eliminated by constraints, i.e., they must be in the solution set. To circumvent this dilemma, a fictitious point GRID 150 was created for  $T_{150}$ , and it was connected to  $T_{100}$  by a super conductor, so that  $T_{100}(t) = T_{150}(t)$ .  $T_{150}$  instead of  $T_{100}$  was then referenced by  $h(T_r)$ , i.e.,  $h(T_r) = h(T_{100}) = h(T_{150})$ , which, in turn, produced the equivalent nonlinear thermal loads. Figure 3(d) shows all relevant cards

and their relationships in detail, and the entered quantities in the nonlinear load tables are easily identifiable in comparison with those values given in Table 4.

The two  $t$ -sets (b) and (c) were again selected to execute the computer runs. Results with the labeling of Cases 14 and 15 are shown in Table 7. Temperatures are still overall in good agreement with each other, but they no longer show the same quality as evidenced in previous cases. A scrutiny of the results reveals that relatively large temperature differences exist between these two temperature profiles in the range of time from 1.0 to 3.0 sec. This is a typical example showing the intricacies of this method, the interplays of the selected time-step sizes with the integration algorithm, and the effect on temperature results due to the presence of the MPC.

Equation (4) shows that an inherent time lag error exists in applying the nonlinear thermal loads. They are computed for  $T$  of the  $(n+1)$ th time step based on temperature values at the  $n$ th and  $(n-1)$ th time steps. The larger the time-step size, the greater the deviation of the nonlinear thermal load from the desired value. When MPC is employed to relate a floating temperature (e.g.,  $T_1$ ) to a reference temperature (e.g.,  $T_{150}$ ) on which values of the nonlinear thermal loads depend indirectly, it introduces an additional step of time lag, and thus, an aggregated error for the nonlinear thermal load value. The errors are now induced by the time lag steps spanning from the  $(n+1)$ th to  $(n-2)$ th steps. Consequently, the selection of a finer time step size is appropriate in applying the nonlinear thermal loads, especially when an MPC is present to average two temperatures.

#### OTHER APPLICATIONS AND RECOMMENDATIONS

The modeling techniques of using the NLEs to simulate the temperature-dependent convective film coefficients have been methodically illustrated. The demonstrated equivalent nonlinear thermal loads are all functions of temperature. If a simulation calls for the nonlinear thermal load to be a function of heat flux, a combination of the EPOINT and TF cards, in addition to the NOLIN1s and TABLED1s, have to be used. In all cases, values for the equivalent nonlinear thermal loads must be prepared by the user manually. This is an error-prone and elaborate process inherent in using the NLEs. This deficiency, however, can be removed with only a relatively simple change made to the NOLIN1. It will further broaden its capability greatly. The recommendation is to modify the computer code pertinent to S, the fifth field, in the NOLIN1 card so that it will accept any time-varying temperature, whether specified or computed values at some grid points in the model. The expanded capabilities will include, for instance, the same demonstrative problem with the hot-gas temperatures being a specified time-dependent temperature function.



## CONCLUSIONS

The nonlinear load elements have been illustrated to simulate nonlinear boundary conditions, in particular the temperature-dependent convective film coefficients in transient thermal analyses. The following remarks also serve as guidelines for those who attempt to use the NLEs:

(1) Prior to applying the nonlinear thermal loads to an NTA model in a transient thermal problem, a trial run on a simplified model is advisable. Only the linear thermal loads equal in magnitude to that of the nonlinear load set are necessary to apply to this model, and a relatively coarse time-step size may be used in the execution. The profile of the temperature results will provide a good indication on which the final selection of appropriate time-step sizes in runs with the real NLEs depends.

(2) Taking into consideration of the time lag errors, very small time-step sizes are always desirable. However, it is not necessary to select the same fine size uniformly over the entire range of time for integration. To maximize the processing efficiency, the selection of nonuniform time-step sizes according to the results of the trial run is a sensible and practical approach. Very fine time-step sizes should be reserved to concentrate at which the temperature profile has the largest slope in its rate change of temperature curve.

