# FEASIBILITY STUDY OF USING A TWO-PLATE MODEL TO APPROXIMATE THE TDRSS SOLAR PRESSURE EFFECTS

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## ABSTRACT

An investigation was performed to determine the feasibility of using a two plate model to approximate the Tracking and Data Relay Satellite (TDRS) in orbit propagation, taking into account the effects of solar radiation pressure. The two-plate model comprises one plate which always points to the earth, and the other which is hinged to an axis normal to the orbital pland and is always rotated so that its normal makes a minimum angle with the direction of the sun.

The results indicate that it is sufficient to take three parameters (i.e., the areas of the two plates and the reflectivity of the earth-pointing plate) to achieve an accuracy of one meter during a 24-hout orbit propagation.

This work was supported by NASA Contract No. NAS5-25139. The authors wish to acknowledge the programming assistance provided by Michael Toporek.

# SECTION 1 - INTRODUCTION

Most of the work involving solar radiation pressure on orbiting satellites has so far been limited to those which are spherical and which have circular nominal orbits. The comparatively few studies which are less restrictive are still based on very simplified models. For example, the work of Eliasberg[1] deals with an elliptically orbiting spherical satellite and is concerned with the first order perturbation effects expressed in terms of Keplerian elements. The work of Fang[2] deals with a circularly orbiting spherical satellite with a perfectly reflecting earth-pointing disk. It is concerned with the first order effects expressed in terms of along-track, cross-track and radial components. Moreover, it also deals with the physical insights into the modeling errors connected with tracking and orbit determination of the Tracking and Data Relay Satellites (TDRS). On the other hand, the work of Georgevic[3] deals only with the computation of solar radiation force on a cylinder and on a parabolic reflector, but does not deal with an orbiting satellite. (Even then, the computation for the parabolic reflector is further simplified by assuming that the ratio of the force on the illuminated area of the reflector and the force on the whole area of the reflector is the same as the ratio of the corresponding projected areas. It is obvious that this assumption, introduced to eliminate the cumbersome self-shadowing effects, is not really correct.)

The present work is concerned with the solar radiation effects on the TDRS illustrated in Figure 1.1, and modeled as comprised of the 69 components listed in Table 1.1. (In the course of the present study, a novel method, simple in comparison to other existing methods, for computing self-shadowing was formulated but this consideration was not included in computing the net solar radiation force on the satellite). The orbit of the TDRS is taken to be representative of a realistic one in that it is not nominally perfectly circular. The study also considers the question of how accurately the 69-component TDRS can be approximated by a two-plate model, i.e., one plate is hinged to an axis normal to the orbital plane and is always rotated to make a maximum angle with the sun, while the other plate is always earth-pointing. This two-plate model has the capability of handling up to four solve-for parameters, i.e., the area and reflectivity of each of the two plates.

Section 2 is concerned with the analysis of a differential correction procedure to obtain the values of these four solve-for parameters. A reference orbit for the 69-component TDRS is first generated. Its orbital position at regular intervals of one hour is then used as epoch elements of the two-plate model to obtain the values of the parameters which yield the best approximating orbit over the next 24 hours.

Section 3 summarizes the numerical results obtained in this feasibility study, and presents tabulated and graphical results for rapid comparisons.

Section 4 discusses the quality of the results, and the applicability of the two-plate model for use in orbit determination purposes.

