FINITE DIFFERENCE GRID GENERATION BY MULTIVARIATE BLENDING FUNCTION INTERPOLATION*

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ABSTRACT

The General Interpolants Method (GIM) code solves the multidimensional Navier-Stokes equations for arbitrary geometric domains. The geometry module in the GIM code generates two- and three-dimensional grids over specified flow regimes, establishes boundary condition information and computes finite difference analogs for use in the GIM code numerical solution module. The technique can be classified as an algebraic equation approach.

The geometry package uses multivariate blending function interpolation of vector-values functions which define the shapes of the edges and surfaces bounding the flow domain. By employing blending functions which conform to the cardinality conditions the flow domain may be mapped onto a unit square (2-D) or unit cube (3-D), thus producing an intrinsic coordinate system for the region of interest. The intrinsic coordinate system facilitates grid spacing control to allow for optimum distribution of nodes in the flow domain.

The GIM formulation is not a finite element method in the classical sense. Rather, finite difference methods are used exclusively but with the difference equations written in general curvilinear coordinates. Transformations are used to locally transform the physical planes into regions of unit cubes. The mesh is generated on this unit cube and local metric-like coefficients generated. Each region of the flow domain is likewise transformed and then blended via the finite element formulation to form the full flow domain. In order to treat "completely-arbitrary" geometric domains, different transformation functions can be employed in different regions. We then transform the blended domain to physical space and solve the Cartesian set of equations for the full region. The geometry part of the problem is thus treated much like a finite element technique while integration of the equations is done with finite difference analogs.

*This work was supported, in part, by NASA Langley Contracts NAS1-15341, 15783, and 15795.

BUILDING BLOCK CONCEPT

The development is done in local curvilinear intrinsic coordinates based on the following concepts:

- Analytical regions such as rectangles, spheres, cylinders, hexahedrals, etc., have intrinsic or natural coordinates.
- Complex regions can be subdivided into a number of smaller regions which can be described by analytic functions. The degenerate case is to subdivide small enough to use very small straight-line segments.
- Intrinsic curvilinear coordinate systems result in constant coordinate lines throughout a simply connected, bounded domain in Euclidean space.
- The intersection of the lines of constant coordinates produce nodal points evenly spaced in the domain.
- Intrinsic curvilinear coordinate systems can be produced by a univalent mapping of a unit cube onto the simply connected bounded domain.

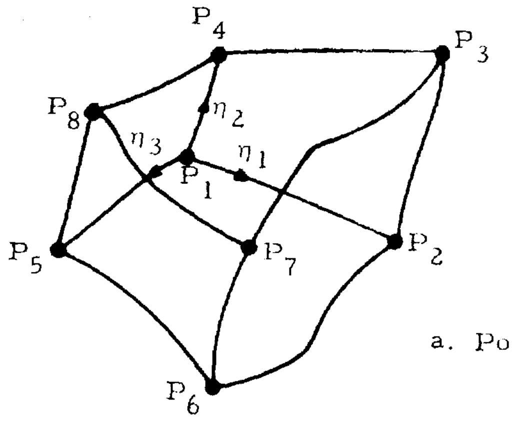
Thus, if a transformation can be found which will map a unit cube univalently onto a general analytical domain, then any complex region can be piecewise transformed and blended using general interpolants.

Consider the general hexahedral configuration shown. The local intrinsic coordinates are η_1, η_2, η_3 with origin at point P_1. The shape of the geometry is defined by

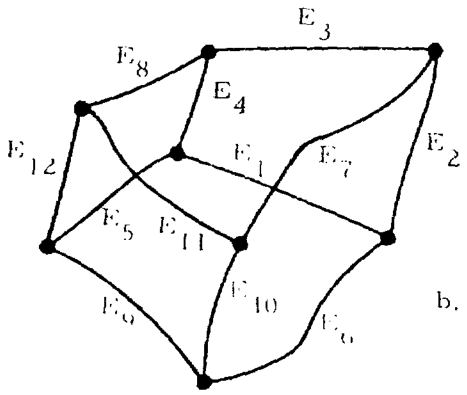
- Eight corner points, $\overline{P}_i$
- Twelve edge functions, $\overline{E}_i$
- Six surface functions, $\overline{S}_i$

This shape is then fully described if the edges and surfaces can be specified as continuous analytic vector functions $\overline{S}_i(x,y,z)$, $\overline{E}_i(x,y,z)$.