(3) When MPC (also TF, if used) is present in the model for establishing a reference temperature that is referenced by the NLEs, such as NOLIN1, a fictitious super thermal conductor has to be used to connect to that original grid point of referencing. A new grid point at the other end of the super thermal conductor is then to be referenced by any NLEs so as not to violate the cardinal rule in applying the nonlinear loads.

(4) For solution stability, which also affects accuracy, the proper selection of a value for the parameter  $\beta$  of the integration algorithm depends on the degree of nonlinearity in presence. However, only the range of  $0.5 < \beta < 1$  should be considered.

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TABLE 1

DIMENSIONS AND THERMOPHYSICAL  
PROPERTIES OF THE COMPOSITE SLAB (PIPE)

Material	Stainless Steel (a)	Insulation-Phenolic (b)
Thermal Conductivity $k$ (W/cm-k)	0.16258	0.00502
Thermal Capacitance $\rho C_p$ (J/cm <sup>3</sup> -k)	4.029	1.725
Thickness $\Delta X$ (cm)	0.30	0.40
Cross-section $A$ (cm <sup>2</sup> )	1.0	1.0

TABLE 2

THE EQUIVALENT NONLINEAR LOADS FOR  
THE CASE OF  $h = 0.467 \text{ W/cm}^2\text{-k}$

T <sub>200</sub>	N <sub>200</sub> = hAT <sub>200</sub>	T <sub>1</sub>	-N <sub>1</sub> = hAT <sub>1</sub>
1422.0	664.074	293	136.831
↓	↓	589	275.063
		867	404.889
		1144	536.248
		1422	664.074

TABLE 3

THE EQUIVALENT NONLINEAR LOADS  
FOR THE CASE OF  $h = h(T_1)$

T <sub>1</sub>	h(T <sub>1</sub> )	-N <sub>1</sub> (T <sub>1</sub> ) = hAT <sub>1</sub>	T <sub>200</sub>	N <sub>200</sub> (T <sub>1</sub> ) = hAT <sub>200</sub>
293	0.132	38.676	1422	187.704
589	0.231	136.059	↓	328.482
867	0.314	272.238		446.508
1144	0.393	449.592		558.846
1422	0.467	664.074		664.074

TABLE 4

THE EQUIVALENT NONLINEAR LOADS  
FOR THE CASE OF  $h = h(T_{ave})$

$T_{200}$	$T_1$	$T_{ave}$	$h(T_{ave})$	$N_{200}(T_{ave}) = hAT_{200}$	$-N_1(T_{ave}) = hAT_1$
1422	293	857.5	0.132	187.704	113.190
↓	589	1005.5	0.231	328.482	136.059
	867	1144.5	0.314	446.508	272.238
	1144	1283.0	0.393	558.846	449.592
	1422	1422.0	0.467	664.074	664.074

TABLE 5

A COMPARISON OF TEMPERATURE RESULTS  
FOR THE CASES WITH A CONSTANT h

Time (sec)	Case No.				
	1	2	3	10	11
0	293.00	293.00	293.00	293.00	293.00
0.25	575.82	580.41	578.73	582.08	581.00
0.50	665.64	668.29	667.90	669.82	668.75
0.75	723.79	725.67	725.68	727.06	726.06
1.00	768.38	769.88	770.07	771.19	770.23
1.25	806.14	807.42	808.22	809.19	807.75
1.50	839.92	841.08	842.04	842.93	841.40
1.75	871.04	872.11	873.17	873.99	872.42
2.00	900.10	901.10	902.23	903.00	901.41
3.00	1000.39	1001.21	1003.56	1004.18	1002.48
4.00	1080.22	1080.90	1083.37	1083.86	1082.23
5.00	1143.93	1144.50	1146.93	1147.32	1145.81
6.00	1194.83	1195.29	1197.61	1197.92	1196.53
7.00	1235.52	1235.90	1238.05	1238.29	1237.04
8.00	1268.08	1268.39	1270.35	1270.55	1269.43
At-set	(A)	(C)	(A)	(B)	(C)
CPU/I-O (min)	0.125/1.170	0.189/1.133	0.154/1.309	0.177/1.368	0.198/1.256

