

FAST GENERATION OF BODY CONFORMING GRIDS FOR 3-D

AXIAL TURBOMACHINERY FLOW CALCULATIONS

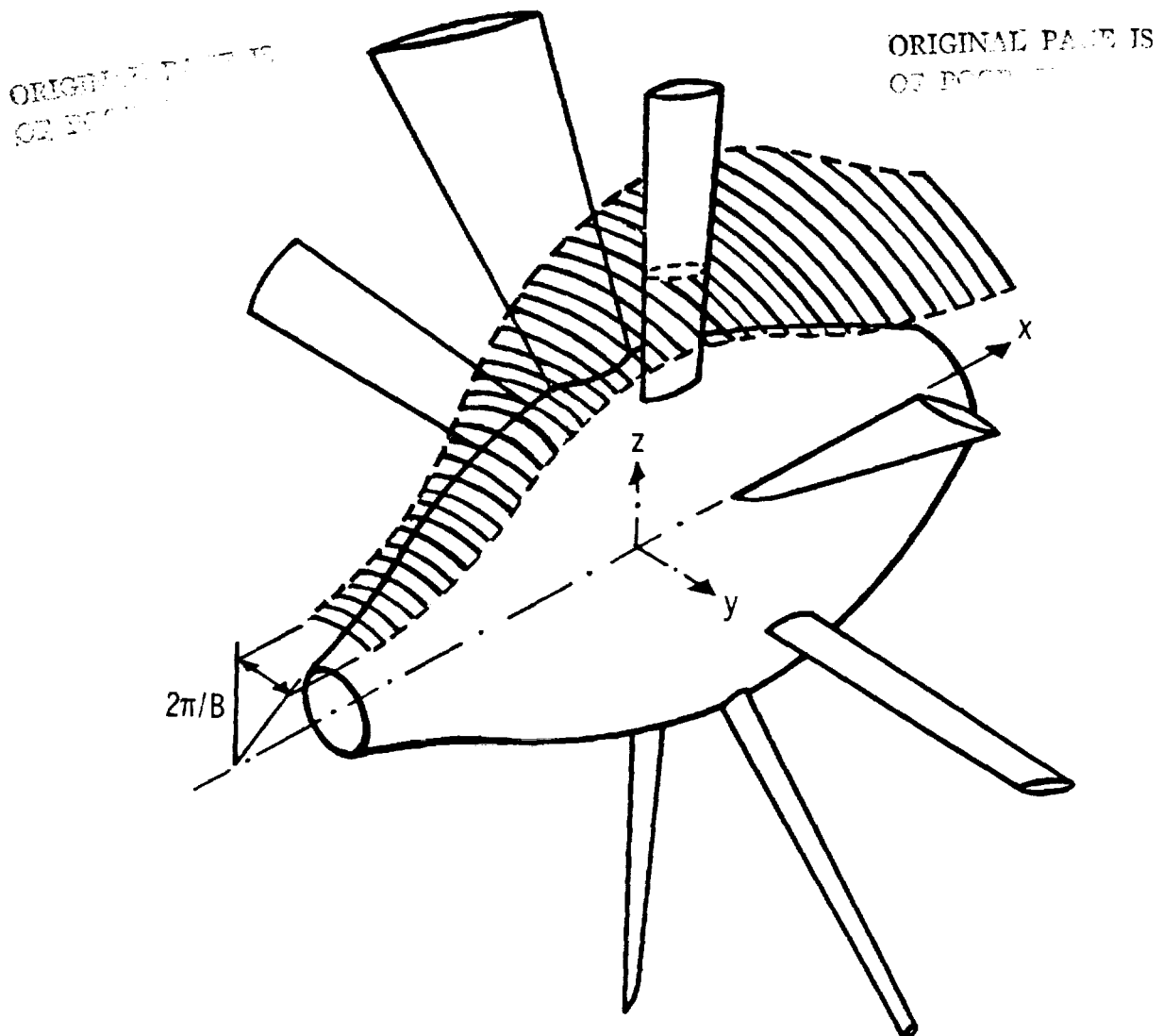
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A fast algorithm has been developed for accurately generating boundary conforming, three-dimensional, consecutively refined, computational grids applicable to arbitrary axial turbomachinery geometry. The method is based on using a single analytic function to generate two-dimensional grids on a number of coaxial axisymmetric surfaces positioned between the hub and the shroud. These grids are of the "O"-type and are characterized by quasi-orthogonality, geometric periodicity, and an adequate resolution throughout the flowfield. Due to the built-in additional nonorthogonal coordinate stretching and shearing, the grid lines leaving the trailing edge of the blade end at downstream infinity, thus simplifying the numerical treatment of the three-dimensional trailing vortex sheet.

