

A SIMPLE NUMERICAL ORTHOGONAL COORDINATE  
GENERATOR FOR FLUID DYNAMIC APPLICATIONS

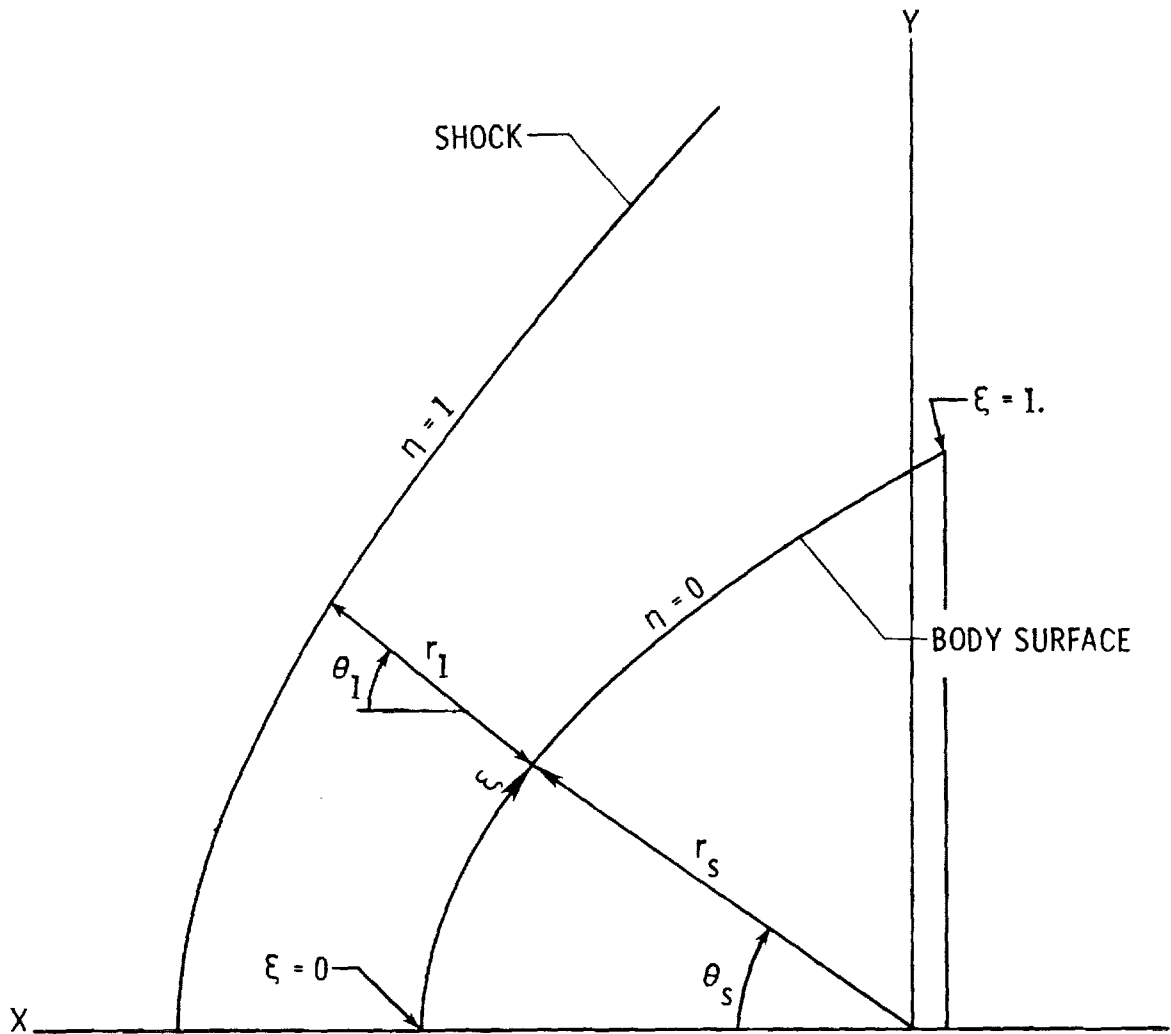
Randolph A. Graves, Jr.  
OAST Aerodynamics Office  
NASA Headquarters  
Washington, DC

Abstract

An application of a simple numerical technique which allows for the rapid construction of orthogonal coordinate systems about two dimensional and axisymmetric bodies is presented. This technique which is based on a "predictor-corrector" numerical method is both simple in concept and easy to program. It can be used to generate orthogonal meshes which have unequally spaced points in two directions. These orthogonal meshes in their transformed computational plane are, however, equally spaced so that the differencing for the metric coefficients and the fluid dynamic equation terms can be easily determined using equally spaced central finite differences. Solutions to the Navier-Stokes equations for flow over blunt bodies with reverse curvature are presented. The coupling of the time dependent fluid dynamic equations and the coordinate generator worked well with no undersirable effects noted.

