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BOUNDARY-FITTED COORDINATES FOR REGIONS WITH HIGHLY CURVED BOUNDARIES AND REENTRANT BOUNDARIES

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A procedure has been developed, using the differential-equation approach, for generating boundary-fitted coordinates for regions with highly curved boundaries as well as reentrant boundaries, such as those encountered in breaking surface waves. The resulting coordinates are nearly orthogonal and can provide adequate resolution even in the reentrant region. Consistent treatment of end boundaries and the use of a systematic initialization scheme and advanced implicit numerical solution techniques make the procedure highly efficient. The method developed for implicit enforcement of the periodicity boundary condition should be beneficial in the analysis of turbomachinery flow applications.

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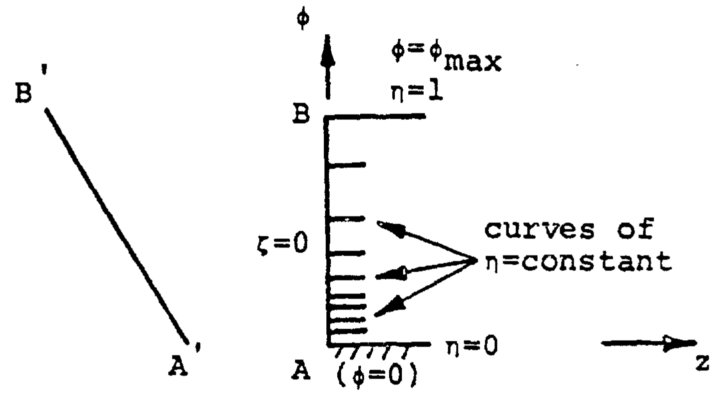
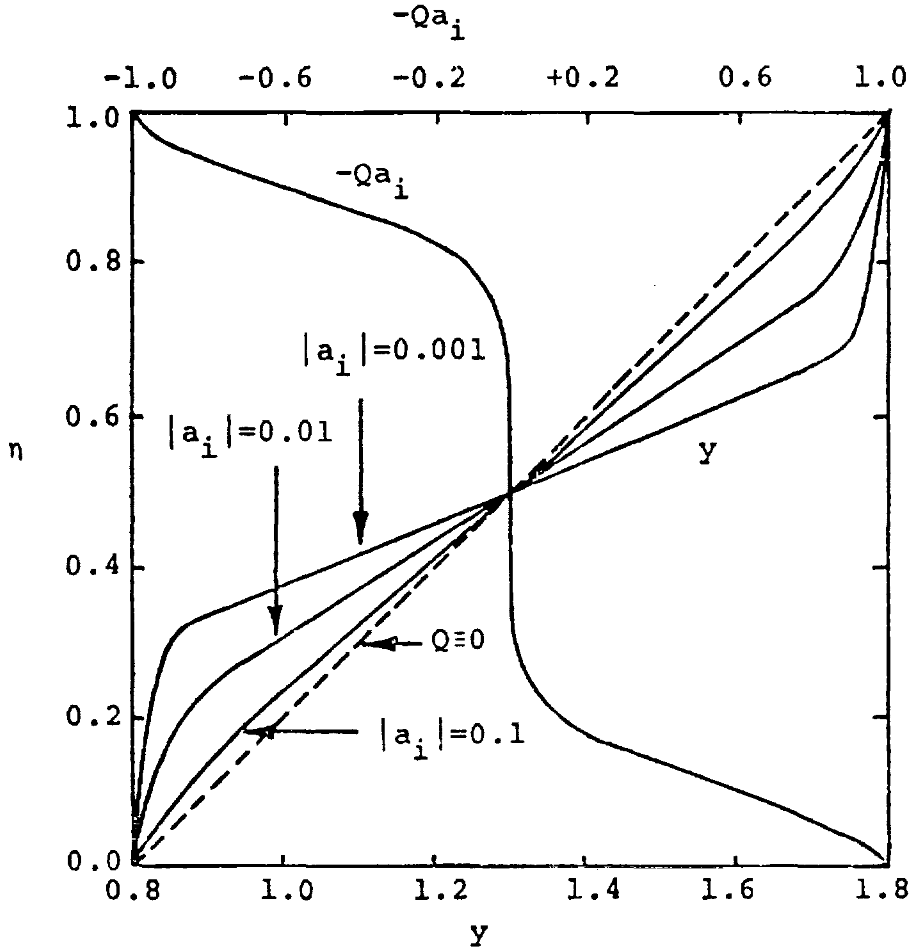
CONSISTENT TREATMENT OF END-BOUNDARIES



A limiting form of the coordinate equations at the end-boundary is solved to determine, prior to the complete solution, the point distribution at this boundary, consistent with the interior distribution. This procedure avoids discontinuities in the transformed-coordinate derivatives near the end-boundaries, while maintaining Dirichlet boundary conditions for the transformation.