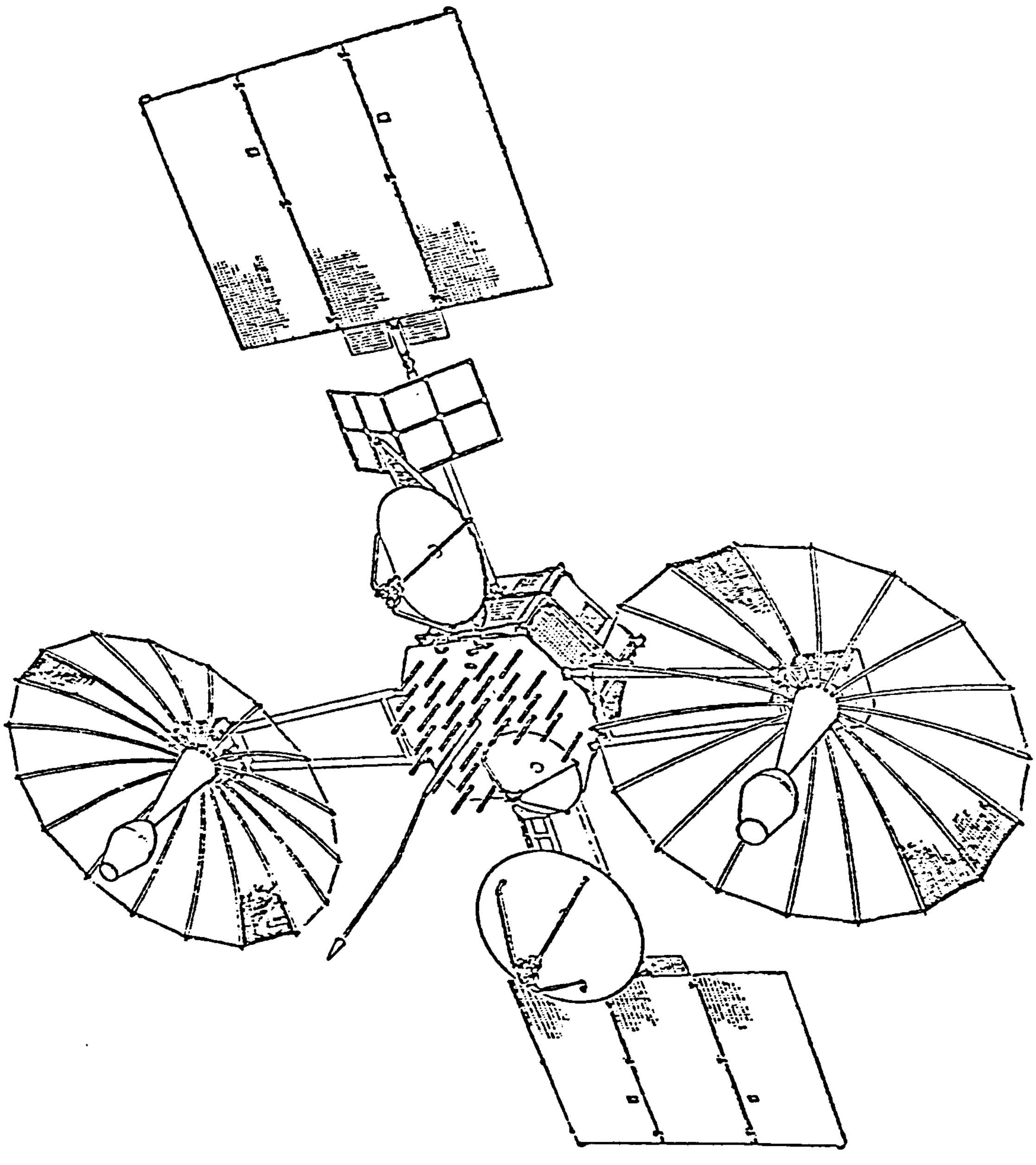


FIGURE 1.1 THE TDRS SATELLITE

TABLE 1.1

DETAILS OF 69-COMPONENT MODEL

| Component # | Description | Area ($m^2$) | Reflectivity | Components of Normal X | Y | Z |
|---|---|---|---|---|---|---|
| 1 | Solar Panel 1 | 14.7553 | 0.0 | Normal makes minimum angle with sun | | |
| 2 | Solar Panel 2 | 14.7553 | 0.0 | " | " | " |
| 3 | SGL Antenna | 3.13761 | 0.5 | Points to White Sands | | |
| 4 | C-Band Antenna | 2.67112 | 0.5 | Points to Los Angeles | | |
| 5 | Solar Sail | 0.90593 | 1.0 | 0 | 0 | 1 |
| 6 | Antenna Feed 1 (top) | 0.00462 | 1.0 | 0 | 0 | 1 |
| 7 | Antenna Feed 2 (top) | 0.00462 | 1.0 | 0 | 0 | 1 |
| 8 | Antenna Feed 1 (bottom) | 0.29570 | 1.0 | 0 | 0 | -1 |
| 9 | Antenna Feed 2 (bottom) | 0.29570 | 1.0 | 0 | 0 | -1 |
| 10 | Antenna Feed 1 (side) | 1.11771 | 1.0 | | | |
| 11 | Antenna Feed 2 (side) | 1.11771 | 1.0 | | | |
| 12 | Main Body (top) | 4.03665 | 0.5 | 0 | 0 | 1 |
| 13 | Main Body (bottom) | 4.03665 | 0.5 | 0 | 0 | -1 |
| 14 | Main Body (side 1) | 1.1599 | 0.5 | 0 | 1 | 0 |
| 15 | Main Body (side 2) | 1.1599 | 0.5 | .866 | .5 | 0 |
| 16 | Main Body (side 3) | 1.1599 | 0.5 | .866 | -.5 | 0 |
| 17 | Main Body (side 4) | 1.1599 | 0.5 | 0 | -1 | 0 |
| 18 | Main Body (side 5) | 1.1599 | 0.5 | -.866 | -.5 | 0 |
| 19 | Main Body (side 6) | 1.1599 | 0.5 | -.866 | .5 | 0 |
| 20-69 | Sections of stationary antennae | Computed by POLYN | 1.0 | Computed by POLYN | | |

## SECTION 2 - ANALYSIS

This section is concerned with the analysis of a differential correction procedure to obtain the values of the four solve-for parameters incorporated into the two-plate model.

For convenience, let us introduce the following notation:

$\vec{f}(t)$ = position vector of 69-component TDRS at time t

$\vec{g}(t,\vec{\alpha})$ = position vector of two-plate model at time t

$\vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ = 4 parameters for two-plate model

Q = loss function defined as $\sum_{i=1}^{25} \left| \vec{f}(t_i) - \vec{g}(t_i,\vec{\alpha}) \right|^2$

The problem is then to obtain the values of $\vec{\alpha}$ such that Q is a minimum. It is obvious that the minimum of Q is given by the necessary condition $\partial Q / \partial \alpha_j = 0$ where j = 1, 2, 3, 4.

The loss function Q may also be written as

$$Q = \sum_{i=1}^{25} \left[ \vec{f}(t_i) - \vec{g}(t_i,\vec{\alpha}) \right] \cdot \left[ \vec{f}(t_i) - \vec{g}(t_i,\vec{\alpha}) \right] \tag{2-1}$$

The necessary condition for minimum is

$$\frac{\partial Q}{\partial \alpha_j} = -2 \sum_{i=1}^{25} \left\{ \left[ \vec{f}(t_i) - \vec{g}(t_i,\vec{\alpha}) \right] \cdot \frac{\partial \vec{g}(t_i,\vec{\alpha})}{\partial \alpha_j} \right\} = 0 \qquad j = 1,2,3,4 \tag{2-2}$$

The function $\vec{g}(t_i,\vec{\alpha})$ may be expanded in a Taylor series about an a priori value $\vec{\alpha}_o$ .

$$\vec{g}(t_i,\vec{\alpha}) = \vec{g}(t_i,\vec{\alpha}_o) + \sum_{k=1}^{4} \left.\frac{\partial \vec{g}(t_i,\vec{\alpha})}{\partial \alpha_k}\right|_{\vec{\alpha}=\vec{\alpha}_o} \Delta\alpha_k + \cdots \tag{2.3}$$

where $\Delta\alpha_k$ is defined as

$$\Delta\alpha_k = \alpha_k - \alpha_{o,k} \qquad k=1,2,3,4 \tag{2.4}$$

Let
$$\Delta\vec{f}(t_i,\vec{\alpha}_o) \equiv \vec{f}(t_i) - \vec{g}(t_i,\vec{\alpha}_o) \tag{2.5}$$

