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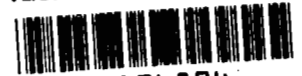
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# Spatial Frequency Response of an Optical Heterodyne Receiver

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# Spatial Frequency Response of an Optical Heterodyne Receiver

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## SUMMARY

The principles of transfer function analysis have been applied to a passive optical heterodyne receiver to obtain the modulation transfer function (MTF). MTF calculations have been performed based on an optical platform which is imaging vertically varying profiles at worst-case shuttle orbit altitudes. An analysis of the derogatory effects of sampling (aliasing) and central obscurations on both resolution and heterodyne efficiency is given.

## INTRODUCTION

One measure of performance of an optical imaging system is its ability to reproduce an object distribution with sufficient signal-to-noise ratio and resolution so as to make the information contained within the image useful. Generally, such a system may be characterized by its optical transfer function (OTF) or, in certain systems by the modulation transfer function (MTF). (See ref. 1).

For conventional imaging systems using either coherent or incoherent illumination, one usually assumes linearity in the imaging process so that the cascading property of transfer function analysis applied (ref. 2). Under this assumption, the MTF's of the individual subsystems (i.e., optics, detector, electronics, etc.) can be multiplied to give the overall transfer function.

In this report, the principles of transfer function analysis have been applied to a passive optical heterodyne receiver which is assumed to be imaging vertically varying spatial profiles at worst-case shuttle orbit altitudes. Results of the analysis show some interesting departures from the properties described above; namely, that the cascading property must be carefully applied and that optical receivers having obscurations, such as Cassegrains, are not optimum for heterodyne-type detection.

## SYMBOLS AND ABBREVIATIONS

$A_g$	source geometrical shape factor
$\hat{A}_g$	Fourier transform of $\hat{A}_g$ , $m^2$
$B_{HIF}$	effective mixing bandwidth, Hz
$B_{SIF}$	effective shot noise bandwidth, Hz
$D_A$	diameter of receiver aperture, m
D.C.	direct current
$d_i$	distance of image from receiver aperture, m
$E_g$	amplitude of geometrical image signal field, $(W/m^2)^{1/2}$
$E_o$	amplitude of local oscillator field, $(W/m^2)^{1/2}$
$E_s$	amplitude of signal field in detector plane, $(W/m^2)^{1/2}$
$e$	electron charge, C
$f$	intermediate frequency, Hz
$G_{det}$	Fourier transform of detector aperture function, $m^2$
$G_g$	Fourier spectrum of geometrical image spectral radiance, $W/str/Hz$
$G_H$	normalized heterodyne transfer function
$G_o$	Fourier spectrum of local oscillator field modulated by detector pupil function, $(Wm^2)^{1/2}$
$H$	coherent optical transfer function
$H_d$	photodetector transfer function
$H_{het}$	total electric heterodyne transfer function
$H_{ifa}$	I.F. amplifier and filter transfer function
$H_{LP}$	low-pass filter transfer function
$H_m$	transfer function representing carrier diffusion and transit time effects
$H_{sq}$	square-law detector transfer function
$H_T$	tangent height

$H_{tr}$	photodetector shot noise transfer function
$h$	Planck's constant, J(sec)
$h(r)$	point spread function, $m^{-2}$
$I_d$	photodetector current, A
$I_{het}^2$	mean-square heterodyne current power, $A^2$
$I_{IF}(I_{IF}^2)$	I.F. amplifier output current (current power); A, ( $A^2$ )
$I_{LO}$	photodetector current induced by L.O. field, A
$I_o, I_{sync}$	synchronous detector output current, A
$I_s$	photodetector current induced by signal field, A
$I_{sync}$	spatial Fourier component of synchronous detector current, $m^2A$
$I_T$	low-pass filter output current, A
I.F.	intermediate frequency, Hz
I.F.O.V.	geometrical instantaneous field of view
$\underline{K}$	spatial frequency vector, $m^{-1}$
$K_c$	coherent cut-off frequency, cycles/rad
$K_{ob}$	coherent cut-on frequency due to central obscuration, cycles/rad
$K_x, K_y$	rectangular components of $\underline{K}$ , $m^{-1}$
$K_\theta, K_\phi$	angular frequency variables corresponding to $K_x, K_y$ , cycles/rad
$KT$	product of Boltzman's constant and source temperature, J
$l_x, l$	detector edge length, m
LHS	laser heterodyne spectrometer
L.O.	local oscillator
MTF	modulation transfer function
$N$	shot noise current power due to L.O., normalization factor, $A^2$
OTF	optical transfer function
$P_g$	geometrical image spectral radiance, $W/m^2/sr/Hz$

$P_O$	total L.O. power absorbed by detector, W
R	orbital height, m
$\underline{r}, \underline{s}$	detector plane spatial vectors, m
S/N	signal-to-noise ratio in shot noise limit
T	optical transmission factor
$T_{det}$	aperture function of detector geometry
$T_O^*$	conjugate of L.O. field modulated by detector aperture, $(W/m^2)^{1/2}$
t	time, sec
$v_O$	vertical component of orbital velocity, m/sec
x,y	rectangular components of $\underline{r}, \underline{s}$ ; m
z	distance of object from receiver aperture, m
$\delta(s)$	Dirac delta function, $m^{-2}$
$\eta$	quantum efficiency
$\eta_{het}$	heterodyne quantum efficiency
$\theta, \phi$	angular coordinates corresponding to x,y; rad
$\theta_F$	instantaneous field of view, rad
$\lambda$	optical wavelength, m
$\nu$	optical signal frequency, Hz
$\nu_O$	local oscillator frequency, Hz
$\tau$	effective integration time of low-pass filter, sec
$\chi$	efficiency factor for uniform extended source
$\chi_{het}$	efficiency factor for arbitrary source distribution
$\omega$	radian frequency, rad/sec
$\langle \rangle$	average over ensemble of signal fields
$\otimes$	convolution operation

Superscript:

\* complex conjugate operation

## THEORETICAL ANALYSIS

### Imaging Considerations

Consider an optical receiver which is imaging an object amplitude distribution as shown in figure 1. Using scalar diffraction theory, the signal amplitude  $E_s$  in the detector plane  $\underline{r}$  is given by (ref. 3)

$$E_s(\underline{r}) = E_g(\underline{r}) \otimes h(\underline{r}) = \int_{-\infty}^{+\infty} d^2s E_g(\underline{s}) h(\underline{r} - \underline{s}) \quad (1)$$

where  $h(\underline{r})$  is the impulse response of the imaging optics and  $E_g(\underline{r})$  is the amplitude of the geometrical image of the object. The shift invariance of  $h(\underline{r})$  can be justified for the heterodyne applications discussed here by a careful examination of the various phase factors appearing in the impulse response function.