SOLUTION OF LIMITING EQUATION AT END-BOUNDARY



$$\phi_{\eta\eta} + Q\,\phi_{\eta}^{3} = 0$$

where

$$Q(\eta) = \sum_{k=1}^{2} \frac{1}{a_k} \exp[-(\eta-\eta_k)^2/(2b_k^2)]$$

$a_1 < 0$, $\qquad |a_1| = a_2$

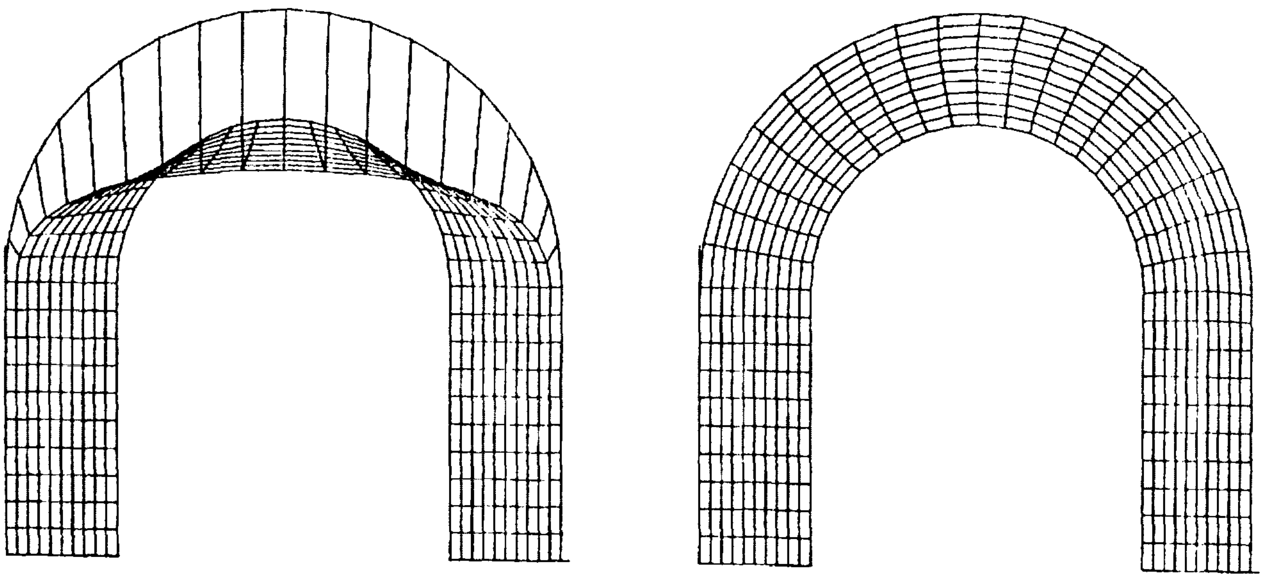
$b_1 = b_2 = 0.1$

$\eta_1 = 0$, $\qquad \eta_2 = 1$

INITIALIZATION PROCEDURE

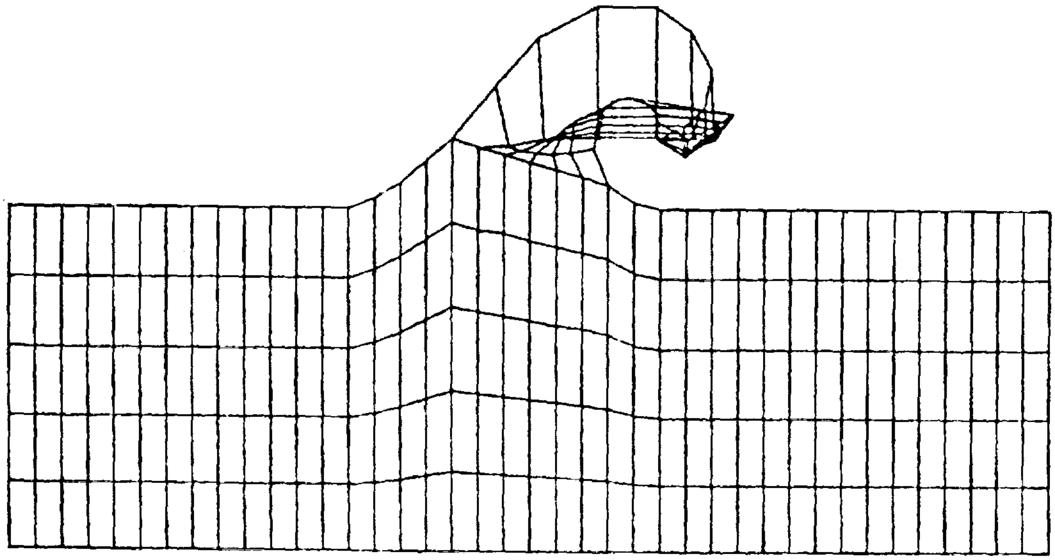