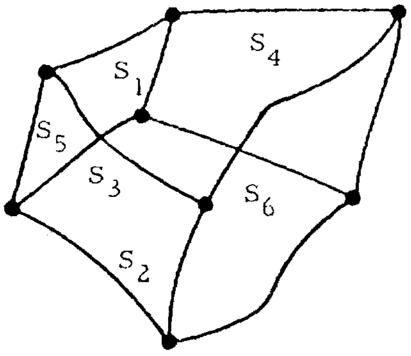
BUILDING BLOCK CONCEPT



a. Point Designations



b. Edge Designations



c. Surface Designations

GENERAL INTERPOLANT FUNCTION

Based on the work of Gordon and Hall we have developed a general relationship between physical Cartesian space and local curvilinear intrinsic coordinates. This relation is given by the general trilinear interpolant shown on the adjacent figure.

In this equation, $\underline{\overline{X}}$ vector is the Cartesian coordinates

$$\underline{\overline{X}}(\eta_1, \eta_2, \eta_3) = \begin{bmatrix} x(\eta_1, \eta_2, \eta_3) \\ y(\eta_1, \eta_2, \eta_3) \\ z(\eta_1, \eta_2, \eta_3) \end{bmatrix}$$

and S_i, E_i are the vector functions defining the surfaces and edges, respectively, and $\overline{P}_i$ are the (x, y, z) coordinates of the corner points. Edge and surface functions that are currently included in the GIM code are the following:

- EDGES (Combinations of up to Five Types)

  Linear Segment
  Circular Arc
  Conic (Elliptical, Parabolic, Hyperbolic)
  Helical Arc
  Sinusoidal Segment

- SURFACES (Bounded by Above Edges)

  Flat Plate
  Cylindrical Surface
  Edge of Revolution

This library of available functions is simply called upon piecewise via input to the computer code.

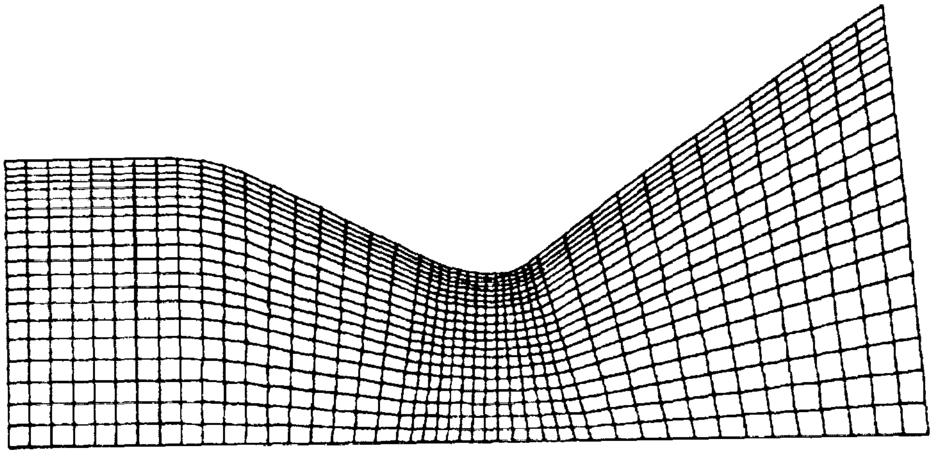
With this transformation, any point in local coordinates η_1, η_2, η_3 can be related to global Cartesian coordinates x, y, z. Likewise any derivatives of functions in local coordinates can be related to that derivative in physical space.

