

THE USE OF THE PRINCIPLE OF SUPERPOSITION IN MEASURING AND PREDICTING
THE THERMAL CHARACTERISTICS OF AN ELECTRONIC EQUIPMENT OPERATED IN A
SPACE ENVIRONMENT

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ABSTRACT

This paper reviews the advantages and possible pitfalls of using a generalized method of measuring and, based on these measurements, predicting the transient or steady-state thermal response characteristics of an electronic equipment designed to operate in a space environment. The method requires generation of a set of thermal influence coefficients by test measurement in vacuo. A simplified thermal mockup is used in this test. Once this data set has been measured, temperatures resulting from arbitrary steady-state or time varying power profiles can be economically calculated with the aid of a digital computer.

INTRODUCTION

The two most common methods of predicting the thermal performance of an electronic equipment are to:

- 1) Make a thermal mathematical model and solve the resulting network equations using finite difference or finite element methods with the aid of a computer
- 2) Build a thermal mockup of the equipment which approximates the geometry and the dissipation profile of the equipment under design and directly measure the thermal performance in a simulated environment.

This paper describes a middle road to the above methods utilizing both measurements to initially thermally characterize an equipment and then a simple computer program to accurately and economically predict either the transient or steady-state thermal response of the equipment for any power versus time or steady-state profiles of interest. The method is of great value when the thermal performance with more than one power profile (duty cycle) is required.

The method requires the construction of a thermal mockup of similar configuration to the equipment under design with a unit power resistor attached within the equipment at each location of an electronic component having a significant heat dissipation. The unit resistors may be all of the same wattage rating and dissipation despite the fact they are located at positions where varying amounts of heat are to be dissipated. A thermocouple is installed in the vicinity of each of the unit resistors as well as at all other locations within the equipment at which temperature information is desired.

A single set of measurements in which each unit resistor is powered, one at a time, and the resulting temperature-time profiles at all points of interest are measured and recorded. This data bank allows the later prediction of temperatures using any power profile accurately, quickly and economically utilizing an easy to write computer program. This simple method is accurate only

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in systems whose characteristic underlying differential equations are linear such as the equations describing the heat conduction within an equipment in a space (no air) environment.

This paper describes how the method was legitimately and successfully used to predict the steady-state thermal performance of an electronic equipment having various duty cycles despite the fact that the equipment case-to-surroundings heat transfer within a spacecraft was by thermal radiation, a mode of heat transfer described by nonlinear equations. A discussion of when it would be permissible to include such nonlinear surface heat transfer is presented.

NOMENCLATURE

A	radiating area of case
C	thermal capacity of conducting material
E	constant which is function of emissivities of equipment case finish and suroud finish in temperature-vacuum chamber
k	thermal conductivity
q''	heat flux per unit area
q'''	heat generation per unit time per unit volume
T	absolute temperature
w	specific weight of conductivity material
x,y,z	spatial coordinates
σ	Stefan-Boltzmann constant

Subscripts

c	cold
h	hot
i	index number of test measurement of temperature field resulting from single source
m	mean
t	total

THEORETICAL JUSTIFICATION OF METHOD

Heat transfer in electronic equipments used in space takes place by conduction and radiation. It is usual that the method of heat transfer internal to an equipment, that is, between heat dissipating components and the outer case of the equipment, is by conduction through structural paths with radiative heat transfer playing a negligible role. This usually happens by design as providing structural paths having high thermal conductances is necessary for good thermal management. Heat transfer from the outer equipment case may be by conduction through the equipment mounts, or thermal radiation from the case to the surroundings or a combination of both of these transfer mechanisms.

When heat transfer within the box is only by conduction, the resulting temperature field must satisfy the general heat conduction equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'''}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial \theta} \quad (1)$$

$$\alpha = \frac{k}{Cw} \quad (2)$$

Since this governing equation is linear, the well-known principle of superposition can be used in the manner described in this report to predict the time varying or steady-state temperature field within the equipment. Reference 1 states: "The principle of superposition can be applied to all cases in which the effect of simultaneous superposed actions is the sum of the effects of each individual action. It can be applied, therefore, to linear system, i.e., to physical systems governed by linear differential or integral equations." Using this method to predict the thermal performance of a particular equipment will now be described.