TABLE 6

A COMPARISON OF TEMPERATURE RESULTS  
FOR THE CASES WITH  $h = h(T_1)$

Time (sec)	Case No.				
	12	13	16	17	18
0	293.00	293.00	293.00	↓	↓
0.25	401.14	401.27	401.06		
0.50	445.89	446.07	445.82		
0.75	478.54	478.76	478.61		
1.00	505.58	505.83	505.82		
1.25	529.69	530.07	530.17		
1.50	552.17	552.62	552.80		
1.75	573.65	574.15	574.40		
2.00	594.44	594.99	595.28		
3.00	673.30	673.96	674.38		
4.00	746.00	746.73	747.09		
5.00	813.37	814.15	814.46		
6.00	875.67	876.52	876.80		
7.00	929.48	930.38	930.66		
8.00	977.03	977.96	978.18		
$\Delta t$ -sec	(C)	(B)	(B)		
CPU/I-O (min)	0.224/1.487	0.200/1.606	0.192/1.615	-	-
Parameter $\beta$	0.55	0.55	1.0	0.25	0

TABLE 7

A COMPARISON OF TEMPERATURE RESULTS  
FOR THE CASES WITH  $h = h(T_{ave})$

Time (sec)	Case No.	
	14	15
0	293.00	293.00
0.25	350.94	351.29
0.50	387.00	387.63
0.75	420.17	420.99
1.00	453.04	454.05
1.25	486.41	488.16
1.50	521.83	524.19
1.75	559.79	562.77
2.00	600.52	603.20
3.00	724.14	725.32
4.00	832.04	833.01
5.00	928.13	928.48
6.00	1006.74	1006.81
7.00	1075.18	1075.07
8.00	1135.05	1134.79
$\Delta t$ -sec	(B)	(C)
CPU/I-O (min)	0.192/1.661	0.235/1.490