GENERAL INTERPOLANT FUNCTION

$$\begin{aligned}
&\underline{\overline{X}}\,(\eta_1, \eta_2, \eta_3) = \\
&(1-\eta_1)\,\vec{S}_5 + \eta_1\,\vec{S}_6 + (1-\eta_2)\,\vec{S}_2 + \eta_2\,\vec{S}_4 \\
&+ (1-\eta_3)\,\vec{S}_1 + \eta_3\,\vec{S}_3 \\
&- (1-\eta_1)(1-\eta_2)\,\vec{E}_5 - (1-\eta_1)\,\eta_2\,\vec{E}_8 - \eta_1\,(1-\eta_2)\,\vec{E}_6 \\
&- \eta_1\,\eta_2\,\vec{E}_7 - (1-\eta_1)(1-\eta_3)\,\vec{E}_4 - (1-\eta_1)\,\eta_3\,\vec{E}_{12} \\
&- \eta_1\,(1-\eta_3)\,\vec{E}_2 - \eta_1\,\eta_3\,\vec{E}_{10} - (1-\eta_2)(1-\eta_3)\,\vec{E}_1 \\
&- (1-\eta_2)\,\eta_3\,\vec{E}_9 - \eta_2\,(1-\eta_3)\,\vec{E}_3 - \eta_2\,\eta_3\,\vec{E}_{11} \\
&+ (1-\eta_1)(1-\eta_2)(1-\eta_3)\,\vec{P}_1 + (1-\eta_1)(1-\eta_2)\,\eta_3\,\vec{P}_5 \\
&+ (1-\eta_1)\,\eta_2\,(1-\eta_3)\,\vec{P}_4 + (1-\eta_1)\,\eta_2\,\eta_3\,\vec{P}_8 \\
&+ \eta_1\,(1-\eta_2)(1-\eta_3)\,\vec{P}_2 + \eta_1\,(1-\eta_2)\,\eta_3\,\vec{P}_6 \\
&+ \eta_1\,\eta_2\,(1-\eta_3)\,\vec{P}_3 + \eta_1\,\eta_2\,\eta_3\,\vec{P}_7
\end{aligned}$$

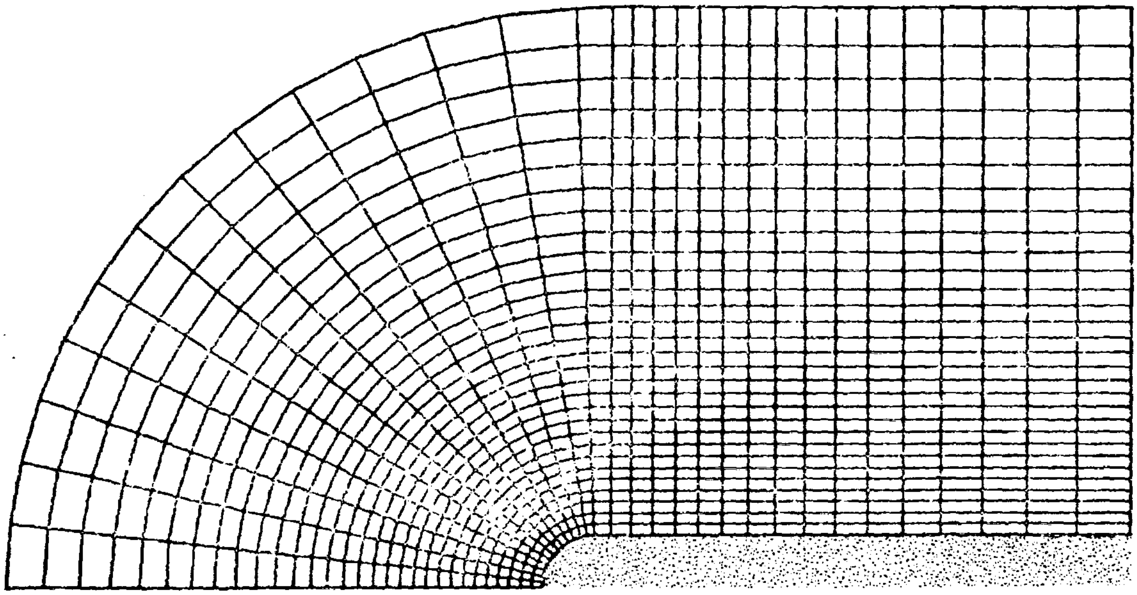
INTERNAL FLOW GRID
(Axisymmetric Rocket Nozzle)

