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ABSTRACT

The latitude and longitude of a point on the earth's surface are found from its x, y grid coordinates in the van der Grinten projection. The latitude is a solution of a cubic equation and the longitude a solution of a quadratic equation. Also, the x, y grid coordinates of a point on the earth's surface can be found if its latitude and longtiude are known by solving two simultaneous quadratic equations.

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Interest in the van der Grinten projection (van der Grinten, 1905) has revived recently because of its use in the geophysical atlas of Lowman and Frey (1979). The maps contained therein are based in turn on the map "The Physical World" (1977 revision), which is produced by the National Geographic Society and uses the van der Grinten projection.

O'Keefe and Greenberg (1977) have given an algorithm for doing the forward problem in the van der Grinten projection: showing how to go from latitude and longitude to x, y grid coordinates. Rubincam (1980b) has made corrections to some of the equations given in O'Keefe and Greenberg (1977). Snyder (1979) also made a correction.

In this paper we will show how to do the inverse problem; going from x, y grid coordinates to latitude and longitude. This is required for some applications. For instance, Rubincam (1980a) computes a density distribution based on the gravity field at points on a rectangular array in grid coordinate space (see Fig. 1) for plotting on the tectonic activity map of P. Lowman (Lowman and Frey, 1979, p. 40) in order to observe correlations between density and tectonics. Since the density distribution is expressed in terms of spherical harmonics, the latitude and longitude are necessary for computing the density at each array point; hence the need for the inversion.

Further, we will give an alternative algebraic algorithm for doing the forward problem, which may be simpler than the trigonometric approach of O'Keefe and Greenberg (1977). Snyder (1979) has also given an alternative algorithm.

In the following we will use the notation of O'Keefe and Greenberg (1977).

We first wish to find latitude ϕ and longitude λ of a point when the grid coordinates (x, y), reference longitude λ_o, and scale factor ρ are known. The origin of the grid coordinate system is at $\phi = 0$ and $\lambda = \lambda_o$.

Let us begin with the latitude ϕ. We will assume that $y > 0$, so that $\phi > 0$ and $b > 0$, where $\phi = b\pi$ and ϕ is measured in radians. It proves convenient to find b from

$$x^2 + (y - y_J)^2 = s^2, \tag{1}$$

which is the equation for a circle (see Fig. 2). Quantities y_J and s can be expressed in terms of b and ρ:

$$y_J = \frac{2\rho\,(1 - b)\sqrt{1 + 2b}}{b(\sqrt{1 + 2b} - \sqrt{1 - 2b})} \tag{2}$$

$$s = \frac{2\rho\,(1 + b)\sqrt{1 - 2b}}{b\,(\sqrt{1 + 2b} - \sqrt{1 - 2b})} \tag{3}$$

These two equations are derived in the Appendix. They agree with those originally given by van der Grinten (1905, p. 360) for $2b = c$ and $2\rho = 1$.

Substituting (2) and (3) in (1) and multiplying by

$$b(\sqrt{1 + 2b} - \sqrt{1 - 2b})^2$$

gives

$$[4\rho(1 - b)y - 2b(x^2 + y^2)]\,\sqrt{1 - 4b^2} = 4\rho(1 - b)(1 + 2b)y - 2b(x^2 + y^2) - 16\rho^2 b^2.$$

Squaring this expression and dividing by 16b gives the cubic equation

$$c_3 b^3 + c_2 b^2 + c_1 b + c_0 = 0 \tag{4}$$

after considerable algebra, where

$$c_3 = 16\rho^4 + 16\rho^3 y + 8\rho^2 y^2 + 4\rho y(x^2 + y^2) + (x^2 + y^2)^2$$

$$c_2 = -8\rho^3 y + 4\rho^2(x^2 + y^2) - 12\rho^2 y^2 - 2\rho y(x^2 + y^2)$$

$$c_1 = -8\rho^3 y - 2\rho y(x^2 + y^2)$$

and $$c_0 = 4\rho^2 y^2.$$

The trigonometric solution is the most convenient way to solve (4). It is (Selby, 1973, pp. 103-104):

$$b = m \cos\left(\theta + \frac{4\pi}{3}\right) - \frac{c_2}{3c_3} \tag{5}$$

where

$$\theta = \frac{1}{3} \text{Arc}\cos\left(\frac{3d}{am}\right)$$

and

$$m = 2\sqrt{-\frac{a}{3}}$$

with

$$d = \frac{1}{27}\left(2\frac{c_2^3}{c_3^3} - 9\frac{c_2 c_1}{c_3^2} + 27\frac{c_0}{c_3}\right)$$

and

$$a = \frac{1}{3}\left(3\frac{c_1}{c_3} - \frac{c_2^2}{c_3^2}\right).$$

Latitude ϕ is then found by multiplying (5) by π. This is the desired result for $y > 0$. The latitude ϕ when $y < 0$ is easily found from the above result, since the van der Grinten projection has reflective symmetry about the equator. In this case we merely set $y = -y$, solve for b from (5), and then set $\phi = -b\pi$. The case for $y = 0$ is trivial; here $\phi = b = 0$.