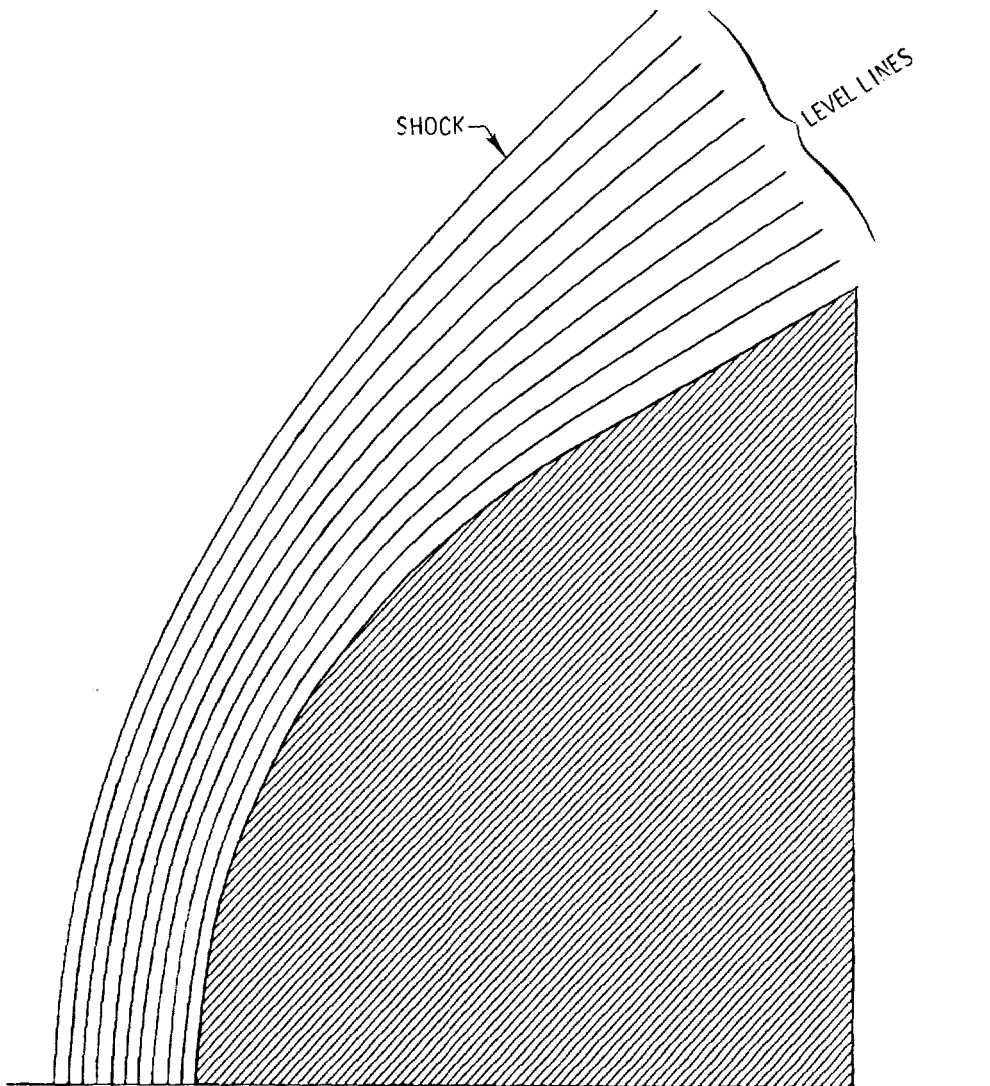
## Flowfield Geometrical Relationships

The numerically generated orthogonal coordinates are determined from the original cartesian coordinate systems description of the body surface and outer boundary. Taking the origin of the  $X, Y$  system as lying inside the body to be described, the surface distance  $\xi$ , which forms one of the transformed orthogonal coordinates, can be easily calculated by defining  $\xi$  as zero at origin of the region of interest and increasing to unity at the end of the region (nondimensionalized surface distance). The other orthogonal coordinate,  $\eta$ , is taken as zero on the body surface and as unity on the outer boundary. Thus the region of interest is transformed into a nondimensional square.