The grid shown was used to compute the flow in a model of the Space Shuttle engine using the GIM code. The mesh is stretched in the radial direction to cluster points near the wall and stretched axially to place points near the throat of the nozzle. Only a portion of the complete grid is shown for clarity and illustration. The grid shows the general format used by the GIM code for internal, two-dimensional flows in non-rectangular shapes.



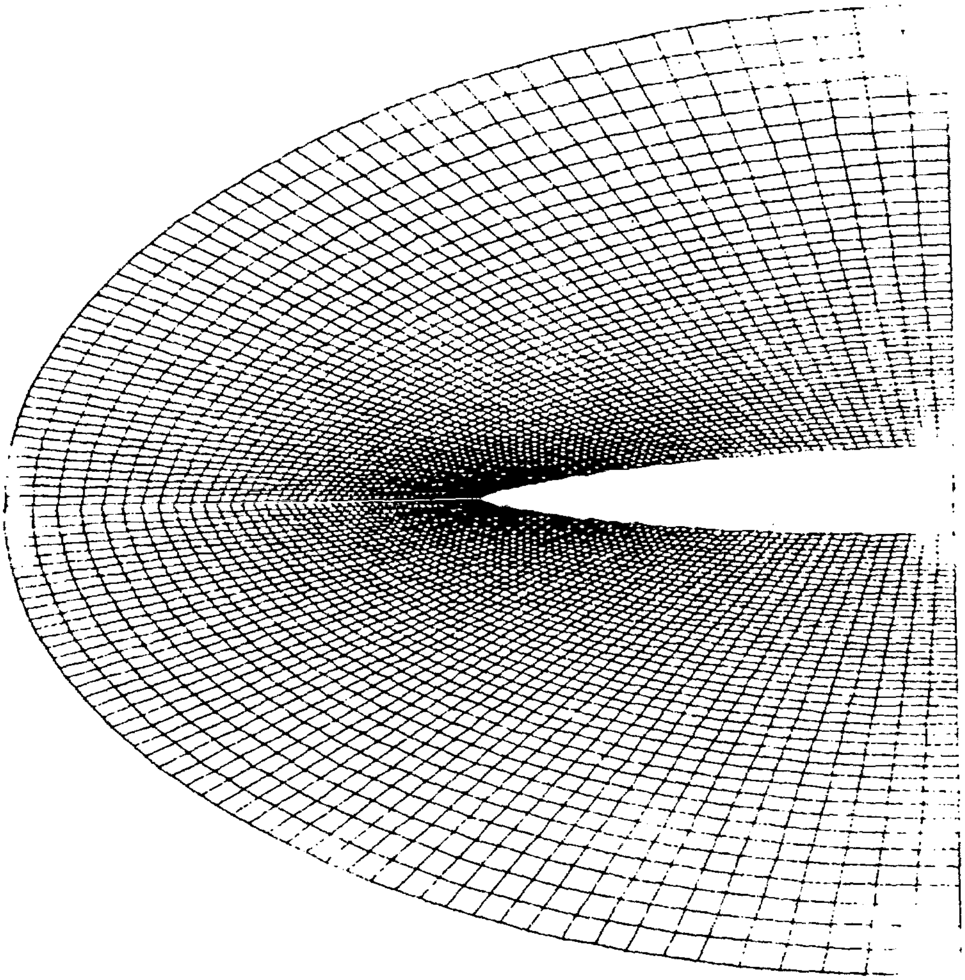
EXTERNAL FLOW GRID
(Two-Dimensional Blunt Body Flow)

This figure shows a polar-like grid used for computing external flow over a blunt body. The body surface is treated inviscidly, and thus does not require an extremely tight mesh. The outer boundary is the freestream flow. The grid illustrates the GIM code technique for two-dimensional external flows using a polar-like coordinate system.