The longitude λ can be found from the equation (see Fig. 2)

$$(x - x_M)^2 + y^2 = r^2, \tag{6}$$

where (O'Keefe and Greenberg, 1977; Rubincam, 1980b):

$$x_M = \rho\left(\ell - \frac{1}{\ell}\right), \tag{7}$$

$$r = \left|\rho\left(\ell + \frac{1}{\ell}\right)\right|, \tag{8}$$

and

$$\ell = \frac{\lambda - \lambda_0}{\pi}. \tag{9}$$

Substituting (7) and (8) in (6) gives an equation quadratic in ℓ, with solution

$$\ell = \frac{x^2 + y^2 - 4\rho^2 + \sqrt{16\rho^4 + 8\rho^2(x^2 - y^2) + (x^2 + y^2)^2}}{4\rho x} \tag{10}$$

when $x \neq 0$. Longitude λ is found from (10) via (9). The trivial case of $x = 0$ gives $\lambda = \lambda_0$.

This completes our derivation of the basic equations for the inversion process. They have been verified through numerical examples.

The forward problem can be done according to the following algorithm. First, find b and ℓ from ϕ, λ, and λ_0. Next, compute y_J by substituting $|b|$ for b in (2) and putting a minus (−) sign in front of the resulting expression if $b < 0$. Then compute s by substituting $|b|$ for b in (3). After that, compute x_M and r from (7) and (8). Finally, find grid coordinates x and y by solving simultaneously the quadratic equations (1) and (6). This gives

$$x = \frac{x_M(x_M^2 + y_J^2 + s^2 - r^2) + \frac{\ell}{|\ell|}\,|y_J|\sqrt{4r^2s^2 - [(x_M^2 + y_J^2) - (r^2 + s^2)]^2}}{2(x_M^2 + y_J^2)} \tag{10}$$

$$y = \frac{y_J(x_M^2 + y_J^2 + r^2 - s^2) - \frac{b}{|b|}\,|x_M|\sqrt{4r^2s^2 - [(x_M^2 + y_J^2) - (r^2 + s^2)]^2}}{2(x_M^2 + y_J^2)} \tag{11}$$

These equations have been verified numerically.

Our approach to the forward problem may be quicker than the trigonometric approach of O'Keefe and Greenberg (1977) when programmed on a computer. It certainly involves less confusion about signs. Further, it differs from Snyder's (1979) approach; his involves some trigonometry, while ours is wholly algebraic.

Summarizing our results on the van der Grinten projection, we have: verified van der Grinten's original equations for y_J and s, which are given by our (2) and (3); solved the inverse problem of going from x, y grid coordinates to latitude and longitude; and given an algebraic algorithm for doing the forward problem of going from latitude and longitude to x, y grid coordinates.

APPENDIX

Derivation of Equations for s and y_J

Here we derive (2) and (3) to find s and y_J in terms of b and ρ. We make constant reference to the geometrical relationships of Fig. 2. Our Fig. 2 is based on O'Keefe and Greenberg's (1977) Fig. 2.

Now $A'O = 2\rho$ and $BO = 4\rho b$ (O'Keefe and Greenberg, 1977). Also, $FO = BO - BF = 4\rho b - BF$. Since BEN is a right isoseles triangle, we have $BE = BN = 2\rho - 4\rho b$. These equations and the ratios of the lengths of sides of similar triangles BEF and A'FO give

$$BF = \frac{2\rho b\,(1 - 2b)}{1 - b} \quad .$$

From right triangle BCO and the Pythagorean theorem we have

$$BC = 2\rho\sqrt{1 - 4b^2} \quad .$$

Also, $DO = 4\rho b - BD$. So by similar triangles BCD and A'DO we get

$$\frac{BD}{BC} = \frac{DO}{2\rho} = \frac{4\rho b - BD}{2\rho}$$

or

$$BD = \frac{4\rho b\sqrt{1 - 4b^2}}{1 + \sqrt{1 - 4b^2}}.$$

Therefore

$$DF = BD - BF = \frac{2\rho b(-1 + 2b + \sqrt{1 - 4b^2})}{(1 - b)\,(1 + \sqrt{1 - 4b^2})}.$$

We have also $FO = 2\rho b/(1 - b)$. From right triangle FGO we obtain

$$FG = \frac{2\rho\sqrt{1 - 2b}}{1 - b},$$

and from right triangle DFG we get

$$(DG)^2 = \frac{8\rho^2(1 + b)\,(1 - 2b)}{(1 - b)(1 + \sqrt{1 - 4b^2})},$$

both by Pythagoras.