IMPLEMENTATION OF METHOD

The particular equipment for which the author successfully used the analysis method described in this paper had the following characteristics:

- 1) In application, it was to be operated with a large number of different "duty cycles". Each duty cycle was defined by the power versus time history of each heat dissipating component within the equipment.
- 2) The equipment was radiatively cooled to its surroundings and, during the operation, the outer case temperature was approximately uniform.
- 3) Paths of high thermal conductance between heat dissipating components and the case were provided by design for heat management purposes; that is, the unit was designed for efficient internal conduction cooling.

TESTING PROCEDURE

The electronic equipment chassis that was later used in the construction of the engineering model was instrumented by the bonding of wire-wound resistors at locations at which there would later be significant thermal dissipations from individual electronic components. Thermocouples were attached at all component locations at which operating temperature information was required. In this particular test, a total of sixty thermocouples and forty resistors were used to characterize the thermal profile of the unit. A thermocouple was always mounted on the chassis structure adjacent to each wire-wound resistor.

A high vacuum, 10^{-5} Torr range, test chamber having a temperature controlled shroud was used in the test. The temperature of the shroud was held at that uniform temperature which had been specified for the design. Each resistor was activated, one at a time, and the resulting temperature-time history at all thermocouple locations was recorded.

DATA PROCESSING

The data was processed in the following manner. The temperature rise above shroud temperature for each location as a result of powering each resistor was recorded in a matrix format at the end of each ten-minute interval. Using the shroud temperature as the baseline or reference temperature meant that the nonlinear radiative equipment-to-surroundings (shroud) thermal resistance was included in the compilation of these thermal "influence coefficients". The error resulting from the inclusion of these radiative resistances will be discussed below where it will be shown to be:

- 1) Small
- 2) On the safe side, i.e., resulting in higher than actual predicted temperatures.

THE COMPUTER CODE

The concept for the computer code is obvious and simple. A program was written which simply added the temperature rises at each thermocouple location resulting from step infusions of heat at each resistor location at the end of incremental units of time. The process is one of summation of effects requiring the direct addition of temperature rises (a noniterative solution) and the computer time (cost) needed to analyze various duty cycles (power vs time) is very small.

Had the unit itself been conduction-cooled through its mounts, the above method would have been completely theoretically correct and the following section on estimating the upper bound of the error resulting from the nonlinearity of the radiative heat transfer path between the equipment case and the surroundings would not need to be considered.

UPPER BOUND OF NONLINEARITY ERROR

The upper bound of the error resulting from treating the radiative conductance from the case to the shroud as independent of temperature level will now be determined.

The basic equation describing radiative heat transfer from each region (node) of the equipment case to the shroud may be written in the following form:

$$q'' = EA\sigma(T_h^4 - T_c^4) \quad (3)$$

On factoring, equation (3) may be written:

$$q'' = EA\sigma(T_h^3 + T_h^2T_c + T_hT_c^2 + T_c^3)(T_h - T_c) \quad (4)$$

References 2 and 3 show that if $T_h - T_c$ is small compared to $(T_h + T_c)/2 = T_m$, equation (4) may be replaced by:

$$q'' = 4EA\sigma T_m^3(T_h - T_c) \quad (5)$$

The criterion for the above approximation was always met when measurement with the individual unit resistors was taking place.

The formula describing the total heat transfer from any case region (or the entire case if the temperature is uniform) is of the same form as equation (3):

$$q_T'' = EA\sigma(T_{ht}^4 - T_c^4) \quad (6)$$

Rewriting equation (6):

$$q_T'' = EA\sigma(T_{ht}^2 + T_c^2)(T_{ht} + T_c)(T_{ht} - T_c) \quad (7)$$

Solving for $(T_{ht} - T_c)$ yields:

$$(T_{ht} - T_c) = \frac{q_T''}{EA\sigma(T_{ht}^2 + T_c^2)(T_{ht} + T_c)} \quad (8)$$

$(T_{ht} - T_c)$ found by summing (superposition) using equation (5) is:

$$(T_{ht} - T_c) = \sum_{i=1}^n (T_{ht} - T_c) = \sum_{i=1}^n \frac{q_i''}{4EA\sigma T_m^3} = \frac{q_t''}{4EA\sigma T_m^3} \quad (9)$$

The ratio of $T_{ht} - T_c$ calculated by superposition using equation (9) to the exact total equation (8) has been calculated for a wide range of sink temperatures and temperature differences, and the results are presented in Figure 1.