For the mixing of two deterministic optical beams in an ideal detector, the mean-square heterodyne current power at the difference frequency,  $f = |\nu - \nu_0|$ , is (ref. 4)

$$I_{\text{het}}^2 = \frac{2\eta^2 e^2}{(h\nu)^2} \left| \int_{\text{det}} d^2r E_o^*(\underline{r}) E_s(\underline{r}) \right|^2 \quad (2)$$

where  $E_o(\underline{r})$  is the local oscillator amplitude distribution in the detector plane,  $\eta$  is the quantum efficiency,  $e$  is the electronic charge, and  $h\nu$  is the photon energy. A simple classical approach is taken to obtain the correct expression from which the spatial frequency analysis may begin. We recognize that the geometrical image field  $E_g(\underline{s}, \nu)$  is a stochastic process which we synthesize by discrete frequency components with random phases. Now, for a deterministic L.O. field and a quasi-monochromatic optical signal, equation (1) and the generalization of equation (2) combine to give

$$I_{\text{het}}^2 = \frac{2\eta^2 e^2}{(h\nu)^2} \sum_f \iint_{-\infty}^{+\infty} d^2s d^2s' \langle E_g(\underline{s}, f+\nu_0) E_g^*(\underline{s}', f+\nu_0) \rangle \left[ \int_{\text{det}} d^2r E_o^*(\underline{r}) h(\underline{r}-\underline{s}) \right] \left[ \int_{\text{det}} d^2r' E_o(\underline{r}') h^*(\underline{r}'-\underline{s}') \right] \quad (3)$$

where  $\langle \rangle$  represents an average over the ensemble of signal fields. It is assumed that the source of the image field on the detector (i.e., the sun) is spatially incoherent. The appropriate substitutions are

$$\langle E_g(\underline{s}, \nu) E_g^*(\underline{s}', \nu) \rangle \rightarrow \Delta f \lambda^2 P_g(\underline{s}, \nu) \delta(\underline{s} - \underline{s}')$$

$$I_{\text{het}}^2 \rightarrow I_{\text{het}}^2(f) \Delta f$$

where  $P_g$  represents the image spectral radiance at the detector plane in  $\text{W/m}^2/\text{sr}/\text{Hz}$  and  $I_{\text{het}}^2(f)$  is the current spectral power density in  $\text{A}^2/\text{Hz}$ .

Equation (3) becomes

$$I_{\text{het}}^2(f) = \frac{2\eta^2 e^2 \lambda^2}{(h\nu)^2} \int_{-\infty}^{+\infty} d^2s [P_g(\underline{s}, \nu_o + f) + P_g(\underline{s}, \nu_o - f)] |T_o^*(\underline{s}) \otimes h(-\underline{s})|^2$$

where  $f \geq 0$  and we note that the L.O. mixes with the signal field components at  $\nu_o + f$  and  $\nu_o - f$ . Here, we have expressed the detector overlap integral of the L.O. field and the impulse response function as

$$\int_{-\infty}^{+\infty} T_{\text{det}}(\underline{r}) E_o^*(\underline{r}) h(\underline{r} - \underline{s}) d^2r = T_o^*(\underline{s}) \otimes h(-\underline{s})$$

where  $T_{\text{det}}(\underline{r})$  is the aperture function of the detector geometry and the product  $T_{\text{det}}(\underline{r}) E_o^*(\underline{r}) = T_o^*(\underline{r})$  is simply that portion of the L.O. that is transmitted by the detector aperture.

Referring to the detector scheme of figure 2, the output current from the synchronous detector is

$$I_{\text{sync}} = \frac{2Te^2 \eta^2 \lambda^2}{(h\nu)^2} \int_{-\infty}^{+\infty} d^2s \int_{(f>0)} df |H_{\text{het}}(f)|^2 [P_g(\underline{s}, \nu_o + f) + P_g(\underline{s}, \nu_o - f)] |T_o^*(\underline{s}) \otimes h(-\underline{s})|^2$$



where  $H_{het}$  is the total heterodyne transfer function defined by

$$|H_{het}(f)|^2 = |H_m|^2 |H_d|^2 |H_{ifa}|^2$$

and  $T$  is the optical transmission factor. The various contributions to  $H_{het}$  are: the signal/L.O. mixing transfer function  $H_m$ , representing carrier diffusion and transit time effects in the detector; the photodetector transfer function  $H_d$ , comprised of contributions due to capacitance, resistance, and inductance; and the I.F. (intermediate frequency) amplifier and filter transfer function  $H_{ifa}$ . The square-law detector is assumed to have a unity transfer function ( $H_{sq} = 1$ ). The shot-noise transfer function  $H_{tr}$  is due only to transit time effects as opposed to  $H_m$ .