Now by similar triangles DHJ and DFG we have

$$\frac{DJ}{\tfrac{1}{2}DG} = \frac{DG}{DF},$$

where DH = DG/2. But DJ = s; so

$$s = \frac{(DG)^2}{2DF} = \frac{2\rho(1+b)\sqrt{1-2b}}{b(\sqrt{1+2b}-\sqrt{1-2b})},$$

which is the desired equation for s.

Finally, y_J = OJ = DJ + DO = s + 4ρb – BD, so that

$$y_J = \frac{2\rho(1-b)\sqrt{1+2b}}{b(\sqrt{1+2b}-\sqrt{1-2b})},$$

which is the desired equation for y_J.

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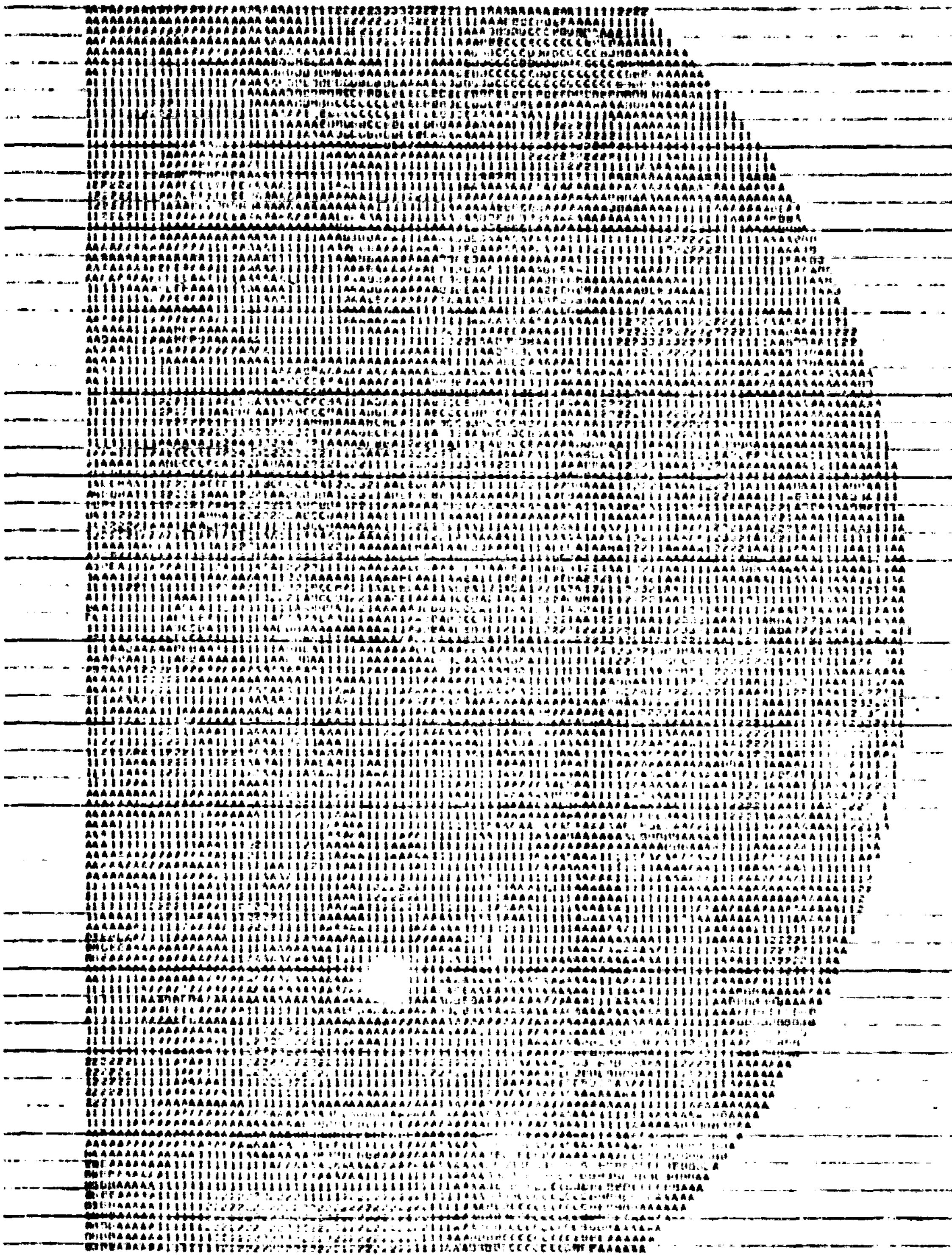

Figure 1. Example of a computer plot of the information theory density distribution derived from the gravity field. Only part of the eastern hemisphere is shown. Letters represent low density and numbers high density.