GEOMETRIC INITIALIZATION

INITIALIZATION BY LOCALLY SELF-SIMILAR SOLUTION

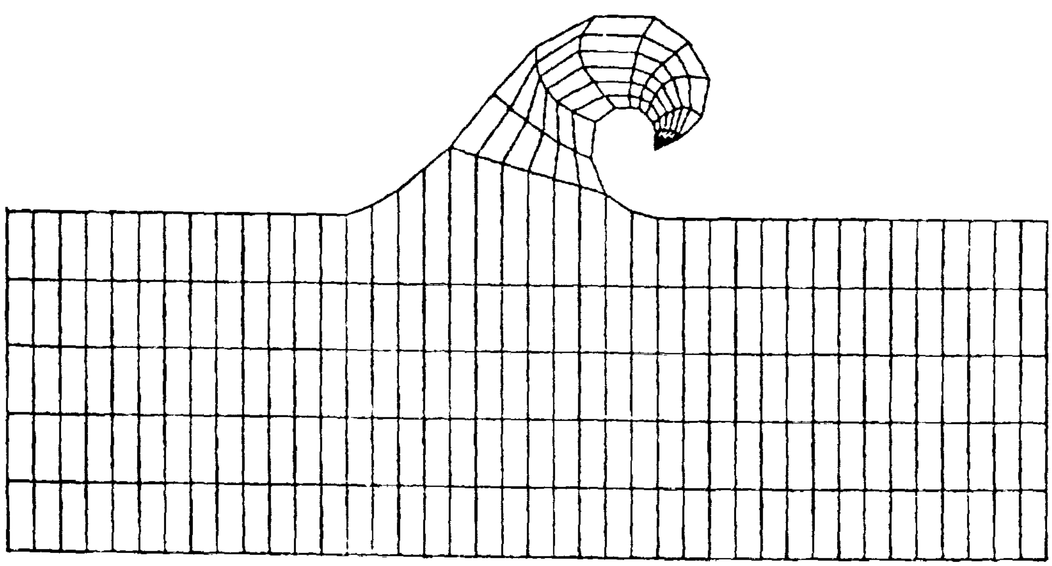


INITIALIZATION PROCEDURE

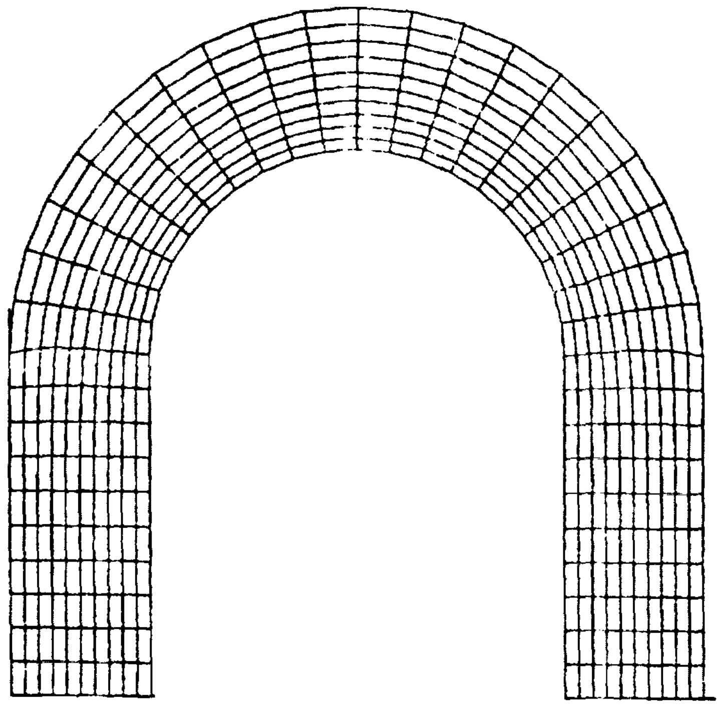
GEOMETRIC INITIALIZATION



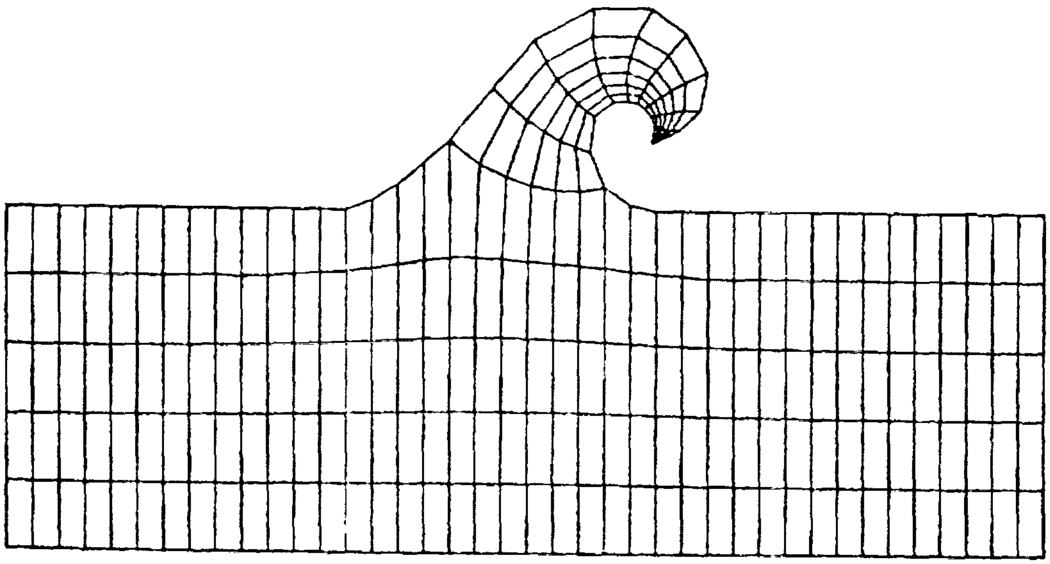
INITIALIZATION BY LOCALLY SELF-SIMILAR SOLUTION



SURFACE-ORIENTED COORDINATES FOR DUCT WITH HIGHLY CURVED BOUNDARIES



BOUNDARY-ORIENTED COORDINATES FOR A TYPICAL SURFACE WAVE WITH REENTRANT BOUNDARIES



IMPLICIT ENFORCEMENT OF PERIODICITY BOUNDARY CONDITION

DIFFERENTIAL EQUATION:

$$\phi'' + a\phi = b$$

PERIODICITY BOUNDARY CONDITIONS:

$$\phi_0 = \phi_1 = A \; ; \qquad \phi_0' = \phi_1' = B$$

where A and B are unknown.