### Heterodyne Transfer Function

In this paper, we are interested in the spatial frequency response of the heterodyne receiver to a vertically varying object profile as shown in figure 1. Other than a knowledge of the total I.F. power, this is different from the I.F. considerations discussed previously. To obtain the spatial frequency response and, ultimately, the system modulation transfer function (MTF), we assume the object scene radiance (and, consequently, the image scene) is linearly translated owing to motion of the optical receiver (e.g., an orbiting platform). This induces a translation of the image coordinates by an amount

$$\underline{s} \rightarrow \underline{s} - \underline{r}(t)$$

and

$$P_g(\underline{s}, \nu_o \pm f) \rightarrow P_g(\underline{s} - \underline{r}, \nu_o \pm f)$$

Further, we define

$$P_g(-\underline{r}) \equiv \int_{(f>0)} df |H_{het}(f)|^2 [P_g(\underline{r}, \nu_o + f) + P_g(\underline{r}, \nu_o - f)]$$

Since the impulse response  $h(\underline{r})$  is invariant, we have the output current from the synchronous detector as

$$I_{sync}(t) = I_{sync}[\underline{r}(t)] = \frac{2Te^2 \eta^2 \lambda^2}{(h\nu)^2} \int_{-\infty}^{+\infty} d^2s P_g(\underline{r} - \underline{s}) |T_o^*(\underline{s}) \otimes h(-\underline{s})|^2 \quad (4)$$

Equation (4) is of the form of a convolution

$$I_{\text{sync}}[\underline{r}(t)] = \frac{2Te^2 n^2 \lambda^2}{(h\nu)^2} P_g(\underline{r}) \otimes |T_o^*(\underline{r}) \otimes h(-\underline{r})|^2$$

Decomposition of  $I_{\text{sync}}$  into its spatial frequency components is obtained by the Fourier transformation

$$\hat{I}_{\text{sync}}(\underline{K}) = \int_{-\infty}^{\infty} e^{-i2\pi\underline{K}\cdot\underline{r}} I_{\text{sync}}(\underline{r}) d^2r$$

By use of the convolution theorem, we have

$$\hat{I}_{\text{sync}}(\underline{K}) = \frac{2Te^2 n^2 \lambda^2}{(h\nu)^2} G_g(\underline{K}) [G_o^*(-\underline{K})H(-\underline{K}) \otimes G_o(\underline{K})H^*(\underline{K})] \quad (5)$$

where  $\underline{K}$  is the spatial frequency vector variable defined by its rectangular components  $(K_x, K_y)$ ,  $G_g(\underline{K})$  is the object, or more specifically, the geometrical image spectrum,  $G_o(\underline{K})$  is the detector pupil function modulated L.O. spectrum, and  $H(\underline{K})$  is the coherent transfer function of the system (ref. 3).

Equation (5) illustrates the departure of the transfer function obtained in a heterodyne system with that obtained in conventional imaging systems. Remembering that the coherent transfer function  $H(\underline{K})$  is equal to the pupil function of the optical receiver (with a suitable change in variables) (ref. 3), the conventional optical transfer function is proportional to

$$G_{\text{det}}(\underline{K}) [H(-\underline{K}) \otimes H^*(\underline{K})]$$

where  $G_{\text{det}}(\underline{K})$  is the Fourier transform of the detector aperture function,  $T_{\text{det}}(\underline{K})$ . In equation (5), we see, however, that  $H(\underline{K})$  is modified by the spectrum of the L.O./detector combination  $G_o(\underline{K})$ . The normalized convolution of the product  $G_o(\underline{K})H^*(\underline{K})$  with its negative argument complex conjugate is defined as the heterodyne transfer function  $G_H$ . Functionally, then, we define a normalized heterodyne transfer function by

$$G_H(\underline{K}) \equiv \frac{G_o^*(-\underline{K})H(-\underline{K}) \otimes G_o(\underline{K})H^*(\underline{K})}{[G_o^*(-\underline{K})H(-\underline{K}) \otimes G_o(\underline{K})H^*(\underline{K})]_{\underline{K}=0}} \quad (6)$$

or

$$G_H(\underline{K}) = \frac{\int d^2K' G_O(\underline{K}') H^*(\underline{K}') G_O^*(\underline{K}' - \underline{K}) H(\underline{K}' - \underline{K})}{\int d^2K' |G_O(\underline{K}')|^2 |H(\underline{K}')|^2}$$

### Heterodyne Efficiency Factor

The denominator of equation (6) indicates that the product  $G_O(\underline{K})H(\underline{K})$  represents the optics/L.O. detector amplitude spectrum that is transferred to the detector. Using Parseval's theorem, the integral

$\int d^2K |G_O(\underline{K})|^2 |H(\underline{K})|^2$  is thus the power available for heterodyning out of a total L.O.-detector power of  $P_O = \int d^2K |G_O(\underline{K})|^2$ . For a uniform extended source, we may thus define an efficiency factor

$$\chi \equiv \frac{\int d^2K |G_O(\underline{K})|^2 |H(\underline{K})|^2}{\int d^2K |G_O(\underline{K})|^2} \leq 1 \quad (7)$$

With this definition, the current spectrum for the synchronous detector (equation (5)) becomes

$$\hat{I}_{\text{sync}}(\underline{K}) = \frac{2Te^2 \eta^2 \lambda^2}{(h\nu)^2} P_O \chi G_H(\underline{K}) G_g(\underline{K}) \quad (8)$$

Equation (8) may be related to a more conventional form of heterodyne efficiency found in the literature (ref. 5). The synchronous detector current is the inverse Fourier transform of equation (8); that is,

$$I_{\text{sync}}(\underline{r}) = \int_{-\infty}^{+\infty} d^2K \hat{I}_{\text{sync}}(\underline{K}) e^{i2\pi \underline{K} \cdot \underline{r}}$$

For a stationary scene (i.e., before translation of the image coordinates), we have  $\underline{r} = 0$  so that

$$I_{\text{sync}}(0) = \int_{-\infty}^{+\infty} \hat{i}_{\text{sync}}(\underline{K}) d^2K \quad (9)$$