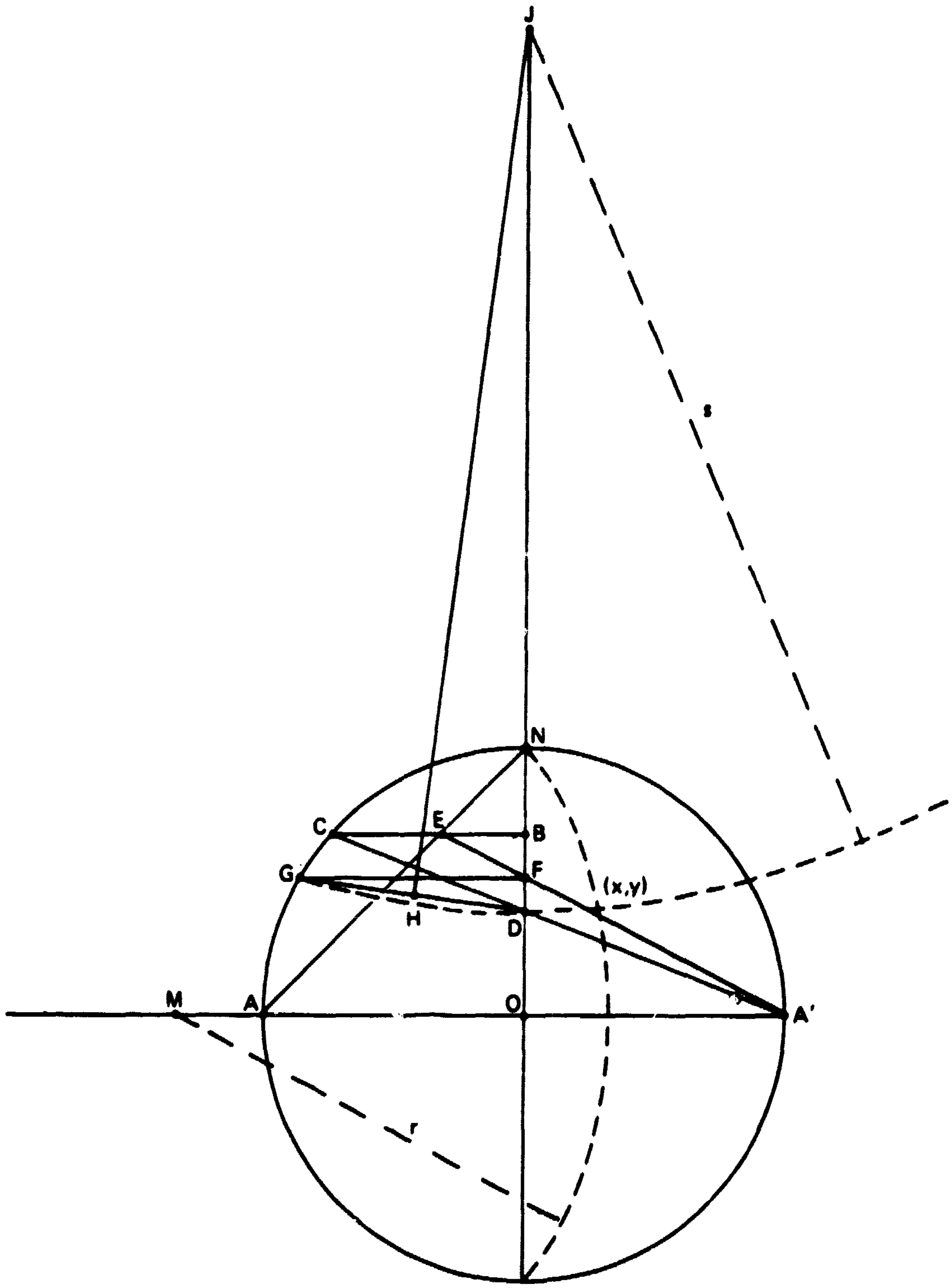

Figure 2. Geometrical relationships in the van der Grinten projection based on O'Keefe and Greenberg (1977). Note that the coordinate point (x, y) is found from the intersection of two circles.

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