SOLUTION PROCEDURE: Let $\phi = Af + Bg + h$

$$f'' + af = 0 \qquad g'' + ag = 0 \qquad h'' + ah = b$$

with

$$f_0 = 1 \qquad g_0 = 0 \qquad h_0 = 0$$

$$f_1' = 0 \qquad g_1' = 1 \qquad h_1' = 0$$

Then,

$$Af_1 + Bg_1 + h_1 = A$$

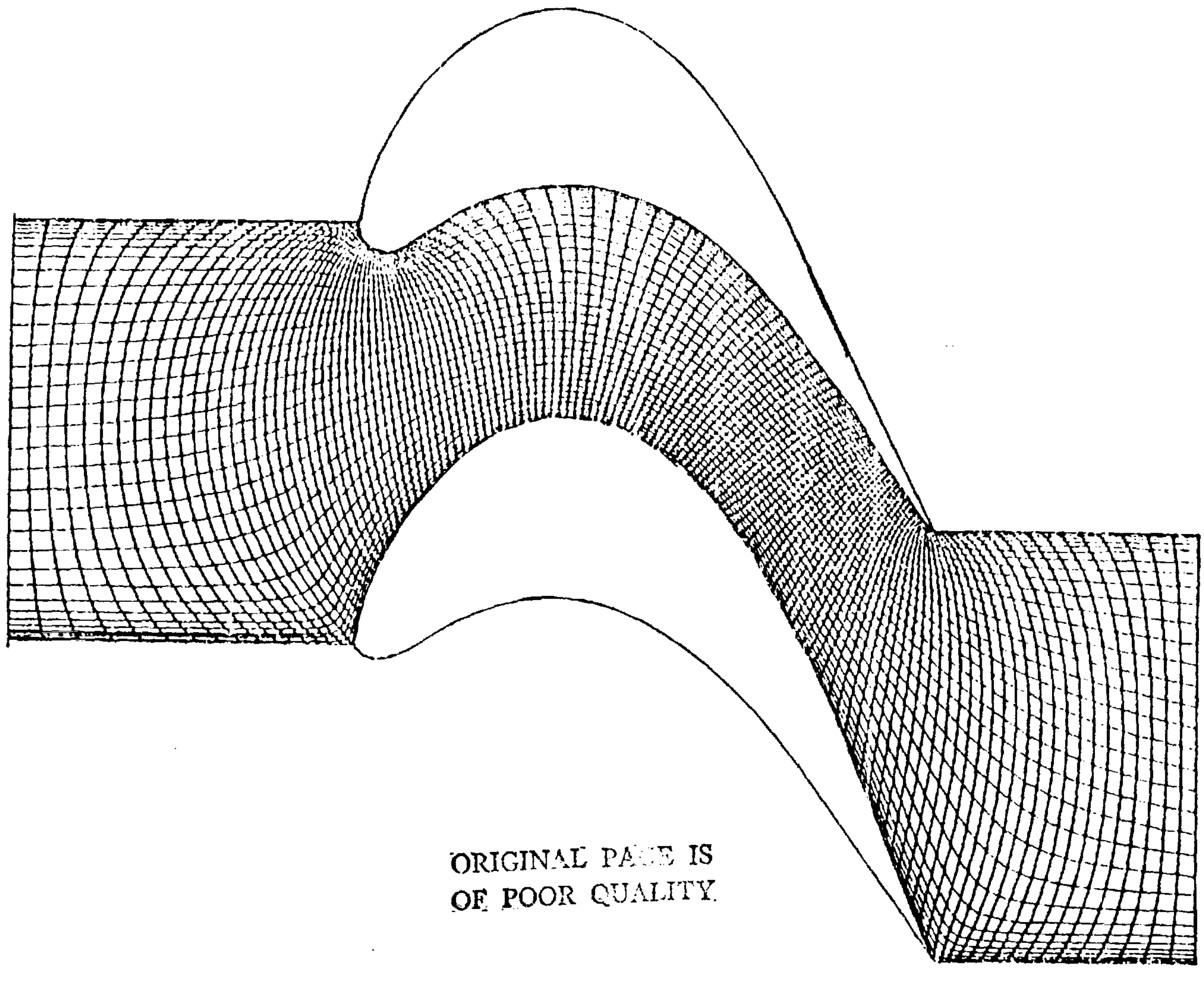
$$Af_0' + Bg_0' + h_0' = B$$

so that

$$A = [h_1(1-g_0') + h_0' g_1]/[(1-f_1)(1-g_0') - f_0' g_1]$$

$$B = [h_0'(1-f_1) + f_0' h_1]/[(1-f_1)(1-g_0') - f_0' g_1]$$

SURFACE-ORIENTED COORDINATES FOR A TURBINE CASCADE - (129 x 33) NONUNIFORM GRID WITH EASILY APPLICABLE PERIODICITY



STREAMWISE-ALIGNED SURFACE-ORIENTED COORDINATES FOR A TYPICAL TURBINE CASCADE - (161 x 33) NONUNIFORM GRID

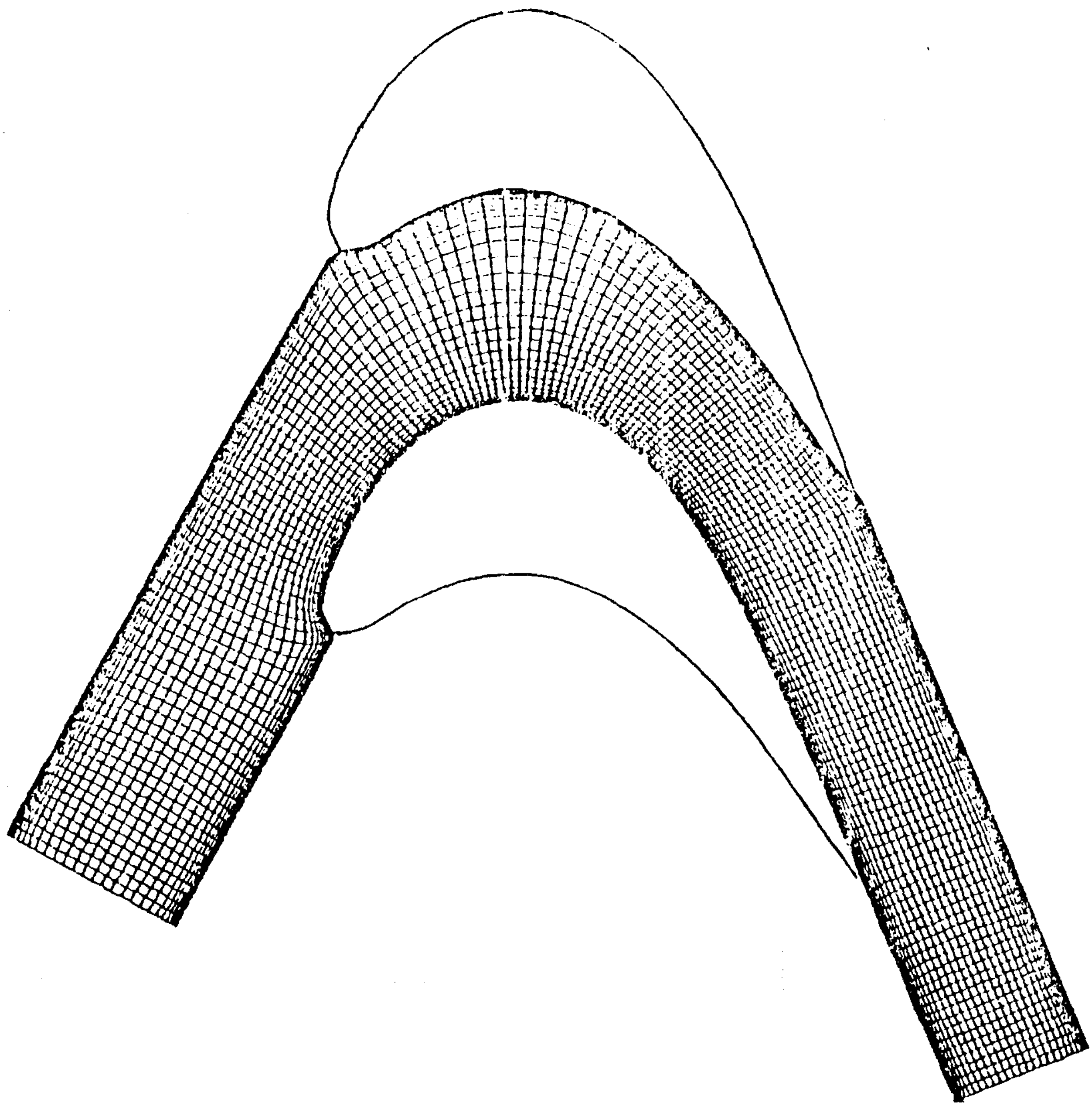


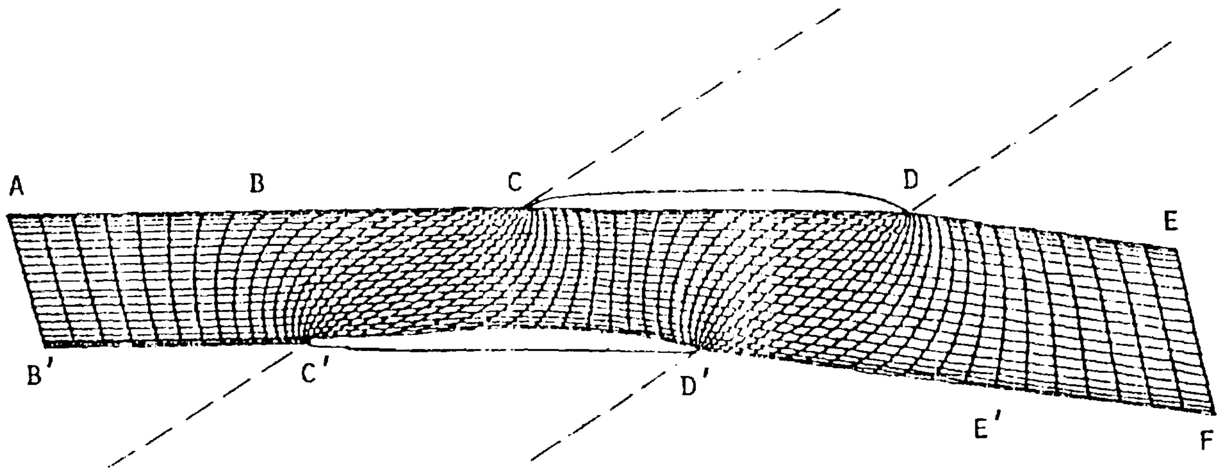
TABLE 1. EFFECT OF MULTIGRID (MG) ITERATION TECHNIQUE ON CONVERGENCE OF COORDINATE SOLUTION FOR CASCADE WITH EASILY APPLICABLE PERIODICITY

Method	Grid	Work Units of Resp. Finest Grid	CPU Seconds	Remarks
ADI	(65 x 17)	100	37.69	uniform spacing
SIP	(65 x 17)	53	11.96	uniform spacing
MG-SIP	(65 x 17)	6.5	2.08	uniform spacing
ADI	(65 x 17)	95	36.67	nonuniform spacing
SIP	(65 x 17)	25	6.33	nonuniform spacing
MG-SIP	(65 x 17)	7.5	2.32	nonuniform spacing
MG-SIP	(129 x 33)	6.4	8.44	nonuniform spacing