Substitution of equation (8) into equation (9) and assuming a blackbody source of geometrical shape factor  $A_g(\underline{r})$  and radiance  $P_g(\underline{r})$ , we have

$$P_g(\underline{r}) = \frac{2h\nu B_{\text{HIF}}}{\lambda^2 (e^{h\nu/KT} - 1)} A_g(\underline{r})$$

and

$$I_{\text{sync}}(\underline{r} = 0) = \frac{4Te^2 \eta^2 P_{\text{O HIF}} B_{\text{HIF}}}{h\nu(e^{h\nu/KT} - 1)} \chi \int_{-\infty}^{+\infty} d^2K \hat{A}_g(\underline{K}) G_H(\underline{K}) \quad (10)$$

where mixing occurs over an effective bandwidth  $2B_{\text{HIF}}$  centered at the L.O. frequency, a polarization loss factor of 0.5 is included, and  $A_g(\underline{K})$  is the Fourier transform of  $A_g(\underline{r})$ . The integral portion of equation (10) has the form of a throughput (i.e., that portion of the image passed by the heterodyne transfer function). The product of  $\chi$  and this integral is an efficiency

$$\chi_{\text{het}} \equiv \chi \int_{-\infty}^{+\infty} d^2K \hat{A}_g(\underline{K}) G_H(\underline{K}) \quad (11)$$

so that equation (10) becomes

$$I_{\text{sync}}(\underline{r} = 0) = \frac{4Te^2 \eta^2 P_{\text{O HIF}} B_{\text{HIF}} \chi_{\text{het}}}{h\nu(e^{h\nu/KT} - 1)}$$

If now, we define a shot-noise level due to the L.O. by

$$N = \frac{2\eta e^2}{(h\nu)} P_o B_{SIF}$$

then the signal-to-noise ratio in the shot-noise limit becomes

$$\frac{S}{N} = \frac{I_{sync}(r=0)}{N} = \frac{2T}{(e^{h\nu/KT} - 1)} \eta_{het} \cdot \frac{B_{HIF}}{B_{SIF}}$$

where we have defined

$$\eta_{het} \equiv \eta \chi_{het} \leq \eta$$

as the heterodyne quantum efficiency and  $B_{SIF}$  is the effective shot noise bandwidth. Note that for a uniform extended source,  $A_g(\underline{K}) = \delta(\underline{K})$  and equation (11) reduces to  $\chi_{het} = \chi$  and  $\eta_{het} = \eta\chi$ . In this case, the efficiency factor  $\chi$ , which we have defined in equation (7), is equivalent (to within the D.C. quantum efficiency  $\eta$ ) to the heterodyne quantum efficiency,  $\eta_{het}$ , found in the literature (ref. 5).

### System Transfer Function

Results from the previous section may now be used to calculate the system transfer function, including the low-pass filter (see fig. 2), for the specific example of imaging a one-dimensional object through an optical receiver which has rectangular symmetry. This has some physical significance since the resolution elements of interest in an orbiting heterodyne receiver are vertically varying stratospheric layers. To avoid scaling difficulties in the calculations, we will use angular coordinates defined by (see fig. 1)

$$\theta = \frac{X}{d_i} \quad \phi = \frac{Y}{d_i} \quad (\text{rad})$$

(12)

$$K_\theta = K_X d_i \quad K_\phi = K_Y d_i \quad (\text{cycles/rad})$$

and

$$\theta_F = \frac{\ell_X}{d_i} \quad (\text{rad})$$

where  $\theta_F$  is the geometrical instantaneous field of view (I.F.O.V.) of the optical receiver.

Using equations (12) and the one-dimensional geometry, equation (6) becomes

$$G_H(K_\theta; K_C, \theta_F) = \frac{[\text{sinc}(\theta_F K_\theta) \text{RECT}(\frac{K_\theta}{2K_C})] \otimes [\text{sinc}(\theta_F K_\theta) \text{RECT}(\frac{K_\theta}{2K_C})]}{\{[\text{sinc}(\theta_F K_\theta) \text{RECT}(\frac{K_\theta}{2K_C})] \otimes [\text{sinc}(\theta_F K_\theta) \text{RECT}(\frac{K_\theta}{2K_C})]\}_{K_\theta=0}} \quad (13)$$

where

$$\text{sinc}(X) \equiv \frac{\sin \pi X}{\pi X} \quad \text{and} \quad \text{RECT}(\frac{X}{2\ell}) \equiv \begin{cases} 1, & |X| \leq \ell \\ 0, & |X| > \ell \end{cases}$$

Equation (13) assumes a plane-wave local oscillator incident on the detector so that the detector/L.O. transfer function becomes simply the Fourier transform of the detector aperture. Further, the coherent transfer function for the optics is the pupil function (rectangular in shape) having a coherent cutoff frequency of  $K_C = D_A/2\lambda$ , where  $D_A$  is the diameter of the receiver aperture and  $\lambda$  the wavelength. This convolution process is shown in figure 3.

Equation (13) along with equation (8) gives the system transfer function up to the low-pass filter. Expressing  $G_H(K_\theta; K_C, \theta_F)$  and  $\chi(K_C, \theta_F)$  (eq. (7)) in integral form, we have

$$G_H(K_\theta; K_C, \theta_F) = \frac{\int_{-\infty}^{+\infty} \text{sinc}(\theta_F K'_\theta) \text{RECT}(\frac{K'_\theta}{2K_C}) \text{sinc}[\theta_P (K'_\theta - K_\theta)] \text{RECT}(\frac{K'_\theta - K_\theta}{2K_C}) dK'_\theta}{\int_{-\infty}^{+\infty} \text{sinc}^2(\theta_F K_\theta) \text{RECT}(\frac{K_\theta}{2K_C}) dK_\theta} \quad (14)$$

and

$$\chi(K_\theta, \theta_P) = \theta_P^2 \left[ \int_{-\infty}^{+\infty} \text{sinc}^2(\theta_F K_\theta) \text{RECT}\left(\frac{K_\theta}{2K_C}\right) dK_\theta \right]^2 \quad (15)$$