Substitution of equations (2.3) and (2.5) into (2.2) yields to first order

$$\sum_{i=1}^{25} \left\{ \left[ \Delta\vec{f}(t_i) - \left( \sum_{k=1}^{4} \left.\frac{\partial \vec{g}(t_i,\vec{\alpha})}{\partial \alpha_k}\right|_{\vec{\alpha}=\vec{\alpha}_o} \Delta\alpha_k \right) \right] \cdot \left.\frac{\partial \vec{g}(t_i,\vec{\alpha})}{\partial \alpha_j}\right|_{\vec{\alpha}=\vec{\alpha}_o} \right\} = 0$$

$$j=1,2,3,4 \tag{2.6}$$

Interchanging the order of summation yields

$$\sum_{k=1}^{4} \left\{ \sum_{i=1}^{25} \left[ \left.\frac{\partial \vec{g}(t_i,\vec{\alpha})}{\partial \alpha_j}\right|_{\vec{\alpha}=\vec{\alpha}_o} \cdot \left.\frac{\partial \vec{g}(t_i,\vec{\alpha})}{\partial \alpha_k}\right|_{\vec{\alpha}=\vec{\alpha}_o} \right] \right\} \Delta\alpha_k$$

$$= \sum_{i=1}^{25} \left[ \left.\frac{\partial \vec{g}(t_i,\vec{\alpha})}{\partial \alpha_j}\right|_{\vec{\alpha}=\vec{\alpha}_o} \cdot \Delta\vec{f}(t_i,\vec{\alpha}_o) \right] \tag{2.7}$$

$$j=1,2,3,4$$

Let
$$a_{jk} \equiv \sum_{i=1}^{25} \left[ \left.\frac{\partial \vec{g}(t_i,\vec{\alpha})}{\partial \alpha_j}\right|_{\vec{\alpha}=\vec{\alpha}_o} \cdot \left.\frac{\partial \vec{g}(t_i,\vec{\alpha})}{\partial \alpha_k}\right|_{\vec{\alpha}=\vec{\alpha}_o} \right] \qquad j,k=1,2,3,4 \tag{2.8}$$

$$b_j \equiv \sum_{i=1}^{25} \left[ \left.\frac{\partial \vec{g}(t_i,\vec{\alpha})}{\partial \alpha_j}\right|_{\vec{\alpha}=\vec{\alpha}_o} \cdot \Delta\vec{f}(t_i,\vec{\alpha}_o) \right] \tag{2.9}$$

Equation (2.7) then becomes

$$\sum_{k=1}^{4} a_{jk} \Delta\alpha_k = b_j \qquad j=1,2,3,4 \qquad (2.10)$$

It remains to solve for $\Delta\alpha_k$ where k=1,2,3,4. This constitutes the first iteration in the differential correction procedure to solve for the value of $\vec{\alpha}$ such that Q is a minimum.

## SECTION 3 - RESULTS

This section summarizes the numerical results in this feasibility study, and presents tabulated and graphical results for rapid comparisons.

Numerous computer runs were made for the TDRS with epoch elements:

$$X_o = 31{,}662{,}513.0 \text{ m}$$
$$Y_o = -27{,}523{,}890.0 \text{ m}$$
$$Z_o = 0.0 \text{ m}$$

$$\dot{X}_o = 2{,}012.15997 \text{ m/sec}$$
$$\dot{Y}_o = 2{,}314.7253 \text{ m/sec}$$
$$\dot{Z}_o = 376.58528 \text{ m/sec}$$

One set of runs had epoch time set at Day 183.0, Year 1980 (i.e., July 1, 1980 which is close to the summer solstice), and another set had epoch time set at Day 275.0, Year 1980 (i.e., October 1, 1980 which is close to the autumnal equinox). Each of these sets of runs was made for the cases of N=1,2,3 and 4 parameters. The parameters were aligned in the following sequence which is probably the order of decreasing importance:

