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Generation of Boundary and Boundary-Layer Fitting Grids*

C. M. Ablow and S. Schechter
SRI International
Menlo Park, CA 94025

ABSTRACT

The details of extended physical processes, such as the gas dynamic flow over an airfoil, the reactive flow through a combustor, or the electric field in a multi-contact transistor, are understood by solving the differential equations of a mathematical model of the process. The accuracy of finite difference methods for the numerical solution of the equations is increased if the underlying mesh fits the region boundaries and is closely spaced in regions where the solution is rapidly varying. Automatic methods for producing a satisfactorily adjusted mesh have been developeed for one-dimensional problems. In one simple, effective scheme of this kind the unknown function and the distribution of mesh modes are found simultaneously, the nodes being placed so that they correspond to points uniformly spaced on the solution curve.

In a two-dimensional generalization, the nodes correspond to points equally spaced on the solution surface in two directions that are as nearly orthogonal as possible. Examples of such meshes are shown for given surfaces in the figures. The meshes fit the circular boundaries and come closer together where the given surface is steeper.

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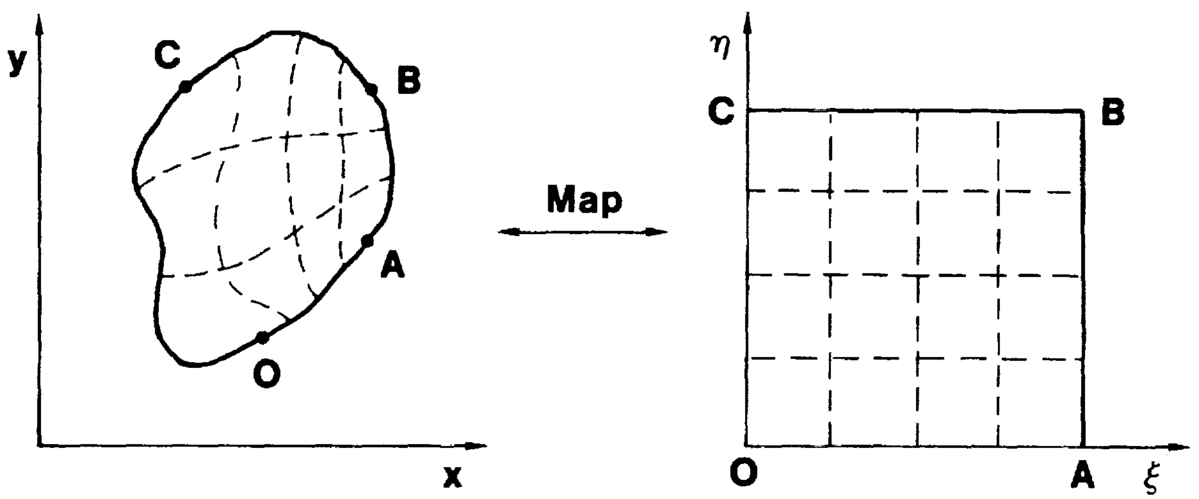
GENERAL GRID GENERATION METHOD

Map problem domain onto unit cube in computational coordinate space. Uniform, rectangular grid in computer coordinates gives curvilinear grid in original domain.



Choose map to reduce truncation error of finite difference solution scheme for problem.

Problem: Find z(x,y) so that

$$(*) \quad z_{xx} + z_{yy} = \rho(x,y) \text{ in } x^2 + y^2 < 1.$$
$$z = b(x,y) \text{ on } x^2 + y^2 = 1.$$

with ρ and b given functions.