It should be emphasized that this figure does not indicate the actual error but the upper bound of the actual error as during the unit resistor tests $T_m = T_c$ for each individual test. It should also be realized that this radiative resistance is never the total resistance between source and sink (i.e., component mounting location and shroud) but merely a series component. Thus, a 10-percent error in calculation of this quantity would result in a lesser total error.

If the possible upper error bound as shown in Figure 1 were too large to be acceptable, the absolute total error could be reduced by including in the computational program a temperature level correction factor, after equation (5), to correct the measured i^{th} case-to-shroud temperature difference to a case temperature more indicative of the case temperature during operation.

The form of this correction factor would be:

$$\frac{T_{mi}^3}{T_{mt}^3} \quad (10)$$

where T_{mi} was the mean absolute temperature resulting from the averaging of case and shroud temperature during the i^{th} test and T_{mt} the temperature resulting from the averaging of case and shroud temperatures with the case temperature predicted using the total unit power.

CONCLUSION

The prediction method described in this paper can be an extremely useful and economical tool for predicting either the steady-state or transient thermal response. It is of particular value in each of the following situations:

- 1) Calculating the responses to equipment to be operated with many different duty cycles, i.e., power time profiles, where it would be impossible (due to time limitations) or uneconomical to run a multitude of tests using the actual power profiles
- 2) The procedure is also a valuable tool for checking the accuracy of finite element or finite difference programs in which internal thermal conductances have been determined by calculation rather than measurement and for which semi-empirical verification is desired.

REFERENCES

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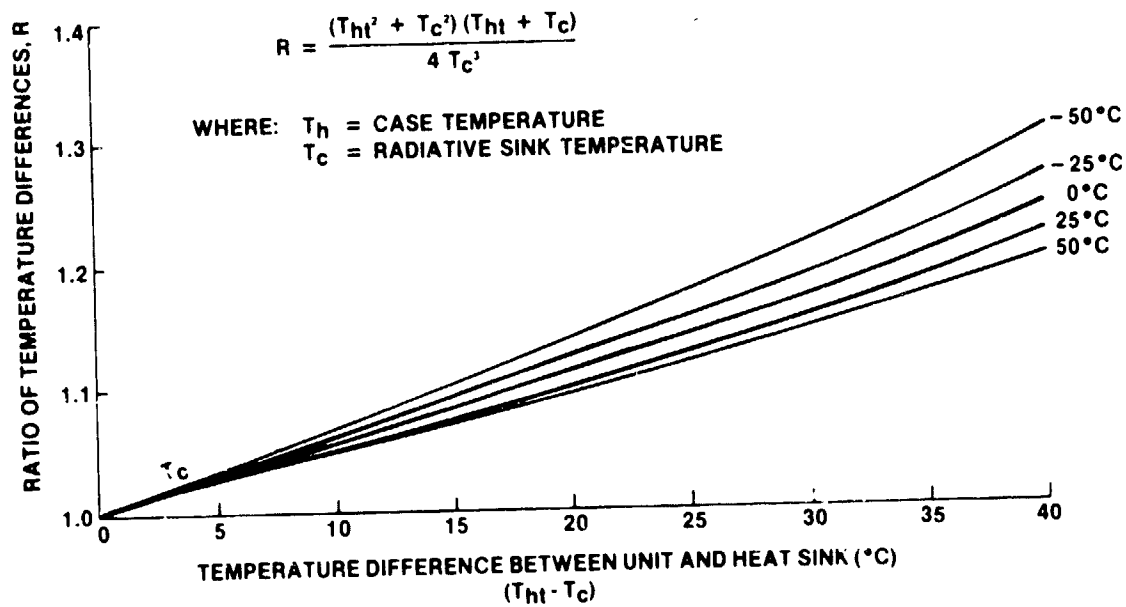


Figure 1. Ratio of Theoretical Maximum Sum of Surface-to-Sink Temperatures to Actual Surface-to-Sink Temperature Difference at the Same Total Unit Power