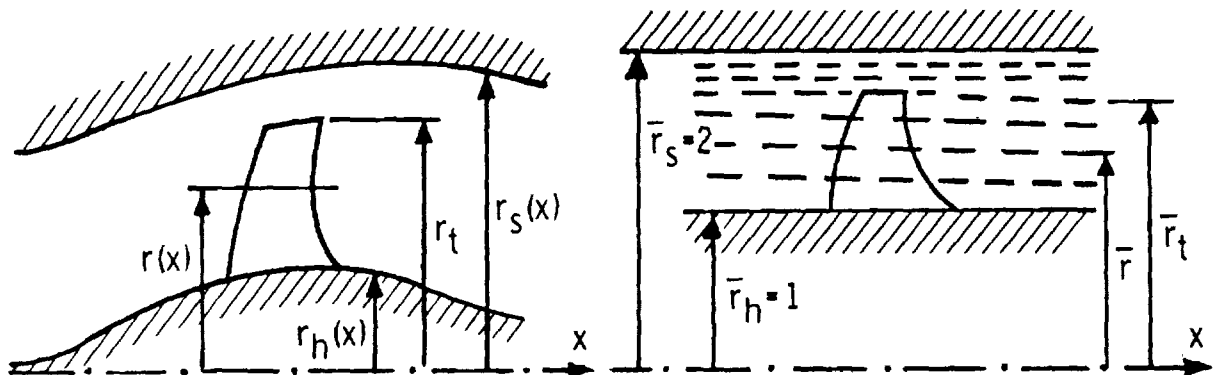
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The main objective of this work is to develop a fast algorithm for generating body-conforming three-dimensional computational grids. An equally important objective is to preserve the high accuracy of the discretized representation of the solid boundaries. When analyzing steady flows through turbomachinery rotors and stators, it is sufficient to consider a single rotationally periodic segment of the flowfield. This segment is a doubly infinite strip stretching in the direction of the axis of rotation. The strip has a constant angular width of $2\pi/B$ where B is the total number of blades. Each blade has an arbitrary spanwise distribution of taper, sweep, dihedral and twist angle. The local airfoil shapes can vary in an arbitrary fashion along the blade span. The rotor hub and the duct (or shroud) can have different arbitrary axisymmetric shapes.

Such an arbitrary three-dimensional physical domain (Fig. 1) is first discretized in the spanwise direction by a number of coaxial axisymmetric surfaces which are irregularly spaced between hub and shroud.