Change coordinates: $(x,y) \rightarrow (\xi, \eta)$

$$(*) \quad \frac{\partial}{\partial \xi} \frac{g z_\xi - f z_\eta}{j} + \frac{\partial}{\partial \eta} \frac{e z_\eta - f z_\xi}{j} = j\rho$$

$$ds^2 \equiv dx^2 + dy^2 = e d\xi^2 + 2f d\xi d\eta + g d\eta^2$$

$$e = x_\xi{}^2 + y_\xi{}^2$$

$$f = x_\xi x_\eta + y_\xi y_\eta$$

$$g = x_\eta{}^2 + y_\eta{}^2$$

$$j = (eg - f^2)^{1/2}$$

Truncation error of centered finite-difference approximation

$$= \frac{1}{12j}\Big[g z_{\xi\xi\xi\xi}\Delta\xi^2 + e z_{\eta\eta\eta\eta}\Delta\eta^2 - 4f(z_{\xi\xi\xi\eta}\Delta\xi^2 + z_{\xi\eta\eta\eta}\Delta\eta^2\Big]$$

Choose boundary-fitting map to
(1) Minimize $(f/j)^2$
(2) Reduce errors in separate ξ and η directions.

One-dimensional monitor function* methods are satisfactory for (2). Boundary adjustment of map used for (1).

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*A. B. White, Jr., SIAM J Num Anal 16 (1979)
C. M. Ablow and S. Schechter, J. Comp Physics 27 (1978)

One-dimensional monitor function takes equally spaced values at mesh modes.

Simplest, effective, problem-dependent monitor is distance on solution surface:

$$\frac{ds}{d\xi} \quad \text{is to be constant as } \xi \text{ varies}$$

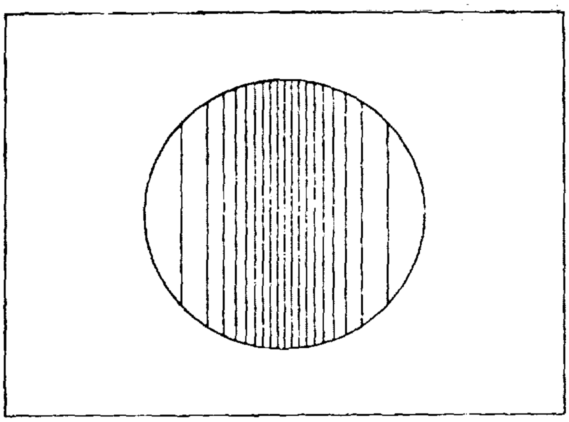
(1) $\frac{\partial}{\partial \xi}(x_\xi{}^2 + y_\xi{}^2 + z_\xi{}^2) = 0$

Same for η direction

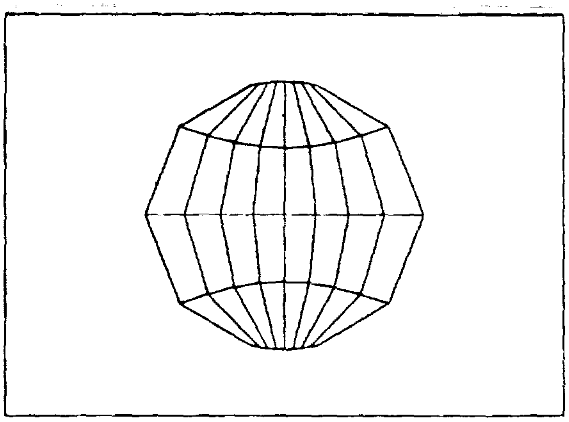
(2) $\frac{\partial}{\partial \eta}(x_\eta{}^2 + y_\eta{}^2 + z_\eta{}^2) = 0$

(1) and (2) plus given differential equation (*) for z determine solution when corner points O,A,B,C have been chosen. Corners are moved to minimize $\sum (f/j)^2$

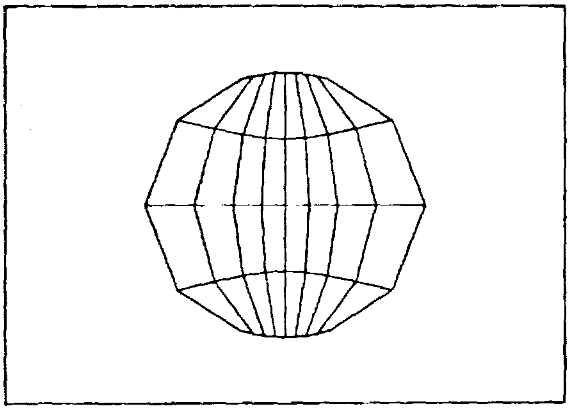
Examples show grids found from (1) and (2) for various given functions z having regions of sharp variation (boundary layers). In practice, function z and the grid mapping would be found by simultaneous solution of the complete set of differential equations (1), (2), and (*).