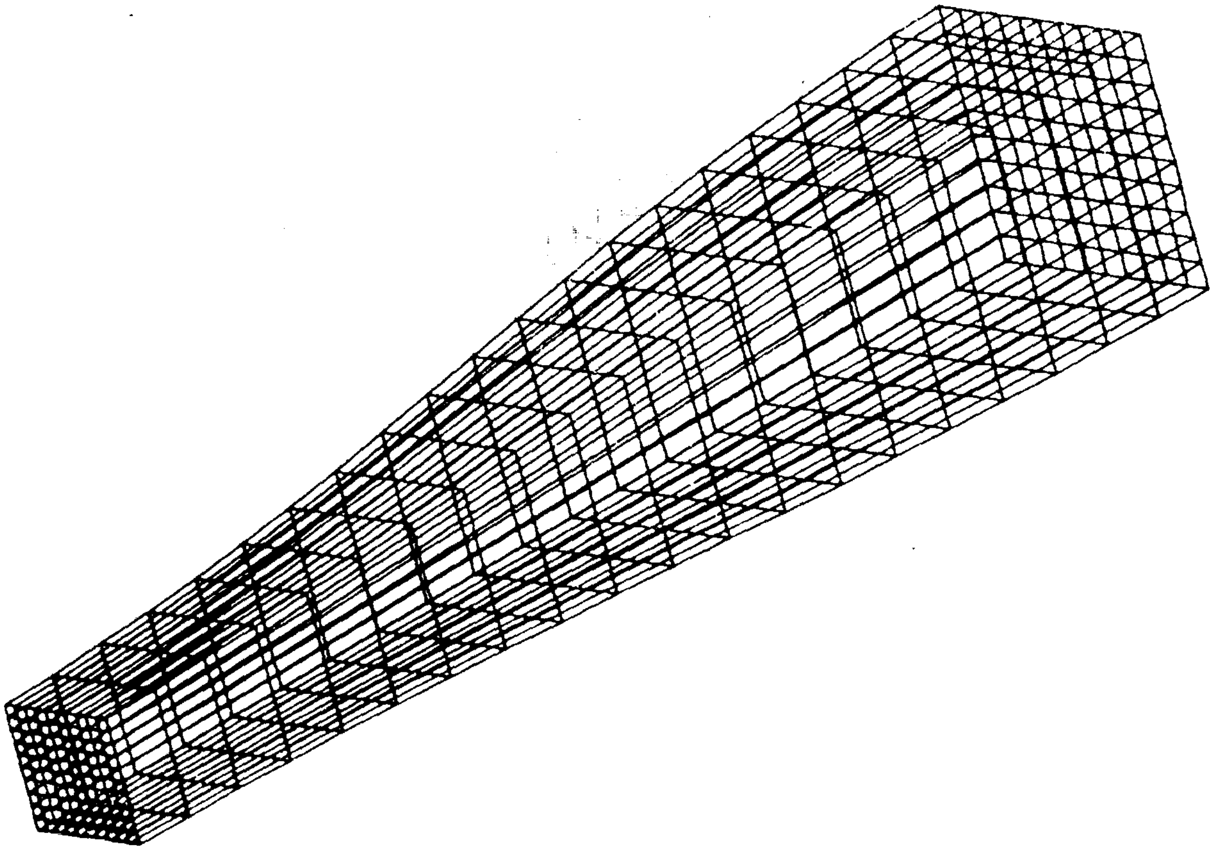
EXTERNAL FLOW GRID
(Non-Orthogonal Curvilinear Coordinates)

The nodal network for the external flow over an ogive cylinder illustrates the capability of the GIM code geometry package to stretch the nodal distribution. The grid is very compact in the leading edge region and greatly expanded in the far field areas. The axial points follow the body surface and could generally be called "body-oriented coordinates" in the nomenclature of the literature. The radial grid lines are not necessarily normal to the lateral lines or to the body surface. The GIM code, through its "nodal-analog" concept can operate on this general non-orthogonal curvilinear grid.