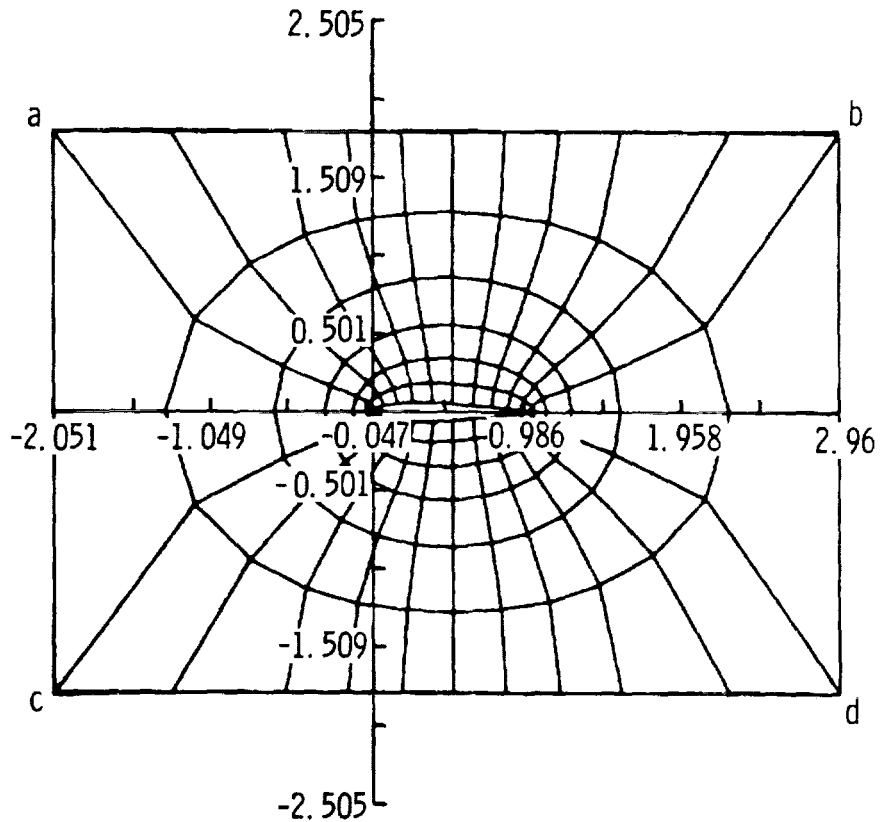


The major problem in generating the spanwise surfaces is an accurate determination of the intersection contours between the irregular blade surface and the coaxial axisymmetric surfaces cutting the blade. The coordinates of the points on these contours are defined by fitting cubic splines along the blade and interpolating at the radial stations corresponding to each axisymmetric surface $\bar{r} = \text{Constant}$.

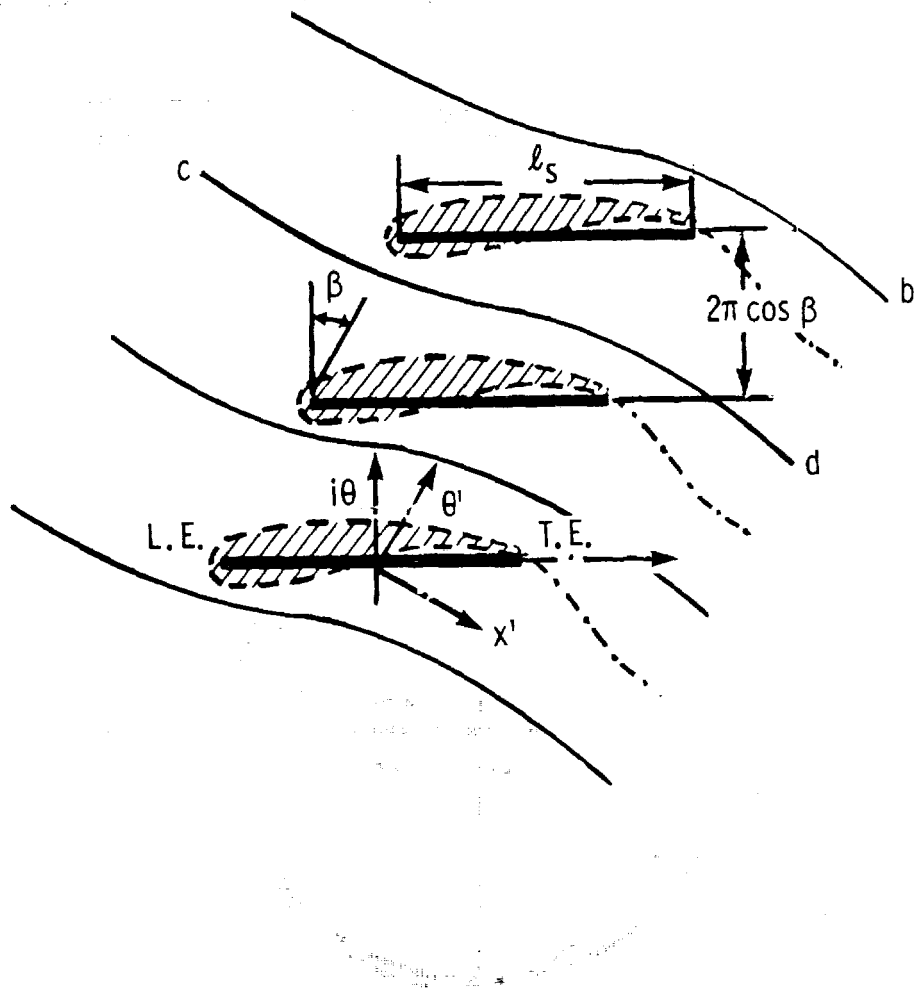


$$\bar{r} = 1 + \frac{r(x) - r_h(x)}{r_s(x) - r_h(x)}$$

The two-dimensional grid should have the following features: (a) grid cells should conform with the contour shape and the shape of the periodic boundaries \bar{ab} and \bar{cd} , (b) grid should be geometrically periodic in the θ' -direction meaning that the grid points along the periodic boundary \bar{ab} must have the same respective x' -coordinates as the grid points along the periodic boundary \bar{cd} , (c) grid lines should not be excessively non-orthogonal in the vicinity of solid boundaries, (d) a grid line emanating from the trailing edge should end at downstream infinity and (e) grid cells should be concentrated in the regions of high flow gradients.