Finally, inclusion of the low-pass filter transfer function  $H_{LP}$  (i.e., the integration time,  $\tau$ ) which is modeled as a running mean integrator, we have

$$H_{LP}(K_\theta) = \text{sinc}\left(\frac{v_0 \tau}{z} K_\theta\right)$$

where  $v_0$  is the vertical component of the orbital velocity,  $z$  the receiver-object distance, and  $\tau$  the integration time. The total transfer function is then the modulation transfer function

$$\text{MTF}(K_\theta) = |G_H(K_\theta) H_{LP}(K_\theta)|$$

Equations (14) and (15) can be evaluated in terms of tabulated functions yielding the following relations which will be used for computational purposes

$$\begin{aligned} \text{MTF}(K_\theta; \theta_F, K_C, \frac{v_0 \tau}{z}) = & (N) \text{RECT}\left(\frac{K_\theta}{4K_C}\right) \left| \text{sinc}\left(\frac{v_0 \tau}{z} K_\theta\right) \left\{ \left( \frac{\cos(\pi \theta_F K_\theta)}{\pi \theta_F K_\theta} \right) \times \right. \right. \\ & [C_{in}(|2\pi \theta_F K_\theta - 2\pi \theta_F K_C|) - C_{in}(2\pi \theta_F K_C)] + \\ & \left. \left. \left( \frac{\sin(\pi \theta_F K_\theta)}{\pi \theta_F K_\theta} \right) [s_i(2\pi \theta_F K_C) - s_i(2\pi \theta_F K_\theta - 2\pi \theta_F K_C)] \right\} \right| \quad (16) \end{aligned}$$

$$\chi(\theta_F, K_C) = \frac{4}{\pi} [s_i(2\pi \theta_F K_C) - \frac{1 - \cos(2\pi \theta_F K_C)}{2\pi \theta_F K_C}]^2 \quad \text{and} \quad N = (\pi \chi^{1/2})^{-1}$$

In the above equations, the functions  $C_{in}(X)$  and  $s_i(X)$  are defined as (ref. 6)

$$s_i(X) = \int_0^X \frac{\sin t}{t} dt \quad \text{and} \quad C_{in}(X) = \int_0^X \frac{1 - \cos t}{t} dt$$

## RESULTS AND DISCUSSION

### MTF Calculations

Equations (16) and (17) may now be evaluated for some specific parameter values which are applicable to the optical receiver in a spacelab type of scenario. A worst-case set of orbital values would be obtained for the receiver platform at an orbital height of  $R = 400$  Km and a tangent height  $H_T$  of 10 km. At these values, we assume that the receiver is operating in a solar occultation mode where the sunrise or sunset velocity due to orbital motion is  $v_o = 2$  km/sec. The MTF and  $\chi$  calculations (equations (16) and (17)) will be done for an I.F.O.V. of  $\theta_F = 0.5$  mrad, an equivalent optical receiver aperture of  $D_A = 50.8$  mm, and values of integration time of  $\tau = 0.2$  sec and 0.4 sec. Further, the value of  $D_A = 50.8$  mm at a wavelength of  $\lambda = 11.152$   $\mu\text{m}$  ( $\text{HNO}_3$  line) corresponds to an optics cutoff frequency of  $K_C = D_A/2\lambda = 2278$  cycles/rad. These parameters are compatible with the values for an LHS type experiment using a tunable diode laser as the L.O. and associated optics for coupling this type radiation to a detector having the required time-frequency response (ref. 7).

The results of the calculations are shown in figure 4. It can be shown that, for values of  $\tau$  greater than roughly 0.2 sec., the optical (heterodyne) transfer function dominates the MTF; and, for  $\tau$  somewhat less than 0.4 sec., the low-pass filter is the dominant frequency limiting factor. Note that the angular frequency values can be converted to linear spatial frequency (cycles/km) by the relationships of equation (12) by appropriately scaling image and object space by the ratio of image distance  $d_i$  to object distance  $z$ . For the orbital values assumed,  $z = 2262$  km and consequently a value of 2262 cycles/rad corresponds to an object spatial frequency of 1 cycle/km. Examination of the MTF curves shows that resolutions of the order of 1.5 to 2.0 km may be expected for the various integration times.

### Efficiency Calculations

The efficiency factor (heterodyne efficiency) given by equation (17) is shown plotted in figure 5 for the example of the plane-wave L.O.. Two geometries are shown: rectangular optics (as has been previously assumed) and circular optics adjusted for equal optics and detector areas. The significance of the abscissa ( $2\pi\theta_F K_C$ ) relative to heterodyne efficiency becomes apparent when



it is noted that at the value of  $2\pi\theta_F K_C \approx 7.7$  corresponds approximately to an image (sun) size filling the detector of one Airy disk of the receiver aperture. In this region, the efficiency is in excess of 80 percent.

### Sampling Error

The calculations shown plotted in figure 5 do not include any sampling errors which may occur. Suppose we sample the output of the low-pass filter, which has been modeled as a running mean integrator, at a rate of the inverse of the integration time. This is equivalent to a sampled mean integration scheme. Under this constraint, it may be shown that for certain values of  $\tau$  the signal is undersampled. This results in an aliasing or foldover error which can be significant relative to the desired signal. For example, shown in figure 6 is the total MTF for the values of orbital and system parameters previously stated. Two integration times are considered:  $\tau = 0.2$  sec and  $\tau = 0.4$  sec. If we define the sampling error as the ratio of the "foldover" amplitude on the MTF plot to the amplitude of the MTF itself (i.e., a white signal spectrum), we see that the error for 0.4 sec. is approximately 40 percent at 0.5 cycle/km frequency and considerably worse for higher values of spatial frequency. Conversely, for  $\tau = 0.2$  sec and the correspondingly higher sampling rate, the sampling error is negligible.