TABLE 2. CONVERGENCE OF COORDINATE SOLUTION FOR CASCADE GEOMETRY WITH PERIODICITY USING A STRONGLY IMPLICIT PROCEDURE (SIP) AND MULTIGRID (MG) TECHNIQUE

Method	Grid	Work Units of Resp. Finest Grid	CPU Seconds	Remarks
SIP	(161 x 33)	81.00	≈100.0	uniform spacing. convergence is one order less than for nonuniform spacing.
MG-SIP	(161 x 33)	7.48	10.79	uniform spacing
MG-SIP	(161 x 33)	8.23	11.49	nonuniform spacing
MG-SIP	(81 x 17)	8.88	4.02	nonuniform spacing

TYPICAL SURFACE-ORIENTED COORDINATES FOR A CASCADE WITH HIGH STAGGER - (65 x 21) NONUNIFORM GRID



This figure shows a multiple-circular-arc supersonic compressor cascade with a large stagger angle and a typical coordinate distribution for such a cascade. The grid lines are concentrated near the surface of both the blades, especially near their leading and trailing edges, in order to provide good resolution for the viscous and shock effects in these regions. In addition to the nonuniform distribution of the grid points, an effort has been made to maintain near-orthogonality wherever possible. The existing non-orthogonality can be easily removed by increasing the number of points in the streamwise direction, although the coordinate distribution shown in this figure may actually be preferred for supersonic cascades. Moreover, the point distribution along the free boundaries is such as to enable enforcement of the periodicity condition, i.e., the point distributions along BC and DE are the same as along B'C' and D'E', respectively. The number of working units required to generate the (65 x 21) coordinates shown was 8.44 using the SIP-multigrid method; the corresponding CPU time was 3.48 seconds.

CONCLUSIONS

- Generation of coordinates for regions with highly curved boundaries requires suitable initial conditions; locally self-similar equations provide an excellent non-iterative initial solution.
- Generation of appropriate Dirichlet boundary conditions even with non-zero forcing functions enhances solution convergence rate.
- Use of implicit numerical solution procedures together with the multigrid iteration technique constitutes an effective method for solution of the nonlinear governing differential equations with large number of grid points.
- An adaptive coordinate distribution is formulated for the breaking surface-wave problem with a reentrant boundary; solutions are presently being obtained for a free surface wave starting from an initial sinusoidal form and undergoing the breaking phenomenon.