Once the shape of the intersection contour on a particular cutting axisymmetric surface is known, the problem becomes one of discretizing a doubly connected two-dimensional domain $\tilde{w} = x + i\theta$.



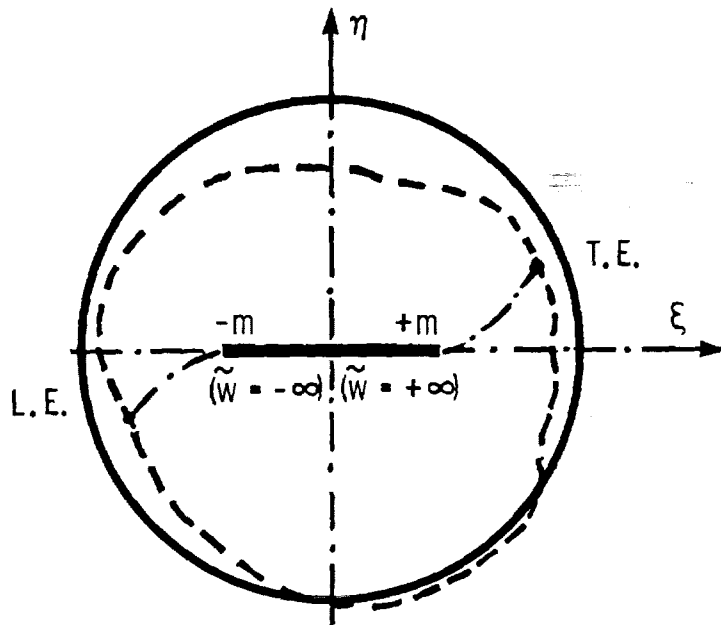
A grid with these properties can be most easily generated with the use of a single analytic function. One such function is

$$\tilde{w} = e^{i\beta} \ln \left(\frac{m - \tilde{z}}{m + \tilde{z}} \right) + e^{-i\beta} \ln \left(\frac{1 - m\tilde{z}}{1 + m\tilde{z}} \right); \quad 0 < m < 1$$

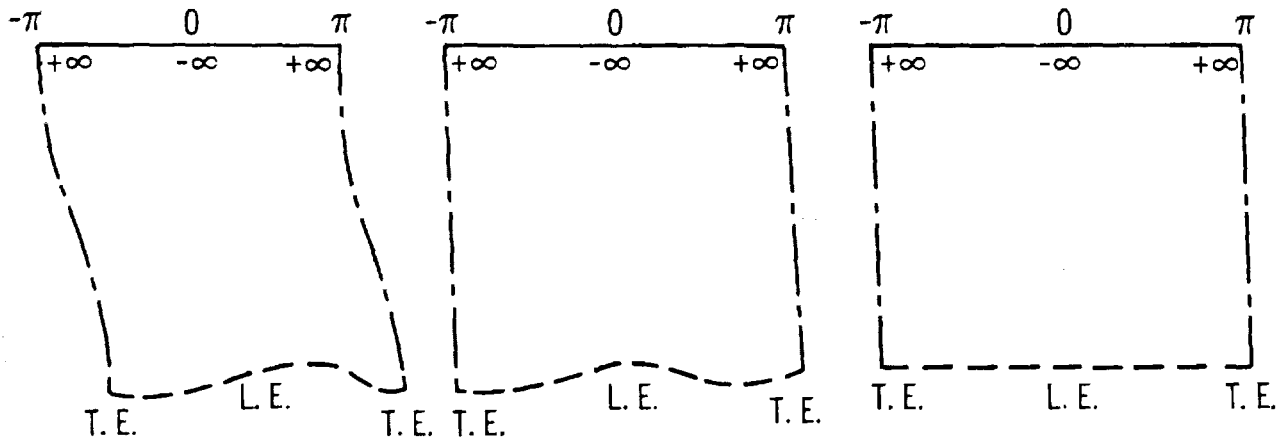
where $\tilde{w} = x + i\theta$ and $z = \xi + i\eta$. This complex function maps conformally a unit circle with a slit in the middle whose end-points are situated at $\tilde{z} = \pm m$ onto the cascade of straight slits in the \tilde{w} -plane. Each slit has a length l_s where

$$l_s = 4 \left(\cos \beta \sinh^{-1} \frac{2m \cos \beta}{1 - m^2} + \sin \beta \sin^{-1} \frac{2m \sin \beta}{1 + m^2} \right)$$