$\alpha_1$ = area of sun-pointing plate ($M^2$)
$\alpha_2$ = area of earth-pointing plate ($M^2$)
$\alpha_3$ = reflectivity of earth-point plate
$\alpha_4$ = reflectivity of sun-pointing plate

The initial values of these parameters used in the first iteration of the differential correction procedure were taken to be the following:

| N | = | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $\alpha_{0,1}$ | = | 36.6 | 29.5 | 29.5 | 29.5 |
| $\alpha_{0,2}$ | = | 0.0 | 18.81 | 18.81 | 18.81 |
| $\alpha_{0,3}$ | = | 0.0 | 0.0 | 0.74 | 0.74 |
| $\alpha_{0,4}$ | = | 0.0 | 0.0 | 0.0 | 0.0 |

The tolerance $\varepsilon$ for testing convergence of the iterations (i.e., $|\Delta\vec{\alpha}|<\epsilon$) was taken to be 0.01. The values for the stepsize $\vec{\delta}$ for computing the partial derivatives was taken to be 0.1. In the runs described above, convergence was achieved after two or three iterations. Figures 3.1 and 3.2 summarize the results of these computer runs. The symbols appearing on the horizontal axis in these figures have the following connotations:

O = The sun is overhead with respect to the satellite
U = The sun is underfoot with respect to the satellite
A = The satellite is moving away from the sun
T = The satellite is moving toward the sun

