GRID AND METRIC GENERATION ON THE ASSEMBLY OF LOCALLY BI-QUADRATIC COORDINATE TRANSFORMATIONS[+]

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ABSTRACT

The generation of metric coefficients of the coordinate transformation from a generally curved-sided domain boundary to the unit square (cube) is required for efficient solution algorithms in computational fluid mechanics. An algebraic procedure is presented for establishment of these data on the union of arbitrarily selected sub-domains of the global solution domain. A uniformly smooth progression of grid refinement is readily generated, including multiple specification of refined grids for a given macro-element domain discretization. The procedure is illustrated as generally applicable to non-simply connected domains in two- and three-dimensions.

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COMPUTATIONAL REQUIREMENT

NAVIER-STOKES EQUATIONS

$$L(q_i) = \frac{\partial q_i}{\partial t} + \frac{\partial}{\partial x_j}\left[u_j q_i + f_{ij}\right] = 0$$

$$\ell(q_i) = a_1 q_i + a_2 \frac{\partial q_i}{\partial x_j}\hat{n}_j + a_3 = 0$$

COORDINATE TRANSFORMATION

$$x_i = x_i(\eta_j) \qquad \frac{\partial}{\partial x_j} = \frac{\partial}{\partial \eta_k}\left[\quad\right]\frac{\partial \eta_k}{\partial x_j}$$

$$J^{-1} = \left[\frac{\partial \eta_k}{\partial x_j}\right] \qquad \bar{u}_k = \frac{\partial \eta_k}{\partial x_j} u_j$$

NUMERICAL SOLUTION ALGORITHM

$$S_e\Big[\{DET\underline{J}\}_e^T[M3000]\{QI\}_e^{\blacktriangledown} - \{UBAR\underline{K}\}_e^T[M30\underline{K}0]\{QI\}_e$$

$$-\{ETA\underline{KL}\}_e^T[M30\underline{K}0]\{F\underline{L}I\}_e\Big] \equiv \{0\}$$

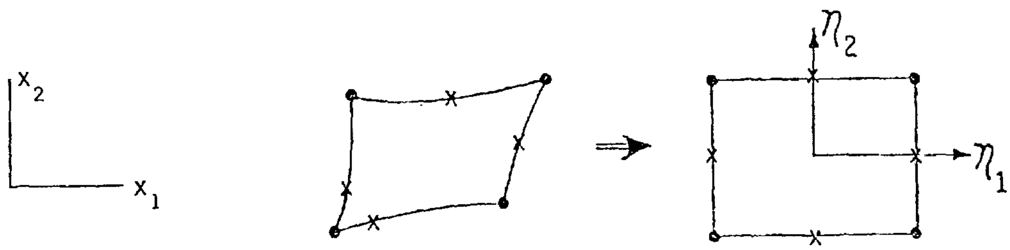
DISCUSSION

The Navier-Stokes equations contain the vector divergence operator. The required transformation projects x_i onto η_j with coordinate surfaces defined coincident with solution domain boundaries. The Cartesian description of dependent variables is retained, while the convection velocity is expressed in contravariant scalar components. The numerical solution implementation requires nodal distributions of components of the forward and inverse Jacobians, and $\underline{J}$, $\underline{K}$, and $\underline{L}$ are tensor summation indices.

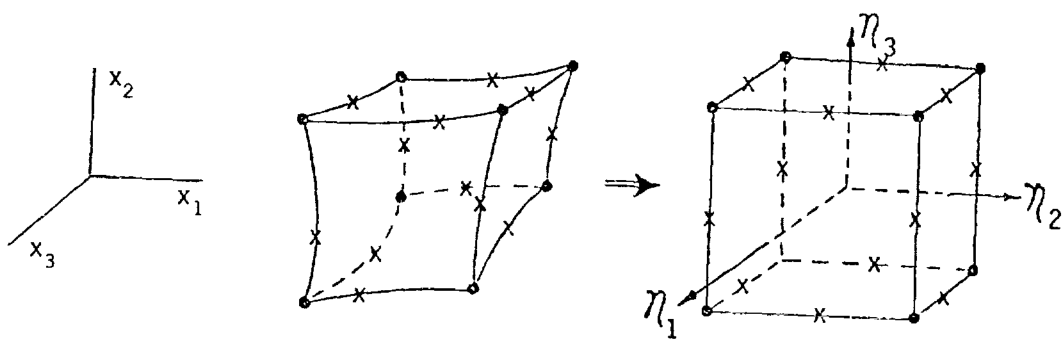
LOCALLY BI-QUADRATIC COORDINATE TRANSFORMATION