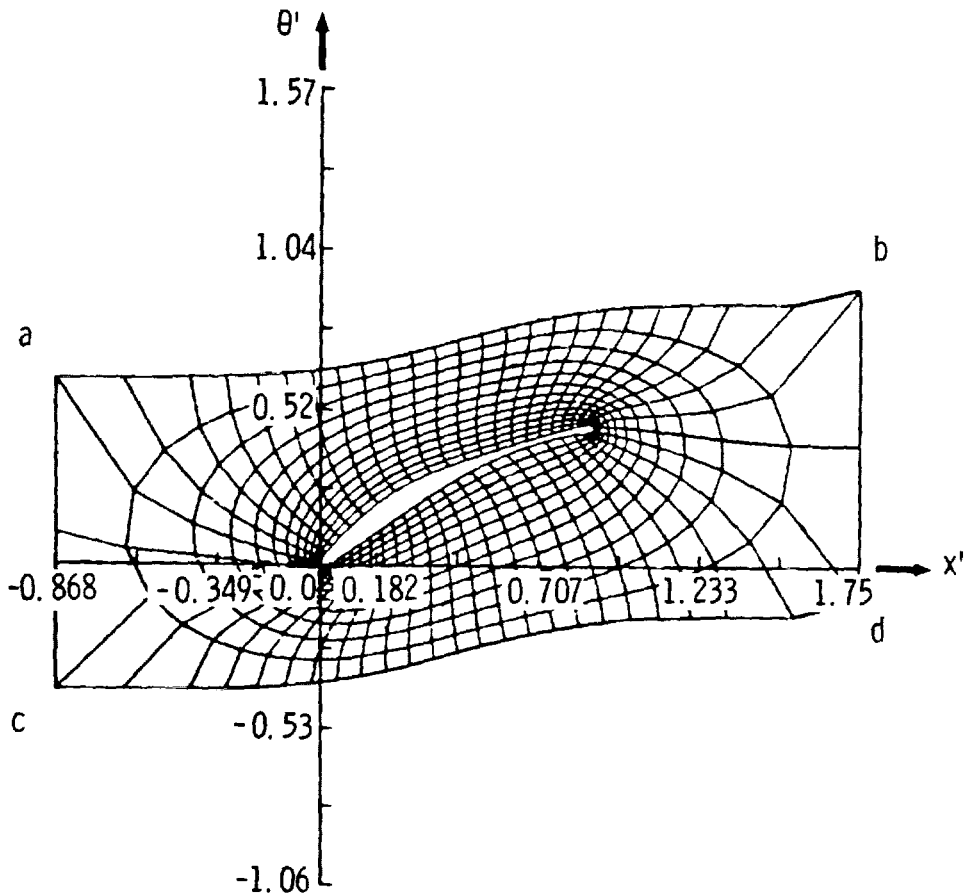
The slits are spaced $2\pi \cos \beta$ distance apart, where β is the stagger angle of the cascade of slits.