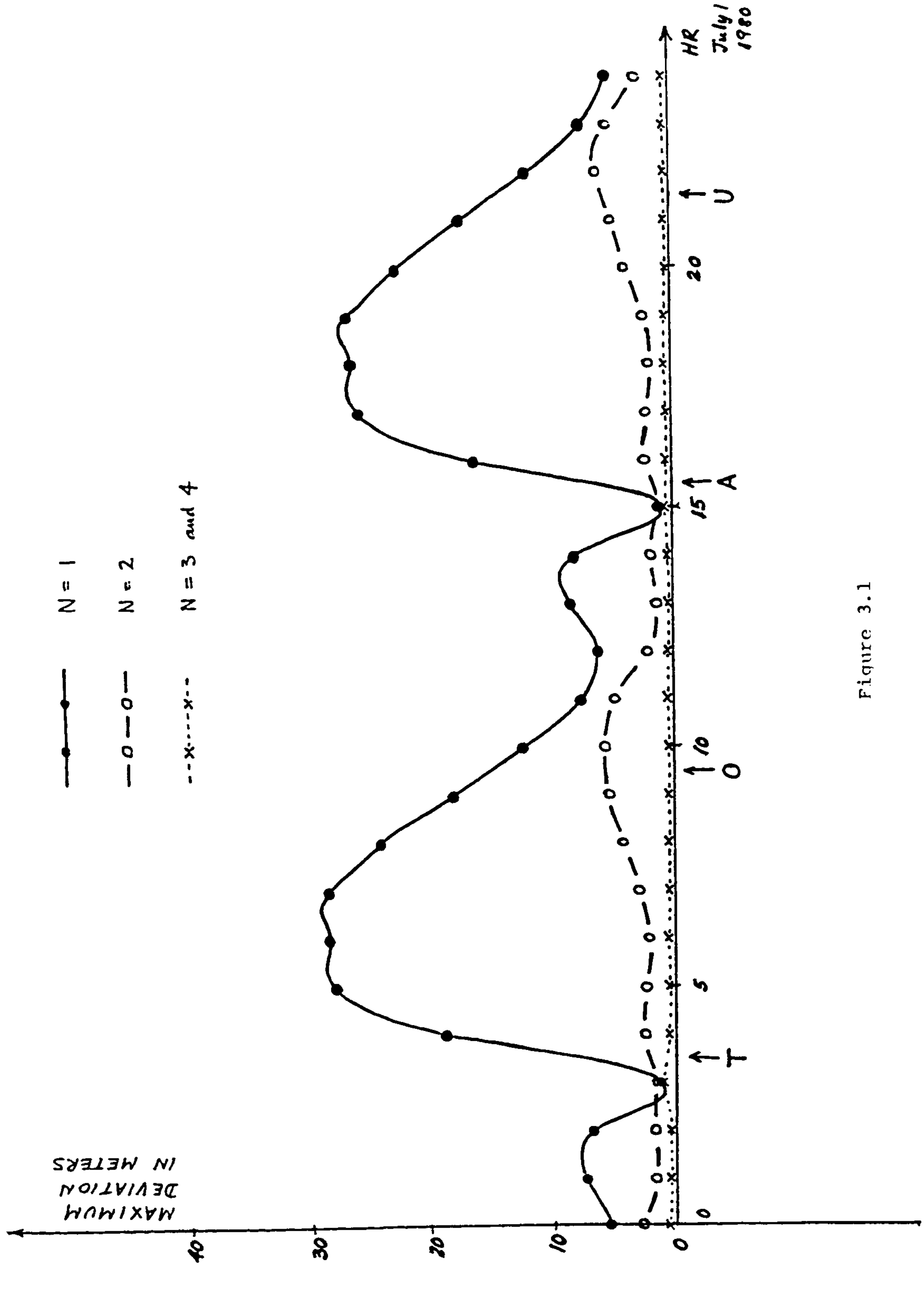

MAXIMUM DEVIATION IN METERS
40
30
20
10
0
N = 1
N = 2
N = 3 and 4
0
5
10
15
20
T
O
A
U
HR
July 1 1980

Figure 3.1

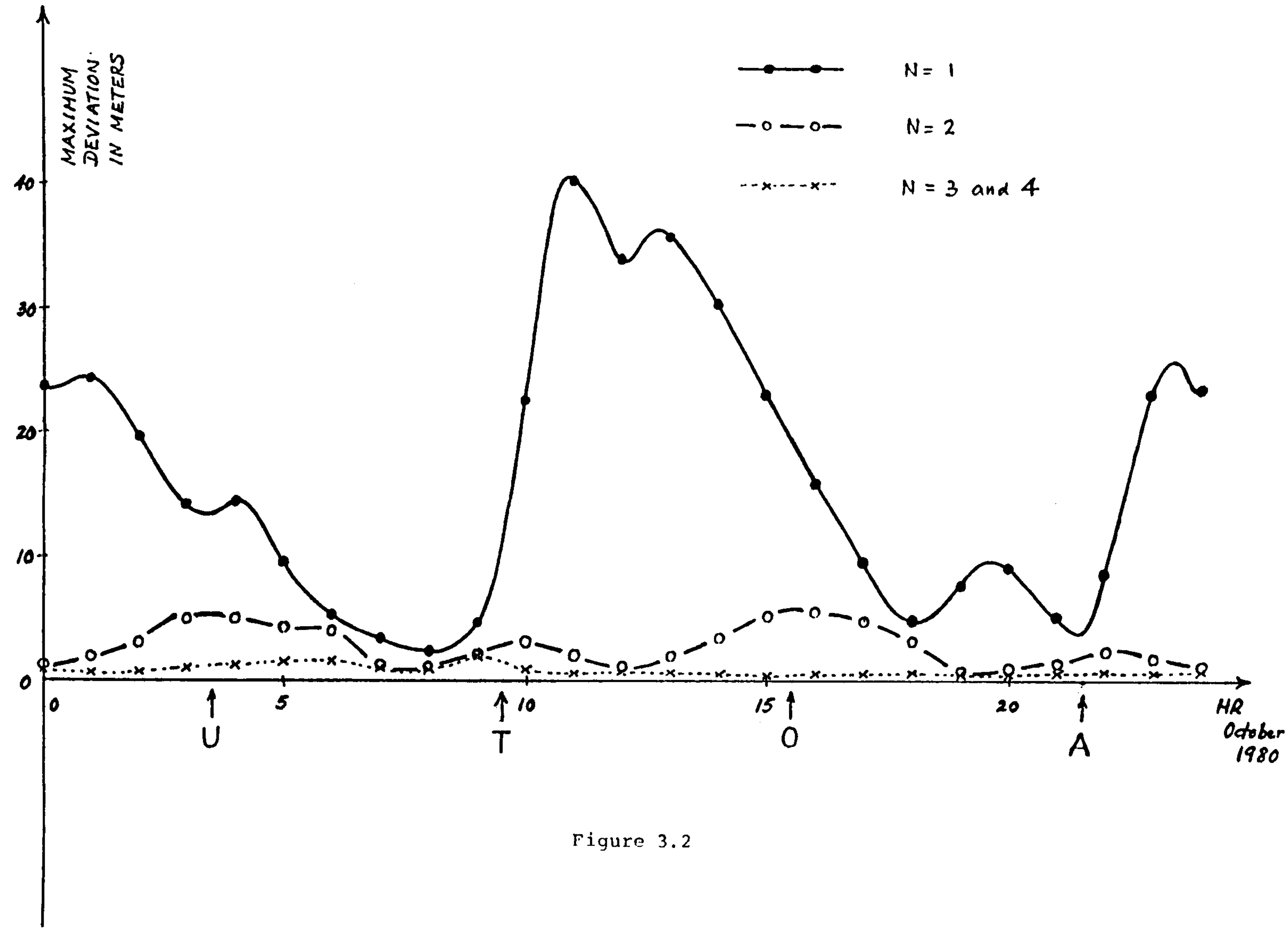

MAXIMUM DEVIATION IN METERS
40
30
20
10
0
0
5
10
15
20
HR October 1980
U
T
O
A
N= 1
N= 2
N = 3 and 4

Figure 3.2

## SECTION 4 - CONCLUSION

This section is concerned with