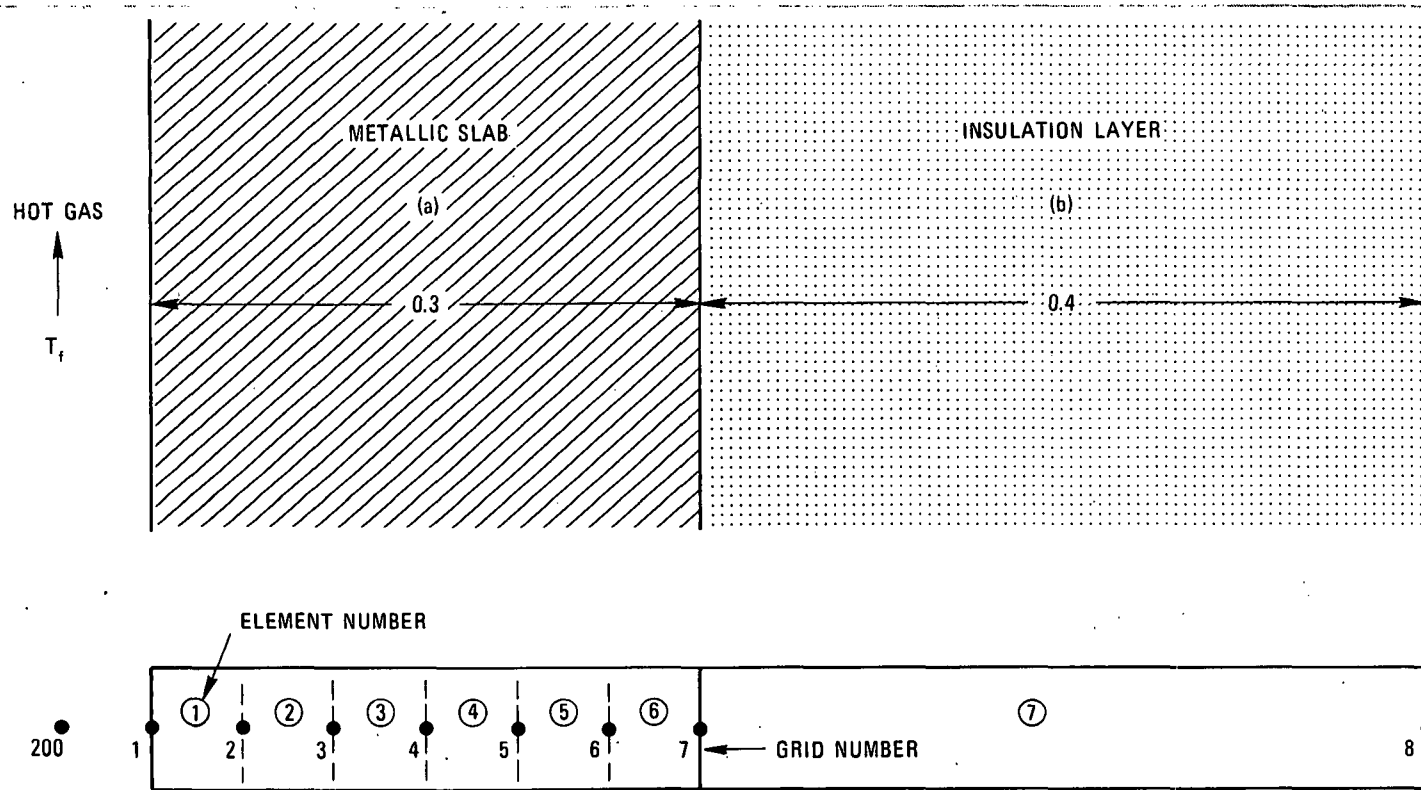


Figure 1 - The Composite Slab (Pipe) and its Finite Element Model.



```

$ EXECUTIVE CONTROL DECK
ID H P LEE, NASA-GSFC, SPACE TECHNOLOGY DIVISION
APP HEAT
SOL 59
CEND
$ CASE CONTROL DECK
ECHO=BOTH
LINES=75
TITLE= DEMONSTRATION OF TEMPERATURE-DEPENDENT CONVECTIVE FILM COEFFICIENT
SUBTITLE= WITH A CONSTANT HOT GAS TEMPERATURE AT GRID 200
MPC=305
DLLOAD=401
NONLINEAK=405
TSTEP=601
IC=701
TEMP(MATERIAL)=705
OUTPUT
SET 20=1,2,200
ULOAD=20
ELFORCE=20
NLLOAD=20
SET 40= 1,4,7,8,100,150,200
THERMAL= 40
BEGIN BULK
$ BULK DATA DECK
$ GRID POINTS
GRID 1 0.00
GRID 2 0.05
GRID 3 0.10
GRID 4 0.15
GRID 5 0.20
GRID 6 0.25
GRID 7 0.30
GRID 8 0.70
GRID 200 -1.0
$ CONDUCTION ELEMENTS
CRUD 1 101 1 2 101 2 3
CRUD 3 101 3 4 101 4 5
CRUD 5 101 5 6 101 6 7
PROD 101 102 1.0 6 6 101 6 7
MAT4 102 0.16258 4.029 8
CRUD 7 103 7 8
PROD 103 104 1.0
MAT4 104 0.00502 1.725
$ INITIAL CONDITIONS
TEMP 701 200 1422.0
TEMPD 701 293.0
$ ESTIMATED TEMPERATURES
TEMPU 705 1422.0
$ SPECIFIED CONSTANT TEMPERATURE FOR THE HOT GAS AT GRID 200
CELAS2 301 1.E6 200 1
DAREA 501 200 1 1422.E6
TLOAD2 401 501 0 0.0 1.E5 0.0 0.0 +LD1
+LD1 0.0 0.0
$ TIME INCREMENTS AND INTEGRATION STEPS
TSTEP 601 25 0.005 10 +TS1
+TS1 35 0.025 5 +TS2
+TS2 20 0.05 5 +TS3
+TS3 120 0.10 10
$ *****
$ *****
ENDDATA
/*