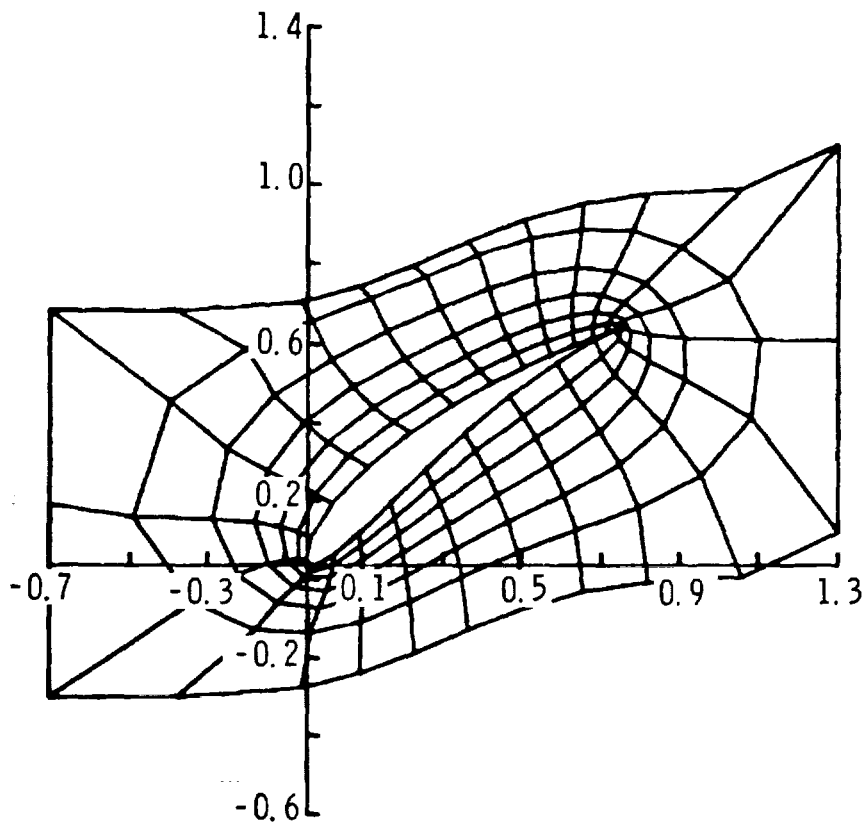
The unit circle is "unwrapped" using elliptic polar coordinates (refs. 1 and 2) resulting in a deformed rhomboidal shape which is then sheared in the horizontal and vertical direction (ref. 2) resulting in a rectangular (X,Y) computational domain.



The transformation of an actual cascade of airfoils will result in a cascade of unit circles which are even more deformed. Consequently, more nonorthogonality will be introduced in the transformation by additional shearing of coordinates. A uniform grid in the (X,Y) plane which is symmetrically spaced with respect to the Y -axis, remaps back into the physical (x,θ) plane as an "O"-type boundary conforming grid. The actual radial coordinates are obtained by fitting cubic splines along the elliptic mesh lines and interpolating at a number of axial stations at which the radius of the corresponding axisymmetric surface is known.

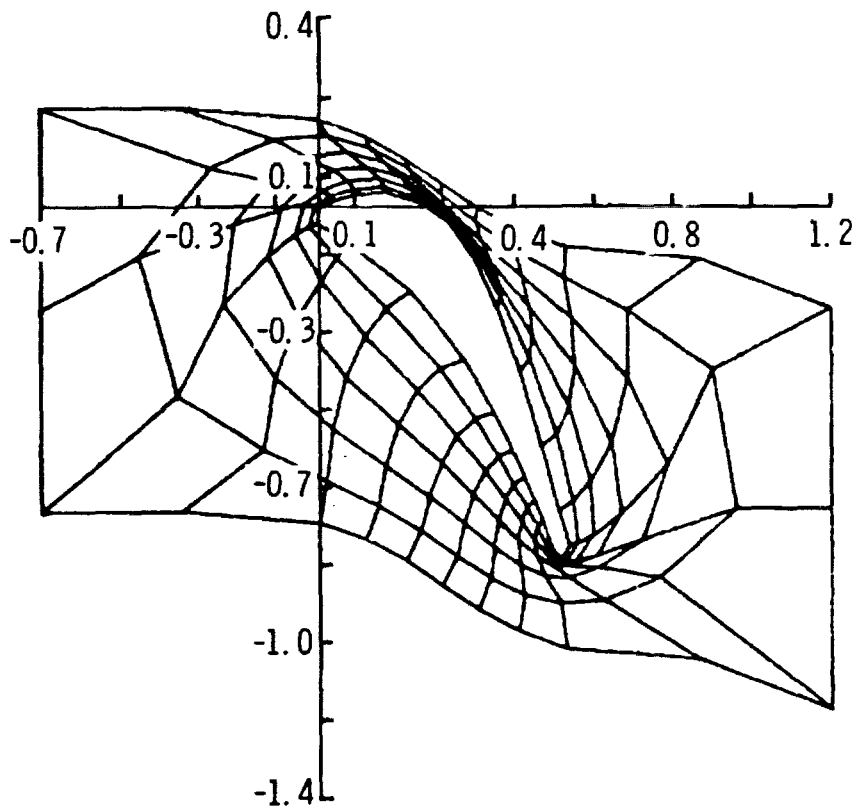


The present method is equally applicable to the blades with blunt (or rounded), wedge and cusp trailing and/or leading edge.