a brief discussion of the results obtained in Section 3, and also an attempt to relate them to results obtained in other investigations.

From Figures 3.1 and 3.2, it is seen that the following observations may be made:

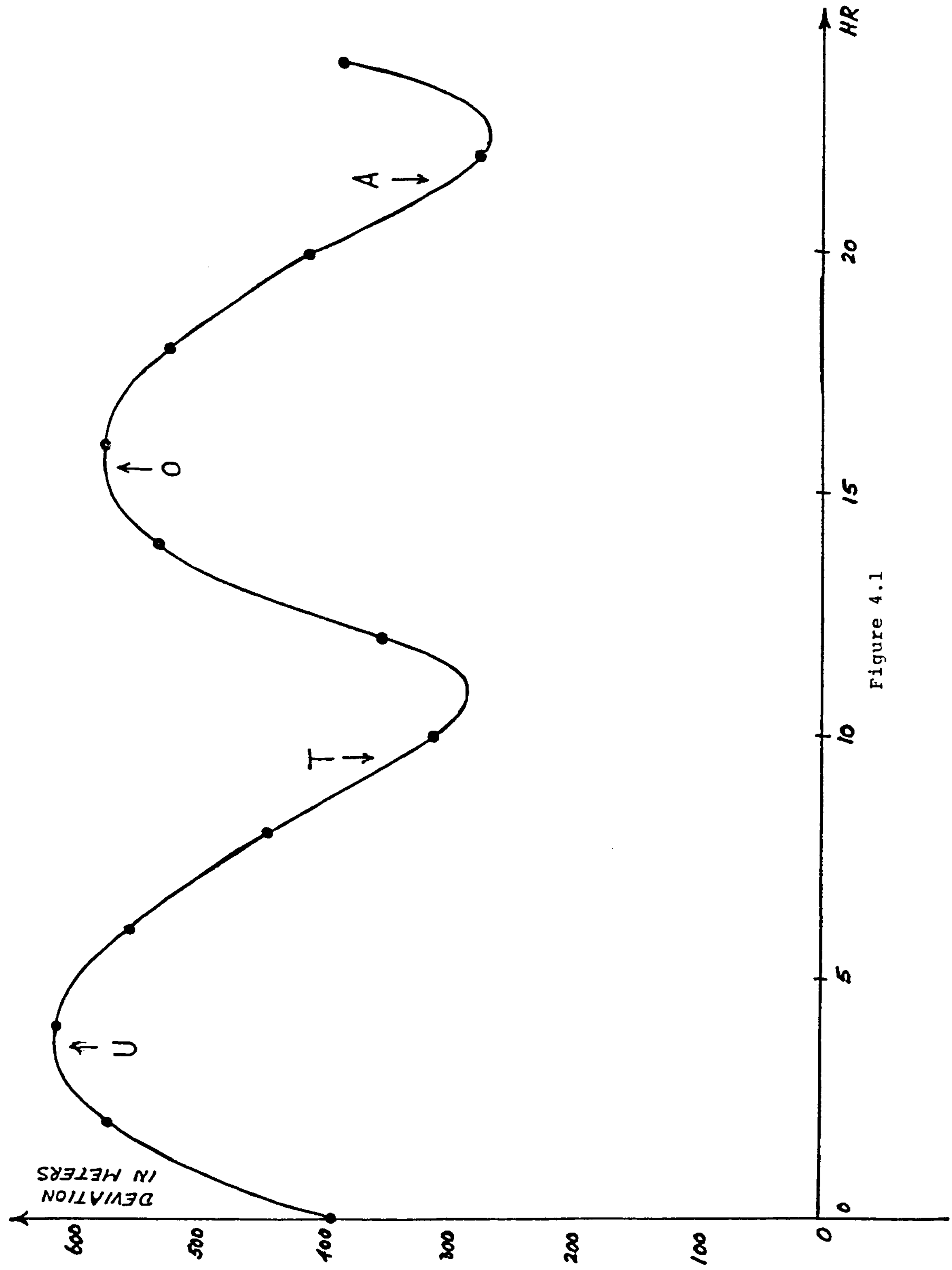
1. The maximum deviation of the two-plate model from the 69-component TDRS is essentially cyclical with a 12-hour period.
2. The magnitude of these deviations decreases as the number N of parameters increases, as would be expected.
3. There is a pronounced phase change of this period curve in going from the case of N=1 to N=2.
4. The curves for the cases of N=3 and N=4 are essentially identical. Moreover, the amplitude of oscillation is so small that they are almost constant.