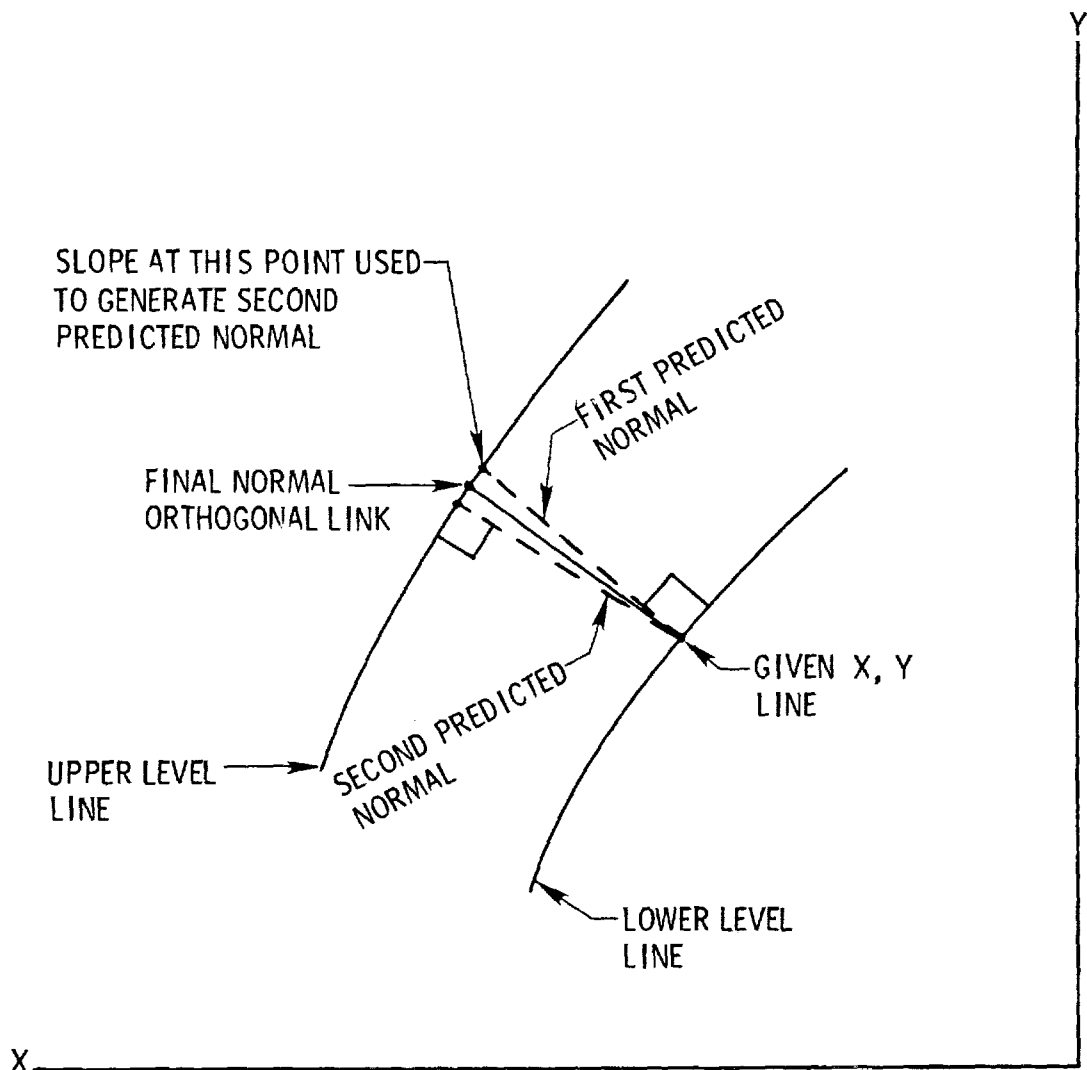
## Level Line Construction

The level lines between the outer boundary and the body surface can be constructed arbitrarily; however, the easiest approach is to construct the level lines along straight lines connecting corresponding points on the body and the outer boundary. The mesh points on the outer boundary are not the final mesh points but initial values used only to set up the level lines. The actual mesh points will result from the numerical generation of the orthogonal normal lines. The spacing of the level lines is arbitrary and highly stretched meshes can be easily constructed.