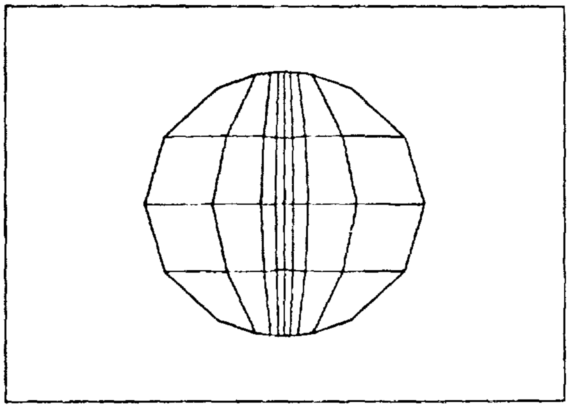
Level lines for
z = tanh px
in unit circle.



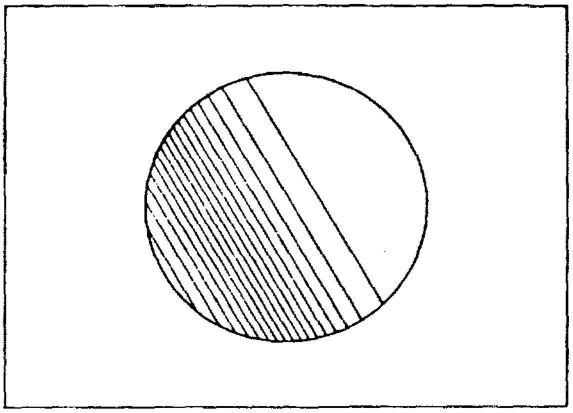
Equidistant mesh
p = 1.0



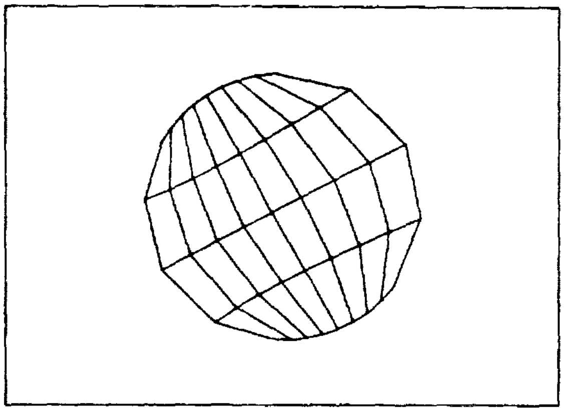
Equidistant mesh
p = 2.0



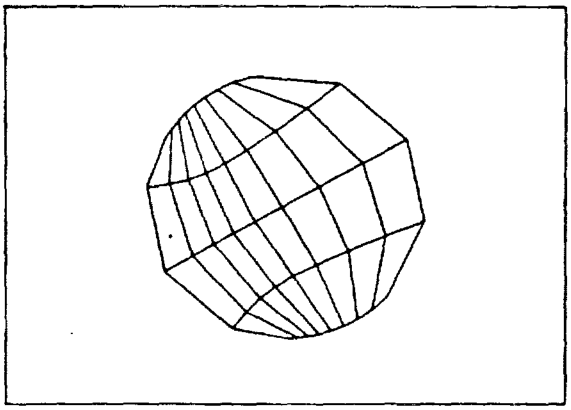
Equidistant mesh
p = 8.0



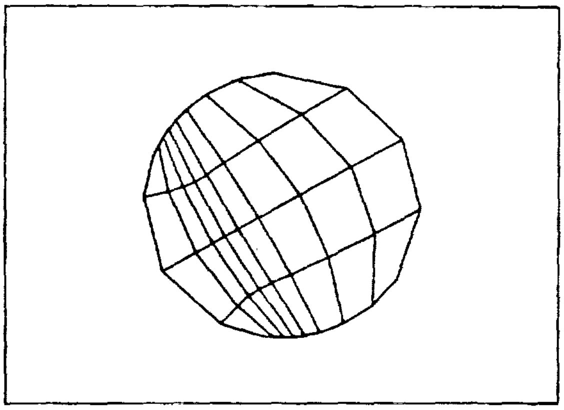
Level lines for z =
tanh[p(x cos 30 + y sin 30) + 0.5]