THREE-DIMENSIONAL GRID
(Simple Rectilinear Coordinates)

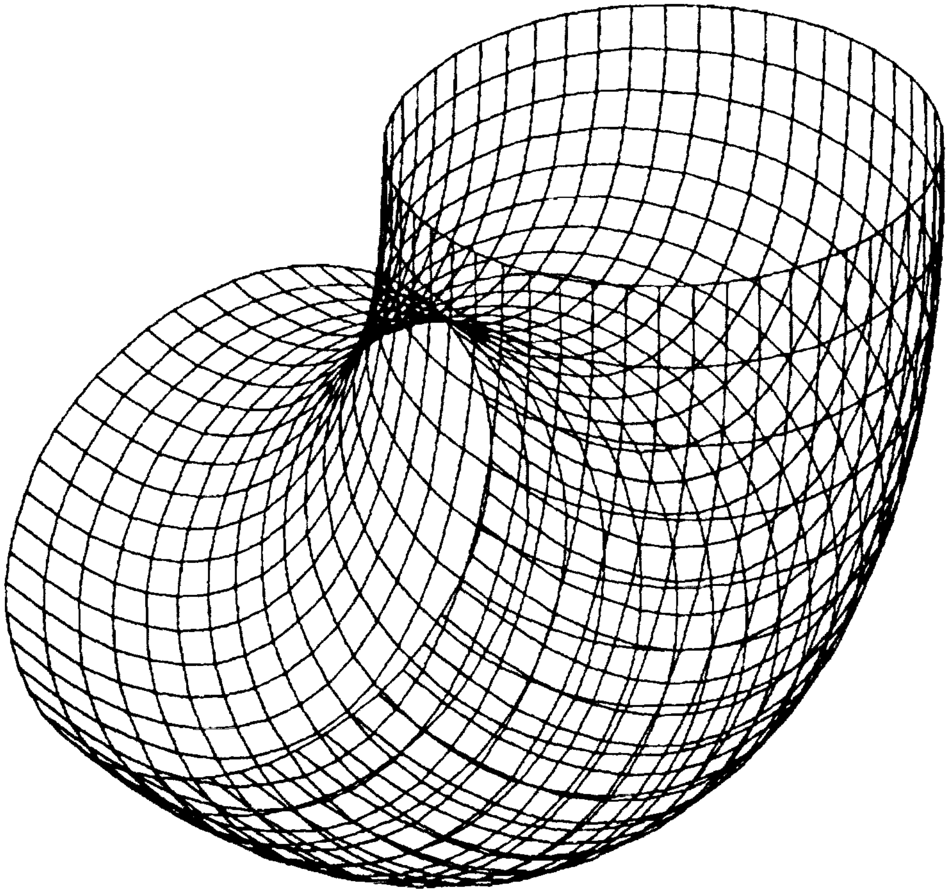
Supersonic flow in expanding ducts of arbitrary cross section is a common occurrence in computational fluid dynamics. This figure illustrates a simple grid for a three-dimensional duct whose cross section varies sinusoidally with the axial coordinate. The "top" wall and the "front" wall have this sinusoidal variation while the "bottom" and "back" walls are flat plates. The grid shown was used to resolve the expanding-recompressing supersonic flow including the intersection of the two shock sheets.



THREE-DIMENSIONAL GRID
(Pipe Flow in a 90 deg Elbow Turn)