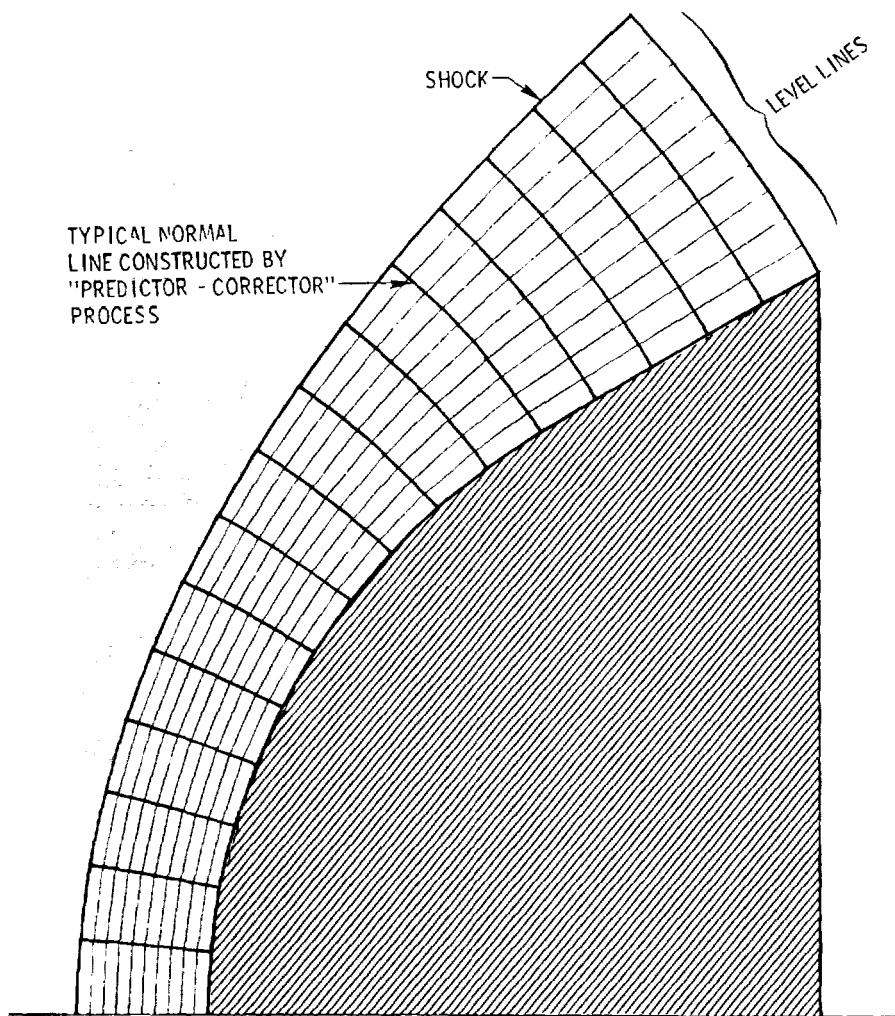
## Normal Line Construction Technique

Once the level lines have been determined, the normal lines are constructed numerically so that an orthogonal system is defined. The approach to the construction of the normal lines is the one given by McNally which uses a simple "predictor-corrector" technique analogous to the trapezoidal integration method of numerical integration. In this technique, the solution is first predicted from the level line at a known point by using the Euler method. Once the predicted point on the next level line is obtained, the slope at that point is calculated and a new predicted point is obtained using this slope. The actual solution is then a combination of these two solutions, i.e. the final X,Y values are an average of the predicted and corrected ones.



## Typical Coordinate Mesh Construction

Starting on the body, the normal line construction technique proceeds point by point along a level line until all normals on that level have been constructed. The solution then proceeds to the next level and the process is continued until the outer boundary is reached. Thus the complete mesh system is numerically generated in a simple straight forward, noniterative process. Since the computational plane  $(\xi, \eta)$  is an equally spaced rectangular region, the metric coefficients can be determined from the completed mesh system using equally spaced finite difference relations. Fourth order accurate difference relations are recommended as they provide for smoothly varying metric coefficients.