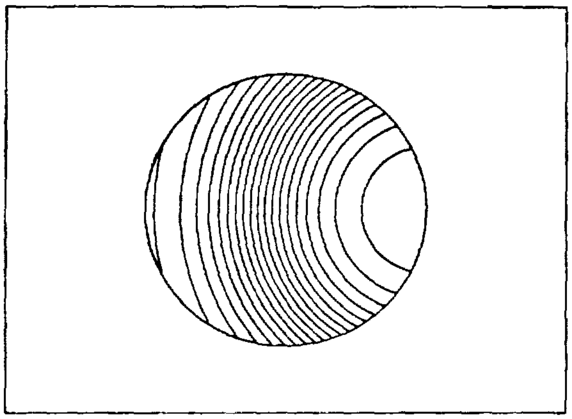
Equidistant mesh
p = 1.0



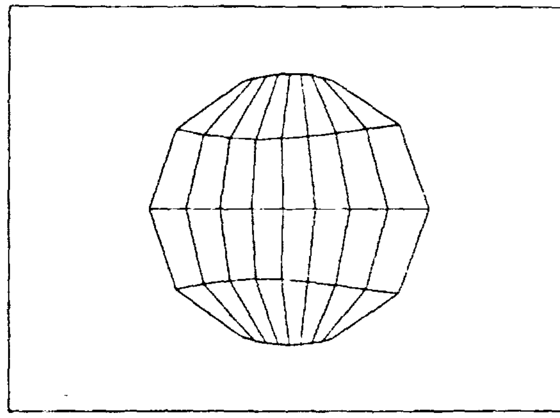
Equidistant mesh
p = 2.0



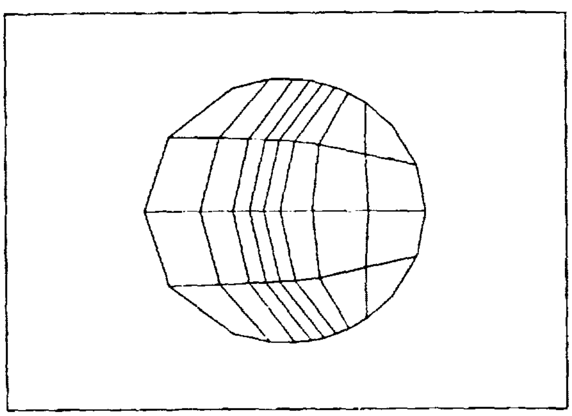
Equidistant mesh
p = 4.0



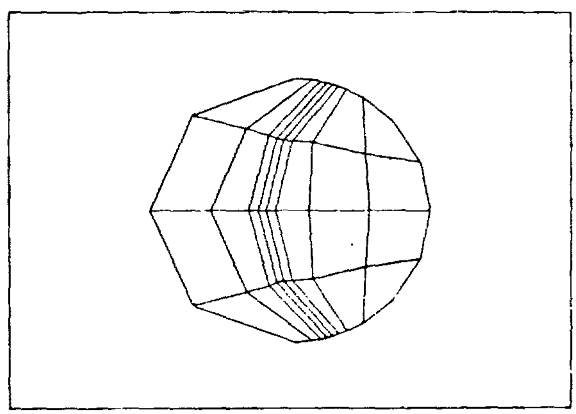
Level lines for
$z = \tanh p\,(R - 1.2)$
$R^2 = (x - 1)^2 + y^2$



Equidistant mesh
$p = 1.0$



Equidistant mesh
$p = 4.0$



Equidistant mesh
$p = 8.0$