PHYSICAL DOMAIN TRANSFORMED DOMAIN

$$x_i \equiv \{N_2(\vec{\eta})\}^T\{XI\}_e$$



Two-Dimensional

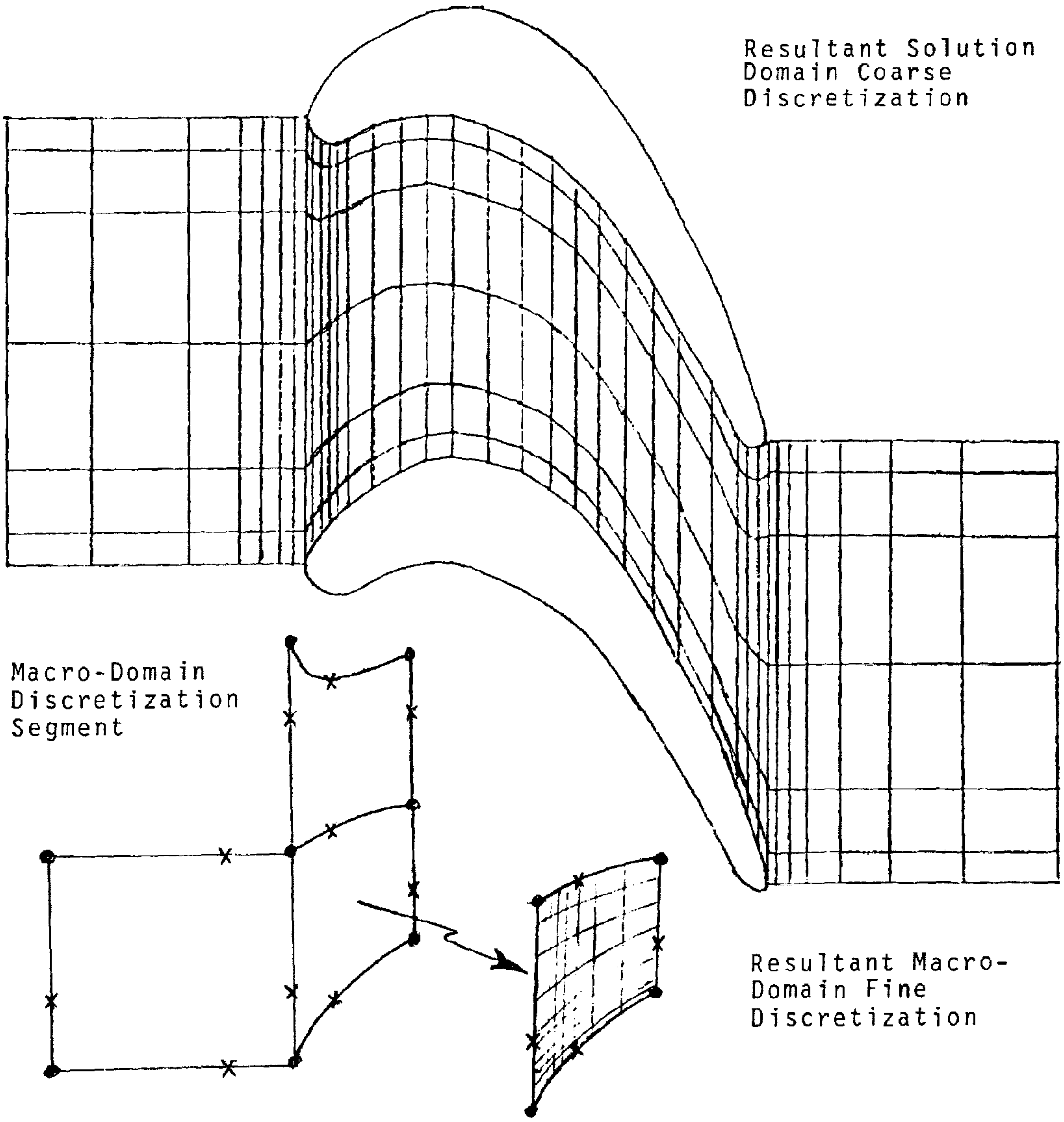


Three-Dimensional

DISCUSSION

The bi-quadratic cardinal basis $\{N_2(\vec{\eta})\}$ transforms the vertex and non-vertex node coordinate description of a smooth region of R^n onto the unit square or cube spanned by the locally rectangular Cartesian coordinate system $\vec{\eta}$. The inverse transformation J^{-1} is non-singular for a range of non-midpoint definitions of the non-vertex node coordinates (x), yielding a non-uniform discretization on R^n.

EXAMPLE: COMPRESSOR BLADE ROW



DISCUSSION

Three of the ten macro-domains, used to form the blade row discretization, are shown. The non-midside location of non-vertex nodes produces the non-uniform grid, only a few gridlines of which are shown. The inset illustrates a fine discretization of one macro-domain. The coordinates of all nodes on boundaries of macro-domains are unique.

DETAILS OF THE COORDINATE TRANSFORMATION

NODAL COORDINATES {XI}:

$$x_i \equiv \{N_2(\eta_j)\}^T\{XI\}_e$$

WHERE:

$$\{N_2(\eta_i)\} \equiv \frac{1}{4}\begin{Bmatrix} (1-\eta_1)(1-\eta_2)(-\eta_1-\eta_2-1) \\ (1+\eta_1)(1-\eta_2)(\eta_1-\eta_2-1) \\ (1+\eta_1)(1+\eta_2)(\eta_1+\eta_2-1) \\ (1-\eta_1)(1+\eta_2)(-\eta_1+\eta_2-1) \\ 2(1-\eta_1^2)(1-\eta_2) \\ 2(1+\eta_1)(1-\eta_2^2) \\ 2(1-\eta_1^2)(1+\eta_2) \\ 2(1-\eta_1)(1+\eta_2^2) \end{Bmatrix}$$

JACOBIANS

$$J \equiv \left[\frac{\partial x_i}{\partial \eta_j}\right] = J(\eta_j,\ XI)$$

$$J^{-1} \equiv \left[\frac{\partial \eta_j}{\partial x_i}\right] = \frac{1}{\det J}\ [\text{cofactors of } J]$$

$$= J^{-1}(\eta_j,\ XI)$$

DISCUSSION

Within a macro-domain, the components of both J and J^{-1} are continuous functions of η_j and the global macro-node coordinate pairs (triples) {XI}, $1 \leq I \leq n$. Each global coordinate x_i possesses an independent transformation; the corresponding Jacobian must be of rank n to assure existence of J^{-1}. Once the matrix elements of {XI} are defined, selection of any coordinate (η_1, η_2) defines a unique coordinate pair (x_1, x_2), i.e., a mesh point on the refined grid in physical space.