A computer run was also made comparing the 69-component TDRS with and without solar radiation pressure effects. The results are plotted in Figure 4.1.

It is noted that in this case the curve is essentially sinusoidal, unlike those in Figure 3.1 and 3.2. Moreover, the maximum deviation occurs when the sun is overhead or underfoot in Figure 4.1, unlike the cases of N=1 and N=2 in Figures 3.1 and 3.2.

Finally, it is interesting to recall the results obtained by Fang[2] who observed that:

1. In orbit propagation, the least perturbation occurs when the sun vector is parallel to the satellite velocity vector in the beginning, and the worst perturbation occurs when the sun is overhead or underfoot in the beginning.
2. TDRS orbits determined from a one-day tracking arc tend to be less sensitive to solar pressure errors if the tracking arc begins when the sun is directly overhead or underfoot. This is contrary to the (previous) result for solar pressure perturbations in the absence of tracking.



DEVIATION IN METERS
600
500
400
300
200
100
0
HR
0
5
10
15
20
U
T
O
A


Figure 4.1

From the numerical results of this feasibility study, it is seen that the 69-component TDRS can be accurately replaced by the two-plate model. It suffices to take only three parameters to achieve an accuracy to within about one meter. Moreover, it is sufficient to use only one approximating orbit throughout the 24 hour period, instead of 24 approximating orbits regularly spaced throughout the day.

## REFERENCES


(1) Eliasberg, P.E., "Introduction to the Theory of Flight of Artificial Earth Satellites," Moscow (1965). Translation, Jerusalem (1967).

(2) Fang, B.T., "Short-Term Solar Pressure Effect and GM Uncertainty on TDRS Orbital Accuracy--A Study of the Interaction of Modeling Error with Tracking and Orbit Determination," EG&G/Washington Analytical Services Center, Inc., Report No. 002-78 (1978).

(3) Georgevic, R.M., "The Solar Pressure Force and Torque Model," J. Astron. Sc., Vol. XX, No. 5, pp. 257-274. (1973).