```

Figure 2 - A Listing of the Input Data Deck of the Demonstrative Problem

```

$ CONVECTIVE BOUNDARY IN CONTACT WITH HOT GAS
CHBDY 1001 1002 POINT 1
+CY 200
PHBDY 1002 1003 1.0
MAT4 1003 0.467
+CY

```

(a) A constant h by the conventional modeling technique

```

NOLIN1 405 1 1 -1.0 1 1 4051
TABLED1 4051
+TB1 293.0 136.831 589.0 275.063 867.0 404.889 1144.0 534.248 +TB1
+TB2 1422.0 664.074 ENDT
NOLIN1 405 1 1 664.074 1 1 4052
TABLED1 4052
+TB52 293.0 1.0 1450.0 1.0 ENDT
+TB52

```

(b) A constant h by the equivalent nonlinear thermal load approach

```

NOLIN1 405 1 1 1.0 150 1 4054
TABLED1 4054
+TB7 293.0 187.704 589.0 328.482 867.0 446.508 1144.0 558.846 +TB7
+TB8 1422.0 664.074 ENDT
NOLIN1 405 1 1 -1.0 150 1 4055
TABLED1 4055
+TB55 293.0 38.676 589.0 136.059 867.0 272.238 1144.0 449.592 +TB55
+TB56 1422.0 664.074 ENDT
+TB56

```

(c)  $h = h(T_1)$  by the equivalent nonlinear thermal load approach

```

$ USING MPC TO CREATE AN AVERAGE TEMPERATURE AT GRID 100 FOR GRIDS 1 AND 200
MPC 305 100 1 1.0 1 1 -0.5 +MC
+MC 200 1 -0.5
$ CREATING GRID 150 TO AVOID GRID 100 BEING DIRECTLY REFERENCED BY NOLIN1
GRID 100 -0.5 0.0 0.0
GRID 150 -0.5 1.0 0.0
CROD 8 105 100 150
PRUD 105 106 1.0
MAT4 106 5.0 0.1
TEMP 701 100 857.5 150 857.5
$ *****
$ SIMULATING TEMPERATURE-DEPENDENT FILM COEFFICIENTS BASED ON T(150)
NOLIN1 405 1 1 1.00 150 1 4061
TABLED1 4061
+TB1 857.0 187.704 1005.5 328.482 1144.5 446.508 1283.0 558.846 +TB1
+TB2 1422.0 664.074 ENDT
NOLIN1 405 1 1 -1.0 150 1 4062
TABLED1 4062
+TB3 857.0 113.19 1005.5 136.059 1144.5 272.238 1283.0 449.592 +TB3
+TB4 1422.0 667.074 ENDT
+TB4

```

(d)  $h = h(T_r)$  by the equivalent nonlinear thermal load approach

Figure 3 - The Images of Packets of Data Cards for Simulating Various Convective Film Coefficients

\$	TIME INCREMENTS	AND	INTEGRATION STEPS	
TSTEP	601	40	0.025	10
+TS3		20	0.05	5
+TS4		60	0.10	10

+TS3  
+TS4

$\Delta t$ -set (a)

\$	TIME INCREMENTS	AND	INTEGRATION STEPS	
TSTEP	601	25	0.005	10
+TS1		35	0.025	5
+TS2		20	0.05	5
+TS3		120	0.10	10

+TS1  
+TS2  
+TS3

$\Delta t$ -set (b)

\$	TIME INCREMENTS	IN INTEGRATIONS	AND PRINT-OUT	
TSTEP	601	400	0.005	50
+T1		120	0.05	20

+T1

$\Delta t$ -set (c)

Figure 4 - Three Sets of Time-Step Size Used in Computer Executions