### Heterodyne Receivers With Obscurations

Telescopes having central obscurations, such as Cassegrains, are often used for imaging a source. If this type receiver is used as a collector for heterodyne-type detection, one needs to compare the efficiency  $\chi$  and the heterodyne transfer function  $G_H$  with that obtained for no obscuration case.

In figure 7, we consider the effects of receiver apertures having obscuration ratios of 0 and 20 percent for  $\theta_F = 0.2$  and 0.5 mrad. Note the enhancement of response in the 2000 cycles/rad region at the expense of that near 1000 cycles/rad for 20 percent obscuration and  $\theta_F = 0.5$  mrad. The effects of obscurations are more pronounced for square as opposed to circular geometries. An unobscured conventional MTF discussed earlier is plotted for  $\theta_F = 0.5$  mrad showing a somewhat reduced frequency response characteristic from the heterodyne MTF. For a smaller detector ( $\theta_F = 0.2$  mrad), a 20 percent obscuration tends to assume the shape of a conventional MTF.

In figure 5, we assume a receiver aperture having obscuration ratios of 20 and 50 percent. For the values of  $\theta_F$  and  $K_C$  used earlier,  $2\pi\theta_F K_C \approx 7.2$ , and comparison of the various efficiency curves at this value shows striking differences. For 50 percent obscuration case, one sees that the heterodyne efficiency is virtually zero while for 20 percent obscuration a relative efficiency of slightly greater than 0.2 is achieved. This compares with a value of greater than 0.8 when unobscured. Note further, there is a "peaking" of the efficiency curves for obscured systems. The rule-of-thumb requirement of one Airy disk on the detector for "good" efficiency no longer holds, but rather the source image needs to be less than this value to achieve the maximum efficiency

for that particular system. The effect can be explained on the basis of the overlap integral (eq. (2)) of the L.O. field and signal field distributions. Thus, the diffracted field due to the central obscuration is out of phase with that of the primary diffracted field and, as the size of the detector and/or optics increases, the cancellation tends to be more complete.

#### CONCLUDING REMARKS

The analysis of a passive heterodyne receiver with respect to its imaging performance (transfer function) and its heterodyne efficiency shows some interesting departures from the results which are obtained in strictly coherent or incoherent imaging systems. For example, the cascading property of MTF analysis must be carefully applied, since the coherent transfer function of the optical receiver and that due to the L.O.-detector combination are not separable but are related by the convolution of their products. Application of these results to a spacelab-type optical heterodyne receiver (LHS) shows that resolutions of the order of 1.5 to 2.0 km are possible for worst-case type orbital scenarios.

Further, comparison of obscured-type receivers (e.g., Cassegrains) with unobscured receivers shows that both resolution and efficiency are severely degraded in an obscured-type receiver and, consequently, should not be used for a passive heterodyne detection scheme.

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November 24, 1980

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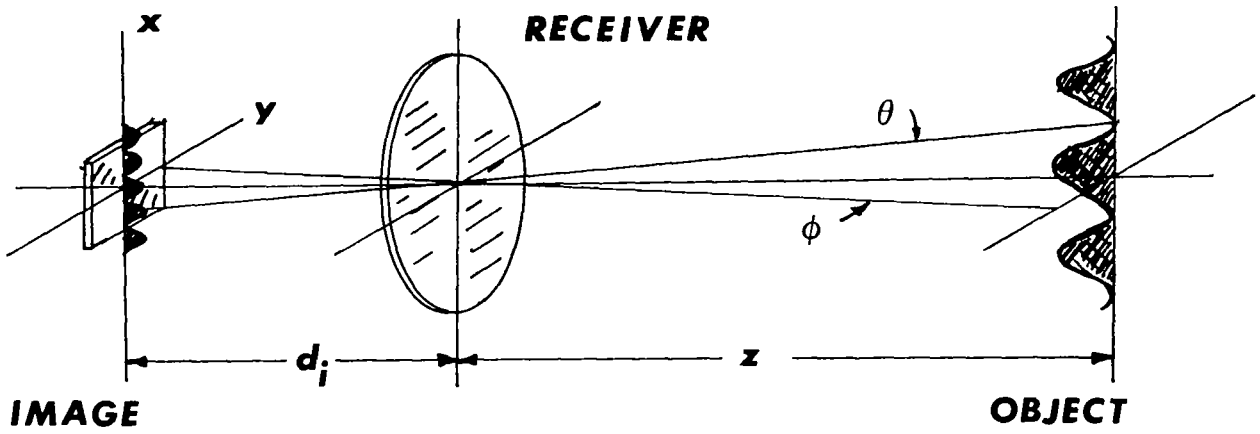


Figure 1.- Imaging geometry.