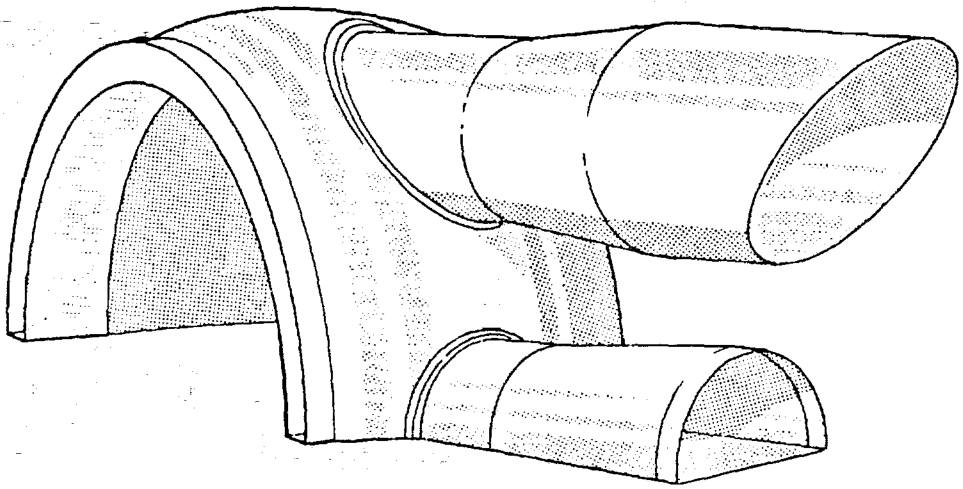
There are numerous flow fields of interest which contain a sharp turn inside a smooth pipe. The GIM code has treated certain of these for application to jet deflector nozzle flow in VTOL aircraft. The portion of a grid shown in the adjacent figure was used for this calculation.

The 90 deg elbow demonstrates the capability to model three-dimensional non-Cartesian geometries. The internal nodes were emitted for clarity. The elbow grid was generated by employing edge-of-revolution surfaces with circular arc segments as the edges being revolved.



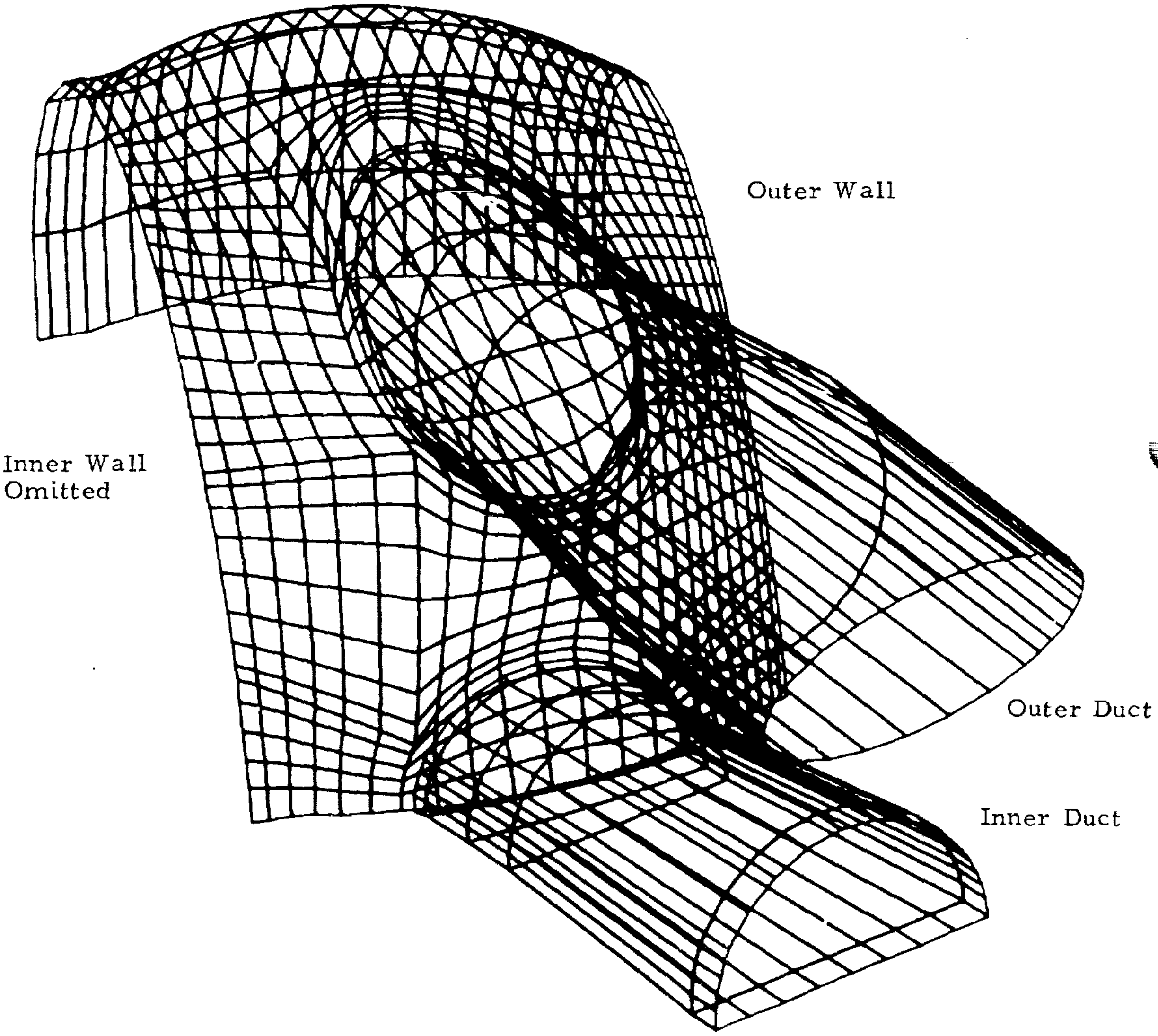
GRID FOR SPACE SHUTTLE MAIN ENGINE
(Hot Gas Manifold Geometry Model)