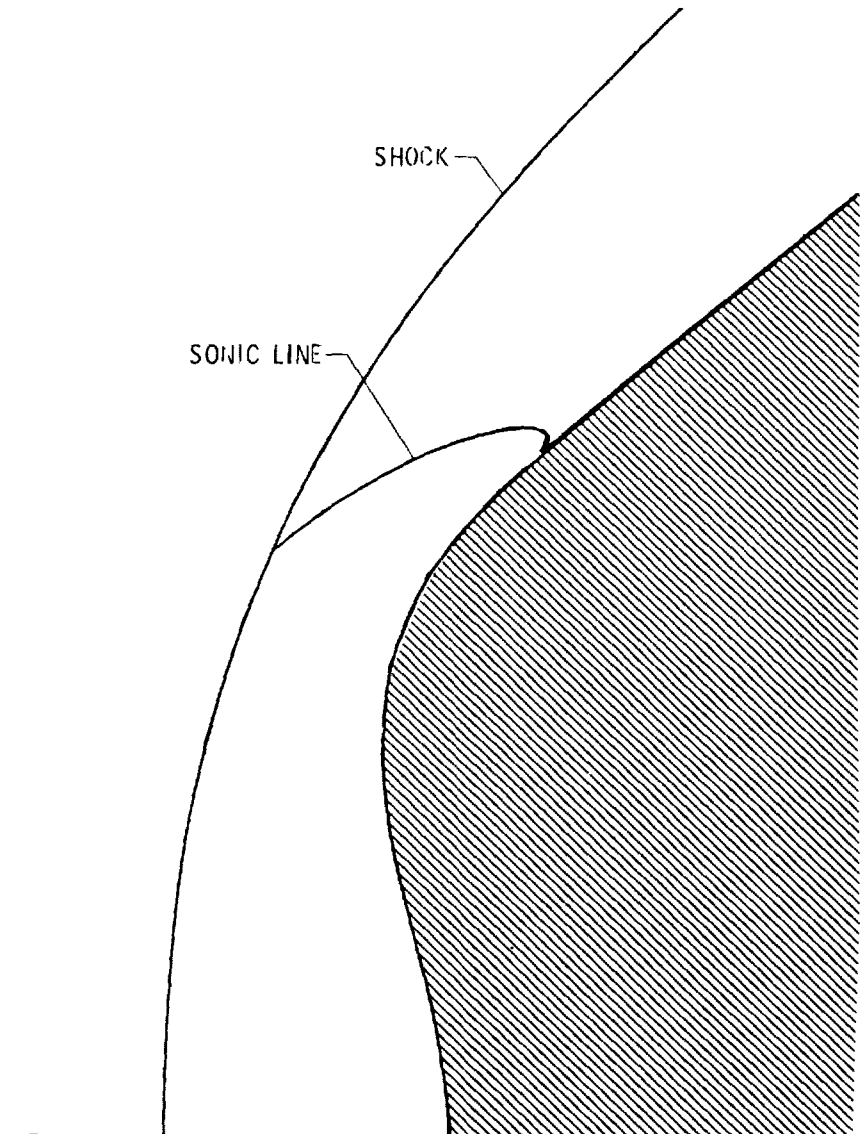


## Shock and Sonic Line

Solutions to the laminar flow Navier-Stokes equations were obtained for flow over bodies with blunted noses, including reverse curvature. These bodies were generated using the following cubic forebody generator,

$$X=X_0 +A_1 y^2 +A_2 y^3$$

where  $X_0$  determines the nose offset while the coefficients  $A_1$  and  $A_2$  are determined such that the forebody nose section joins smoothly to the conical flank. This solution was run for a free stream Mach number of 10.33 and  $X_0=.4$ . The shock shape and sonic line are typical of the solution for bodies with very blunt nose regions.



## Converged Coordinate System

The converged coordinate system shown for  $X_0=.4$  is composed of 15 transverse stations and 31 normal stations. The normal direction spacing is highly stretched to provide resolution for the boundary layer. There is only mild stretching in the transverse direction to provide for improve stagnation region resolution. There were no undesirable effects noted in the coupling of the viscous flow calculations with the coordinate generation.