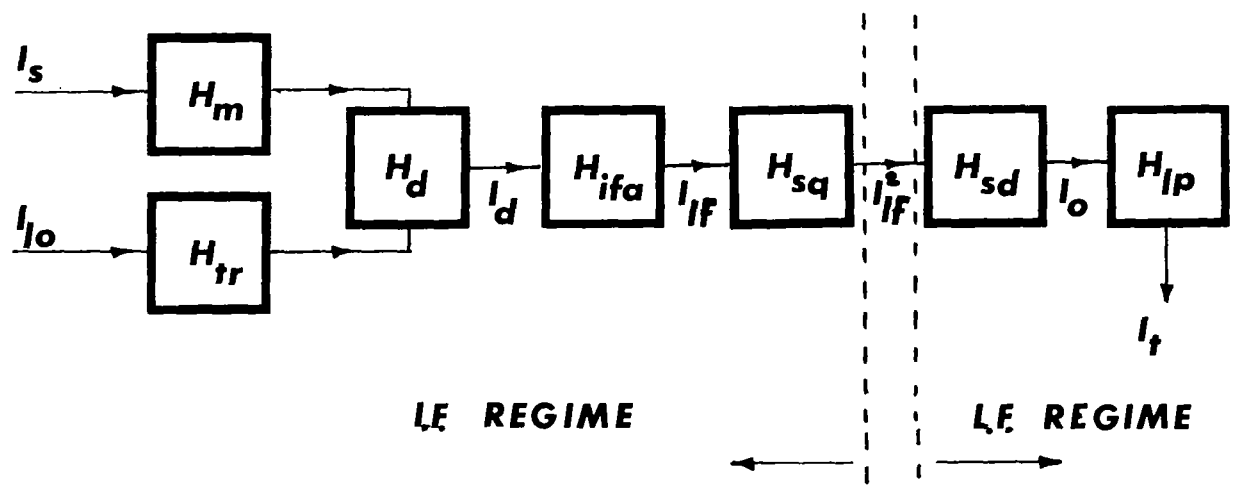


Figure 2.- System transfer functions.

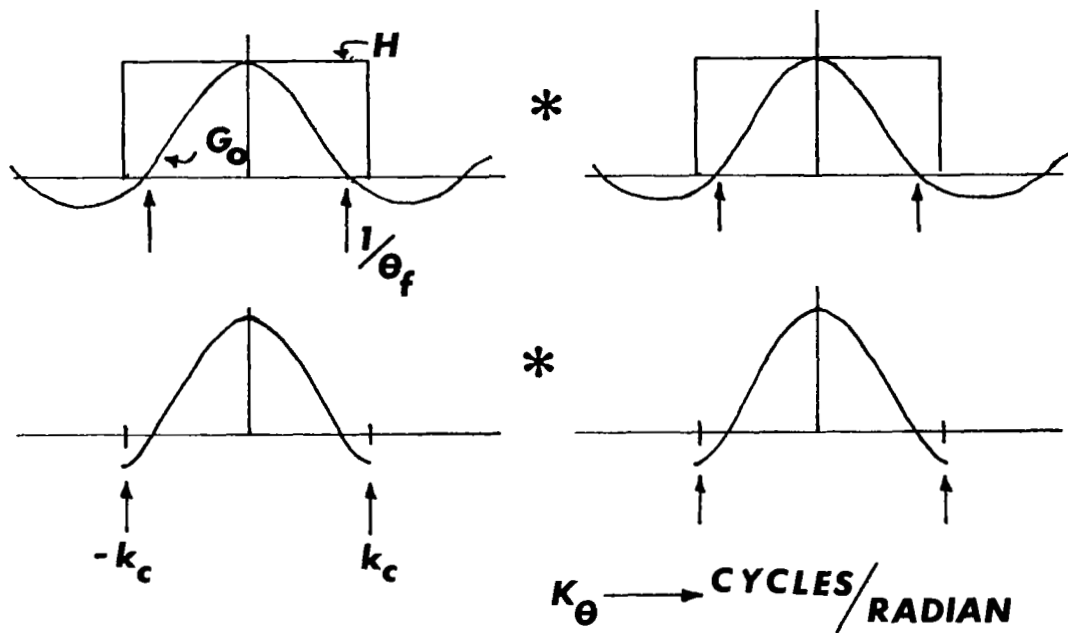


Figure 3.- Graphical interpretation of heterodyne transfer function.

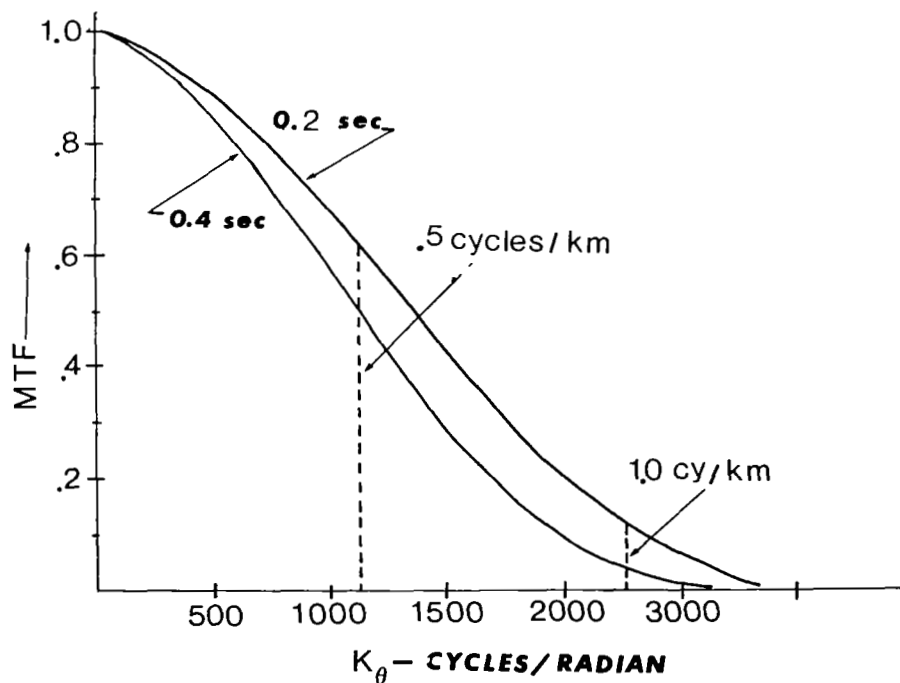


Figure 4.- Total MTF for worst-case shuttle orbit and two values of integration time.