The recent problems encountered with the Space Shuttle main engine tests have resulted in a GIM code analysis of the system. The "hot gas manifold" is a portion of this analysis for the high pressure turbopump system. The grid shown in the adjacent figure was used for this calculation. Only a small number of nodes are shown for clarity; the full model consists of approximately 14,000 nodes. The extreme complexity of this geometry illustrates the necessity of using a GIM-like technique. Transforming this case to a square box computational domain is, of course, impossible. The results of the GIM code analysis agree qualitatively with flow tests that have been run on the hot gas manifold.



Hot Gas Manifold Configuration

GRID FOR SPACE SHUTTLE MAIN ENGINE
(Hot Gas Manifold Geometry Model)



SUMMARY

- Finite difference grids can be generated for very general configurations by using multivariate blending function interpolation.

- The GIM code difference scheme operates on general non-orthogonal curvilinear coordinate grids.

- This scheme does not require a single transformation of the flow domain onto a square box. Thus, GIM routines can indeed treat arbitrary three-dimensional shapes.

- Grids generated for both internal and external flows in two and three dimensions have shown the versatility of the algebraic approach.

- The GIM code integration module has successfully computed flows on these complex grids, including the Space Shuttle main engine turbopump system.

- Plans for future application of the code include supersonic flow over missiles at angle of attack and three-dimensional, viscous, reacting flows in advanced aircraft engines. Plans for future grid generation work include schemes for time-varying networks which adapt themselves to the dynamics of the flow.

BIBLIOGRAPHY

Spradley, L.W., J.F. Stalnaker and A.W. Ratliff, "Computation of Three-Dimensional Viscous Flows with the Navier-Stokes Equations," AIAA Paper 80-1348, July 1980.

Spradley, L.W., and M.L. Pearson, "GIM Code User's Manual for the STAR-100 Computer," NASA-CR-3157, Langley Research Center, Hampton, Va., 1979.

Spradley, L.W., P.G. Anderson and M.L. Pearson, "Computation of Three-Dimensional Nozzle-Exhaust Flows with the GIM Code," NASA CR-3042, Langley Research Center, Hampton, Va., August 1978.

Prozan, R.J., L.W. Spradley, P.G. Anderson and M.L. Pearson, "The General Interpolants Method," AIAA Paper 77-642, June 1977.

Gordon, W.J., and C.A. Hall, "Construction of Curvilinear Coordinate Systems and Applications to Mesh Generation," <u>J. Numer. Math.</u>, Vol. 1, 1973, pp. 461-477.