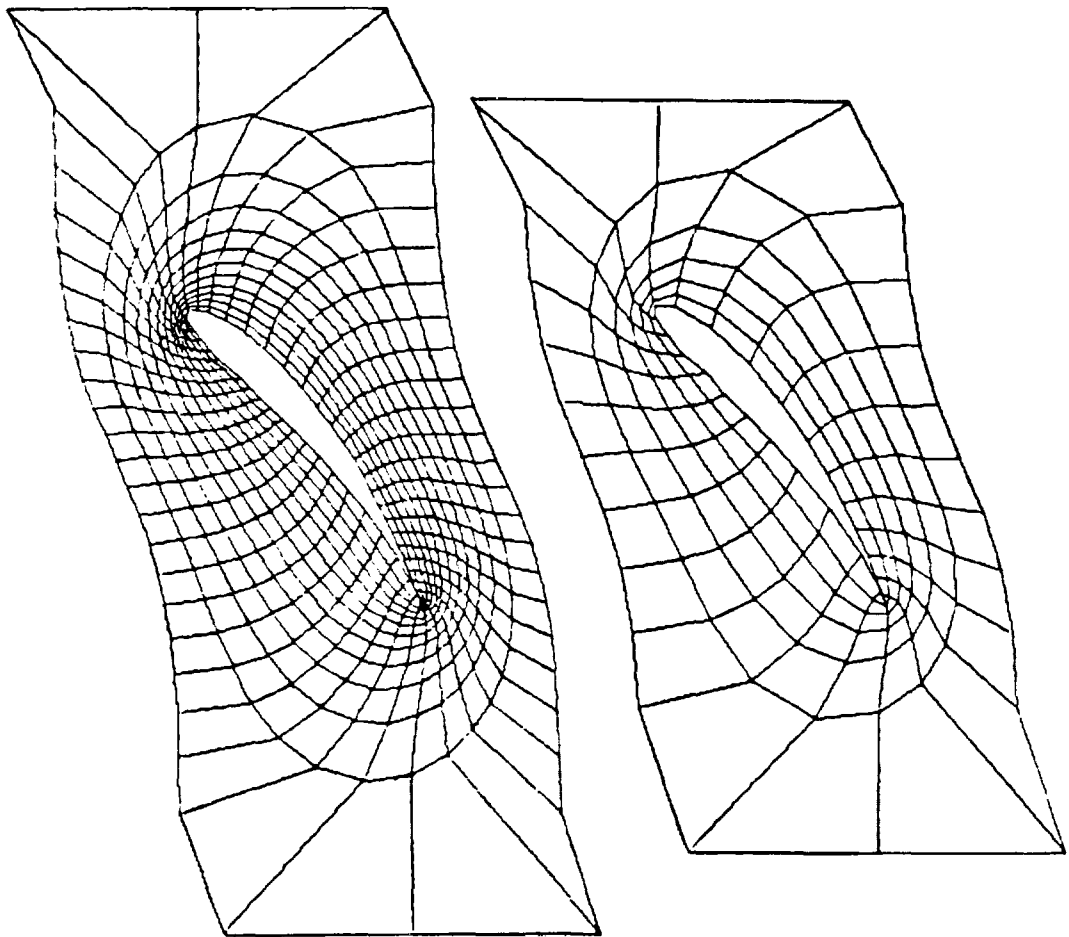


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A disadvantage of the present method is that it is not applicable for the very thick, highly staggered blades which are very closely spaced. This problem can be resolved by using a different form of the mapping function; for example, one which maps a cascade of circles into a cascade of circular arcs instead of a cascade of straight slots.



A sample run shows that it takes 7.3 seconds of CPU time on an IBM 3033 to generate (x, y, z) coordinates of two 3-D grids and to write them on two separate disks. The first (coarse) grid consisted of 27x9x9 points and the second (refined) grid has 51x15x17 points.



REFERENCES

1. Dulikravich, D. S.: Numerical Calculation of Inviscid Transonic Flow Through Rotors and Fans. Ph.D. Thesis, Cornell Univ., 1979.
2. Spiegel, M. R.: Mathematical Handbook of Formulas and Tables. Schaum's Outline Series, McGraw-Hill Book Co., 1968, p. 127.