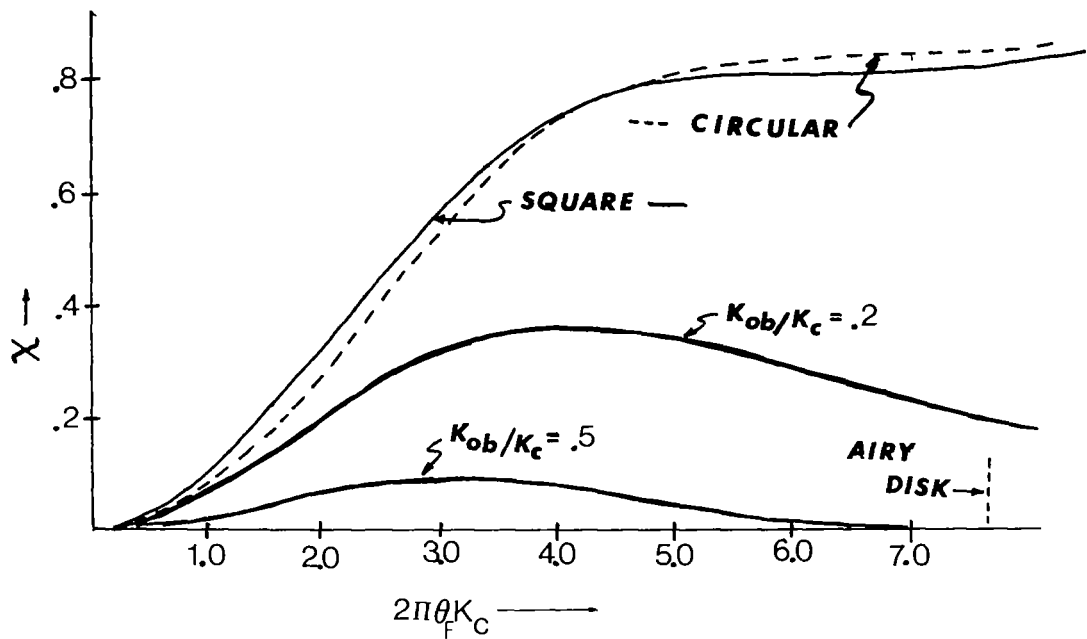


Figure 5.- Efficiency factor,  $\chi$ , versus system parameter  $2\pi\theta_F K_C$  for various receiver geometries.

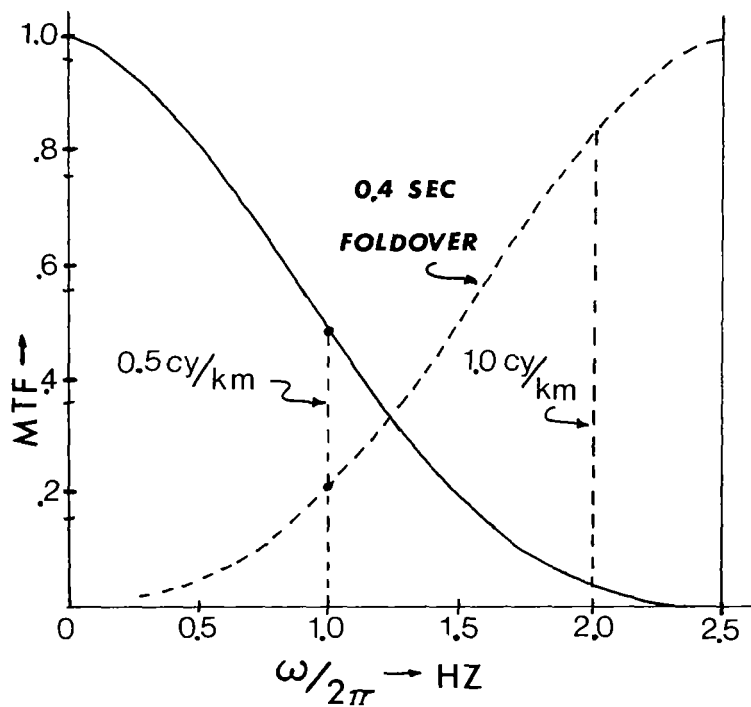


Figure 6.- Aliasing error for 2.5-Hz sampling rate at worst-case shuttle orbit.

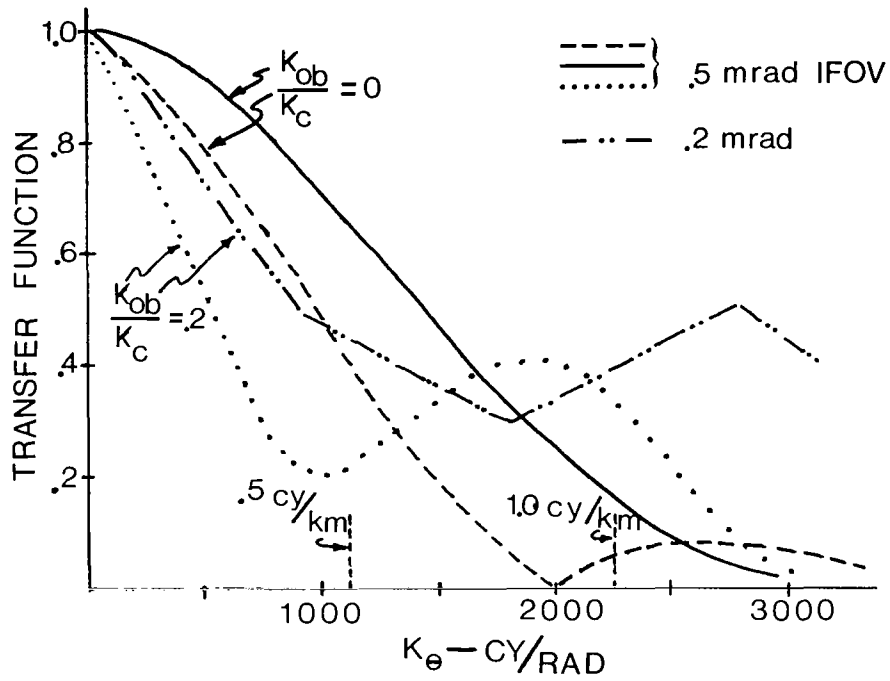


Figure 7.- Comparison of transfer functions for various geometries. Dashed curve represents conventional MTF with 0.5 mrad I.F.O.V.

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